Table A.1 Cumulative Binomial Probabilities

$$B(x; n, p) = \sum_{y=0}^{x} b(y; n, p)$$

a. $n = 5$

x	p = 0.01	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95	0.99
0	.951	.774	.590	.328	.237	.168	.078	.031	.010	.002	.001	.000	.000	.000	.000
1	.999	.977	.919	.737	.633	.528	.337	.188	.087	.031	.016	.007	.000	.000	.000
2	1.000	.999	.991	.942	.896	.837	.683	.500	.317	.163	.104	.058	.009	.001	.000
3	1.000	1.000	1.000	.993	.984	.969	.913	.812	.663	.472	.367	.263	.081	.023	.001
4	1.000	1.000	1.000	1.000	.999	.998	.990	.969	.922	.832	.763	.672	.410	.226	.049

b. $n = 10$

x	p = 0.01	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95	0.99
0	.904	.599	.349	.107	.056	.028	.006	.001	.000	.000	.000	.000	.000	.000	.000
1	.996	.914	.736	.376	.244	.149	.046	.011	.002	.000	.000	.000	.000	.000	.000
2	1.000	.988	.930	.678	.526	.383	.167	.055	.012	.002	.000	.000	.000	.000	.000
3	1.000	.999	.987	.879	.776	.650	.382	.172	.055	.011	.004	.001	.000	.000	.000
4	1.000	1.000	.998	.967	.922	.850	.633	.377	.166	.047	.020	.006	.000	.000	.000
5	1.000	1.000	1.000	.994	.980	.953	.834	.623	.367	.150	.078	.033	.002	.000	.000
6	1.000	1.000	1.000	.999	.996	.989	.945	.828	.618	.350	.224	.121	.013	.001	.000
7	1.000	1.000	1.000	1.000	1.000	.998	.988	.945	.833	.617	.474	.322	.070	.012	.000
8	1.000	1.000	1.000	1.000	1.000	1.000	.998	.989	.954	.851	.756	.624	.264	.086	.004
9	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	.994	.972	.944	.893	.651	.401	.096

c. $n = 15$

x	p = 0.01	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95	0.99
0	.860	.463	.206	.035	.013	.005	.000	.000	.000	.000	.000	.000	.000	.000	.000
1	.990	.829	.549	.167	.080	.035	.005	.000	.000	.000	.000	.000	.000	.000	.000
2	1.000	.964	.816	.398	.236	.127	.027	.004	.000	.000	.000	.000	.000	.000	.000
3	1.000	.995	.944	.648	.461	.297	.091	.018	.002	.000	.000	.000	.000	.000	.000
4	1.000	.999	.987	.836	.686	.515	.217	.059	.009	.001	.000	.000	.000	.000	.000
5	1.000	1.000	.998	.939	.852	.722	.403	.151	.034	.004	.001	.000	.000	.000	.000
6	1.000	1.000	1.000	.982	.943	.869	.610	.304	.095	.015	.004	.001	.000	.000	.000
7	1.000	1.000	1.000	.996	.983	.950	.787	.500	.213	.050	.017	.004	.000	.000	.000
8	1.000	1.000	1.000	.999	.996	.985	.905	.696	.390	.131	.057	.018	.000	.000	.000
9	1.000	1.000	1.000	1.000	.999	.996	.966	.849	.597	.278	.148	.061	.002	.000	.000
10	1.000	1.000	1.000	1.000	1.000	.999	.991	.941	.783	.485	.314	.164	.013	.001	.000
11	1.000	1.000	1.000	1.000	1.000	1.000	.998	.982	.909	.703	.539	.352	.056	.005	.000
12	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.996	.973	.873	.764	.602	.184	.036	.000
13	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.995	.965	.920	.833	.451	.171	.010
14	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.995	.987	.965	.794	.537	.140

(continued)

Table A.1 Cumulative Binomial Probabilities *(cont.)*

$$B(x; n, p) = \sum_{y=0}^{x} b(y; n, p)$$

d. $n = 20$

x	p = 0.01	0.05	0.10	0.20	0.25	0.30	0.40	0.50	0.60	0.70	0.75	0.80	0.90	0.95	0.99
0	.818	.358	.122	.012	.003	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000
1	.983	.736	.392	.069	.024	.008	.001	.000	.000	.000	.000	.000	.000	.000	.000
2	.999	.925	.677	.206	.091	.035	.004	.000	.000	.000	.000	.000	.000	.000	.000
3	1.000	.984	.867	.411	.225	.107	.016	.001	.000	.000	.000	.000	.000	.000	.000
4	1.000	.997	.957	.630	.415	.238	.051	.006	.000	.000	.000	.000	.000	.000	.000
5	1.000	1.000	.989	.804	.617	.416	.126	.021	.002	.000	.000	.000	.000	.000	.000
6	1.000	1.000	.998	.913	.786	.608	.250	.058	.006	.000	.000	.000	.000	.000	.000
7	1.000	1.000	1.000	.968	.898	.772	.416	.132	.021	.001	.000	.000	.000	.000	.000
8	1.000	1.000	1.000	.990	.959	.887	.596	.252	.057	.005	.001	.000	.000	.000	.000
9	1.000	1.000	1.000	.997	.986	.952	.755	.412	.128	.017	.004	.001	.000	.000	.000
10	1.000	1.000	1.000	.999	.996	.983	.872	.588	.245	.048	.014	.003	.000	.000	.000
11	1.000	1.000	1.000	1.000	.999	.995	.943	.748	.404	.113	.041	.010	.000	.000	.000
12	1.000	1.000	1.000	1.000	1.000	.999	.979	.868	.584	.228	.102	.032	.000	.000	.000
13	1.000	1.000	1.000	1.000	1.000	1.000	.994	.942	.750	.392	.214	.087	.002	.000	.000
14	1.000	1.000	1.000	1.000	1.000	1.000	.998	.979	.874	.584	.383	.196	.011	.000	.000
15	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.994	.949	.762	.585	.370	.043	.003	.000
16	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	.984	.893	.775	.589	.133	.016	.000
17	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.996	.965	.909	.794	.323	.075	.001
18	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	.992	.976	.931	.608	.264	.017
19	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	.997	.988	.878	.642	.182

(continued)

Table A.1 Cumulative Binomial Probabilities *(cont.)*

$$B(x; n, p) = \sum_{y=0}^{x} b(y; n, p)$$

e. $n = 25$

		p														
		0.01	**0.05**	**0.10**	**0.20**	**0.25**	**0.30**	**0.40**	**0.50**	**0.60**	**0.70**	**0.75**	**0.80**	**0.90**	**0.95**	**0.99**
	0	.778	.277	.072	.004	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
	1	.974	.642	.271	.027	.007	.002	.000	.000	.000	.000	.000	.000	.000	.000	.000
	2	.998	.873	.537	.098	.032	.009	.000	.000	.000	.000	.000	.000	.000	.000	.000
	3	1.000	.966	.764	.234	.096	.033	.002	.000	.000	.000	.000	.000	.000	.000	.000
	4	1.000	.993	.902	.421	.214	.090	.009	.000	.000	.000	.000	.000	.000	.000	.000
	5	1.000	.999	.967	.617	.378	.193	.029	.002	.000	.000	.000	.000	.000	.000	.000
	6	1.000	1.000	.991	.780	.561	.341	.074	.007	.000	.000	.000	.000	.000	.000	.000
	7	1.000	1.000	.998	.891	.727	.512	.154	.022	.001	.000	.000	.000	.000	.000	.000
	8	1.000	1.000	1.000	.953	.851	.677	.274	.054	.004	.000	.000	.000	.000	.000	.000
	9	1.000	1.000	1.000	.983	.929	.811	.425	.115	.013	.000	.000	.000	.000	.000	.000
	10	1.000	1.000	1.000	.994	.970	.902	.586	.212	.034	.002	.000	.000	.000	.000	.000
	11	1.000	1.000	1.000	.998	.980	.956	.732	.345	.078	.006	.001	.000	.000	.000	.000
x	**12**	1.000	1.000	1.000	1.000	.997	.983	.846	.500	.154	.017	.003	.000	.000	.000	.000
	13	1.000	1.000	1.000	1.000	.999	.994	.922	.655	.268	.044	.020	.002	.000	.000	.000
	14	1.000	1.000	1.000	1.000	1.000	.998	.966	.788	.414	.098	.030	.006	.000	.000	.000
	15	1.000	1.000	1.000	1.000	1.000	1.000	.987	.885	.575	.189	.071	.017	.000	.000	.000
	16	1.000	1.000	1.000	1.000	1.000	1.000	.996	.946	.726	.323	.149	.047	.000	.000	.000
	17	1.000	1.000	1.000	1.000	1.000	1.000	.999	.978	.846	.488	.273	.109	.002	.000	.000
	18	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.993	.926	.659	.439	.220	.009	.000	.000
	19	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.998	.971	.807	.622	.383	.033	.001	.000
	20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.991	.910	.786	.579	.098	.007	.000
	21	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.998	.967	.904	.766	.236	.034	.000
	22	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.991	.968	.902	.463	.127	.002
	23	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.998	.993	.973	.729	.358	.026
	24	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	.999	.996	.928	.723	.222

Table A.2 Cumulative Poisson Probabilities

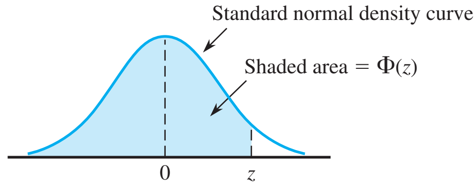
$$F(x; \mu) = \sum_{y=0}^{x} \frac{e^{-\mu}\mu^{y}}{y!}$$

		μ									
		.1	**.2**	**.3**	**.4**	**.5**	**.6**	**.7**	**.8**	**.9**	**1.0**
	0	.905	.819	.741	.670	.607	.549	.497	.449	.407	.368
	1	.995	.982	.963	.938	.910	.878	.844	.809	.772	.736
	2	1.000	.999	.996	.992	.986	.977	.966	.953	.937	.920
x	**3**		1.000	1.000	.999	.998	.997	.994	.991	.987	.981
	4				1.000	1.000	1.000	.999	.999	.998	.996
	5							1.000	1.000	1.000	.999
	6										1.000

(continued)

Table A.3 Standard Normal Curve Areas

$\Phi(z) = P(Z \leq z)$



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0038
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0352	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0722	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3482
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

(continued)

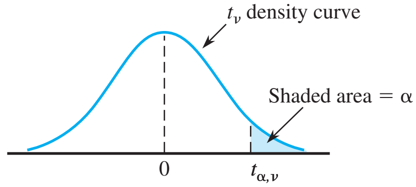
Table A.3 Standard Normal Curve Areas *(cont.)* $\Phi(z) = P(Z \leq z)$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9278	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Table A.5 Critical Values for t Distributions

Example 5: $\alpha = 0.025$, $n = 10$ $(v = 9)$

$t \approx 2.262$



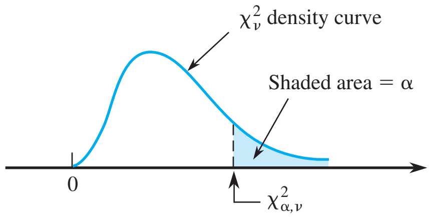
$v = n-1$

ν \ α	.10	.05	.025	.01	.005	.001	.0005
1	3.078	6.314	12.706	31.821	63.657	318.31	636.62
2	1.886	2.920	4.303	6.965	9.925	22.326	31.598
3	1.638	2.353	3.182	4.541	5.841	10.213	12.924
4	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	1.319	1.714	2.069	2.500	2.807	3.485	3.767
24	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	1.310	1.697	2.042	2.457	2.750	3.385	3.646
32	1.309	1.694	2.037	2.449	2.738	3.365	3.622
34	1.307	1.691	2.032	2.441	2.728	3.348	3.601
36	1.306	1.688	2.028	2.434	2.719	3.333	3.582
38	1.304	1.686	2.024	2.429	2.712	3.319	3.566
40	1.303	1.684	2.021	2.423	2.704	3.307	3.551
50	1.299	1.676	2.009	2.403	2.678	3.262	3.496
60	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	1.289	1.658	1.980	2.358	2.617	3.160	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.090	3.291

Standard Normal Values

Table A.7 Critical Values for Chi-Squared Distributions

Ex) $\chi^2_{.025,9} = 19.022$

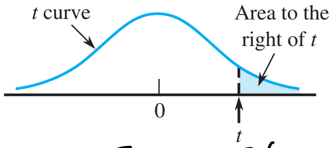


$\nu = n-1$

ν \ α	.995	.99	.975	.95	.90	.10	.05	.025	.01	.005
1	0.000	0.000	0.001	0.004	0.016	2.706	3.843	5.025	6.637	7.882
2	0.010	0.020	0.051	0.103	0.211	4.605	5.992	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.344	12.837
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.832	15.085	16.748
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.440	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.012	18.474	20.276
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.534	20.090	21.954
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.022	21.665	23.587
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.724	26.755
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.041	19.812	22.362	24.735	27.687	29.817
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.600	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.577	32.799
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.407	7.564	8.682	10.085	24.769	27.587	30.190	33.408	35.716
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.843	7.632	8.906	10.117	11.651	27.203	30.143	32.852	36.190	38.580
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.033	8.897	10.283	11.591	13.240	29.615	32.670	35.478	38.930	41.399
22	8.643	9.542	10.982	12.338	14.042	30.813	33.924	36.781	40.289	42.796
23	9.260	10.195	11.688	13.090	14.848	32.007	35.172	38.075	41.637	44.179
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.558
25	10.519	11.523	13.120	14.611	16.473	34.381	37.652	40.646	44.313	46.925
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.807	12.878	14.573	16.151	18.114	36.741	40.113	43.194	46.962	49.642
28	12.461	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.993
29	13.120	14.256	16.147	17.708	19.768	39.087	42.557	45.772	49.586	52.333
30	13.787	14.954	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
31	14.457	15.655	17.538	19.280	21.433	41.422	44.985	48.231	52.190	55.000
32	15.134	16.362	18.291	20.072	22.271	42.585	46.194	49.480	53.486	56.328
33	15.814	17.073	19.046	20.866	23.110	43.745	47.400	50.724	54.774	57.646
34	16.501	17.789	19.806	21.664	23.952	44.903	48.602	51.966	56.061	58.964
35	17.191	18.508	20.569	22.465	24.796	46.059	49.802	53.203	57.340	60.272
36	17.887	19.233	21.336	23.269	25.643	47.212	50.998	54.437	58.619	61.581
37	18.584	19.960	22.105	24.075	26.492	48.363	52.192	55.667	59.891	62.880
38	19.289	20.691	22.878	24.884	27.343	49.513	53.384	56.896	61.162	64.181
39	19.994	21.425	23.654	25.695	28.196	50.660	54.572	58.119	62.426	65.473
40	20.706	22.164	24.433	26.509	29.050	51.805	55.758	59.342	63.691	66.766

For $\nu > 40$, $\chi^2_{\alpha,\nu} \approx \nu\left(1 - \dfrac{2}{9\nu} + z_\alpha\sqrt{\dfrac{2}{9\nu}}\right)^3$

Table A.8 *t* Curve Tail Areas

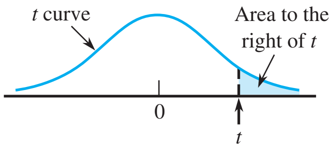


Ex 4 10/26

t \ ν	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
0.0	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500
0.1	.468	.465	.463	.463	.462.	.462	.462	.461	.461	.461	.461	.461	.461	.461	.461	.461	.461	.461
0.2	.437	.430	.427	.426	.425	.424	.424	.423	.423	.423	.423	.422	.422	.422	.422	.422	.422	.422
0.3	.407	.396	.392	.390	.388	.387	.386	.386	.386	.385	.385	.385	.384	.384	.384	.384	.384	.384
0.4	.379	.364	.358	.355	.353	.352	.351	.350	.349	.349	.348	.348	.348	.347	.347	.347	.347	.347
0.5	.352	.333	.326	.322	.319	.317	.316	.315	.315	.314	.313	.313	.313	.312	.312	.312	.312	.312
0.6	.328	.305	.295	.290	.287	.285	.284	.283	.282	.281	.280	.280	.279	.279	.279	.278	.278	.278
0.7	.306	.278	.267	.261	.258	.255	.253	.252	.251	.250	.249	.249	.248	.247	.247	.247	.247	.246
0.8	.285	.254	.241	.234	.230	.227	.225	.223	.222	.221	.220	.220	.219	.218	.218	.218	.217	.217
0.9	.267	.232	.217	.210	.205	.201	.199	.197	.196	.195	.194	.193	.192	.191	.191	.191	.190	.190
1.0	.250	.211	.196	.187	.182	.178	.175	.173	.172	.170	.169	.169	.168	.167	.167	.166	.166	.165
1.1	.235	.193	.176	.167	.162	.157	.154	.152	.150	.149	.147	.146	.146	.144	.144	.144	.143	.143
1.2	.221	.177	.158	.148	.142	.138	.135	.132	.130	.129	.128	.127	.126	.124	.124	.124	.123	.123
1.3	.209	.162	.142	.132	.125	.121	.117	.115	.113	.111	.110	.109	.108	.107	.107	.106	.105	.105
1.4	.197	.148	.128	.117	.110	.106	.102	.100	.098	.096	.095	.093	.092	.091	.091	.090	.090	.089
1.5	.187	.136	.115	.104	.097	.092	.089	.086	.084	.082	.081	.080	.079	.077	.077	.077	.076	.075
1.6	.178	.125	.104	.092	.085	.080	.077	.074	.072	.070	.069	.068	.067	.065	.065	.065	.064	.064
1.7	.169	.116	.094	.082	.075	.070	.065	.064	.062	.060	.059	.057	.056	.055	.055	.054	.054	.053
1.8	.161	.107	.085	.073	.066	.061	.057	.055	.053	.051	.050	.049	.048	.046	.046	.045	.045	.044
1.9	.154	.099	.077	.065	.058	.053	.050	.047	.045	.043	.042	.041	.040	.038	.038	.038	.037	.037
2.0	.148	.092	.070	.058	.051	.046	.043	.040	.038	.037	.035	.034	.033	.032	.032	.031	.031	.030
2.1	.141	.085	.063	.052	.045	.040	.037	.034	.033	.031	.030	.029	.028	.027	.027	.026	.025	.025
2.2	.136	.079	.058	.046	.040	.035	.032	.029	.028	.026	.025	.024	.023	.022	.022	.021	.021	.021
2.3	.131	.074	.052	.041	.035	.031	.027	.025	.023	.022	.021	.020	.019	.018	.018	.018	.017	.017
2.4	.126	.069	.048	.037	.031	.027	.024	.022	.020	.019	.018	.017	.016	.015	.015	.014	.014	.014
2.5	.121	.065	.044	.033	.027	.023	.020	.018	.017	.016	.015	.014	.013	.012	.012	.012	.011	.011
2.6	.117	.061	.040	.030	.024	.020	.018	.016	.014	.013	.012	.012	.011	.010	.010	.010	.009	.009
2.7	.113	.057	.037	.027	.021	.018	.015	.014	.012	.011	.010	.010	.009	.008	.008	.008	.008	.007
2.8	.109	.054	.034	.024	.019	.016	.013	.012	.010	.009	.009	.008	.008	.007	.007	.006	.006	.006
2.9	.106	.051	.031	.022	.017	.014	.011	.010	.009	.008	.007	.007	.006	.005	.005	.005	.005	.005
3.0	.102	.048	.029	.020	.015	.012	.010	.009	.007	.007	.006	.006	.005	.004	.004	.004	.004	.004
3.1	.099	.045	.027	.018	.013	.011	.009	.007	.006	.006	.005	.005	.004	.004	.004	.003	.003	.003
3.2	.096	.043	.025	.016	.012	.009	.008	.006	.005	.005	.004	.004	.003	.003	.003	.003	.003	.002
3.3	.094	.040	.023	.015	.011	.008	.007	.005	.005	.004	.004	.003	.003	.002	.002	.002	.002	.002
3.4	.091	.038	.021	.014	.010	.007	.006	.005	.004	.003	.003	.003	.002	.002	.002	.002	.002	.002
3.5	.089	.036	.020	.012	.009	.006	.005	.004	.003	.003	.002	.002	.002	.002	.002	.001	.001	.001
3.6	.086	.035	.018	.011	.008	.006	.004	.004	.003	.002	.002	.002	.002	.001	.001	.001	.001	.001
3.7	.084	.033	.017	.010	.007	.005	.004	.003	.002	.002	.002	.002	.001	.001	.001	.001	.001	.001
3.8	.082	.031	.016	.010	.006	.004	.003	.003	.002	.002	.001	.001	.001	.001	.001	.001	.001	.001
3.9	.080	.030	.015	.009	.006	.004	.003	.002	.002	.001	.001	.001	.001	.001	.001	.001	.001	.001
4.0	.078	.029	.014	.008	.005	.004	.003	.002	.002	.001	.001	.001	.001	.001	.001	.001	.000	.000

(continued)

Table A.8 *t* Curve Tail Areas *(cont.)*



t \ ν	19	20	21	22	23	24	25	26	27	28	29	30	35	40	60	120	∞ (= z)
0.0	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500	.500
0.1	.461	.461	.461	.461	.461	.461	.461	.461	.461	.461	.461	.461	.460	.460	.460	.460	.460
0.2	.422	.422	.422	.422	.422	.422	.422	.422	.421	.421	.421	.421	.421	.421	.421	.421	.421
0.3	.384	.384	.384	.383	.383	.383	.383	.383	.383	.383	.383	.383	.383	.383	.383	.382	.382
0.4	.347	.347	.347	.347	.346	.346	.346	.346	.346	.346	.346	.346	.346	.346	.345	.345	.345
0.5	.311	.311	.311	.311	.311	.311	.311	.311	.311	.310	.310	.310	.310	.310	.309	.309	.309
0.6	.278	.278	.278	.277	.277	.277	.277	.277	.277	.277	.277	.277	.276	.276	.275	.275	.274
0.7	.246	.246	.246	.246	.245	.245	.245	.245	.245	.245	.245	.245	.244	.244	.243	.243	.242
0.8	.217	.217	.216	.216	.216	.216	.216	.215	.215	.215	.215	.215	.215	.214	.213	.213	.212
0.9	.190	.189	.189	.189	.189	.189	.188	.188	.188	.188	.188	.188	.187	.187	.186	.185	.184
1.0	.165	.165	.164	.164	.164	.164	.163	.163	.163	.163	.163	.163	.162	.162	.161	.160	.159
1.1	.143	.142	.142	.142	.141	.141	.141	.141	.141	.140	.140	.140	.139	.139	.138	.137	.136
1.2	.122	.122	.122	.121	.121	.121	.121	.120	.120	.120	.120	.120	.119	.119	.117	.116	.115
1.3	.105	.104	.104	.104	.103	.103	.103	.103	.102	.102	.102	.102	.101	.101	.099	.098	.097
1.4	.089	.089	.088	.088	.087	.087	.087	.087	.086	.086	.086	.086	.085	.085	.083	.082	.081
1.5	.075	.075	.074	.074	.074	.073	.073	.073	.073	.072	.072	.072	.071	.071	.069	.068	.067
1.6	.063	.063	.062	.062	.062	.061	.061	.061	.061	.060	.060	.060	.059	.059	.057	.056	.055
1.7	.053	.052	.052	.052	.051	.051	.051	.051	.050	.050	.050	.050	.049	.048	.047	.046	.045
1.8	.044	.043	.043	.043	.042	.042	.042	.042	.042	.041	.041	.041	.040	.040	.038	.037	.036
1.9	.036	.036	.036	.035	.035	.035	.035	.034	.034	.034	.034	.034	.033	.032	.031	.030	.029
2.0	.030	.030	.029	.029	.029	.028	.028	.028	.028	.028	.027	.027	.027	.026	.025	.024	.023
2.1	.025	.024	.024	.024	.023	.023	.023	.023	.023	.022	.022	.022	.022	.021	.020	.019	.018
2.2	.020	.020	.020	.019	.019	.019	.019	.018	.018	.018	.018	.018	.017	.017	.016	.015	.014
2.3	.016	.016	.016	.016	.015	.015	.015	.015	.015	.015	.014	.014	.014	.013	.012	.012	.011
2.4	.013	.013	.013	.013	.012	.012	.012	.012	.012	.012	.012	.011	.011	.011	.010	.009	.008
2.5	.011	.011	.010	.010	.010	.010	.010	.010	.009	.009	.009	.009	.009	.008	.008	.007	.006
2.6	.009	.009	.008	.008	.008	.008	.008	.008	.007	.007	.007	.007	.007	.007	.006	.005	.005
2.7	.007	.007	.007	.007	.006	.006	.006	.006	.006	.006	.006	.006	.005	.005	.004	.004	.003
2.8	.006	.006	.005	.005	.005	.005	.005	.005	.005	.005	.005	.004	.004	.004	.003	.003	.003
2.9	.005	.004	.004	.004	.004	.004	.004	.004	.004	.004	.004	.003	.003	.003	.003	.002	.002
3.0	.004	.004	.003	.003	.003	.003	.003	.003	.003	.003	.003	.003	.002	.002	.002	.002	.001
3.1	.003	.003	.003	.003	.003	.002	.002	.002	.002	.002	.002	.002	.002	.002	.001	.001	.001
3.2	.002	.002	.002	.002	.002	.002	.002	.002	.002	.002	.002	.002	.001	.001	.001	.001	.001
3.3	.002	.002	.002	.002	.002	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.000
3.4	.002	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.000	.000
3.5	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.000	.000	.000
3.6	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.001	.000	.000	.000	.000	.000
3.7	.001	.001	.001	.001	.001	.001	.001	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000
3.8	.001	.001	.001	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
3.9	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000
4.0	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000	.000

Confidence Interval Formula Sheet

	Case 1	Case 2	Case 3	Case 4
Dist	Normal	General	General	Binomial
# observations	Anything	Large	Large	Large
μ	?	?	?	?
σ^2	Known	Known	?	?
Statistic	$\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$	$\frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$	$\frac{\bar{x}-\mu}{s/\sqrt{n}}$	$(x/n-p)/\sqrt{\frac{p(1-p)}{n}}$
CI	$\bar{x} \pm z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}$	$\bar{x} \pm z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}$	$\bar{x} \pm z_{\alpha/2}\cdot\frac{s}{\sqrt{n}}$	$\frac{x}{n} \pm z_{\alpha/2}\cdot\sqrt{\frac{\frac{x}{n}(1-\frac{x}{n})}{n}}$ $\hat{p} = \frac{x}{n}$

	Case 5
Dist	Normal
# observations	Anything
μ	?
σ^2	?
Statistic	$\frac{\bar{x}-\mu}{s/\sqrt{n}} = T$ $t_{\alpha,v}$: $v = n-1$ deg. freedom
CI	$\bar{x} \pm t_{\frac{\alpha}{2};n-1}\cdot\frac{s}{\sqrt{n}}$

Case 6 (one sided)

- Estimate population standard deviation
- Upper bound only (change $\alpha/2 \Rightarrow \alpha$)

$$\sqrt{\frac{(n-1)s^2}{\chi^2_{1-\alpha,n-1}}}$$

Case 6 (two sided)

- Estimate σ^2 (variance)
- To estimate σ, take root of results

$$\left[\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}}, \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}\right] \Rightarrow \sigma^2$$

$$\left[\sqrt{\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}}}, \sqrt{\frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}}\right] \Rightarrow \sigma$$

One-Sample Hypothesis Testing: Case 1/2 (Z)

11/7

Test Statistic

- Population is Normal and σ^2 is known (similar to Chapter 7.1)

Null hypothesis: $H_0 : \mu = \mu_0$

Case 1: σ is known

Test statistic: $Z = \frac{\overline{X}-\mu_0}{\sigma/\sqrt{n}}$

Case 2: σ is unknown (replace with S)

Test statistic value: $z = \frac{\overline{x}-\mu_0}{\sigma/\sqrt{n}}$

Alternative Hypothesis	P-value Determination
$H_a : \mu > \mu_0$	Area under the standard normal curve to the right of z
$H_a : \mu < \mu_0$	Area under the standard normal curve to the left of z
$H_a : \mu \neq \mu_0$	2(area under the standard normal curve to the right of $\|z\|$)

Type II Error

Alternative Hypothesis	Type II Error Probability $\beta(\mu')$ for a Level α Test
$H_a : \mu > \mu_0$	$\Phi\left(z_\alpha + \frac{\mu_0-\mu'}{\sigma/\sqrt{n}}\right)$
$H_a : \mu < \mu_0$	$1 - \Phi\left(-z_\alpha + \frac{\mu_0-\mu'}{\sigma/\sqrt{n}}\right)$
$H_a : \mu \neq \mu_0$	$\Phi\left(z_{\frac{\alpha}{2}} + \frac{\mu_0-\mu'}{\sigma/\sqrt{n}}\right) - \Phi\left(-z_{\frac{\alpha}{2}} + \frac{\mu_0-\mu'}{\sigma/\sqrt{n}}\right)$

Sample Size

The sample size n for which a level α test also has $\beta(\mu') = \beta$ at the alternative value μ'

$$n = \begin{cases} \left[\frac{\sigma(z_\alpha+z_\beta)}{\mu_0-\mu'}\right]^2 & \text{for a one-tailed (upper or lower test),} \\ \left[\frac{\sigma(z_{\alpha/2}+z_\beta)}{\mu_0-\mu'}\right]^2 & \text{for a two-tailed test.} \end{cases}$$

- A **type I error** consists of rejecting the null hypothesis H_0 when it is true.
- A **type II error** involves not rejecting the null hypothesis H_0 when it is false.

Decision	H_0 is True	H_0 is False
Fail to Reject H_0	No Error	Type II Error
Reject H_0	Type I Error	No Error

Decision	H_0 is True	H_0 is False	
Fail to Reject H_0	$1-\alpha$	β	Power of
Reject H_0	α	$1-\beta$	a test

Case 3 (t)

Chapter 8.3: The One-Sample t Tests for Hypotheses about a Population Mean

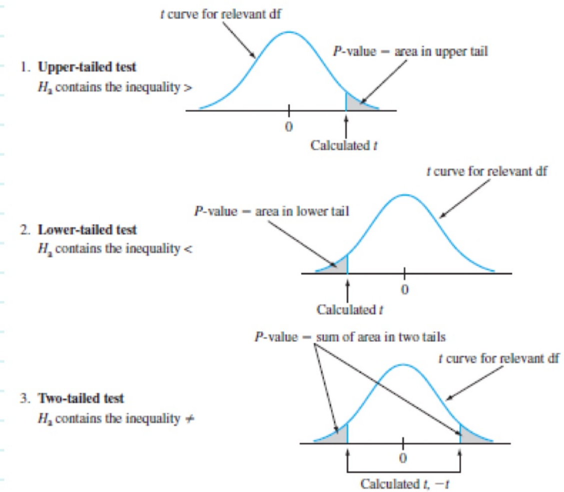
- Population is Normal and σ^2 is not known (similar to Chapter 7.3)

Null hypothesis: $H_0 : \mu = \mu_0$

Test statistic: $T = \frac{\overline{X}-\mu_0}{S/\sqrt{n}}$ $\quad T = \frac{\overline{X}-\mu_0}{S/\sqrt{n}} \sim T_{n-1}$

Test statistic value: $t = \frac{\overline{x}-\mu_0}{s/\sqrt{n}}$ $\quad t = \frac{\overline{x}-\mu_0}{s/\sqrt{n}}$

Alternative Hypothesis	P-value Determination
$H_a : \mu > \mu_0$	Area under the t_{n-1} curve to the right of t
$H_a : \mu < \mu_0$	Area under the t_{n-1} curve to the left of t
$H_a : \mu \neq \mu_0$	2(area under the t_{n-1} curve to the right of $\|t\|$)



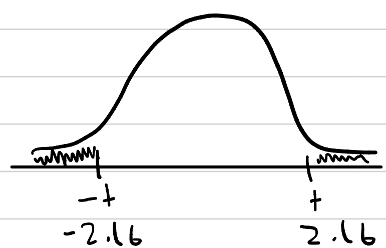
P-values for t tests

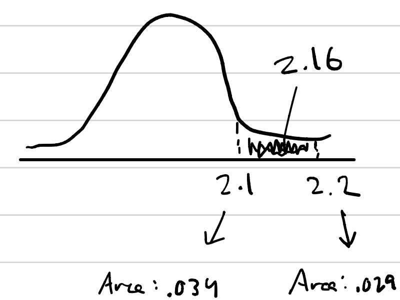
$$t = \frac{\overline{x}-\mu_0}{s/\sqrt{n}} = \frac{\overline{x}-130}{s/\sqrt{9}}$$

$= 2.16$

$\Phi(2.16)$ ← Table A.8

$\approx 0.031 \cdot 2 = \boxed{0.062}$





$p = .062 > .01 \therefore$ Fail to reject H_0

Case 4 ($\hat{p}$)

Chapter 8.4: Tests Concerning a Population Proportion

- We have a large sample (similar to Chapter 7.2). These test procedures are valid provided that $np_0 \geq 10$ and $n(1-p_0) \geq 10$.

Null hypothesis: $H_0 : p = p_0$

Test statistic value: $z = \frac{\hat{p}-p_0}{\sqrt{p_0(1-p_0)/n}}$

Alternative Hypothesis	P-value Determination
$H_a : p > p_0$	Area under the standard normal curve to the right of z
$H_a : p < p_0$	Area under the standard normal curve to the left of z
$H_a : p \neq p_0$	2(area under the standard normal curve to the right of $\lvert z \rvert$)

Type II Error

Alternative Hypothesis	Type II Error Probability $\beta(p')$ for a Level α Test
$H_a : p > p_0$	$\Phi\left(\frac{p_0-p'+z_\alpha\sqrt{p_0(1-p_0)/n}}{\sqrt{p'(1-p')/n}}\right)$
$H_a : p < p_0$	$1-\Phi\left(\frac{p_0-p'-z_\alpha\sqrt{p_0(1-p_0)/n}}{\sqrt{p'(1-p')/n}}\right)$
$H_a : p \neq p_0$	$\Phi\left(\frac{p_0-p'+z_{\frac{\alpha}{2}}\sqrt{p_0(1-p_0)/n}}{\sqrt{p'(1-p')/n}}\right) - \Phi\left(\frac{p_0-p'-z_{\frac{\alpha}{2}}\sqrt{p_0(1-p_0)/n}}{\sqrt{p'(1-p')/n}}\right)$

Sample Size

The sample size n for which a level α test also has $\beta(p') = \beta$ at the alternative value p'

$$n = \begin{cases} \left[\frac{z_\alpha\sqrt{p_0(1-p_0)}+z_\beta\sqrt{p'(1-p')}}{p'-p_0}\right]^2 & \text{for a one-tailed test,} \\ \left[\frac{z_{\frac{\alpha}{2}}\sqrt{p_0(1-p_0)}+z_\beta\sqrt{p'(1-p')}}{p'-p_0}\right]^2 & \text{for a two-tailed test.} \end{cases}$$

Case 5 (χ^2) – NOT exam material

Additional: Hypothesis Tests on the Variance

- Similar to Chapter 7.4

Null hypothesis: $H_0 : \sigma^2 = \sigma_0^2$

Test statistic: $\chi^2 = \frac{(n-1)S^2}{\sigma_0^2}$

Test statistic value: $\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$

Alternative Hypothesis	P-value Determination
$H_a : \sigma^2 > \sigma_0^2$	A_R =Area under the χ^2_{n-1} curve to the right of χ^2
$H_a : \sigma^2 < \sigma_0^2$	A_L =Area under the χ^2_{n-1} curve to the left of χ^2
$H_a : \sigma^2 \neq \sigma_0^2$	$2\min(A_R, A_L)$

Two-Sample Hypothesis Testing

11/7/22

Case 1: Two Population Means (known σ)

$$Z = \frac{\bar{x} - \bar{y} - \Delta_0}{\sqrt{\frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}}}$$

$$\sigma_{\bar{x}-\bar{y}} = \sqrt{\frac{\sigma_1^2}{m} + \frac{\sigma_2^2}{n}}$$

Square result for variance

Type II Error	Formula
$H_a: \mu_1 - \mu_2 > \Delta_0$	$\Phi\left(z_\alpha + \frac{\Delta_0 - \Delta'}{\sigma}\right)$
$H_a: \mu_1 - \mu_2 < \Delta_0$	$1 - \Phi\left(-z_\alpha + \frac{\Delta_0 - \Delta'}{\sigma}\right)$
$H_a: \mu_1 - \mu_2 \neq \Delta_0$	$\Phi\left(z_{\alpha/2} + \frac{\Delta_0 - \Delta'}{\sigma}\right) - \Phi\left(-z_{\alpha/2} + \frac{\Delta_0 - \Delta'}{\sigma}\right)$

Sample Size $(m=n)$	Formula
One-Tail	$\frac{(\sigma_1^2 + \sigma_2^2)(z_\alpha + z_\beta)^2}{(\Delta' - \Delta_0)^2}$
Two-Tail	$\frac{(\sigma_1^2 + \sigma_2^2)(z_{\alpha/2} + z_\beta)^2}{(\Delta' - \Delta_0)^2}$

Alt. Hypothesis	p-value
$H_a: \mu_1 - \mu_2 > \Delta_0$	Area to right
$H_a: \mu_1 - \mu_2 < \Delta_0$	Area to left
$H_a: \mu_1 - \mu_2 \neq \Delta_0$	2(area to right)

Case 2: Non-normal, Unknown Variance, Large Sample ($m > 40$, $n > 40$)

$H_0: \mu_1 - \mu_2 = \Delta_0$

$H_a: \mu_1 - \mu_2 <, \neq, > \Delta_0$

$$z = \frac{\bar{X} - \bar{Y} - \Delta_0}{\sqrt{\frac{S_1^2}{m} + \frac{S_2^2}{n}}}$$

* Sub in σ if pop SD is known *

Confidence Interval: $\bar{X} - \bar{Y} \pm z_{\alpha/2} \sqrt{\frac{S_1^2}{m} + \frac{S_2^2}{n}}$

$H_a: \mu > \#$: Lower Confidence Bound $\bar{X} - \bar{Y} + z_{\alpha} \sqrt{\frac{S_1^2}{m} + \frac{S_2^2}{n}}$

$H_a: \mu < \#$: Upper Confidence Bound $\bar{X} - \bar{Y} - z_{\alpha} \sqrt{\frac{S_1^2}{m} + \frac{S_2^2}{n}}$

Alt. Hypothesis	p-value
$H_a: \mu_1 - \mu_2 > \Delta_0$	Area to right of z
$H_a: \mu_1 - \mu_2 < \Delta_0$	Area to left of z
$H_a: \mu_1 - \mu_2 \neq \Delta_0$	2(area to right) of z

Case 3: Two - Sample t - test

$H_0: \mu_1 - \mu_2 = \Delta_0$
$H_a: \mu_1 - \mu_2 <, \neq, > \Delta_0$

Test Statistic

$$t = \frac{(\bar{X} - \bar{Y}) - \Delta_0}{\sqrt{\frac{S_1^2}{m} + \frac{S_2^2}{n}}}$$

Critical value

$t_{\alpha/2, v}$

Two-Sample CI: $\bar{X} - \bar{Y} \pm t_{\alpha/2, v} \sqrt{\frac{S_1^2}{m} + \frac{S_2^2}{n}}$

$H_a: \mu > \#$: Lower Confidence Bound
$H_a: \mu < \#$: Upper Confidence Bound

$$v = \frac{\left(\frac{S_1^2}{m} + \frac{S_2^2}{n}\right)^2}{\frac{(S_1^2/m)^2}{m-1} + \frac{(S_2^2/n)^2}{n-1}}$$

Alt. Hypothesis	p-value
$H_a: \mu_1 - \mu_2 > \Delta_0$	Area to right of $t_{\alpha, v}$
$H_a: \mu_1 - \mu_2 < \Delta_0$	Area to left of $t_{\alpha, v}$
$H_a: \mu_1 - \mu_2 \neq \Delta_0$	2 (area to right) of $t_{\alpha, v}$

Case 4: Pooled t-test (both populations have the same variance)

$$t = \frac{(\bar{X} - \bar{Y}) - (\mu_1 - \mu_2)}{\sqrt{S_p^2 \left(\frac{1}{m} + \frac{1}{n}\right)}}$$

σ^2 estimator $S_p^2 = \frac{m-1}{m+n-2} S_1^2 + \frac{n-1}{m+n-2} S_2^2$

CI: $\bar{X} - \bar{Y} \pm t_{\alpha/2,\, n+m-2} \sqrt{S_p^2 \left(\frac{1}{m} + \frac{1}{n}\right)}$

Ha: $\mu > \#$: Lower Confidence Bound
Ha: $\mu < \#$: Upper Confidence Bound

Alt. Hypothesis	p-value
Ha: $\mu_1 - \mu_2 > \Delta_0$	Area to right of t_{m+n-2}
Ha: $\mu_1 - \mu_2 < \Delta_0$	Area to left of t_{m+n-2}
Ha: $\mu_1 - \mu_2 \neq \Delta_0$	2(area to right of t_{m+n-2})

Case 5: Paired t-test Procedure

$$t = \frac{\bar{d} - \Delta_0}{S_D / \sqrt{n}}$$

$H_0: \mu_D = \Delta_0$
$H_a: \mu_D <, \neq, > \Delta_0$

$$\bar{d} = \bar{x} - \bar{y}$$

$$S_D = \frac{1}{n-1} \sum_{i=1}^{n} (d_i - \bar{d})^2$$

CI: $\bar{d} \pm t_{\alpha/2,\, n-1} \, S_D / \sqrt{n}$

$H_a: \mu > \#$: Lower Confidence Bound
$H_a: \mu < \#$: Upper Confidence Bound

Alt. Hypothesis	p-value
$H_a: \mu_1 - \mu_2 > \Delta_0$	Area to right of t_{n-1}
$H_a: \mu_1 - \mu_2 < \Delta_0$	Area to left of t_{n-1}
$H_a: \mu_1 - \mu_2 \neq \Delta_0$	2(area to right of t_{n-1}

Paired vs Unpaired Experiments

1. If there is great heterogeneity between experimental units and a large correlation within experimental units (large positive ρ), then the loss in degrees of freedom will be compensated for by the increased precision associated with pairing, so a paired experiment is preferable to an independent-samples experiment.
2. If the experimental units are relatively homogeneous and the correlation within pairs is not large, the gain in precision due to pairing will be outweighed by the decrease in degrees of freedom, so an independent-samples experiment should be used.

Case 6: Difference in Proportions (Large Sample)

$$z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q}\left(\frac{1}{m} + \frac{1}{n}\right)}}$$

$$\hat{p} = \frac{m}{m+n}\hat{p}_1 + \frac{n}{m+n}\hat{p}_2$$

$$CI: \hat{p}_1 - \hat{p}_2 \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{m} + \frac{\hat{p}_2\hat{q}_2}{n}}$$

$$\text{Sample Size: } n = \frac{\left(z_\alpha\sqrt{(p_1+p_2)(q_1+q_2)/2} + z_\beta\sqrt{p_1q_1 + p_2q_2}\right)^2}{d^2} \rightarrow d = p_1 - p_2$$

Type II Error	Formula
$H_a: p_1 - p_2 > 0$	$\Phi\left(\frac{z_\alpha\sqrt{\bar{p}\bar{q}\left(\frac{1}{m}+\frac{1}{n}\right)} - (p_1 - p_2)}{\sigma}\right)$
$H_a: p_1 - p_2 < 0$	$1 - \Phi\left(\frac{-z_\alpha\sqrt{\bar{p}\bar{q}\left(\frac{1}{m}+\frac{1}{n}\right)} - (p_1 - p_2)}{\sigma}\right)$
$H_a: p_1 - p_2 \neq 0$	$\Phi\left(\frac{z_{\alpha/2}\sqrt{\bar{p}\bar{q}\left(\frac{1}{m}+\frac{1}{n}\right)} - (p_1 - p_2)}{\sigma}\right) - \Phi\left(\frac{-z_{\alpha/2}\sqrt{\bar{p}\bar{q}\left(\frac{1}{m}+\frac{1}{n}\right)} - (p_1 - p_2)}{\sigma}\right)$

$$\bar{p} = \frac{mp_1 + np_2}{m+n} \qquad \bar{q} = \frac{mq_1 + nq_2}{m+n} \qquad \sigma = \sqrt{\frac{p_1q_1}{m} + \frac{p_2q_2}{n}}$$

Alt. Hypothesis	p-value
$H_a: \mu_1 - \mu_2 > \Delta_0$	Area to right of z
$H_a: \mu_1 - \mu_2 < \Delta_0$	Area to left of z
$H_a: \mu_1 - \mu_2 \neq \Delta_0$	2 (area to right) of z

Key words from Exam 2 Review

5. The desired percentage of SiO_2 in a certain type of aluminous cement is 5.5. To test whether the true average percentage is 5.5 for a particular production facility, 16 independently obtained samples are analyzed. Suppose that the percentage of SiO_2 in a sample is normally distributed with $\sigma = 0.3$ and that $\bar{x} = 5.25$.

   a. Does this indicate conclusively that the true average percentage differs from 5.5?

   b. If the true average percentage is $\mu = 5.6$ and a level $\alpha = 0.01$ test based on n = 16 is used, what is the probability of detecting this departure from H_0?

   c. What value of n is required to satisfy $\alpha = 0.01$ and $\beta(5.6) = 0.01$?

5b: Probability of detecting departure = 1 - beta (1 - P(type II error))

1. Tensile-strength tests were carried out on two different grades of wire rod, resulting in the accompanying data.

Grade	Sample Size	Sample Mean (kg/mm^2)	Sample SD
AISI 1064	$m = 129$	$\bar{x} = 107.6$	$s_1 = 1.3$
AISI 1078	$n = 129$	$\bar{y} = 123.6$	$s_2 = 2.0$

   Assume that we have large samples in both populations.

   a. Does the data provide compelling evidence for concluding that true average strength for the 1078 grade exceeds that for the 1064 grade by more than 10 kg/mm2? Test the appropriate hypotheses using a significance level of 0.01.

   b. Estimate a confidence interval on the difference between true average strengths for the two grades in a way that provides information about precision and reliability.

1b: Ha: mu > 10
Since the ">" sign was used in this test, the confidence interval required is a one-sided LOWER confidence bound

6. Many freeways have service (or logo) signs that give information on attractions, camping, lodging, food, and gas services prior to off-ramps. These signs typically do not provide information on distances. In a study was reported that in one investigation, six sites along Virginia interstate highways where service signs are posted were selected. For each site, crash data was obtained for a three-year period before distance information was added to the service signs and for a one-year period afterward. The number of crashes per year before and after the sign changes were as follows:

Before: 15 26 66 115 62 64

After: 16 24 42 80 78 73

   a. The study included the statement "A paired t test was performed to determine whether there was any change in the mean number of crashes before and after the addition of distance information on the signs." Carry out such a test. [*Note: Assume that data follows a normal distribution.*]

   b. If a seventh site were to be randomly selected among locations bearing service signs, between what values would you predict the difference in number of crashes to lie?

6b: "Between what values would you PREDICT = prediction interval!!!

Linear Regression & Analysis

12/14/22

Formulas from chapter 12.1:

$$y = \beta_0 + \beta_1 x + \epsilon$$

$$\epsilon \sim N(0, \sigma^2)$$

$$E[aX + b] = aE[X] + b$$

$$Var[aX + b] = a^2 Var[X]$$

Chapter 12.2:

The least squares estimate of the slope coefficient β_1 of the true regression line is

$$b_1 = \hat{\beta}_1 = \frac{\sum(x_i - \overline{x})(y_i - \overline{y})}{\sum(x_i - \overline{x})^2} = \frac{S_{xy}}{S_{xx}},$$

where

$$S_{xy} = \sum x_i y_i - \frac{(\sum x_i)(\sum y_i)}{n}, \qquad S_{xx} = \sum x_i^2 - \frac{(\sum x_i)^2}{n}.$$

The least squares estimate of the slope coefficient β_0 of the true regression line is

$$b_0 = \hat{\beta}_0 = \frac{\sum y_i - \hat{\beta}_1 \sum x_i}{n} = \overline{y} - \hat{\beta}_1 \overline{x}.$$

SSE

The **error sum of squares** (equivalently, residual sum of squares), denoted by SSE, is

① $SSE = S_{yy} - \hat{\beta}_1 S_{xy}$

② $$\text{SSE} = \sum (y_i - \hat{y}_i)^2 = \sum [y_i - (\hat{\beta}_0 + \hat{\beta}_1 x_i)]^2,$$

and the estimate of σ^2 is

$$\hat{\sigma}^2 = s^2 = \frac{\text{SSE}}{n-2} = \frac{\sum(y_i - \hat{y}_i)^2}{n-2}.$$

We can calculate SSE as

$$\text{SSE} = \sum y_i^2 - \hat{\beta}_0 \sum y_i - \hat{\beta}_1 \sum x_i y_i = S_{yy} - \hat{\beta}_1 S_{xy}.$$

Chapter 12.2 Continued...

The **coefficient of determination**, denoted by r^2, is given by

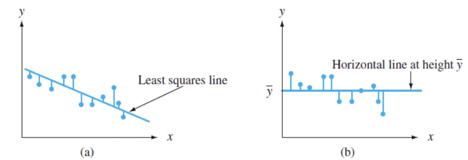
$$r^2 = 1 - \frac{\text{SSE}}{\text{SST}}.$$

It is interpreted as the proportion of observed y variation that can be explained by the simple linear regression model.

SST

A quantitative measure of the total amount of variation in observed y values is given by the **total sum of squares**, denoted by SST is

$$\text{SST} = S_{yy} = \sum (y_i - \bar{y})^2 = \sum y_i^2 - \frac{\left(\sum y_i\right)^2}{n}$$



$\hat{\beta}_0$ $\hat{\beta}_1$

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 75.21243    2.98363  25.208 9.22e-12 ***
iodine      -0.20939    0.03109  -6.734 2.09e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.564 on 12 degrees of freedom
Multiple R-squared:  0.7908,    Adjusted R-squared:  0.7733
F-statistic: 45.35 on 1 and 12 DF,  p-value: 2.091e-05
```

$100r^2$

SSR= $\sum(\hat{y}_i - \bar{y})^2$ =SST-SSE

Regression sum of squares is interpreted as the amount of total variation that is explained by the model.

$$r^2 = 1 - \frac{\text{SSE}}{\text{SST}} = \frac{(\text{SST}-\text{SSE})}{\text{SST}} = \frac{\text{SSR}}{\text{SST}}$$

Chapter 12.3:

- The mean value of $\hat{\beta}_1$ is $E[\hat{\beta}_1] = \beta_1$, so $\hat{\beta}_1$ is an unbiased estimator for β_1 is (the distribution of β_1 is always centered at the value of β_1).
- The variance and standard deviation of $\hat{\beta}_1$ are

$$\text{Var}(\hat{\beta}_1) = \sigma^2_{\hat{\beta}_1} = \frac{\sigma^2}{S_{xx}}, \qquad \sigma_{\hat{\beta}_1} = \frac{\sigma}{\sqrt{S_{xx}}}.$$

Replacing σ by its estimate s gives an estimate for $\sigma_{\hat{\beta}_1}$ (the estimated standard deviation, i.e., estimated standard error, of $\hat{\beta}_1$):

$$s_{\hat{\beta}_1} = \frac{s}{\sqrt{S_{xx}}}.$$

- The estimator $\hat{\beta}_1$ has a normal distribution.
- $T = \frac{\hat{\beta}_1 - \beta_1}{S_{\hat{\beta}_1}} = \frac{\hat{\beta}_1 - \beta_1}{S/\sqrt{S_{xx}}}$ has a t distribution with $(n-2)$ df.

Hypothesis Testing for β_1

$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}}$

Null hypothesis: $H_0 : \beta_1 = \beta_{10}$

Test statistic: $T = \frac{\hat{\beta}_1 - \beta_{10}}{S_{\hat{\beta}_1}}$ $\quad S_{\hat{\beta}_1} = \frac{S}{\sqrt{S_{xx}}}$ $\quad s = \sqrt{\frac{SSE}{n-2}}$

Test statistic value: $t = \frac{\hat{\beta}_1 - \beta_{10}}{s_{\hat{\beta}_1}}$ $\quad S_{xx} = \sum x_i^2 - \frac{(\sum x_i)^2}{n}$

Alternative Hypothesis	P-value Determination
$H_a : \beta_1 > \beta_{10}$	Area under the t_{n-2} curve to the right of t
$H_a : \beta_1 < \beta_{10}$	Area under the t_{n-2} curve to the left of t
$H_a : \beta_1 \neq \beta_{10}$	2(area under the t_{n-2} curve to the right of $\|t\|$)

A CI for the slope β_1 of the true regression line with a confidence level of $100(1-\alpha)\%$ is

$$\hat{\beta}_1 \pm t_{\frac{\alpha}{2}, n-2} s_{\hat{\beta}_1},$$

where $-$ gives the lower limit and $+$ the upper limit of the interval. An upper or lower confidence bound can also be obtained by retaining the appropriate sign ($+$ or $-$) and replacing $\frac{\alpha}{2}$ by α.

$\hat{\beta}_0$ $\hat{\beta}_1$ $S_{\hat{\beta}_1}$ P value $t = \frac{\hat{\beta}_1}{S_{\hat{\beta}_1}}$

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 75.21243    2.98363  25.208 9.22e-12 ***
iodine      -0.20939    0.03109  -6.734 2.09e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.564 on 12 degrees of freedom
Multiple R-squared:  0.7908,    Adjusted R-squared:  0.7733
F-statistic: 45.35 on 1 and 12 DF,  p-value: 2.091e-05
```

$100r^2$

Chapter 12.4

Let $\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 x^*$, where x^* is some fixed value of x.

1. The mean value of $\hat{Y}$ is $E[\hat{Y}] = E[\hat{\beta}_0 + \hat{\beta}_1 x^*] = \beta_0 + \beta_1 x^*$. Thus, $\hat{\beta}_0 + \hat{\beta}_1 x^*$ is an unbiased estimator for $\beta_0 + \beta_1 x^*$ (i.e., for $\mu_{Y|x^*}$).
2. The variance of $\hat{Y}$ is

$$\mathrm{Var}(\hat{Y}) = \sigma_{\hat{Y}}^2 = \sigma^2 \left[\frac{1}{n} + \frac{(x^* - \overline{x})^2}{S_{xx}} \right],$$

The estimated standard deviation of $\hat{Y}$, denoted by $s_{\hat{Y}}$, is:

$$s_{\hat{Y}} = s_{\hat{\beta}_0 + \hat{\beta}_1 x^*} = s\sqrt{\frac{1}{n} + \frac{(x^* - \overline{x})^2}{S_{xx}}}.$$

3. The estimator $\hat{Y}$ has a normal distribution.
4. $T = \frac{\hat{\beta}_0 + \hat{\beta}_1 x^* - (\beta_0 + \beta_1 x^*)}{s_{\hat{\beta}_0 + \hat{\beta}_1 x^*}} = \frac{\hat{Y} - (\beta_0 + \beta_1 x^*)}{s_{\hat{Y}}}$ has a t distribution with $(n-2)$ df.

A CI for $\mu_{Y|x^*}$, the expected value of Y when $x = x^*$, with a confidence level of $100(1-\alpha)\%$ is

$$\hat{\beta}_0 + \hat{\beta}_1 x^* \pm t_{\frac{\alpha}{2}, n-2} s_{\hat{\beta}_0 + \hat{\beta}_1 x^*} = \hat{y} \pm t_{\frac{\alpha}{2}, n-2} s_{\hat{Y}},$$

where $-$ gives the lower limit and $+$ the upper limit of the interval. An upper or lower confidence bound can also be obtained by retaining the appropriate sign ($+$ or $-$) and replacing $\frac{\alpha}{2}$ by α.

A PI for a future Y observation to be made when $x = x^*$, with a confidence level of $100(1-\alpha)\%$ is

$$\hat{\beta}_0 + \hat{\beta}_1 x^* \pm t_{\frac{\alpha}{2}, n-2} s \sqrt{1 + \frac{1}{n} + \frac{(x^* - \overline{x})^2}{S_{xx}}}$$
$$= \hat{\beta}_0 + \hat{\beta}_1 x^* \pm t_{\frac{\alpha}{2}, n-2} \sqrt{s^2 + s_{\hat{\beta}_0 + \hat{\beta}_1 x^*}^2}$$
$$= \hat{y} \pm t_{\frac{\alpha}{2}, n-2} \sqrt{s^2 + s_{\hat{Y}}^2}.$$