PAATHSHALA PHYSICS | M | TWT 01

Max. Marks : 120 **SYLLABUS :** Complex Numbers And Quadratic Equations **Time : 60 min.**

1. The smallest positive integer n for which $(1+i)^{2n} = (1-i)^{2n}$ is :

(a) 1 (b) 2
(c) 3 (d) 4

2. If α and β be the roots of $x^2 + px + q = 0$, then $\dfrac{(\omega\alpha + \omega^2\beta)(\omega^2\alpha + \omega\beta)}{\dfrac{\alpha^2}{\beta} + \dfrac{\beta^2}{\alpha}}$ is equal to (ω, ω^2 are complex cube roots of unity)

(a) $-\dfrac{q}{p}$ (b) $\alpha\beta$
(c) $-\dfrac{p}{q}$ (d) ω

3. If α, β be the roots of the equation $x^2 - px + q = 0$, then the equation whose roots are

$$\alpha^2\left(\frac{\alpha^2}{\beta} - \beta\right) \text{ and } \beta^2\left(\frac{\beta^2}{\alpha} - \alpha\right) \text{ is}$$

(a) $qx^2 - p(p^2 - q)(p^2 - 4q)x - p^2q^2(p^2 - 4q) = 0$
(b) $px^2 - q(p^2 - p)(p^2 - 4q)x + p^2q^2(p^2 - 4q) = 0$
(c) $px^2 - qx + p = 0$
(d) None of these

Space for Rough Work

4. If α and β be the values of x in $m^2(x^2-x)+2mx+3=0$ and m_1 and m_2 be two values of m for which α and β are connected by the relation $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{4}{3}$. Then the value of $\frac{m_1^2}{m_2}+\frac{m_2^2}{m_1}$ is

(a) 6 (b) 68

(c) $\frac{3}{68}$ (d) $-\frac{68}{3}$

5. If $z_1=\sqrt{3}+i\sqrt{3}$ and $z_2=\sqrt{3}+i$, then in which quadrant $\left(\frac{z_1}{z_2}\right)$ lies?

(a) I (b) II

(c) III (d) IV

6. The root of the equation $2(1+i)x^2-4(2-i)x-5-3i=0$ which has greater modulus is

(a) $\frac{3-5i}{2}$ (b) $\frac{5-3i}{2}$

(c) $\frac{3-i}{2}$ (d) None of these

7. Value of $\frac{(\cos\theta+i\sin\theta)^4}{(\cos\theta-i\sin\theta)^3}$ is

(a) $\cos 5\theta+i\sin 5\theta$ (b) $\cos 7\theta+i\sin 7\theta$

(c) $\cos 4\theta+i\sin 4\theta$ (d) $\cos\theta+i\sin\theta$

8. Number of solutions of the equation, $z^3+\frac{3|z|^2}{z}=0$, where z is a complex number and $|z|=\sqrt{3}$ is

(a) 2 (b) 3

(c) 6 (d) 4

9. If $z=x+iy$ is a variable complex number such that $\arg\frac{z-1}{z+1}=\frac{\pi}{4}$ then :

(a) $x^2-y^2-2x=1$ (b) $x^2+y^2-2x=1$

(c) $x^2+y^2-2y=1$ (d) $x^2+y^2+2x=1$

10. Let $a>0$, $b>0$ and $c>0$. Then both the roots of the equation $ax^2+bx+c=0$

(a) are real and negative

(b) have negative real parts

(c) are rational numbers

(d) None of these

11. Let z lies on the circle centred at the origin. If area of the triangle whose vertices are z, ωz and z+ωz, where ω is the cube root of unity is $4\sqrt{3}$ sq. unit. Then radius of the circle is :

(a) 1 unit (b) 2 units

(c) 4 units (d) None of these

12. For a complex number z, the minimum value of $|z|+|z-2|$ is

(a) 1 (b) 2

(c) 3 (d) None of these

13. The complex number z satisfying the equations $|z|-4=|z-i|=|z+5i|=0$, is

(a) $\sqrt{3}-i$ (b) $2\sqrt{3}-2i$

(c) $-2\sqrt{3}+2i$ (d) 0

14. If α,β,γ and a, b, c are complex numbers such that $\frac{\alpha}{a}+\frac{\beta}{b}+\frac{\gamma}{c}=1+i$ and $\frac{a}{\alpha}+\frac{b}{\beta}+\frac{c}{\gamma}=0$, then the value of $\frac{\alpha^2}{a^2}+\frac{\beta^2}{b^2}+\frac{\gamma^2}{c^2}$ is equal to

(a) -1 (b) $2i$

(c) 0 (d) +1

Space for Rough Work

15. If $\left(7-4\sqrt{3}\right)^{x^2-4x+3}+\left(7+4\sqrt{3}\right)^{x^2-4x+3}=14$, then the value of x is given by

(a) $2, 2\pm\sqrt{2}$ (b) $2\pm\sqrt{3}, 3$
(c) $3\pm\sqrt{2}, 2$ (d) None of these

16. If α, β be the roots of $ax^2+bx+c=0$ and γ, δ those of $lx^2+mx+n=0$, then the equation whose roots are $\alpha\gamma+\beta\delta$ and $\alpha\delta+\beta\gamma$ is

(a) $a^2l^2x^2-ablmx+b^2ln+acm^2-4acln=0$

(b) $alx^2-ablmx+(a+b-c)(l+m-n)=0$

(c) $a^2l^2x^2+(a^2+b^2)(l^2+m^2)x-(a+b-c)(l+m-n)=0$

(d) None of these

17. $\left(\dfrac{-1+\sqrt{-3}}{2}\right)^{100}+\left(\dfrac{-1-\sqrt{-3}}{2}\right)^{100}$ is equal to

(a) 2 (b) zero
(c) – 1 (d) 1

18. If x is real, the maximum value of $\dfrac{3x^2+9x+17}{3x^2+9x+7}$ is

(a) $\dfrac{1}{4}$ (b) 41

(c) 1 (d) $\dfrac{17}{7}$

19. $\left(\dfrac{1}{1-2i}+\dfrac{3}{1+i}\right)\left(\dfrac{3+4i}{2-4i}\right)$ is equal to :

(a) $\dfrac{1}{2}+\dfrac{9}{2}i$ (b) $\dfrac{1}{2}-\dfrac{9}{2}i$

(c) $\dfrac{1}{4}-\dfrac{9}{4}i$ (d) $\dfrac{1}{4}+\dfrac{9}{4}i$

20. If p, q, r are non-zero real numbers, the two equation, $2a^2x^2-2abx+b^2=0$ and $p^2x^2+3pqx+q^2=0$ have :

(a) no common root
(b) one common root if $2a^2+b^2=p^2+q^2$
(c) two common roots if $3pq=2ab$
(d) two common roots if $3qb=2ap$

21. The centre of a regular hexagon is at the point z = i. If one of its vertices is at 2 + i, then the adjacent vertices of 2 + i are at the points

(a) $1\pm 2i$ (b) $i+1\pm\sqrt{3}$

(c) $2+i(1\pm\sqrt{3})$ (d) $1+i(1\pm\sqrt{3})$

22. If a, b, c are real numbers $a\neq 0$. If α, is a root of $a^2x^2+bx+c=0$, β is a root of $a^2x^2-bx-c=0$ and $0<\alpha<\beta$, then the equation $a^2x^2+2bx+2c=0$ has a γ root that always satisfies:

(a) $\gamma=\dfrac{\alpha+\beta}{2}$ (b) $\gamma=\dfrac{\alpha-\beta}{2}$

(c) $\gamma=\alpha$ (d) $\alpha<\gamma<\beta$

23. If the roots of the equation $(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$ are equal, then $a^2+b^2+c^2=$

(a) $a+b+c$ (b) $2a+b+c$
(c) $3abc$ (d) $ab+bc+ca$

24. If $|a+ib|=1$, then the simplified form of $\dfrac{1+b+ai}{1+b-ai}$ is

(a) $b+ai$ (b) $a+bi$
(c) $(1+b)^2+a^2$ (d) ai

Space for Rough Work

25. Let a, b, c, p, q be real numbers. Suppose α, β are the roots of the equation $x^2 + 2px + q = 0$ and $\alpha, \dfrac{1}{\beta}$ are the roots of the equation $x^2 + 2bx + c = 0$, where $\beta^2 \notin (-1, 0, 1)$

Statement-1: $(p^2 - q)(b^2 - ac) \geq 0$

Statement-2: $b \neq pa$ or $c \neq qa$

(a) Statement -1 is true, Statement-2 is true; Statement -2 is a correct explanation for Statement-1

(b) Statement -1 is true, Statement-2 is true; Statement -2 is NOT a correct explanation for Statement-1

(c) Statement -1 is false, Statement-2 is true

(d) Statement -1 is true, Statement-2 is false

26. If ω is a non-real cube root of unity, then

$\dfrac{1+2\omega+3\omega^2}{2+3\omega+\omega^2} + \dfrac{2+3\omega+3\omega^2}{3+3\omega+2\omega^2}$ is equal to

(a) -2ω (b) 2ω

(c) ω (d) 0

27. If α, β are the roots of the equation $ax^2 + bx + c = 0$ such that $\beta < \alpha < 0$, then the quadratic equation whose roots are $|\alpha|, |\beta|$, is given by

(a) $|a|x^2 + |b|x + |c| = 0$ (b) $ax^2 - |b|x + c = 0$

(c) $|a|x^2 - |b|x + |c| = 0$ (d) $a|x|^2 + b|x| + |c| = 0$

28. If $z = 2 + i$, then $(z-1)(\overline{z}-5) + (\overline{z}-1)(z-5)$ is equal to

(a) 2 (b) 7

(c) –1 (d) –4

29. If α, β are the roots of the equation $2x^2 + 6x + b = 0, (b < 0)$ then $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha}$ is less than :

(a) 1 (b) –1

(c) 2 (d) –2

30. A + iB form of $\dfrac{(\cos x + i \sin x)(\cos y + i \sin y)}{(\cot u + i)(1 + i \tan v)}$ is equal to :

(a) $\sin u \cos v\, [\cos(x + y - u - v) + i \sin(x + y - u - v)]$

(b) $\sin u \cos v\, [\cos(x + y + u + v) + i \sin(x + y + u + v)]$

(c) $\sin u \cos v\, [\cos(x + y + u + v) - i \sin(x + y - u + v)]$

(d) None of these

Space for Rough Work