# **Attention & Transformers**

Ivo Verhoeven | Advanced Topics in Computational Semantics

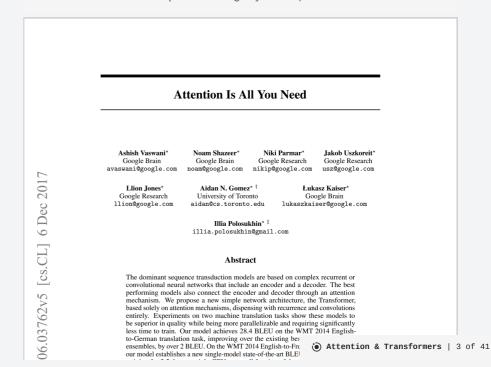
#### **About Me**



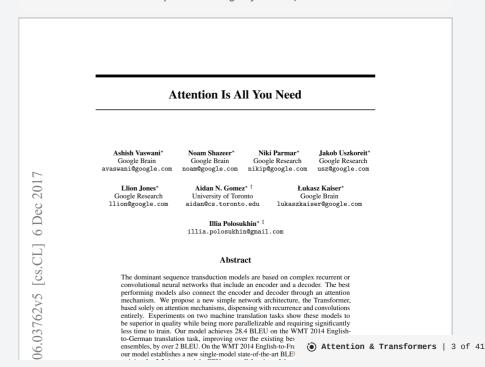
- 2017 2020: BSc. Liberal Arts & Sciences
- 2020 2022: MSc. Al at University of Amsterdam
  - Thesis on meta-learning, morphology and translation
  - Took ATCS in 2021

- 2022 ???: PhD at ILLC
  - Misinformation detection and generalisation with Katia Shutova

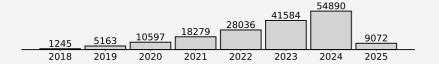
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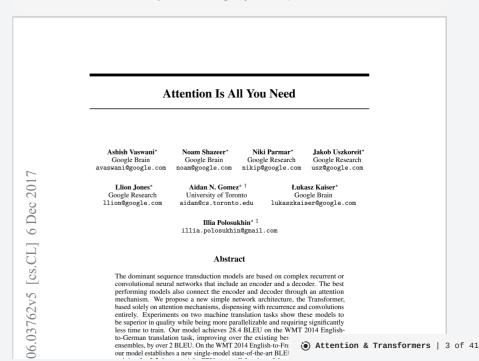


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  - Or ~64 citations a day

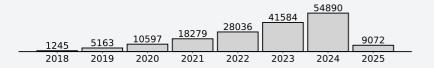


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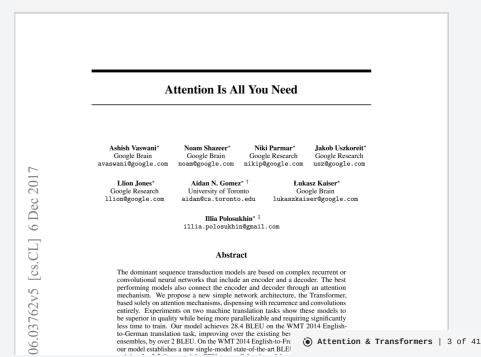


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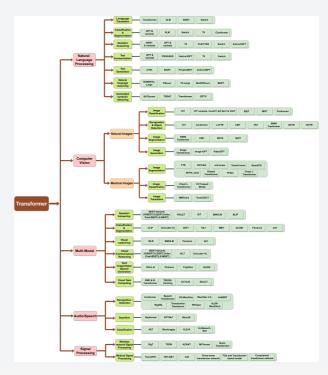


Most cited paper ever has 233 829 citations

Lowry et al. (1951) Protein measurement with the folin phenol reagent.

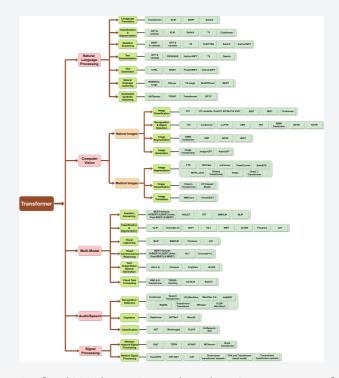


 It's hard to think of an Al area that hasn't been affected by the Transformer



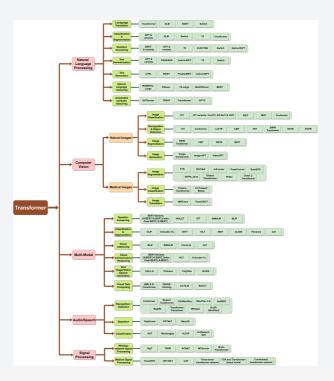
Islam, et al. (2023). A Comprehensive Survey on Application Transformers for Deep Learning Tasks. arXiv:2306.07303.

- It's hard to think of an Al area that hasn't been affected by the Transformer
  - NLP: Transformer > RNN
    - Seq-to-seq: what it was designed for
    - Classification: encoder-only transformers
    - Generation: decoder-only transformers



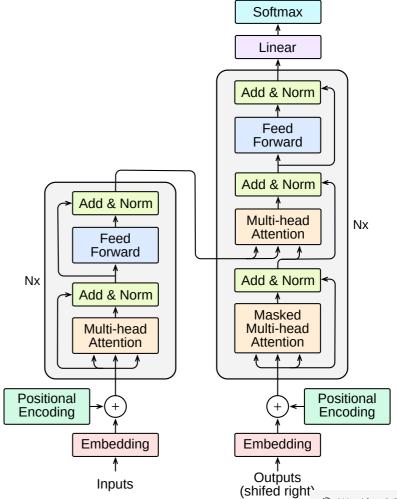
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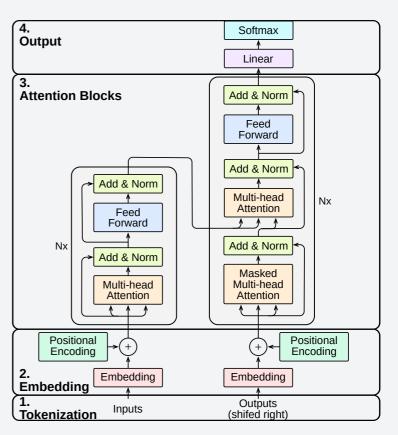
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  - NLP: Transformer > RNN
    - Seq-to-seq: what it was designed for
    - Classification: encoder-only transformers
    - Generation: decoder-only transformers
  - CV: ViT > CNN
  - Multi-modal: Transformer > different architectures
  - Speech: Transformer > CNN
  - Graphs: Transformer/Attention > GCN



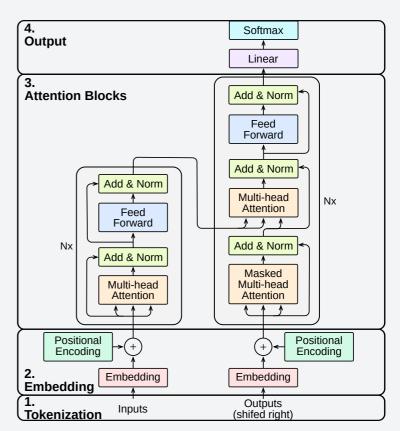
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# The Transformer

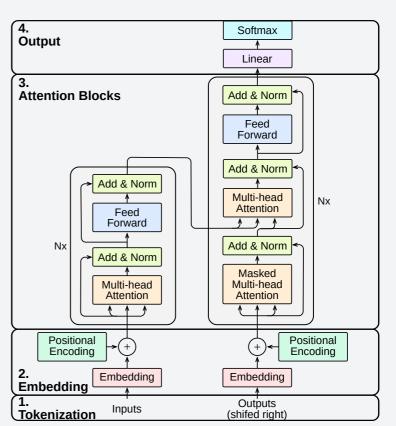




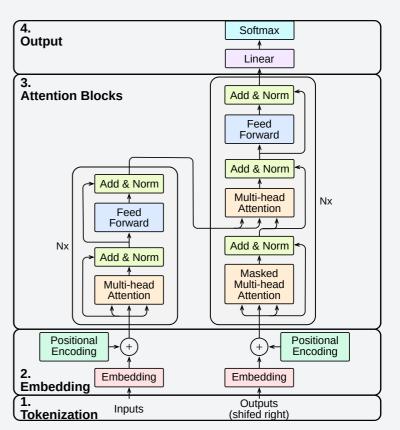
- 4. Output
  - Softmax
  - Linear



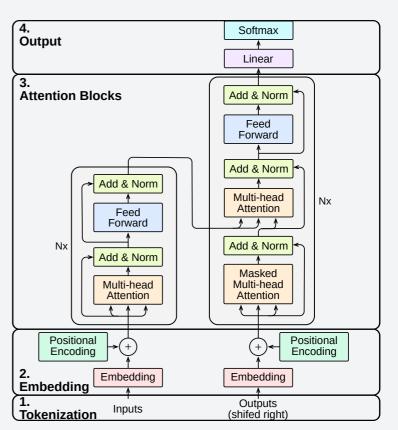
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- 3. Attention Blocks
  - Multi-head Attention
  - Add & Norm
  - Feed Forward



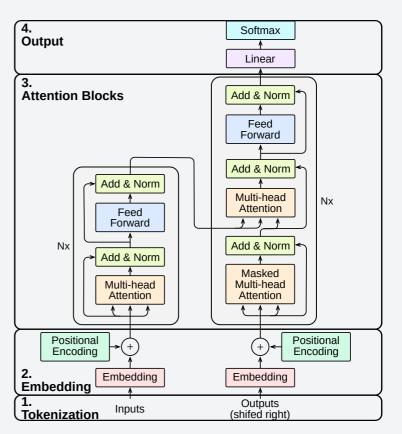
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- 1. Tokenization
  - (Not pictured)

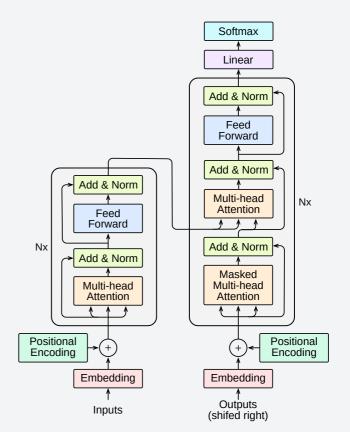


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#### **Table of Contents**

- 1. Encoders & Decoders
- 2. Attention Blocks
  - 1. Multi-head Attention
    - 1. Definition & Properties
    - 2. Non-Transformer Examples
    - 3. Attention in Transformers
    - 4. Multi-head Attention
  - 2. Add & Norm

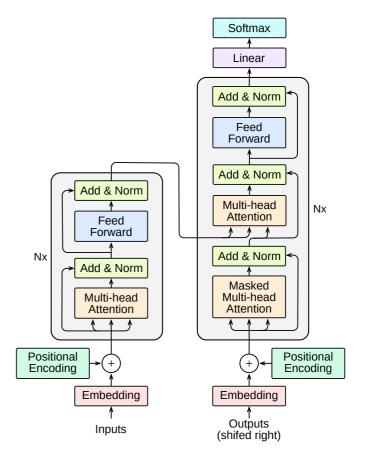


# **Encoders & Decoders**

Text comes in, text goes out

# **Attention Blocks**

What makes the Transformer what it is — and where it came from



- Let V be a matrix of (word) vectors
  - ullet It has a sequence length of  $t_V$
  - ullet It has a dimensionality of  $d_V$

$$\mathbf{Attention}(?,?,\mathbf{V}) = \mathbf{AV}$$
  $\mathbf{A} \in (0,1)^{[t_V imes t_V]}$   $\mathbf{V} \in \mathbb{R}^{[t_V imes d_V]}$ 

- lacktriangle Let  ${f V}$  be a matrix of (word) vectors
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- Attention is just a matrix product of V
   with an attention matrix A
  - lacksquare f A is a square matrix of size  $t_V imes t_V$
  - It's elements are all between (0,1)
  - It's rows sum to 1

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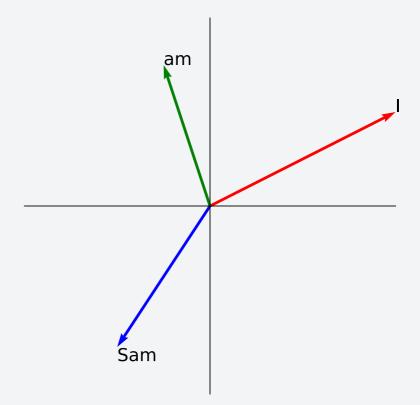
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#### Multi-head Attention

■ The result of Attention is just a convex combination of **V** 

$$\begin{bmatrix} 0.6 & 0.1 & 0.3 \\ 0.3 & 0.5 & 0.2 \\ 0.2 & 0.1 & 0.7 \end{bmatrix} \begin{bmatrix} 2.0 & 1.0 \\ -0.5 & 2.0 \\ -1.0 & -0.5 \end{bmatrix} \begin{bmatrix} \mathbf{I} \\ \mathbf{am} \\ \mathbf{Sam} \end{bmatrix}$$

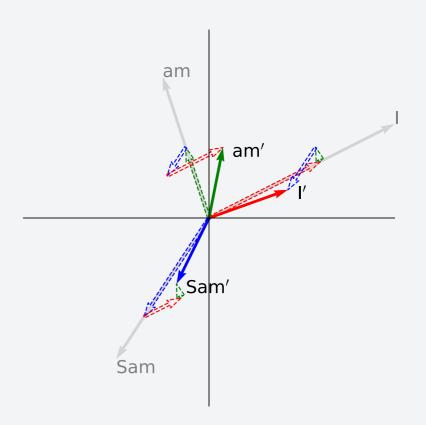


#### Multi-head Attention

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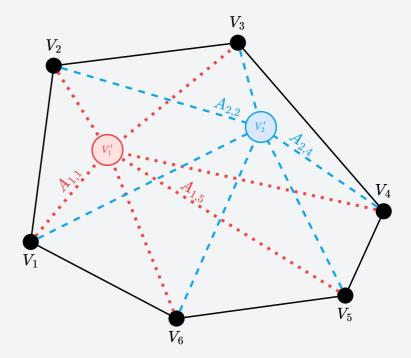
$$= \begin{bmatrix} 0.6 * \mathbf{I} + 0.1 * am + 0.3 * \mathbf{Sam} \\ 0.3 * \mathbf{I} + 0.5 * am + 0.2 * \mathbf{Sam} \\ 0.2 * \mathbf{I} + 0.1 * am + 0.7 * \mathbf{Sam} \end{bmatrix}$$



Multi-head Attention

#### Convex Combination

The elements of  $V^\prime$  will lie inside the convex hull of all of the elements in V



#### Multi-head Attention

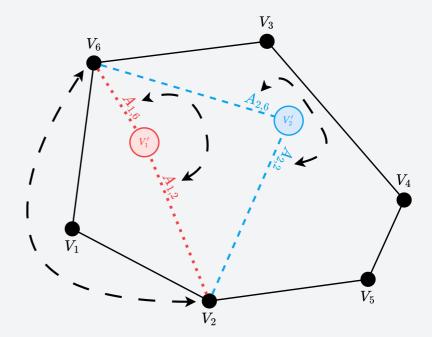
#### **▶** Convex Combination

The elements of  $V^\prime$  will lie inside the convex hull of all of the elements in V

#### Permutation Equivariance

The elements of V' are *equivariant* to a change in the order of the columns of A and the rows of V

- Attention does not care about word order
  - 'I am Sam' ~ 'Sam I am'



Multi-head Attention

So is **Attention** just a linear map?

Not quite

Linear maps are:

Multi-head Attention

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Linear maps are:

Inflexible in terms of sequence length

Multi-head Attention

So is **Attention** just a linear map?

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#### Linear maps are:

- Inflexible in terms of sequence length
- Parameter inefficient

Multi-head Attention

So is **Attention** just a linear map?

Not quite

#### Linear maps are:

- Inflexible in terms of sequence length
- Parameter inefficient
- Invariant to the input content

- Let V be a matrix of value vectors
  - ullet It has a sequence length of  $T_V$
  - ullet It has a dimensionality of  $d_V$
- Let  $\mathbf{K}$  be a matrix of **key** vectors
  - lacktriangle It has a sequence length of  $t_V$
  - ullet It has a dimensionality of  $d_K$
- Let Q be a matrix of query vectors
  - lacktriangle It has a sequence length of  $t_Q$
  - lacksquare It has a dimensionality of  $d_K$

$$ext{Attention}(\mathbf{Q}, \mathbf{K}, \mathbf{V}) = \underbrace{ ext{softmax}\left(f\left(\mathbf{Q}, \mathbf{K}
ight)
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$$\mathbf{A} \in (0,1)^{[t_Q imes t_V]}$$

$$\mathbf{V} \in \mathbb{R}^{[t_V imes d_v]}$$

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  - Read: similarity function

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## Non-Transformer Examples

- **V** contains information
- **K** contains information about information (i.e, metadata)
- $lackbox{ } \mathbf{Q}$  contains metadata about what we want from  $\mathbf{V}$
- $f(\mathbf{Q}, \mathbf{K})$  is high when  $\mathbf{Q}$  is similar to  $\mathbf{K}$

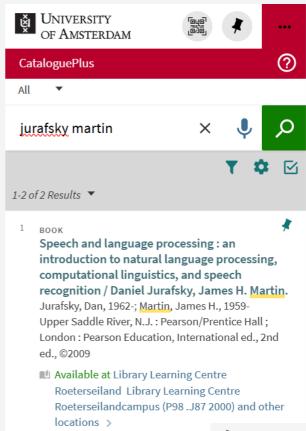
#### Non-Transformer Examples

#### Multi-head Attention

- V contains information
- **K** contains information about information (i.e, metadata)
- $lackbox{f Q}$  contains metadata about what we want from f V
- $f(\mathbf{Q}, \mathbf{K})$  is high when  $\mathbf{Q}$  is similar to  $\mathbf{K}$

#### **E** Soft lookup

We want to find a textbook about NLP in the library (V). We search for titles (K) with "jurafsky" and "martin" as authors (Q). The computer returns books with similar titles (f)



- $f(\mathbf{Q}, \mathbf{K})$  is high when  $\mathbf{Q}$  is similar to  $\mathbf{K}$
- lacktriangle The output of f must a matrix of size  $\mathbf{A} \in (0,1)^{[T_Q imes T_V]}$

#### Multi-head Attention

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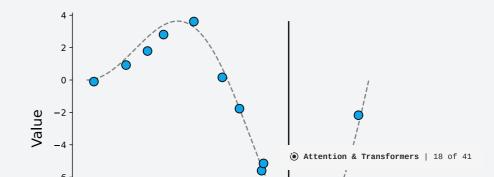
#### **E** Nadaraya-Watson Kernel Regression

We have some sequence of values  $\mathcal{D} = [(1.36, 1.79), (3.40, -1.77) \dots, (6.05, -2.17)]$ 

We want to predict a new sample at x=4.25

We compute the negative Euclidean distance of our new sample with all training samples (f). We normalize the outputs to lie between (0,1)

We compute our predicted value as the mean of the seen values, weighted by the computed similarities



#### Multi-head Attention

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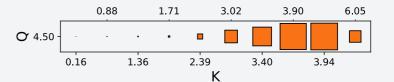
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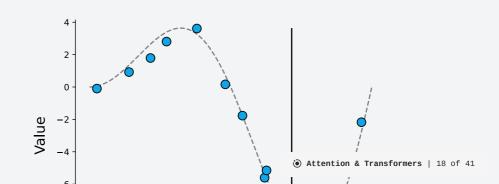
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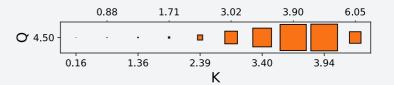
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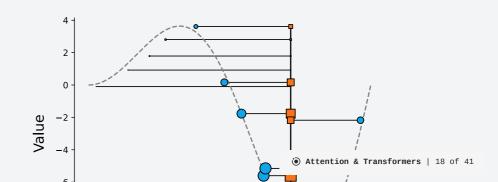
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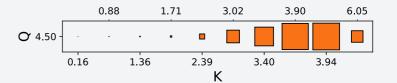
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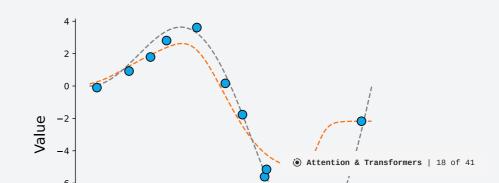
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Multi-head Attention

Attention & Transformers | 19 of 41

 $\Box$ 

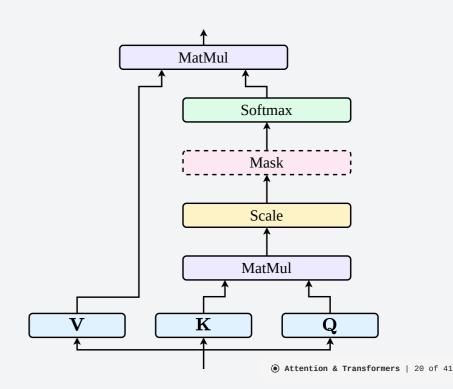
#### Multi-head Attention

 Transformer attention uses a scaled dotproduct kernel function

$$f(\mathbf{Q}, \mathbf{K}) = rac{\mathbf{Q} \mathbf{K}^{\intercal}}{\sqrt{d_k}}$$

- lacksquare  $\mathbf{Q}$  is of size  $t_Q imes d_K$
- lacksquare lacksquare is of size  $t_V imes d_K$
- ullet Attention matrix is thus of size  $t_Q imes t_V$

 $\mathtt{attention}(\mathbf{Q}, \mathbf{K}, \mathbf{V}) = \mathtt{softmax}\left(f(\mathbf{Q}, \mathbf{K})\right) \mathbf{V}$ 



#### Multi-head Attention

 Transformer attention uses a scaled dotproduct kernel function

$$f(\mathbf{Q}, \mathbf{K}) = rac{\mathbf{Q} \mathbf{K}^{\intercal}}{\sqrt{d_k}}$$

- Why scale?
  - Assume the elements in Q and K come from *independent* normal distributions:

$$\mathbf{q},\mathbf{k}\sim\mathcal{N}(0,1)$$

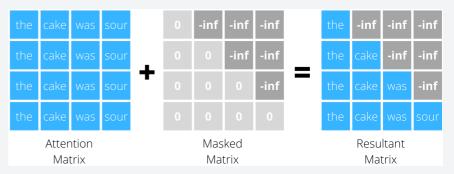
The distribution of their dot-product is:

$$\mathbf{q}^{\intercal}\mathbf{k} \sim \mathcal{N}(0,\sqrt{d_k})$$

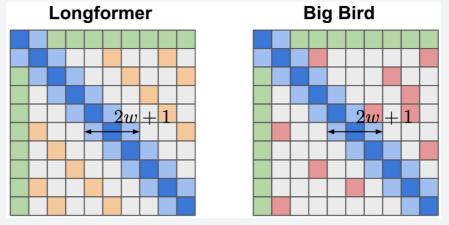
$$egin{aligned} extsf{var}\left[\mathbf{q}^{\intercal}\mathbf{k}
ight] &= extsf{var}\left[\sum_{i}^{d_{k}}q_{i}k_{i}
ight] \ &= \sum_{i}^{d_{k}} extsf{var}\left[q_{i}
ight] extsf{var}\left[k_{i}
ight] \ &= \sum_{i}^{d_{k}}1\cdot 1 \ &= d_{k} \end{aligned}$$

#### Multi-head Attention

- Why mask?
  - Currently all tokens are treated equally
  - Causal masking: decoder tokens should never attend to future tokens, only to the past
  - Local masking: sometimes local attention is all you need



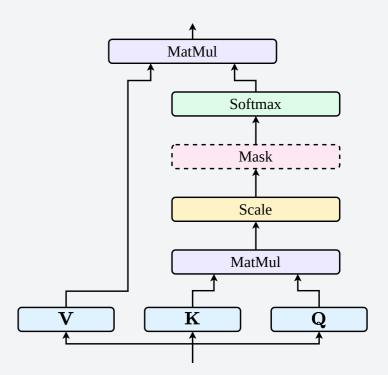
https://krypticmouse.hashnode.dev/attention-is-all-you-need



https://lilianweng.github.io/posts/2023-01-27-the-transformer-family-v2/

Multi-head Attention

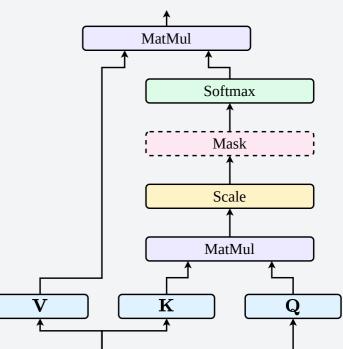
 $\bullet \quad \text{Where do } \mathbf{V},\,\mathbf{K},\,\mathbf{Q} \text{ come from?}$ 



Multi-head Attention

# **Self-attention** MatMul Softmax Mask Scale MatMul

## Cross-attention

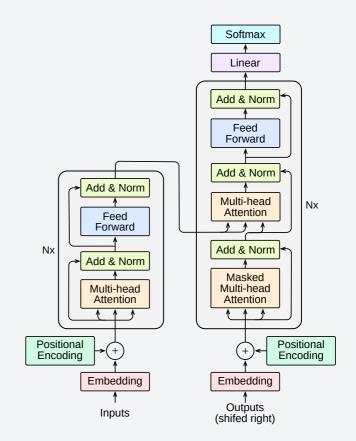


- Where do V, K, Q come from?
  - Self-attention: everything comes from the same sequence
  - Cross-attention: V, K come from source sequence, Q comes from target sequence
  - All components constructed from a projection of the token embeddings

1. 
$$\mathbf{V} = \mathbf{X}\mathbf{W}_V$$

2. 
$$\mathbf{K} = \mathbf{X}\mathbf{W}_K$$

3. 
$$\mathbf{Q} = \mathbf{X}\mathbf{W}_Q$$
 or  $\mathbf{Q} = \mathbf{Y}\mathbf{W}_Q$ 



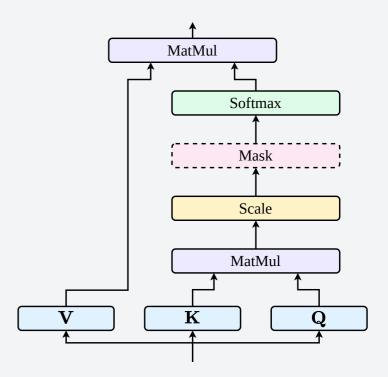
#### Multi-head Attention

Even in self-attention, attention matrix is **not** symmetric

$$egin{aligned} rac{\mathbf{Q}\mathbf{K}^{\intercal}}{\sqrt{d_k}} &= rac{\mathbf{X}\mathbf{W}_Q(\mathbf{X}\mathbf{W}_K)^{\intercal}}{\sqrt{d_k}} \ &= rac{\mathbf{X}\mathbf{W}_Q\mathbf{W}_K^{\intercal}\mathbf{X}^{\intercal}}{\sqrt{d_k}} \end{aligned}$$

#### Asymmetry

The contribution of token  $\mathbf{x}_i$  to  $\mathbf{x}_j$ , is **not** the same as the contribution of tokn  $\mathbf{x}_j$  to  $\mathbf{x}_i$ 



#### Multi-head Attention

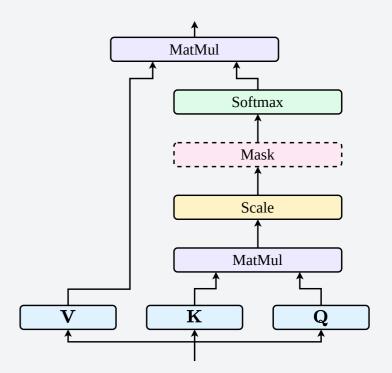
 Transformer attention between two sequences, X and Y has a computational cost of (excluding projections):

$$\mathcal{O}\left(\underbrace{t_x \cdot t_y \cdot d_k}_{ ext{MatMul 1}} + \underbrace{t_x \cdot t_y \cdot d_v}_{ ext{MatMul 2}}
ight)$$

But RNNs have linear time complexity...

$$\mathcal{O}\left(t_x\cdot d_k^2+t_x\cdot d_q^2
ight)$$

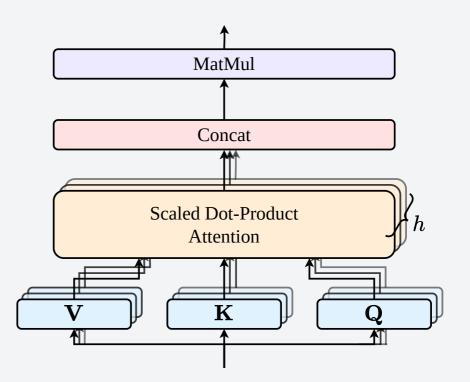
- RNNs are serial, Attention is parallel
  - GPUs *looove* parallelism



Jakob Uszkoreit (August 31, 2017). Transformer: A Novel Neural Network Architecture for Language Understanding. https://research.google/blog/transformer-a-novel-neural-network-architecture-for-language-understanding/

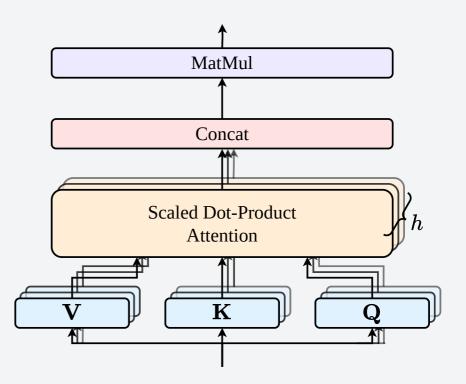
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  - Can only process 1 query type
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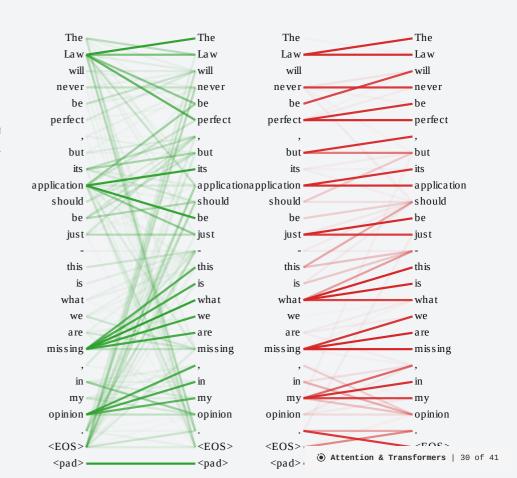
```
self.attention_heads = [
  AttentionHead(d=self.d // self.h) for i in range(self
]
self.mha_proj = nn.Linear(self.d, self.d)
mha = torch.concat([
  attention_heads[i](x) for i in range(self.h)
])
out = self.mha_proj(mha)
```



#### Multi-head Attention

Multi-head attention allows the model to jointly attend to information from different representation subspaces at different positions. [One] attention head, averaging inhibits this.

Vaswani et al. (2017). Attention is all you need. Advan in neural information processing systems, 30. (p. 5 & 1



#### Multi-head Attention

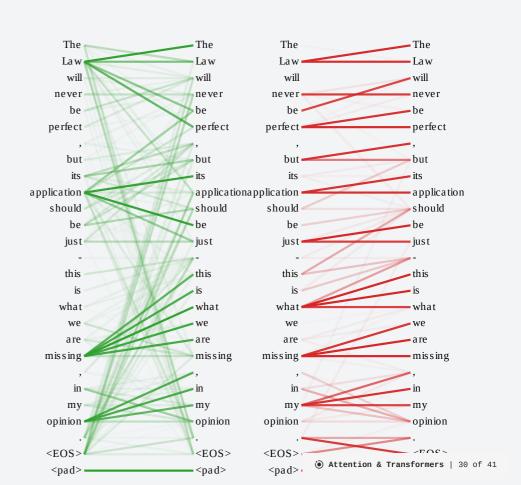
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## Multiple heads, multiple different queries processed in parallel

- Positional heads
- Syntactic heads
- Rare words?

Voita et al. (2019). Analyzing Multi-Head Self-Attentic Specialized Heads Do the Heavy Lifting, the Rest Can Be Pruned. Association for Computational Linguistics.



Multi-head Attention

Do different heads attend to different concepts?

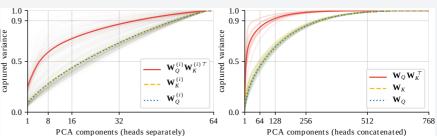
Multi-head Attention

#### Do different heads attend to different concepts?

■ Individual heads = high rank, concantenated head = low rank = low rank

Cordonnier, Loukas & Jaggi (2020). Multi-head attention:

Collaborate instead of concatenate, arXiv:2006.16362.

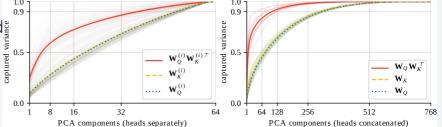


#### Multi-head Attention

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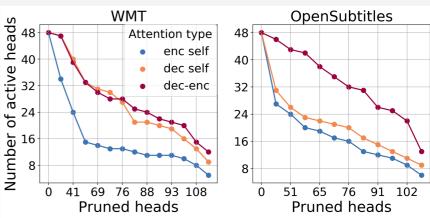
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Cordonnier, Loukas & Jaggi (2020). Multi-head attention: according to the Collaborate instead of concatenate. arXiv:2006.16362.

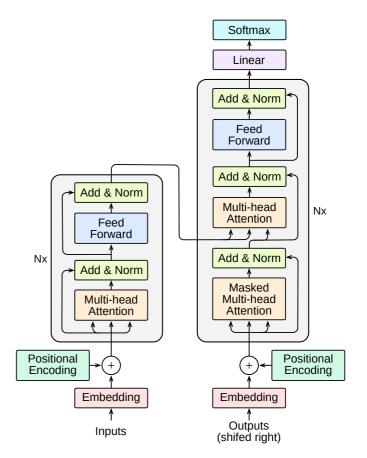


- Most heads can be pruned away
- Enc-Dec heads are more important than Enc-Enc heads

Voita et al. (2019). Analyzing Multi-Head Self-Attention Specialized Heads Do the Heavy Lifting, the Rest Can Be Pruned. Association for Computational Linguistics.



## Add & Norm



## Add & Norm

## **Residual Connections**

Add & Norm

## LayerNorm

Add & Norm

These are the equations

```
egin{array}{lll} \mathbf{X}_l &= 	exttt{LayerNorm} \ \mathbf{X}_{l-1} + 	exttt{SubLayer} \left( \mathbf{X}_{l-1} 
ight) \ \end{array}
```

## **Feed Forward**

# **Embedding**

## **Position Encoding**

# Tokenization

# **Training Transformers**

## The End