Appendix

A. Proof of Proposition 3

Proof. For a point p in bucket $B[u]$, its r-neighbors must locate in $[p.v-r, p.v+r]$. Referring to Definition 8, the points in bucket $B[u]$ must fall into $[v_{\min}+(u-1)\gamma, v_{\min}+u\gamma)$, i.e., $v_{\min}+(u-1)\gamma \le p.v < v_{\min}+u\gamma$.

Since $(\lambda-\frac{1}{2})\gamma < r \le \lambda\gamma$, the following inequality can be derived,

$$v_{\min}+(u-\lambda-1)\gamma \le p.v-r < v_{\min}+(u-\lambda+\frac{1}{2})\gamma,$$
$$v_{\min}+(u+\lambda-\frac{3}{2})\gamma < p.v+r < v_{\min}+(u+\lambda)\gamma.$$

That is,

$$[p.v-r, p.v+r] \subseteq [v_{\min}+(u-\lambda-1)\gamma, v_{\min}+(u+\lambda)\gamma),$$

which corresponds to the union of the ranges of buckets $B[u-\lambda], B[u-\lambda+1], \dots, B[u+\lambda]$, referring to Definition 8. Thus we derive the upper bound $|N(p,r)| \le \sum_{i=u-\lambda}^{u+\lambda} |B[i]|$. □

B. Proof of Proposition 4

Proof. Similar to the proof of Proposition 3, we also have $v_{\min}+(u-1)\gamma \le p.v < v_{\min}+u\gamma$. Since $(\lambda-1)\gamma < r \le (\lambda-\frac{1}{2})\gamma$, we have,

$$v_{\min}+(u-\lambda-\frac{1}{2})\gamma \le p.v-r < v_{\min}+(u-\lambda+1)\gamma.$$

Intuitively, with the constraint $(2\lambda-2)\gamma < 2r \le (2\lambda-1)\gamma$, $N(p,r)$ at most overlaps with 2λ consecutive buckets.

Thus, for the case with $v_{\min}+(u-\lambda-\frac{1}{2})\gamma \le p.v-r < v_{\min}+(u-\lambda)\gamma$, the r-neighbor of p overlaps with 2λ buckets $B[u-\lambda], B[u-\lambda+1] \dots, B[u+\lambda-1]$, leading to

$$|N(p,r)| \le \sum_{i=u-\lambda}^{u+\lambda-1} |B[i]|.$$

Otherwise, for the case with $v_{\min}+(u-\lambda)\gamma \le p.v-r < v_{\min}+(u-\lambda+1)\gamma$, the r-neighbor of p overlaps with 2λ buckets $B[u-\lambda+1], B[u-\lambda+2] \dots, B[u+\lambda]$, leading to

$$|N(p,r)| \le \sum_{i=u-\lambda+1}^{u+\lambda} |B[i]|.$$

Combining the above cases, the upper bound can be obtained as the maximum, i.e.,

$$|N(p,r)| \le \max\left(\sum_{i=u-\lambda}^{u+\lambda-1} |B[i]|, \sum_{i=u-\lambda+1}^{u+\lambda} |B[i]|\right).$$

□

C. Proof of Proposition 5

Proof. For a point p in bucket $B[u]$, its r-neighbors must locate in $[p.v-r, p.v+r]$. Referring to Definition 8, the points in bucket $B[u]$ must fall into $[v_{\min}+(u-1)\gamma, v_{\min}+u\gamma)$, and we thus have $v_{\min}+(u-1)\gamma \le p.v < v_{\min}+u\gamma$. Since $\ell\gamma \le r < (\ell+1)\gamma$, we can derive

$$v_{\min}+(u-\ell-2)\gamma < p.v-r < v_{\min}+(u-\ell)\gamma,$$
$$v_{\min}+(u+\ell-1)\gamma \le p.v+r < v_{\min}+(u+\ell+1)\gamma.$$

That is,

$$[p.v-r, p.v+r] \supseteq [v_{\min}+(u-\ell)\gamma, v_{\min}+(u+\ell-1)\gamma),$$

which leads to the lower bound $|N(p,r)| \ge \sum_{i=u-\ell+1}^{u+\ell-1} |B[i]|$. □

D. Proof of Proposition 6

Proof. Referring to the proofs of Propositions 3 and 5, for a point p in bucket $B[u]$, we have

$$\begin{aligned}[v_{\min}+(u-\ell)\gamma, v_{\min}+(u+\ell-1)\gamma) &\subseteq [p.v-r, p.v+r] \\ &\subseteq [v_{\min}+(u-\lambda-1)\gamma, v_{\min}+(u+\lambda)\gamma),\end{aligned}$$

Combining with Definitions 1 and 8, we can further derive

$$\bigcup_{i=u-\ell+1}^{u+\ell-1} B[i] \subseteq N(p,r) \subseteq \bigcup_{i=u-\lambda}^{u+\lambda} B[i].$$

With the lower bound obtained, we only need to further check the points contained in $\bigcup_{i=u-\lambda}^{u+\lambda} B[i]$ but not contained in $\bigcup_{i=u-\ell+1}^{u+\ell-1} B[i]$, that is $\bigcup_{i=u-\lambda}^{u-\ell} B[i] \cup \bigcup_{i=u+\ell}^{u+\lambda} B[i]$. In particular, since

$$\ell = \lfloor r/\gamma \rfloor \le \lceil r/\gamma \rceil = \lambda \le \lfloor r/\gamma \rfloor + 1 = \ell+1,$$

we have $\lambda = \ell$ or $\ell+1$. For $\lambda = \ell+1$, only the points in 4 additional buckets $B[u-\lambda], B[u-\ell], B[u+\ell]$ and $B[u+\lambda]$ need to be checked. For $\lambda = \ell$, only the points in 2 additional buckets $B[u-\lambda], B[u+\lambda]$ need to be checked. □

E. Proof of Proposition 7

Proof. If all the points in $F_1, \dots, F_\rho$ do not overwrite each other, then the bucket sizes can be simply aggregated when merging, i.e., the maximum size $\sum_{h=1}^{\rho} |B^{(h)}[u]|$. On the other hand, if all the points in $F_1, \dots, F_{\rho-1}$ are overwritten by points in F_ρ, then the bucket size $|B[u]|$ should be the same as $|B^{(\rho)}[u]|$, the minimum size. □

F. Proof of Propositions 8, 9, 10

Proof. Propositions 8, 9, 10 can be easily derived by combining Propositions 3 and 7, Propositions 4 and 7, Propositions 5 and 7, respectively. □

G. Evaluation on Supporting Various Queries

Figures 15, 16, 17, 18 presents the evaluation on 6 datasets under different (1) neighbor distance threshold r, (2) neighbor count threshold k, (3) window size w, and (4) slide size s, respectively.

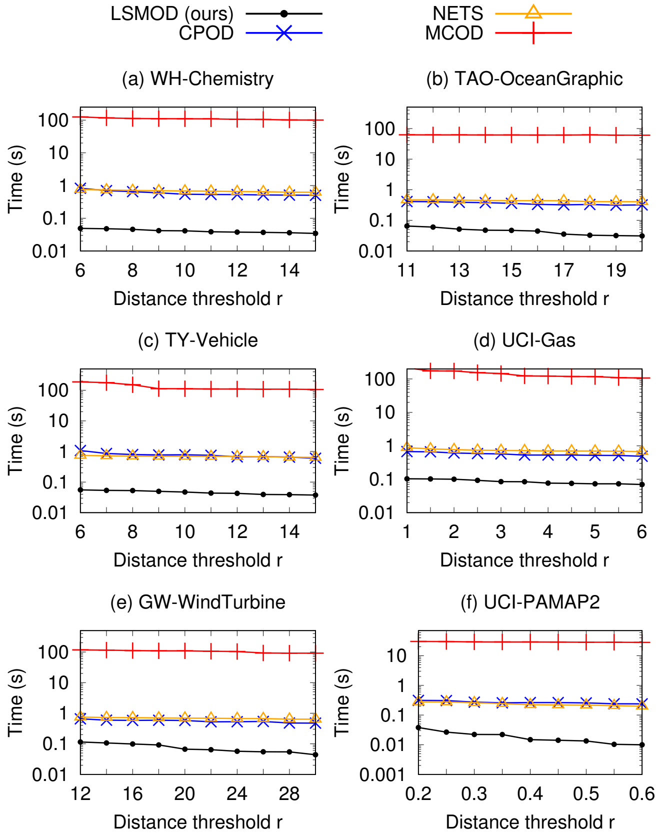

Fig. 15. Varying neighbor distance threshold r of query

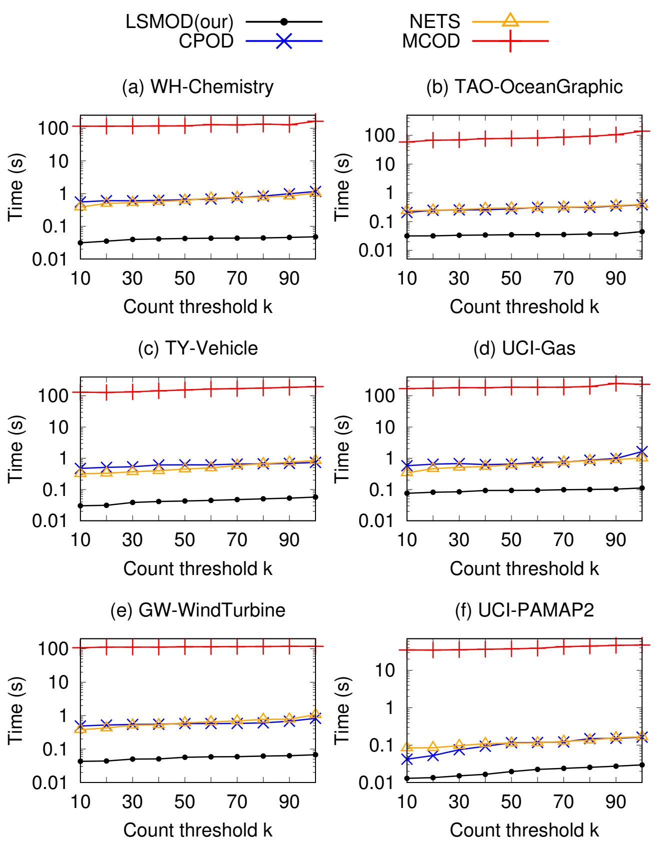

Fig. 16. Varying neighbor count threshold k of query

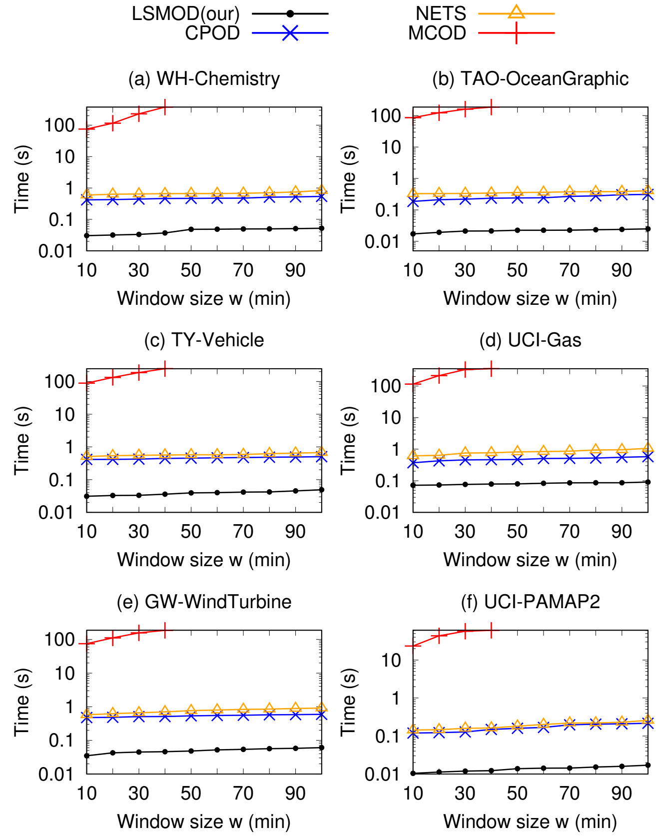

Fig. 17. Varying window size w of query

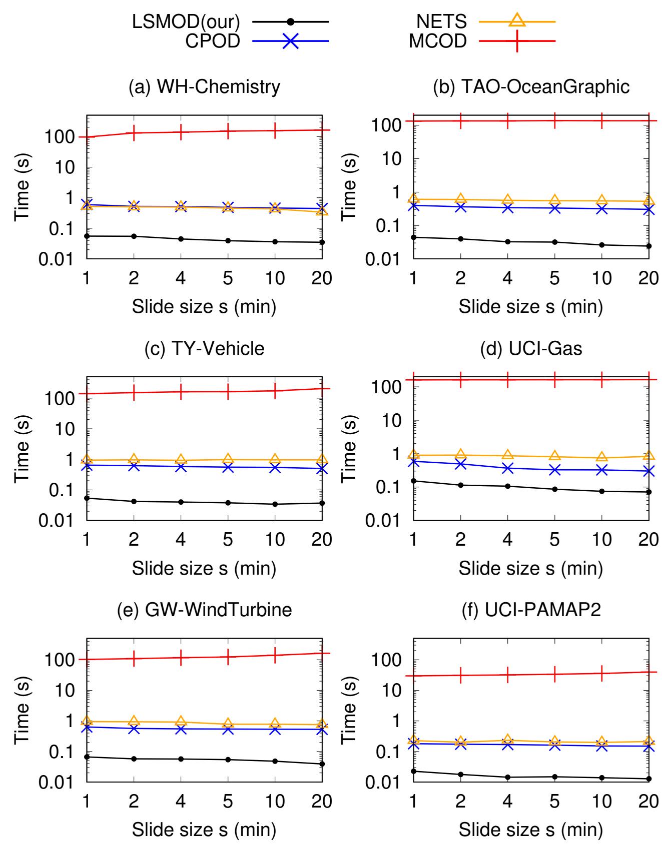

Fig. 18. Varying window slide s of query