

$$\ddot{x} + 5\dot{x} + 4x = 0 \Rightarrow \ddot{x} = -5\dot{x} - 4x$$

$$\left\{ \begin{array}{l} x \triangleq y_1 \xrightarrow{\text{def}} \dot{x} = \dot{y}_1 = y_2 \\ \dot{x} \triangleq y_2 \xrightarrow{\text{def}} \dot{x} = \dot{y}_2 \end{array} \right\} \Rightarrow \dot{y}_2 = 5y_2 - 4y_1 \Rightarrow \begin{cases} \begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -4 & -5 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \\ \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \end{cases}$$

$$\lambda^2 + 5\lambda + 4 = 0 \Rightarrow (D+1)(D+4)$$

$$\ker[(D+1)(D+4)] = \{ e^{-t}, e^{-4t} \} \quad \text{c.i.}$$

$$x(t) = A e^{-t} + B e^{-4t} \Rightarrow x(0) = A + B = 0$$

$$\dot{x}(t) = -A e^{-t} - 4B e^{-4t} \Rightarrow \dot{x}(0) = -A - 4B = -A + 4A = 3A = 3$$

$$\boxed{\begin{array}{l} A = 1 \\ B = -1 \end{array}}$$

$$x(t) = e^{-t} - e^{-4t}$$

$$x(0) = 0$$

$$\dot{x}(0) = 3$$