Computer Vision 1

21 March, 2016, 14.00-17.00

Question 1: Reflection Models

We consider the representation of (object) colors. Given is a light source with spectral power distribution $e(\lambda)$. The color (i.e. spectral reflection) of an object is denoted by $\rho(\lambda)$. The color matching functions of CIE X, Y and Z primary colors are given in figure 1 with 10 nm interval. Further, let $\cos\theta = \vec{n} \cdot \vec{s}$ be the angle between the surface normal $\vec{n}$ and light source direction $\vec{s}$. Then, the Lambertian reflection model is given by:

$$X = \cos\theta \sum_{\lambda} e(\lambda)\rho(\lambda)\bar{\mathrm{x}}(\lambda), \tag{1}$$

$$Y = \cos\theta \sum_{\lambda} e(\lambda)\rho(\lambda)\bar{\mathrm{y}}(\lambda), \tag{2}$$

$$Z = \cos\theta \sum_{\lambda} e(\lambda)\rho(\lambda)\bar{\mathrm{z}}(\lambda). \tag{3}$$

- (a) Assuming a white object (i.e. $\rho(\lambda) = 1$), what is the color of the reflected light? (1 pts)
- (b) Is the amount of the reflected light influenced by the viewpoint? Please explain. (1 pts)
- (c) What are the conditions to obtain the maximum intensity of reflected light? (2 pts)

For the following, assume that $e(\lambda)$ is white light i.e. $e(\lambda) = 1$.

- (d) Consider an object color A where color values are 1 for 550-600 nm and 0 elsewhere i.e. $\rho_A(\lambda) = 1$ from 550 to 600 nm. Compute the X, Y and Z values from figure 1 for $\theta = 45°$ and $\theta = 0°$. Further, compute the chromaticity coordinates x, y and z for A. (2pts)
- (e) Given another object color B from 650 nm to 700 nm i.e. $\rho_B(\lambda) = 1$ from 650 to 700 nm and 0 elsewhere. For $\theta = 0°$, calculate X, Y and Z and the chromaticity coordinates x, y and z. (2 pts)
- (f) With $x = 0.33$ and $y = 0.33$ as reference white W, do A and B differ with respect to their saturation and hue? Briefly explain in words. (1 pts)

λ	$\overline{x}$	$\overline{y}$	$\overline{z}$
480	0.0956	0.1390	0.8130
490	0.0320	0.2080	0.4652
500	0.0049	0.3230	0.2720
510	0.0093	0.5030	0.1582
520	0.0633	0.7100	0.0782
530	0.1655	0.8620	0.0422
540	0.2904	0.9540	0.0203
550	0.4334	0.9950	0.0087
560	0.5945	0.9950	0.0039
570	0.7621	0.9520	0.0021
580	0.9163	0.8700	0.0017
590	1.0263	0.7570	0.0011
600	1.0622	0.6310	0.0008
610	1.0026	0.5030	0.0003
620	0.8544	0.3810	0.0002
630	0.6424	0.2650	0.0000
640	0.4479	0.1750	0.0000
650	0.2835	0.1070	0.0000
660	0.1649	0.0610	0.0000
670	0.0874	0.0320	0.0000
680	0.0468	0.0170	0.0000
690	0.0227	0.0082	0.0000
700	0.0114	0.0041	0.0000

Figure 1: *Tristimulus values of spectral colors X, Y and Z.*

- (g) Prove that $L = \frac{X}{Y}$ (at a pixel) is independent of the (intensity) light source, object geometry and the direction of the light source. (1 pts)
- (h) A simple color invariant is given by X_{x_1}/X_{x_2}, where X_{x_1} and X_{x_2} are the measured red quantities at neighboring positions x_1 and x_2. It is assumed that the light source is constant over neighboring locations. Prove that X_{x_1}/X_{x_2} is independent of the light source. (1 pts)
- (i) Is this color invariant independent of the object geometry? (1 pts)

Color invariants become unstable for certain imaging conditions. One way to handle instabilities is by error propagation:

$$\sigma_q = \sqrt{(\frac{\partial_q}{\partial_u}\sigma_u)^2 + ... + (\frac{\partial_q}{\partial_w}\sigma_w)^2} \tag{4}$$

to compute a function $q(u, ..., w)$ for variables $u, ..., w$ and their corresponding uncertainties $\sigma_u, ..., \sigma_w$. Consider a pixel P having the following values X=20, Y=40, Z=60 with $\sigma = 4$.

- (j) Calculate the uncertainty for color model $2Y + 3Z$ at P. Is color model $2Y + 3Z$ stable? (1 pts)
- (k) Calculate the uncertainty for color model $2X/Y$. Does this color model become unstable when the intensity is decreasing? (2 pts)

Question 2: Image Features and Descriptors

Edges and corners are important features from which image descriptors can be extracted.

- (a) A simple filter that approximates the derivative in x-direction f_x is given by $h = \begin{pmatrix} \underline{-1} & 1 \end{pmatrix}$. Show how higher order derivatives, f_{xx} and f_{xxx} can be estimated using h. (1 pts)

- (b) Given the following filter:

$$g = \alpha \; \begin{array}{|c|c|c|}\hline 1 & 2 & 1 \\ \hline\end{array}$$

  Define what is the simplest filter that we can use to apply multiple convolutions with in order to achieve the same result as above. What is the value of α? (2 pts)

- (c) When convolved with an image f, which of the filters, h, g or $g * h$, correspond to a low-pass, high-pass or band-pass filter? Please explain for the different filters. (2 pts)

Consider the following image patches:

$P =$

0	0	0	0
0	0	0	0
2	2	2	2
2	2	2	2

$Q =$

0	0	2	2
0	0	2	2
0	0	2	2
0	0	0	0

Figure 2: *Intensity values of two small image patches P and Q.*

- (d) Compute the results after convolving image P by a 3x3 filter U, where all elements at the image boundaries (i.e. outside image P) are mirrored. The elements outside filter U are all zero. (2 pts)

$U =$

1	-2	1
-2	$\underline{4}$	-2
1	-2	1

- (e) What kind of features are detected in P? (1 pts)

- (f) Show that the 3x3 filter U is separable. (2 pts)

- (g) Compute the second moment matrix $M = \begin{pmatrix} f_x^2 & f_x f_y \\ f_x f_y & f_y^2 \end{pmatrix}$ for image patch P using a simple derivative filter $h_x = \begin{pmatrix} \underline{-1} & 1 \end{pmatrix}$ in the x-direction and $h_y = \begin{pmatrix} -1 \\ \underline{1} \end{pmatrix}$ in the y-direction. (1 pts)

- (h) Compute the Harris corner response of image patch P by $R = \mathrm{Det}M - \mathrm{k}(\mathrm{Trace}(M))^2$ where $k = 0.04$. Considering R, is there an edge or a corner in the image? (1 pts)

- (i) Compute the eigenvalues of M for image Q in figure 2. How can you use these eigenvalues to detect a corner? (3 pts)
- (j) Compute the histogram of oriented gradients for patch Q. (1 pts)
- (k) Name two photometric image transformations that the SIFT descriptor is invariant to. (1 pts)
- (l) Is the intensity-based SIFT descriptor invariant under a change in intensity of the light source? Please explain.(1 pts)
- (m) What is a color SIFT descriptor? Give an example.(1 pts)

Question 3: Object Detection

In general, a sliding-window approach is used to detect objects.

- (a) How does the sliding-window approach work? (1 pts)
- (b) What are the pros and cons? (1 pts)
- (c) Do you know another approach to limit the number of window candidates? (1 pts)

The sliding-window approach can also be applied to face analysis. After face detection, facial features (e.g. deformations from a neutral expression) are extracted and tracked over time to classify the corresponding emotion.

- (d) What are typical facial features? (1 pts)
- (e) What are the six basic emotions of humans? (1 pts)
- (f) What are the facial characteristics of a happy person? (1 pts)

Given is an image database of 10.000 images. A programmer has designed and implemented two different image retrieval systems S_1 and S_2. A company is interested in the performance of the two systems. To this end, the company has formulated two different search queries Q_1 and Q_2. The number of relevant images with respect to all search queries is 10 composed of the following set (A, B, C, D, E, F, G, H, I, J). The number of images shown to the user is 20 (Answer Set). Further, the order of the 20 highest ranked images of the two different image retrieval systems S_1 and S_2 for query Q_1 is as follows:

S_1	S_2
1. A	1. A
2. L	2. M
3. N	3. B
4. O	4. O
5. S	5. K
6. P	6. C
7. Q	7. D
8. B	8. R
9. T	9. S
10. C	10. E

11. X	11. N
12. V	12. V
13. W	13. T
14. D	14. X
15. Y	15. Q
16. E	16. L
17. Z	17. P
18. K	18. U
19. M	19. Y
20. R	20. Z

and for query Q_2:

S_1	S_2
1. A	1. K
2. B	2. M
3. L	3. Q
4. C	4. N
5. P	5. O
6. D	6. S
7. E	7. A
8. F	8. R
9. O	9. B
10. T	10. T
11. S	11. C
12. V	12. D
13. W	13. E
14. X	14. X
15. Y	15. F
16. K	16. X
17. P	17. L
18. Z	18. Y
19. Q	19. W
20. N	20. P

- (g) Compute the precision and recall for the systems S_1 and S_2 for Q_1 and Q_2 respectively. (1 pts)
- (h) Generate the precision-recall graph for the systems S_1 and S_2 for Q_1. (1 pts)
- (i) Generate the precision-recall graph for the systems S_1 and S_2 for Q_2. (1 pts)
- (j) What conclusion can you draw from the precision-recall graphs? (1 pts)