# Computer Vision 1

## C0.05, 11:00-14:00, October 24th, 2014

### Question 1: Reflection Models

Consider the color of a glossy (shiny) surface:

$$R = Ik_R(N \cdot L) + k_s(V \cdot R)^{n_s} \tag{1}$$
$$G = Ik_G(N \cdot L) + k_s(V \cdot R)^{n_s} \tag{2}$$
$$B = Ik_B(N \cdot L) + k_s(V \cdot R)^{n_s} \tag{3}$$

where $I$ is the intensity of a white light source, $k_R$, $k_G$ and $k_B$ are the amount of red, green and blue reflected by the surface (i.e. color of the surface - albedo). Furthermore, vector $N$ is the surface normal, $L$ is the direction of the light source, $V$ is the viewer direction and $R$ is the reflected light direction. Finally, $k_s$ and $n_s$ are constants.

- (a) Explain the two main terms in this equation. (no need to explain each individual symbol, just the two additive terms) (1 pts)
- (b) What assumptions does the Lambertian illumination model make, and how would this change the equation? (1 pts)
- (c) Given is the figure below (Figure 1), where each arrow denotes an incoming ray of light and the gray box is a gray surface. Redraw the figure on your answer sheet with the light rays as reflected by a Lambertian surface. (1 pts)
- (d) Redraw again to show how the light rays respond to a refractive surface; and again for a specularity. (1 pts)
- (e) Assuming a Lambertian surface, what is the color of the reflected light? (1 pts)
- (f) Assuming a specular surface, what is the color of the reflected light? (1 pts)
- (g) Assuming a specular surface and assuming that the color of the light source is blue, what is the color of the reflected light? (1 pts)
- (h) Assuming a Lambertian surface, prove that $r = \frac{B^2}{RG}$ (at a pixel) is independent of the (intensity) light source, object geometry and the direction of the light source. (2 pts)
- (i) Assuming a glossy surface, prove that $r = \frac{(R-B)^2}{(B-G)^2}$ (at a pixel) is only dependent on $k_R$, $k_B$ and $k_G$ and hence a full invariant. (3 pts)

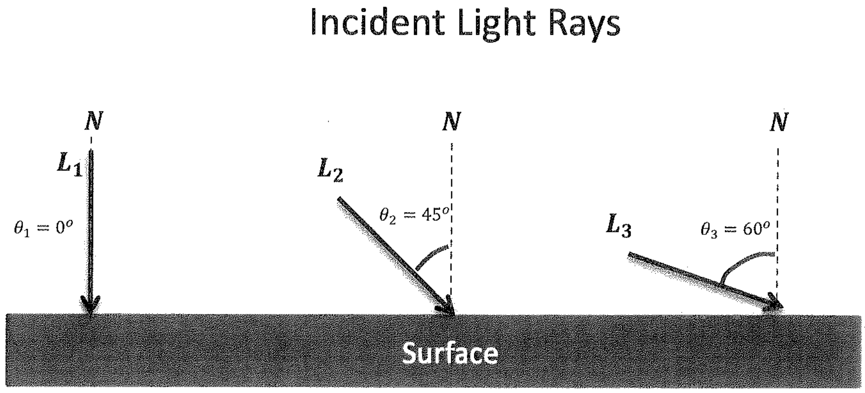


Figure 1: Figure illustrates the incoming light arrays $L_1$ , $L_2$ and $L_3$. $\Theta$ represents the angle between incident light array ($L_i$) and the surface normal ($N$).

## Question 2: Image Features and Descriptors

Edges and corners are important features from which image descriptors can be extracted. Consider the following image patches:

$P =$

| | | | | |
|---|---|---|---|---|
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 |

$Q =$

| | | | | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 |

Figure 2: *Intensity values of two small image patches $P$ and $Q$.*

- (a) Consider the patches $P$ and $Q$ (in Figure 2), show the full results after convolving the images with a 3x3 filter $U$, where all elements at the boundaries (outside the image) are mirrored. (2 pts)

$U =$

| | | |
|---|---|---|
| 1 | 0 | 1 |
| 1 | 0 | 1 |
| 1 | 0 | 1 |

- (b) Is the 3x3 filter $U$ separable? If so, show an example. (2 pts)
- (c) Compute the gradient $\bigtriangledown f = \sqrt{f_x^2 + f_y^2}$ of image patch $P$ using a simple derivative filter $H_x = \begin{pmatrix} -1 & 1 \end{pmatrix}$ in the $x$-direction and $H_y = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ in the $y$-direction. (1 pts)
- (d) Compute the second moment matrix $M = \begin{pmatrix} \Sigma f_x^2 & \Sigma f_x f_y \\ \Sigma f_x f_y & \Sigma f_y^2 \end{pmatrix}$ for image patch $P$. (1 pts)

- (e) Compute the eigenvalues of $M$ for $P$. What do these eigenvalues mean? (2 pts)
- (f) Compute the second moment matrix also for $Q$. (1 pts)
- (g) Compute the eigenvalues of $M$ for $Q$. (2 pts)
- (h) Compute the eigenvectors of $M$ for $Q$. What do these eigenvectors mean? (3 pts)
- (i) Compute the histogram of oriented gradients for patch P and Q. Which histogram is more discriminative? (1 pts)
- (j) Using the histogram of oriented gradients, how do you compute a SIFT descriptor? (1 pts)
- (k) Is the SIFT descriptor invariant under a change in (in-plane) rotation of the object? Please explain. (1 pts)
- (l) Is the intensity-based SIFT descriptor invariant under a change in intensity of the light source? Please explain.(1 pts)

## Question 3: Object Colors and Invariants

Consider three colors $A$ with $R = 60$, $G = 80$, and $B = 40$, $B$ with $R = 45$, $G = 60$, and $B = 30$, and $C$ with $R = 180$, $G = 40$ and $B = 70$.

- (a) Are $A$, $B$ and $C$ different with respect to their intensity? ( 1pts)
- (b) To what extent do $A$, $B$ and $C$ differ with respect to their chromaticity? (1 pts)
- (c) With $W$ as reference white, to what extent do $A$, $B$ and $C$ differ with respect to their hue? (1 pts)

Color invariants become unstable for certain imaging conditions. One way to handle instabilities is by error propagation:

$$\sigma_q = \sqrt{(\frac{\partial_q}{\partial_u}\sigma_u)^2 + ... + (\frac{\partial_q}{\partial_w}\sigma_w)^2} \tag{4}$$

to compute a function $q(u, ..., w)$ for variables $u, ..., w$ and their corresponding uncertainties $\sigma_u, ..., \sigma_w$. Consider a pixel $P$ having the following values R=40, G=40, B=60 with $\sigma_R = \sigma_G = \sigma_B = 2$.

- (d) Calculate the uncertainty for color model $R^2 + G$ at $P$. Is color model $R^2 + G$ stable? (1 pts)
- (e) Calculate the uncertainty for color model $\frac{R}{G^2}$. Does this color model become unstable when the intensity is decreasing? (2 pts)
- (f) Calculate the uncertainty for color model $\frac{2R-G}{G-B}$. Does this color model become unstable when the saturation is decreasing? (3 pts)

## Question 4: Object Recognition and Retrieval Effectiveness

The bag-of-features is an important approach in object detection.

- (a) What is the difference between dense and interest point sampling? (1 pts)
- (b) What are the basic steps of the bag-of-feature approach? (1 pts)
- (c) What are spatial pyramids and why are they useful? (1 pts)

Given is an image database of 10.000 images. A programmer has designed and implemented two different image retrieval systems $S_1$ and $S_2$. A company is interested in the performance of the two systems. To this end, the company has formulated two different search queries $Q_1$ and $Q_2$. The number of relevant images with respect to all search queries is 10 composed of the following set (A, B, C, D, E, F, G, H, I, J). The number of images shown to the user is 15 (Answer Set). Further, the order of the 15 highest ranked images of the two different image retrieval systems $S_1$ and $S_2$ for query $Q_1$ is as follows:

| $S_1$ | $S_2$ |
|---|---|
| 1. $A$ | 1. $A$ |
| 2. $L$ | 2. $M$ |
| 3. $N$ | 3. $O$ |
| 4. $B$ | 4. $B$ |
| 5. $S$ | 5. $K$ |
| 6. $P$ | 6. $C$ |
| 7. $Q$ | 7. $D$ |
| 8. $C$ | 8. $R$ |
| 9. $T$ | 9. $S$ |
| 10. $O$ | 10. $T$ |
| 11. $D$ | 11. $N$ |
| 12. $V$ | 12. $V$ |
| 13. $W$ | 13. $W$ |
| 14. $X$ | 14. $X$ |
| 15. $Y$ | 15. $Q$ |

and for query $Q_2$:

| $S_1$ | $S_2$ |
|---|---|
| 1. $A$ | 1. $A$ |
| 2. $B$ | 2. $M$ |
| 3. $L$ | 3. $Q$ |
| 4. $C$ | 4. $N$ |
| 5. $P$ | 5. $O$ |
| 6. $D$ | 6. $S$ |
| 7. $E$ | 7. $K$ |
| 8. $F$ | 8. $R$ |
| 9. $O$ | 9. $B$ |
| 10. $T$ | 10. $T$ |

| | |
|---|---|
| 11. $S$ | 11. $C$ |
| 12. $V$ | 12. $D$ |
| 13. $W$ | 13. $E$ |
| 14. $X$ | 14. $X$ |
| 15. $Y$ | 15. $F$ |

- (d) Generate the precision-recall graph for the systems $S_1$ and $S_2$ for $Q_1$. (1 pts)
- (e) Generate the precision-recall graph for the systems $S_1$ and $S_2$ for $Q_2$. (1 pts)
- (f) What conclusion can you draw from the precision-recall graphs? (1 pts)
- (g) Compute the R-Precision for $S_1$ and $S_2$ for $Q_1$ and $Q_2$ respectively. (1 pts)