

Let  $E(x) : P \rightarrow C$  be some black-box encryption function that is homomorphic on a plaintext ring  $(P, +, \cdot)$  to a cyphertext ring  $(C, +, \cdot)$ . Define some operator  $\lambda$  bijective on some  $K$ , a subset of  $C$ , to the same subset. Then  $\lambda \circ E^{-1}$  is bijective on  $E^{-1}(K) \rightarrow E^{-1}(K)$  because  $E(x)$  is a homomorphism. So if you can find some  $\lambda_2 : Q \rightarrow Q$  for some  $Q \subseteq P$  that maps a plaintext cookie into an authenticated plaintext cookie, then  $\lambda_2 \circ E$  maps a cyphertext cookie in  $K$  to an authenticated cyphertext cookie.

So if we let  $\lambda_{k,l} : C^{95} \rightarrow C^{95}$  (95-bit ciphertext, because the encrypted cookie is 95 bits) be an operator flipping  $k$  bits starting at the  $l$ -th position, we can iterate over every possible  $k, l$  to find a  $\lambda_{k,l}$  that works. An arbitrary bitflip is trivially bijective on  $C^{95}$ . To test a  $\lambda_{k,m}$ , we can send a request to the site with a cookie set to  $\lambda_{k,m}$ (original cookie). If we get a 500 Internal Server Error, then the plaintext cookie is invalid. Eventually, we find  $\lambda_{1,79}$  is the appropriate linear operator, and requesting the server with  $\lambda_{1,79}$ (original cookie), we get the flag.