https://tinyurl.com/dhcp7h3w

Computer Vision 1

Reflectance & Photometric Stereo Image Processing Basics

Dr. Martin Oswald, Dr. Dimitris Tzionas, Dr. Arun Mukundan,
(m.r.oswald, d.tzionas, a.mukundan)@uva.nl

University of Amsterdam

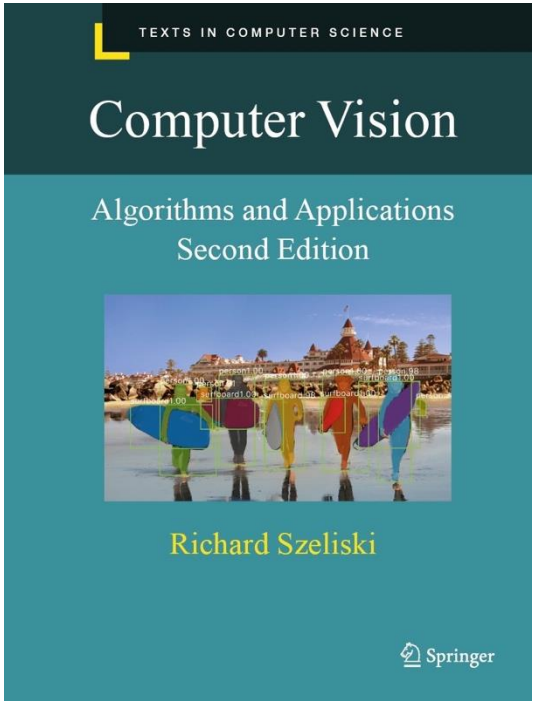
computer vision lab

Outline

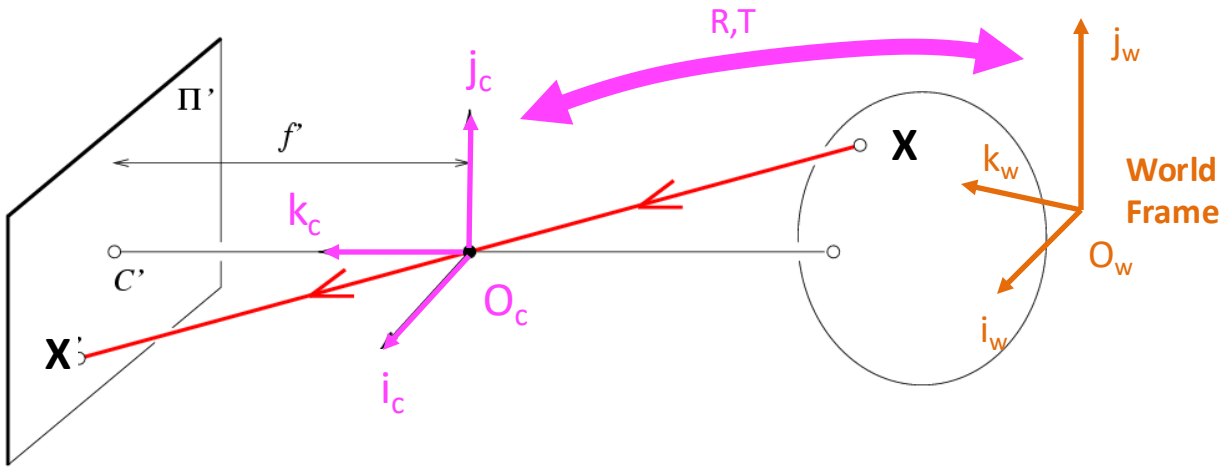
- Reflection and Photometric Stero
  - Reflection Models
  - Shape from Shading
  - Photometric Stereo
  - Intrinsic Decomposition
  - Applications
- Image Processessing Basics
  - Histogram
  - Histogram Modification
  - Histogram Equalization

Textbook

- 3.1.1
- 3.1.2
- 3.1.4
- 2.2.2



Camera Projection Matrix



Camera Frame posed (R, T) w.r.t. World Frame

$$\mathbf{x} = \mathbf{K}\begin{bmatrix}\mathbf{I} & \mathbf{0}\end{bmatrix}\mathbf{X}$$

Intrinsic Matrix

Extrinsic Matrix

O_w: Center of World Frame

O_c: Center of Camera Frame

X: 3D point in World Coordinates: (X,Y,Z,1)

x: 2D Image Coordinates: (u,v,1), up to scale w

K: Intrinsic Matrix (3x3)

R: Rotation (3x3)

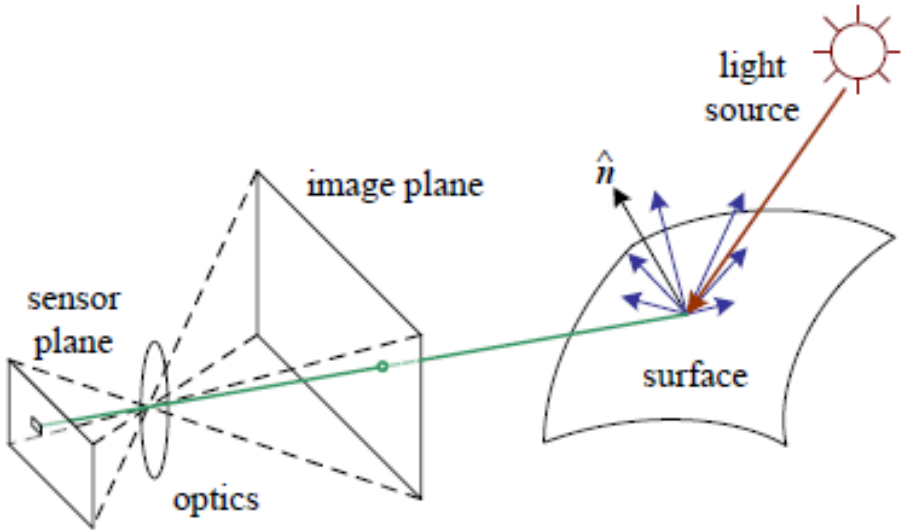
t: Translation (3x1)

Note: different books use different notation!

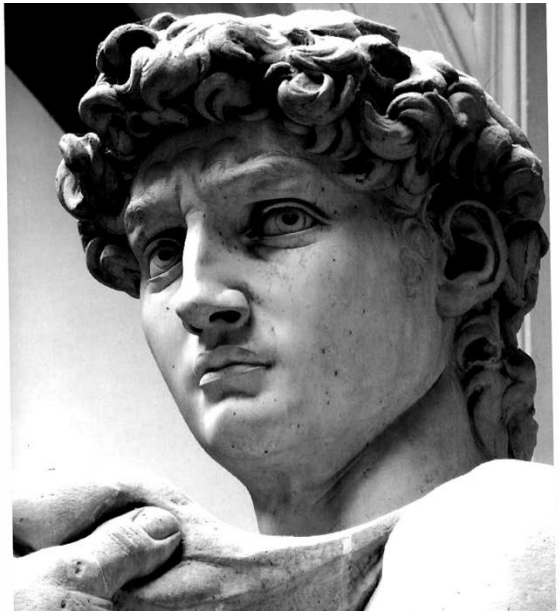
Slide Credit: S. Saverese

4 main factors which define intensity values

- the geometry of the scene
- the reflectances of the visible surface
- the illumination of the scene
- the viewpoint



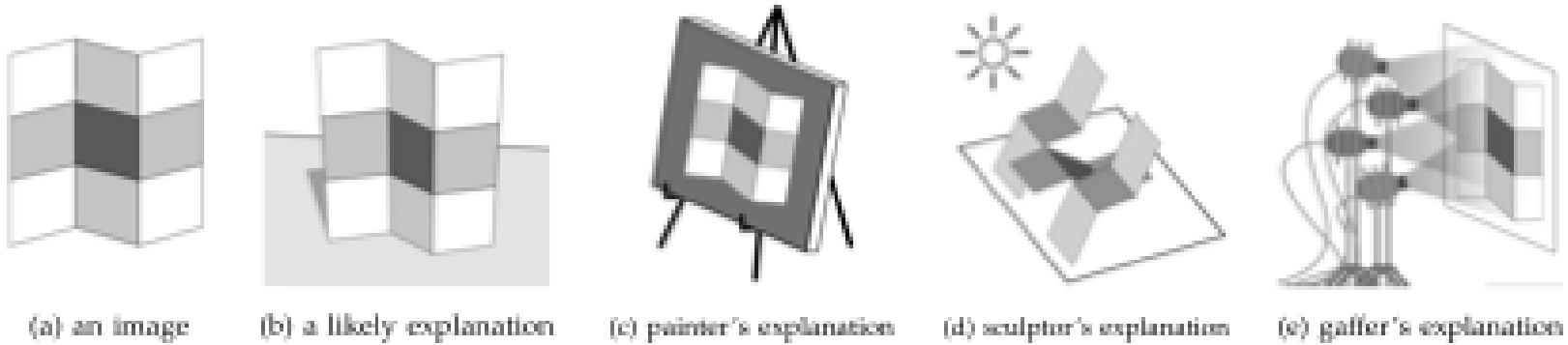
Can we recover shape from shading?



What is the relation between **intensity** and **shape**? Can we recover shape?

[Slide credit: Andreas Geiger]

Adelson and Pentland's Workshop Metaphor



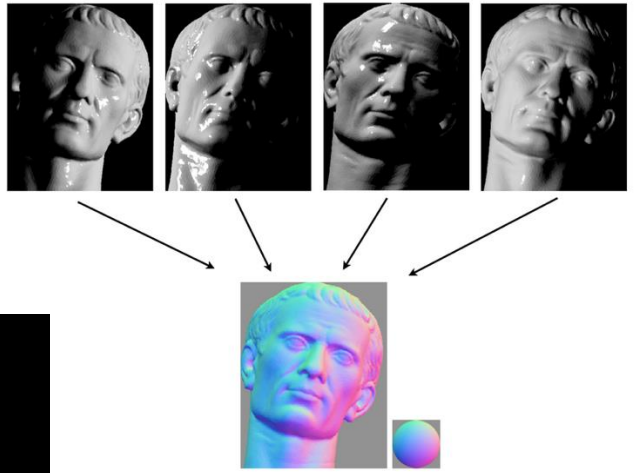
Can we recover **3D from a single image?**

- Inverting the graphics problem is very challenging
- The space of shapes, paint, and light that exactly reproduce an image is vast
- The image in (a) very likely corresponds to (b), but it could be a painting (c), sculpture (d), or an arrangement of lights (e)
- We are looking for a **simple explanation** and must use **prior knowledge**

[Slide credit: Andreas Geiger]

Motivation: Photometric Stereo

- Photometric stereo captures **multiple** images from the **same viewpoint**, but with **different light settings**
- Per-pixel estimation of **normal** and **albedo** or **material**
- Assumes far cameras/lights

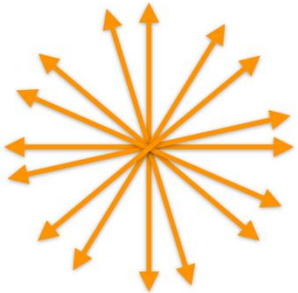


[Esper light stage]

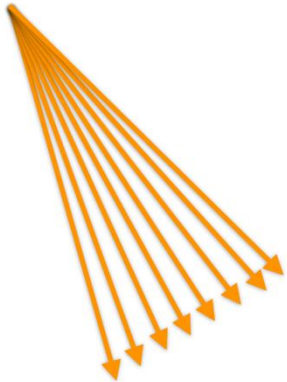
Total Relighting

[Pandey et al, SIGGRAPH'21, YouTube]

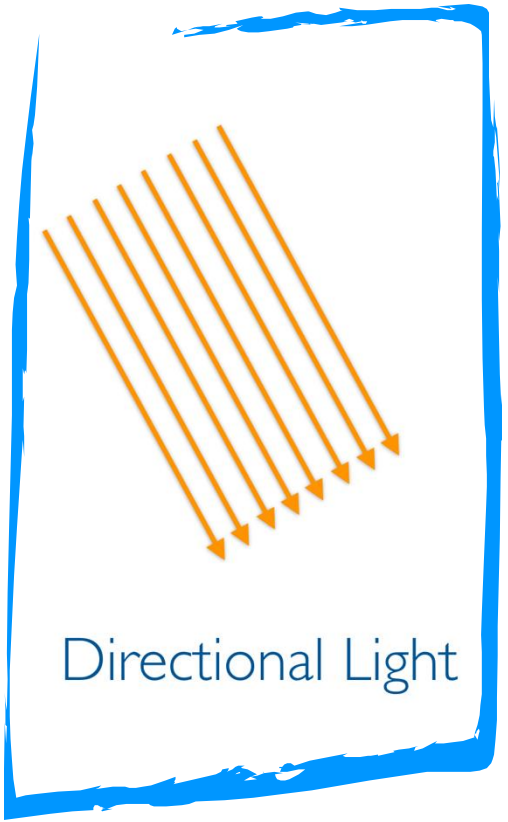
Light sources



Point Light



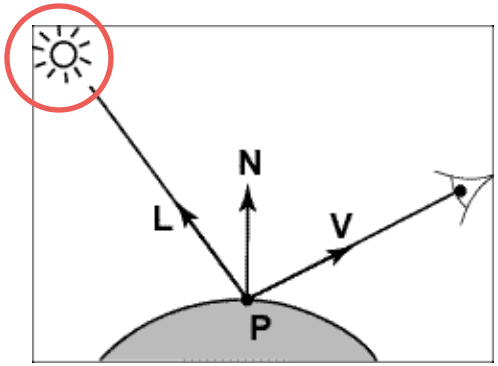
Spot Light



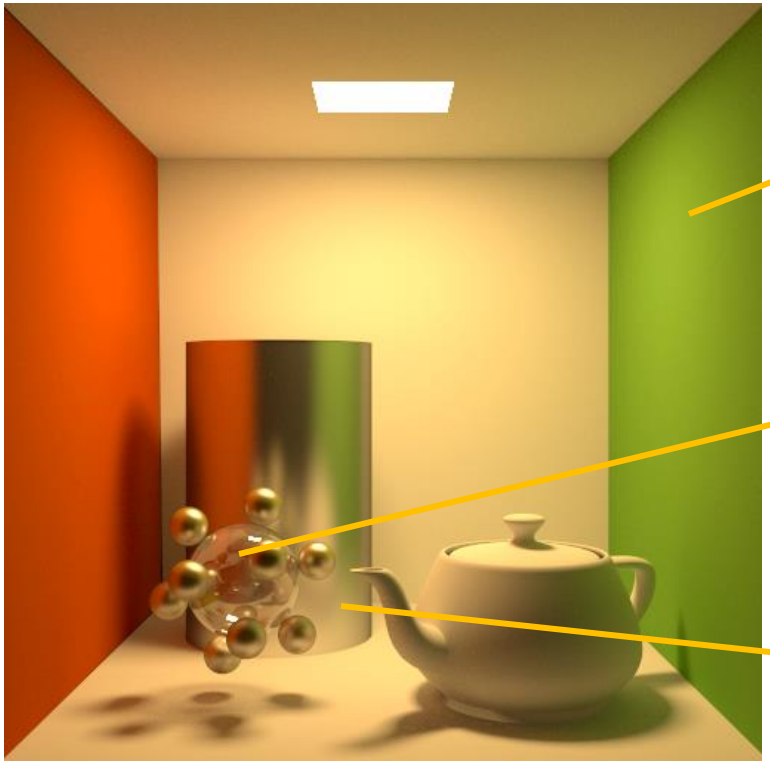
Directional Light

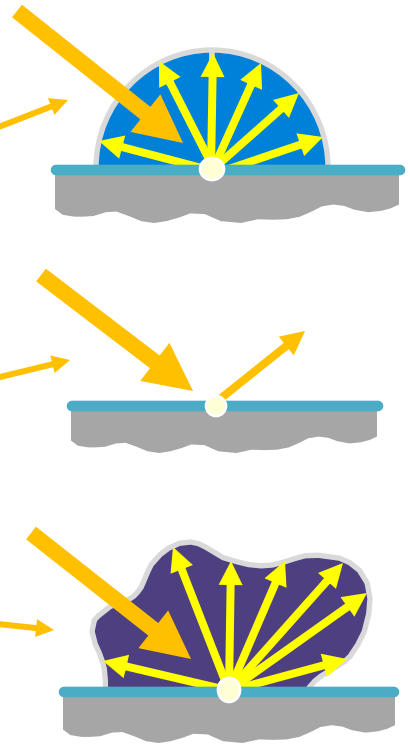
Directional Lighting and Reflectance

- Key property: all rays are parallel
- Equivalent to an infinitely distant point source



Materials - Three Forms





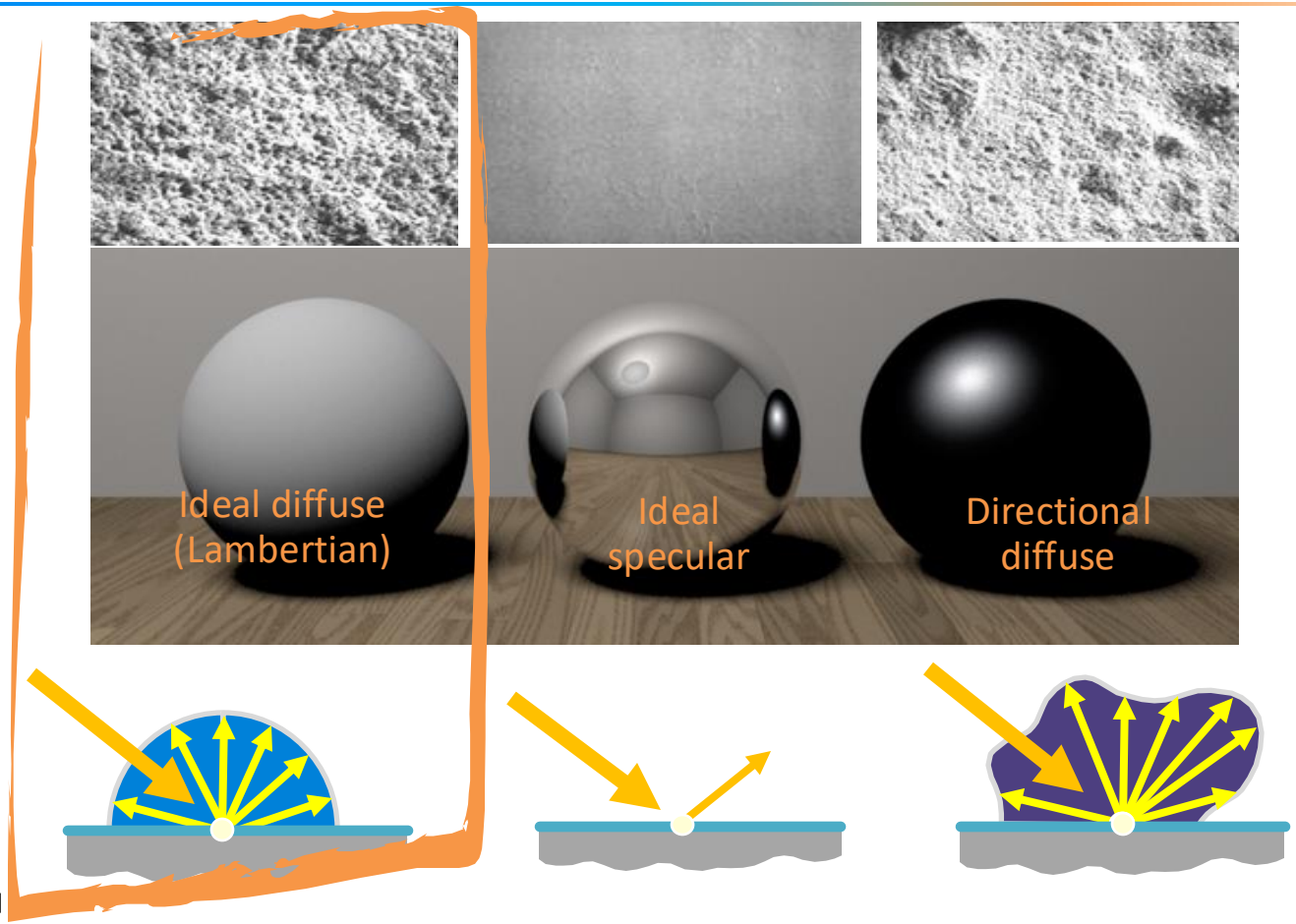
Ideal diffuse (Lambertian)

Ideal specular

Directional diffuse

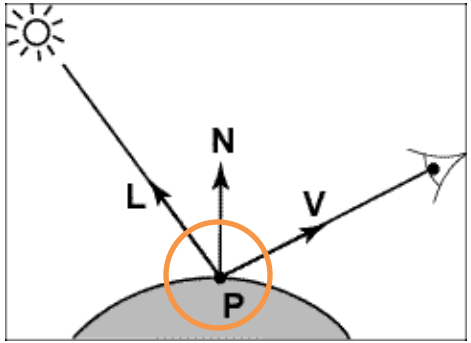
[Slide credit: Kavita Bala]

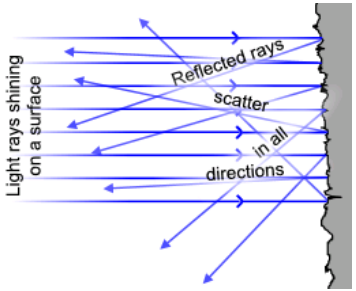
Reflectance - Three Forms

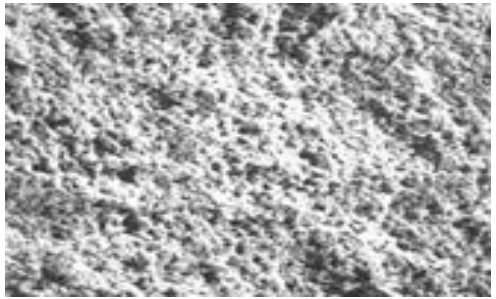


[Slide credit: Kavita Bala]

Reflectance - Three Forms



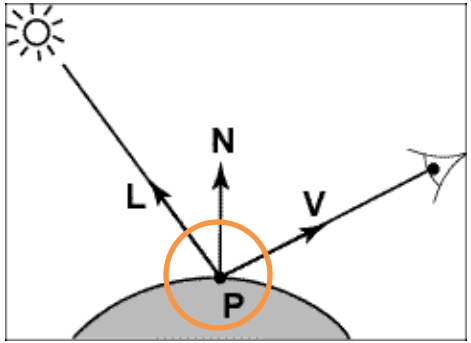


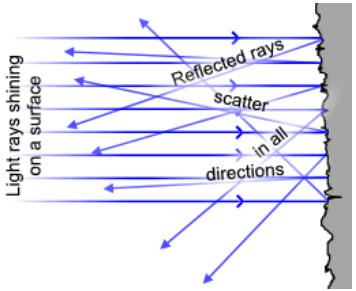


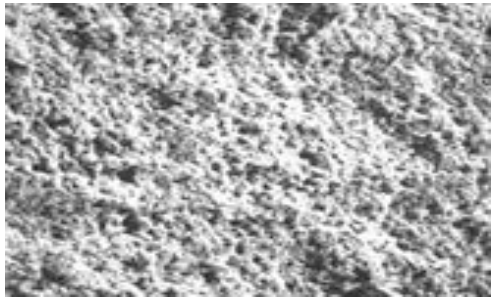
1. Reflected energy is proportional to cosine of angle between L and N **(incoming)**

2. Measured intensity is viewpoint-independent **(outgoing)**

Reflectance - Three Forms







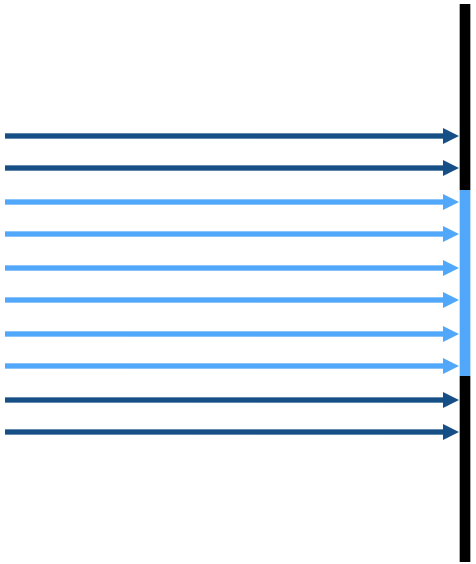
$$I \propto N \cdot L$$

Image intensity $\propto$ Surface normal $\cdot$ Light direction

Image intensity $\propto$ cos(angle between N and L)

Lambertian Reflectance: Incoming

Reflected energy is proportional to cosine of angle between L and N.



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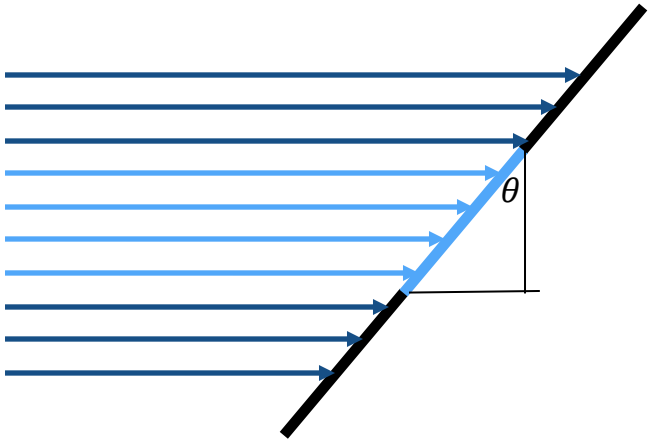
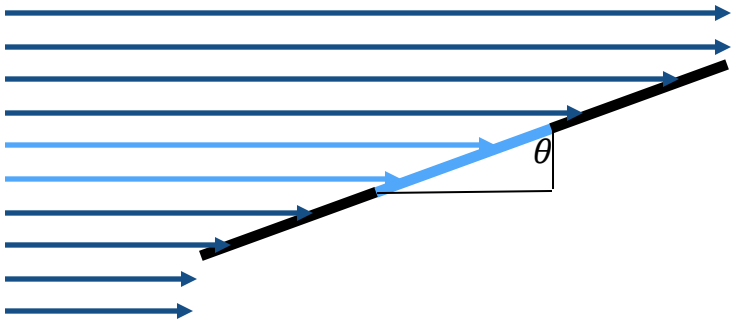


$$I \propto \underbrace{N \cdot L}_{\cos(\theta)}$$

Lambertian Reflectance: Incoming

Reflected energy is proportional to cosine of angle between L and N.

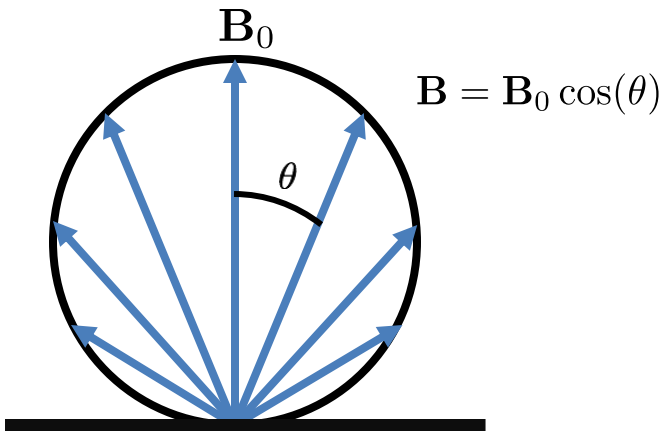


$$I \propto \underbrace{N \cdot L}_{\cos(\theta)}$$

Light hitting surface is proportional to the **cosine**.

Lambertian appearance is view-independent

- Number of photons reflected to a given angle θ is proportional to $\cos(\theta)$



Lambert's cosine law: $\mathbf{B} = \mathbf{B}_0 \cos(\theta)$

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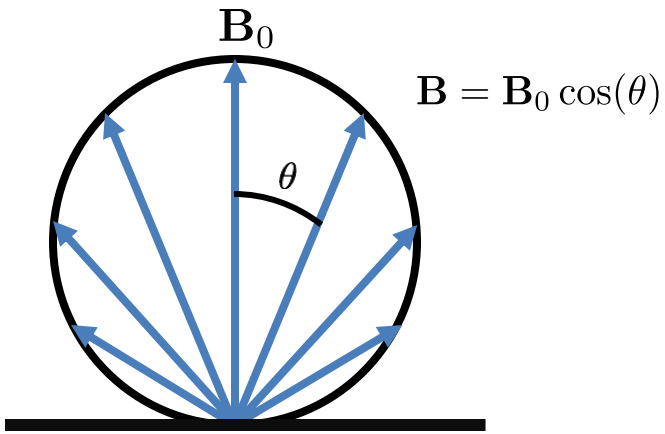
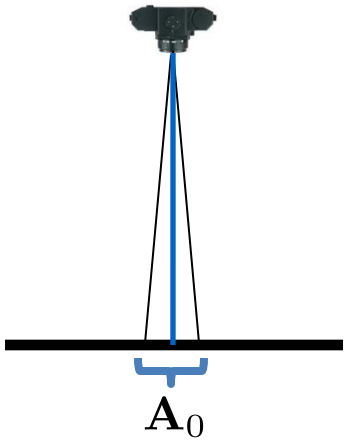


Lambert's cosine law: $\mathbf{B} = \mathbf{B}_0 \cos(\theta)$

- But appearance is the same from every angle due to larger pixel footprint at larger angles



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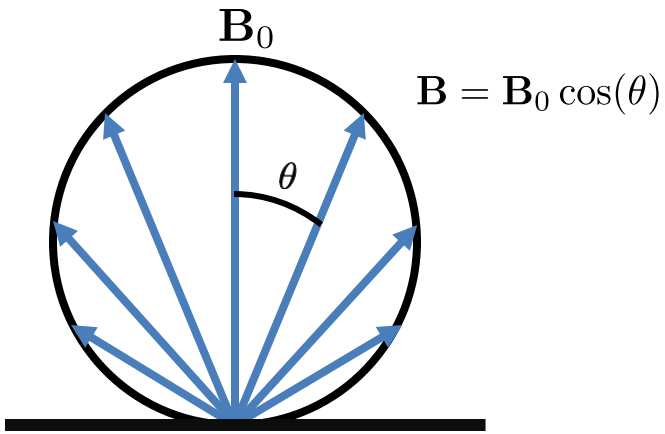

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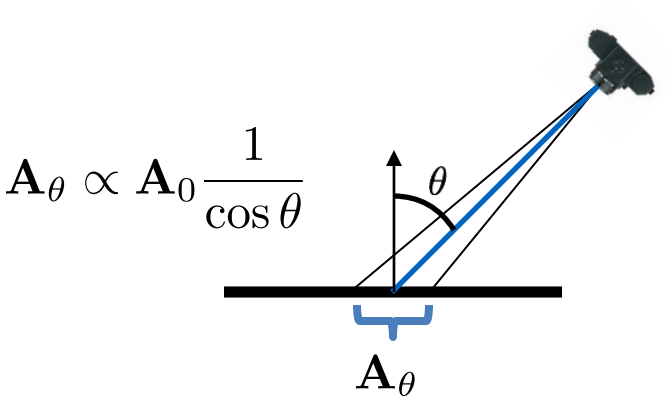

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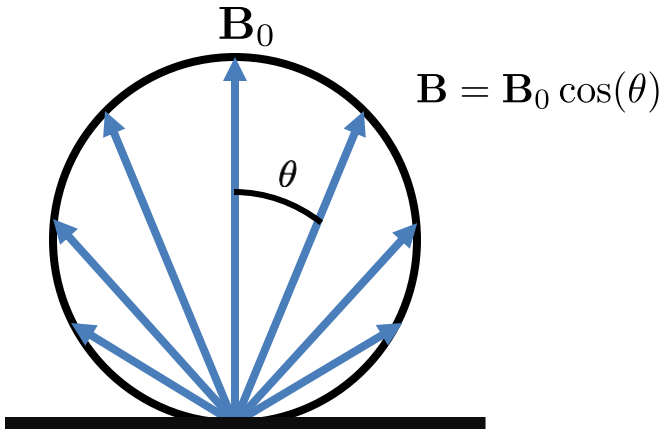

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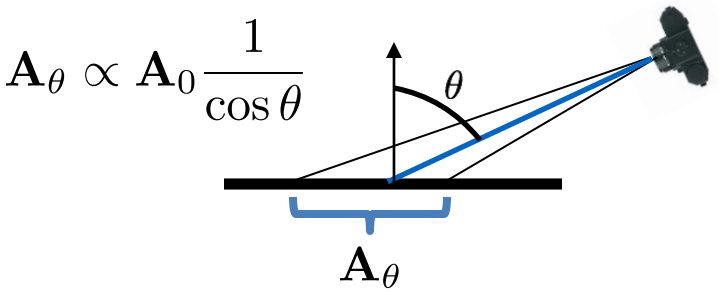
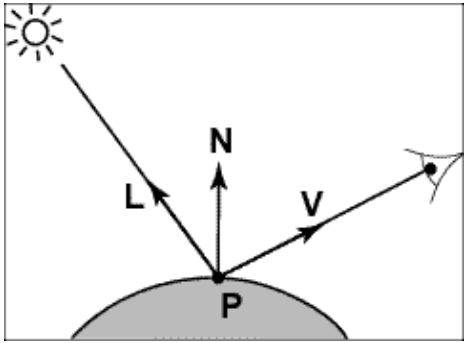
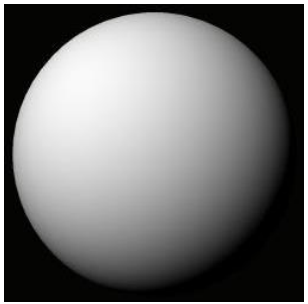


Radiance (what eye sees) $\propto \mathbf{B}_0 \mathbf{A}_0 \cancel{\cos(\theta)} \frac{1}{\cancel{\cos(\theta)}}$

Final Lambertian Image Formation Model

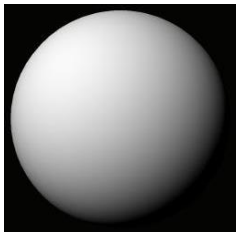


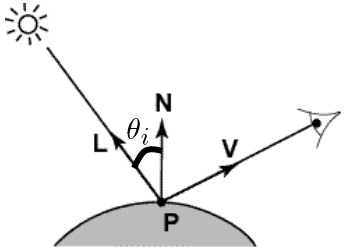
$$I = k_d \mathbf{N} \cdot \mathbf{L}$$



1. Diffuse **albedo**: what fraction of incoming light is reflected?
   - Introduce scale factor k_d
2. Light intensity: how much light is arriving?
   - Compensate with camera exposure (global scale factor)
3. Camera response function
   - Assume pixel value is linearly proportional to incoming energy (perform radiometric calibration if not)

A Single Image: Shape from Shading





Suppose (for now) $k_d = 1$

$$
\begin{aligned}
I &= k_d \mathbf{N} \cdot \mathbf{L} \\
&= \mathbf{N} \cdot \mathbf{L} \\
&= cos\ \theta_i
\end{aligned}
$$

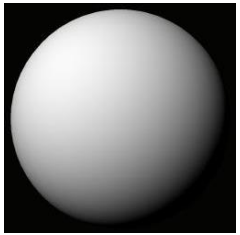
You can directly measure angle between normal and light source

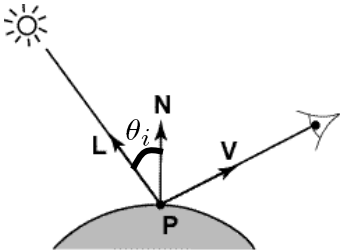
- Not quite enough information to compute surface shape
- But can be if you add some additional info, for example
  - assume a few of the normals are known (e.g., along silhouette)
  - constraints on neighboring normals—"integrability"
  - smoothness
- Hard to get it to work well in practice
  - plus, how many real objects have constant albedo?
  - But, deep learning can help

Shape from Shading

Shape from Shading assumptions:

- Diffuse material with spatially constant albedo
  - Reduces the number of material parameters to 1
- Known point light source at infinity
  - Keeps light direction L constant across all pixels
  - Makes L independent of geometry/depth
- Known camera at infinity (orthography)
  - Keeps view direction V constant across all pixels
  - Makes V independent of geometry/depth





[Slide credit: Andreas Geiger]

Lightsource Position Ambiguity

Plates and bowls or upside down or not?

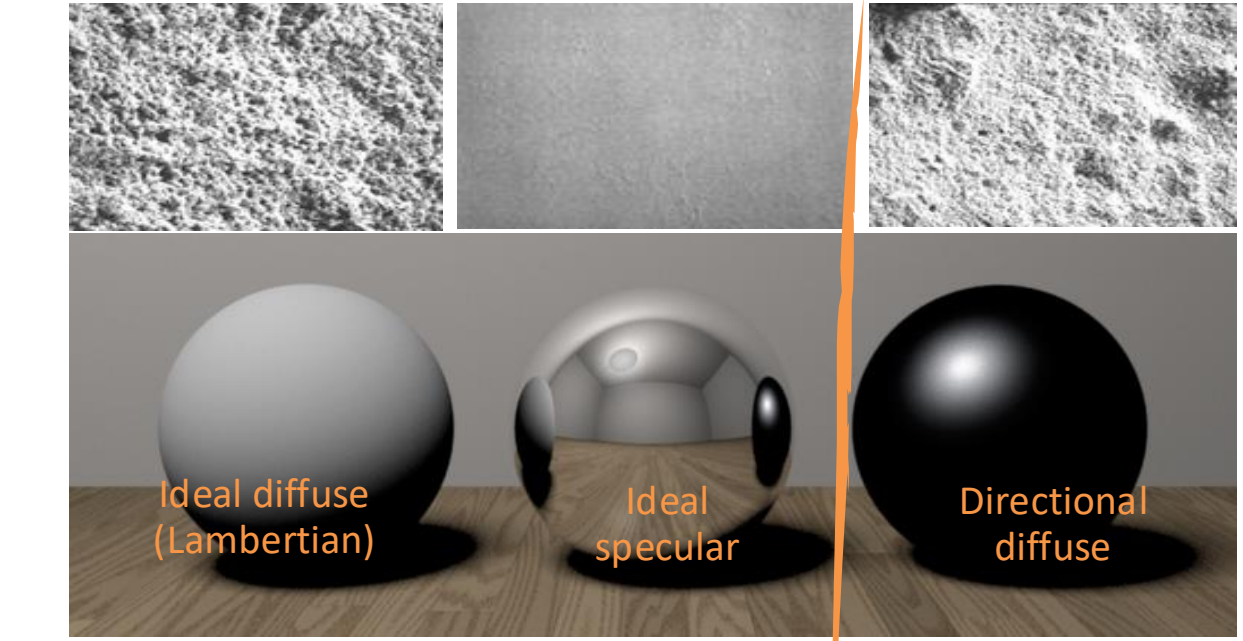
[https://www.good.is/optical-illusion-plates-and-bowls-upside-down-or-not]

Application: Detecting composite photos

Fake photo

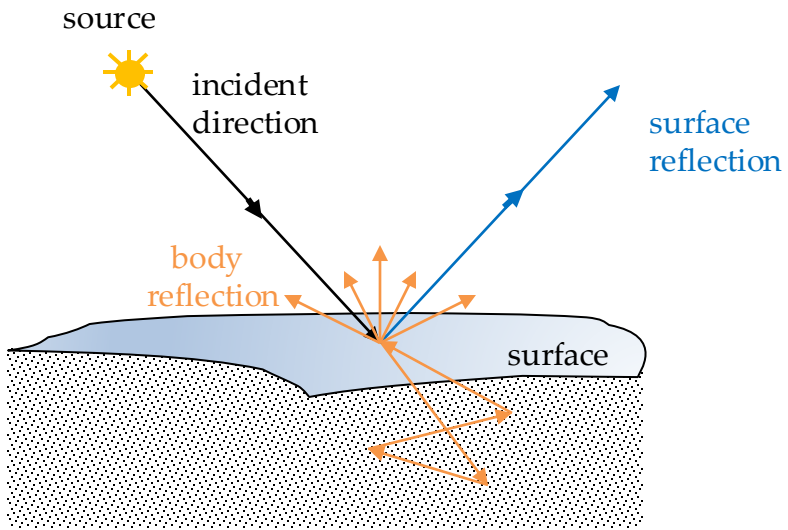
Real photo

Reflectance - Three Forms



[Slide credit: Kavita Bala]

Mechanisms of Surface Reflection



Body Reflection:

- Diffuse Reflection
- Matte Appearance
- Non-Homogeneous Medium
- Clay, paper, etc.

Surface Reflection:

- Specular Reflection
- Glossy Appearance
- Highlights
- Dominant for Metals

Image Intensity = Body Reflection + Surface Reflection

Mechanisms of Surface Reflection

Body Reflection:

- Diffuse Reflection
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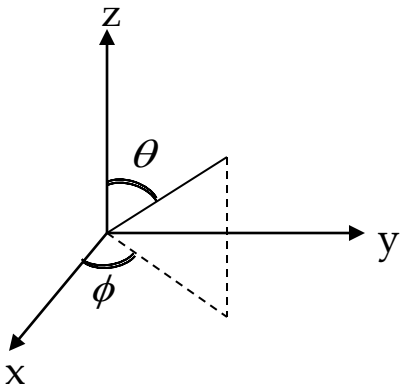
Surface Reflection:

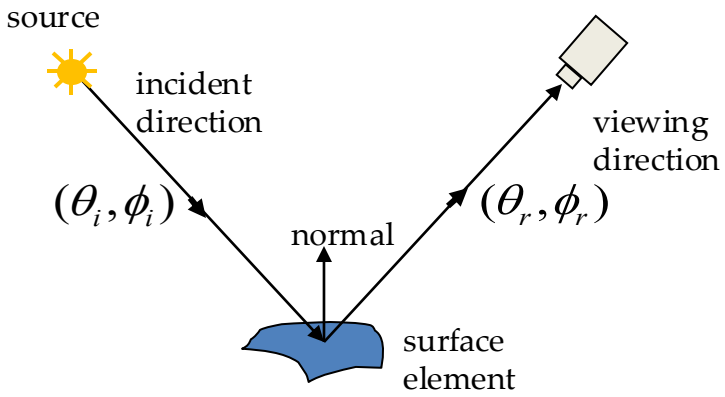
- Specular Reflection
- Glossy Appearance
- Highlights
- Dominant for Metals

Many materials exhibit both Reflections:

BRDF: Bidirectional Reflectance Distribution Function

θ : zenith
ϕ : azimuth



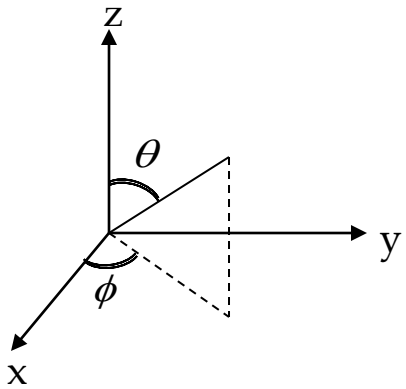


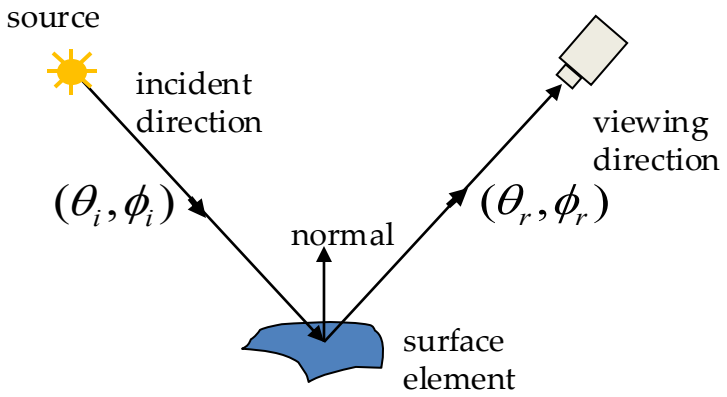
$E^{surface}(\theta_i, \phi_i)$ Irradiance due to source in direction (θ_i, ϕ_i)

$L^{surface}(\theta_r, \phi_r)$ Radiance of Surface in direction (θ_r, ϕ_r)

$$\text{BRDF}: \quad f(\theta_i, \phi_i; \theta_r, \phi_r) = \frac{L^{surface}(\theta_r, \phi_r)}{E^{surface}(\theta_i, \phi_i)}$$

BRDF Properties

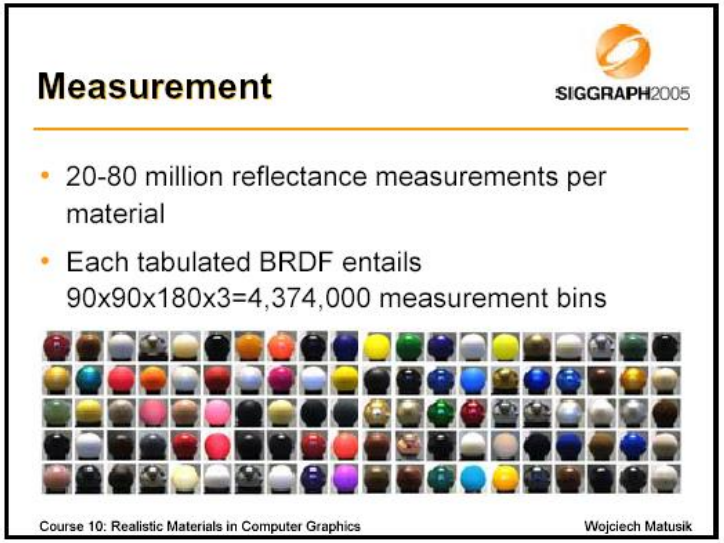




- BRDF is a 4D function
- Non-negative: $f(\theta_i, \phi_i, \theta_r, \phi_r) > 0$
- Helmholtz Reciprocity: $f(\theta_i, \phi_i, \theta_r, \phi_r) = f(\theta_r, \phi_r, \theta_i, \phi_i)$

BRDF databases

- MERL (Matusik et al.): 100 isotropic, 4 nonisotropic, dense



- CURET (Columbia-Utrect): 60 samples, more sparsely sampled, but also bidirectional texure functions (BTF)

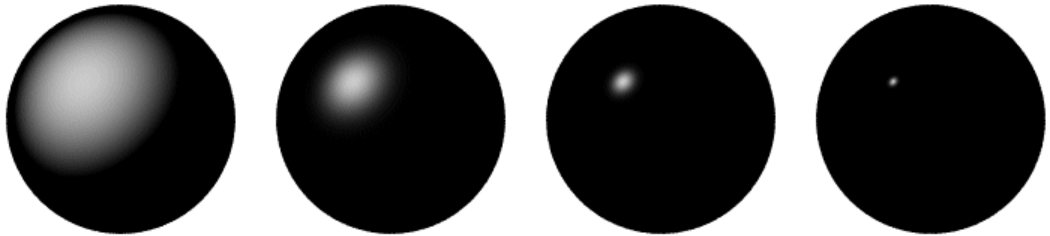
Sample albedos

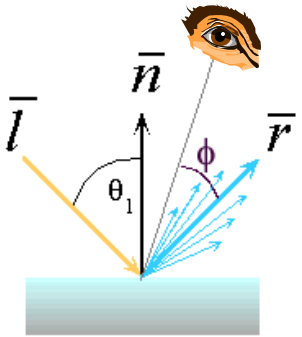
Surface	Typical albedo
Fresh asphalt	0.04[6]
Open ocean	0.06[7]
Worn asphalt	0.12[6]
Conifer forest, summer	0.08,[8] 0.09 to 0.15[9]
Deciduous forest	0.15 to 0.18[9]
Bare soil	0.17[10]
Green grass	0.25[10]
Desert sand	0.40[11]
New concrete	0.55[10]
Ocean ice	0.50 to 0.70[10]
Fresh snow	0.80[10]
Aluminium	0.85[12][13]

[Wikipedia: Albedo]

Phong Model

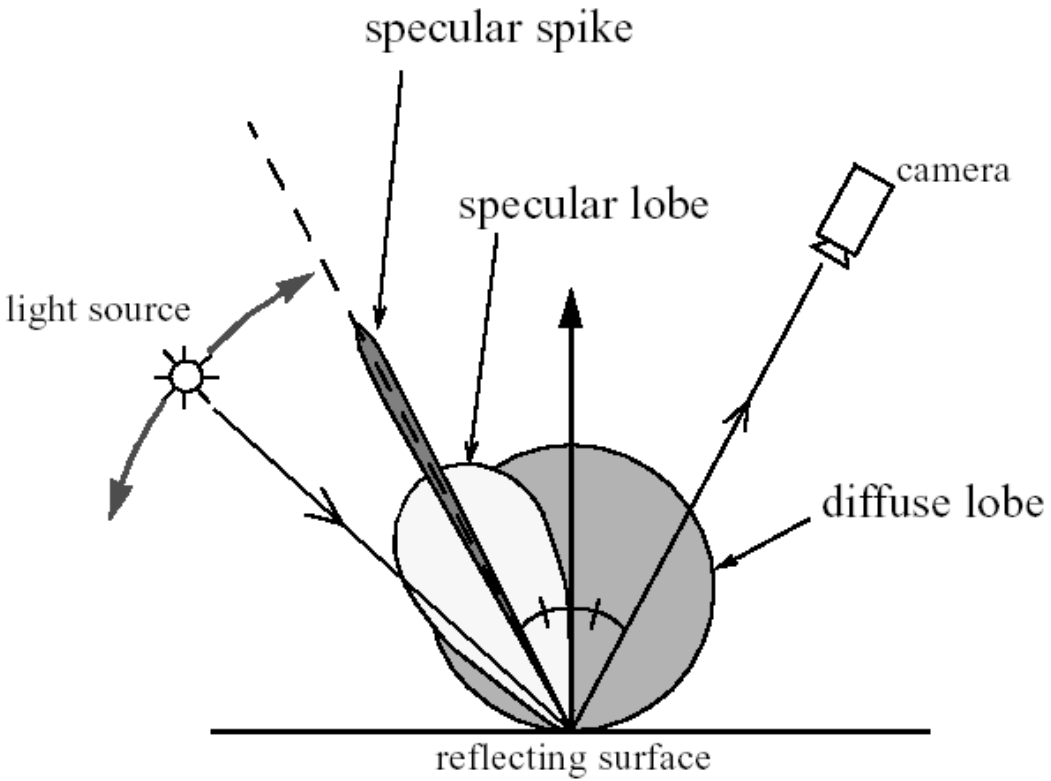
- These spheres illustrate the Phong model as *lighting direction* and n_{shiny} are varied:



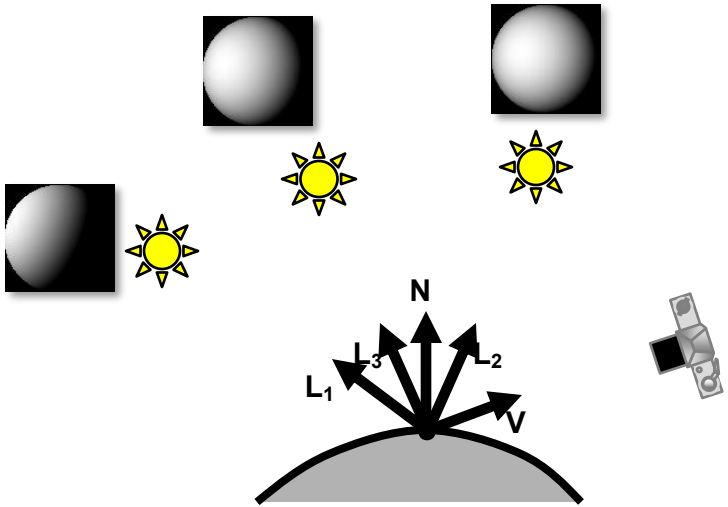


$$L = I\ \rho_s\ (\cos\phi)^{n_{shiny}}$$

All components of Surface Reflection



Multiple Images: Photometric Stereo



$$
\begin{aligned}
I_1 &= k_d \mathbf{N} \cdot \mathbf{L_1} \\
I_2 &= k_d \mathbf{N} \cdot \mathbf{L_2} \\
I_3 &= k_d \mathbf{N} \cdot \mathbf{L_3}
\end{aligned}
$$

Write this as a matrix equation:

$$
\begin{bmatrix} I_1 & I_2 & I_3 \end{bmatrix} = k_d \mathbf{N}^T \begin{bmatrix} \mathbf{L_1} & \mathbf{L_2} & \mathbf{L_3} \end{bmatrix}
$$

Solving the Equations

$$\underbrace{\begin{bmatrix} I_1 & I_2 & I_3 \end{bmatrix}}_{\mathbf{I}} = \underbrace{k_d \mathbf{N}^T}_{\mathbf{G}} \underbrace{\begin{bmatrix} \mathbf{L_1} & \mathbf{L_2} & \mathbf{L_3} \end{bmatrix}}_{\boldsymbol{L}}$$

$\mathbf{I}$: 1×3 $\quad$ $\mathbf{G}$: 1×3 $\quad$ $\boldsymbol{L}$: 3×3

Solving:

$$\mathbf{G} = \mathbf{I}\mathbf{L}^{-1}$$

Normalized the G, separate it into mode k_d and normal N

$$k_d = \|\mathbf{G}\|$$

$$\mathbf{N} = \frac{1}{k_d}\mathbf{G}$$

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Solving the Equations

$$\underbrace{\begin{bmatrix} I_1 & I_2 & I_3 \end{bmatrix}}_{\mathbf{I}} = \underbrace{k_d \mathbf{N}^T}_{\mathbf{G}} \underbrace{\begin{bmatrix} \mathbf{L_1} & \mathbf{L_2} & \mathbf{L_3} \end{bmatrix}}_{\mathcal{L}}$$

$\mathbf{I}$: 1×3 $\quad$ $\mathbf{G}$: 1×3 $\quad$ $\mathcal{L}$: 3×3

$$\mathbf{G} = \mathbf{I}\mathbf{L}^{-1}$$

- When is L nonsingular (invertible)?
  - >= 3 light directions are linearly independent, or:
  - All light direction vectors cannot lie in a plane.
- What if we have more than one pixel?
  - Stack them all into one big system.

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More than Three Lights

$$\begin{bmatrix} I_1 & \dots & I_n \end{bmatrix} = k_d \mathbf{N}^T \begin{bmatrix} \mathbf{L_1} & \dots & \mathbf{L_n} \end{bmatrix}$$

- Solve using least squares (normal equations):

$$\mathbf{I} = \mathbf{GL}$$

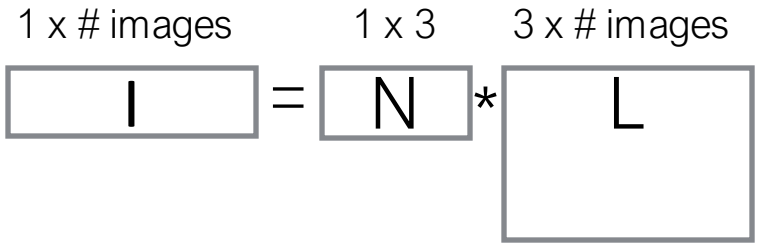
Find $\boldsymbol{G}$ s.t. minimizes $\|\boldsymbol{I} - \boldsymbol{GL}\|^2$

$$\mathbf{IL}^{\mathbf{T}} = \mathbf{GLL}^{\mathbf{T}}$$
$$\mathbf{G} = (\mathbf{IL}^{\mathbf{T}})(\mathbf{LL}^{\mathbf{T}})^{-1}$$

- Given G, solve for N and k_d as before.

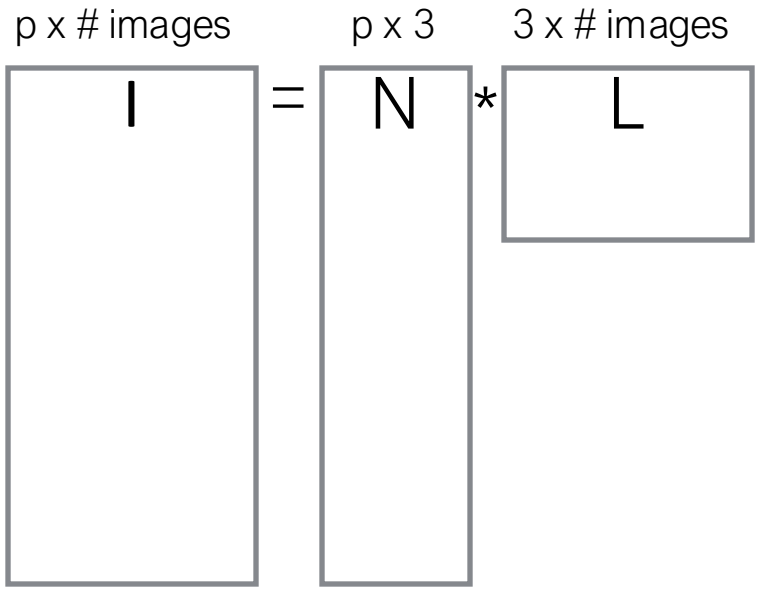
More than one pixel

Previously:



More than one pixel

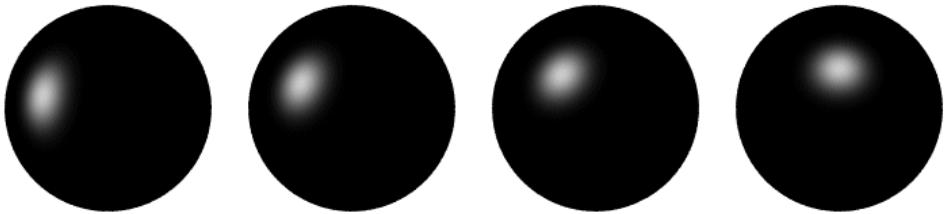
Stack all pixels into one system:



Solve as before.

Computing Light Source Directions

- Trick: Place a mirror ball in the scene.

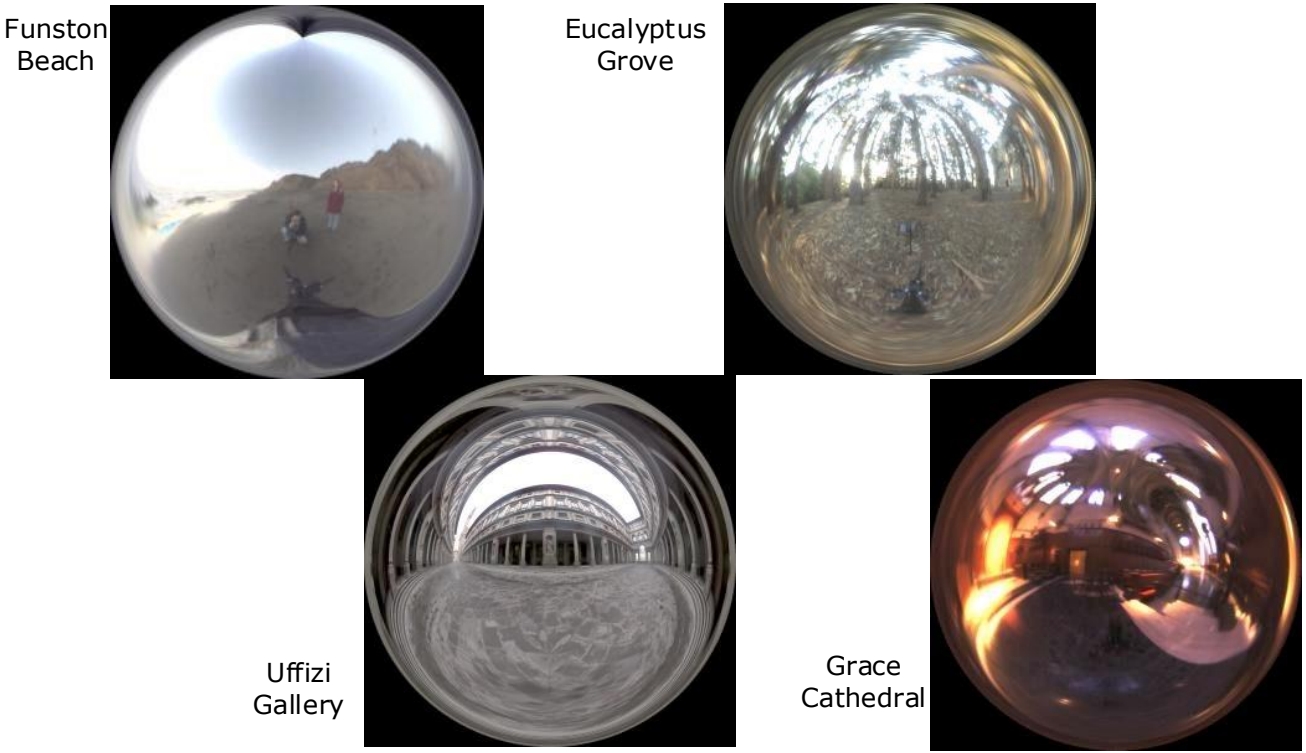


- The location of the highlight is determined by the light source direction.

HDRI Sky Probe

[Slide credit: Kavita Bala]

Real-World HDR Lighting Environments

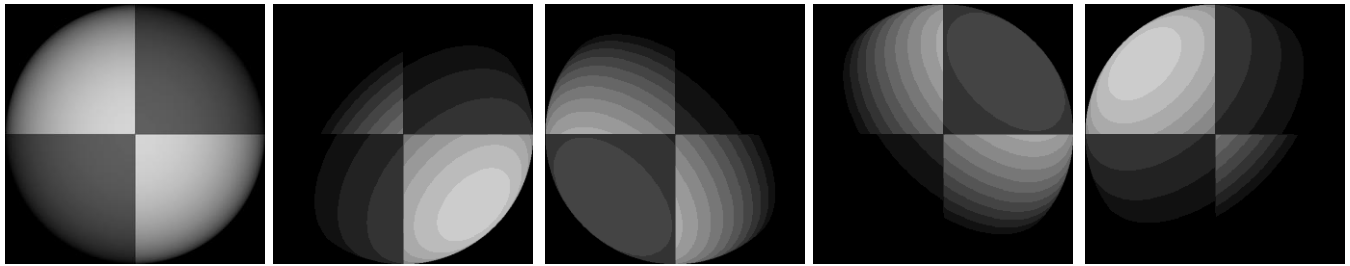


Lighting Environments from the Light Probe Image Gallery:
http://www.debevec.org/Probes/

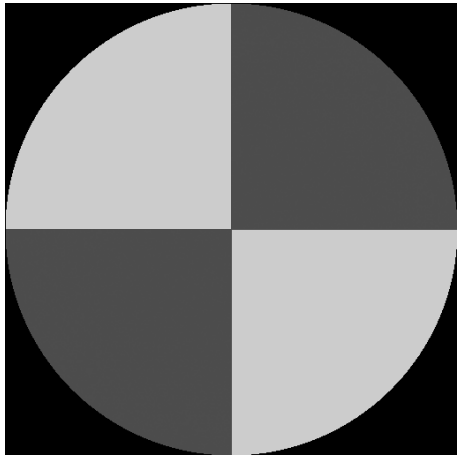
[Slide credit: Kavita Bala]

Example

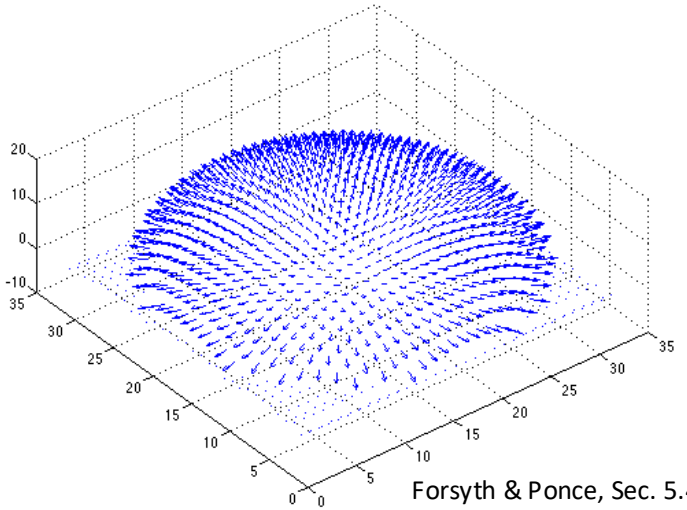
Input views



Recovered albedo



Recovered normal field



Forsyth & Ponce, Sec. 5.4

Depth from normals

- Solving the linear system per-pixel gives us an estimated surface normal for each pixel
- How can we compute depth from normals?
  - Normals are like the “derivative” of the true depth

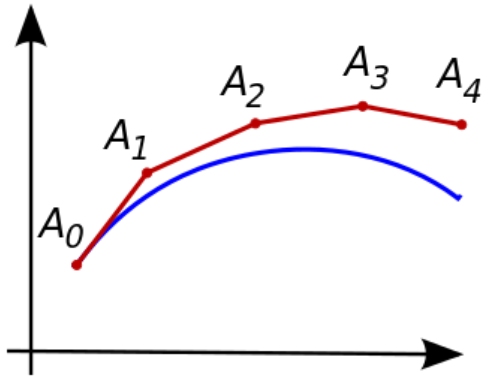
Input photo

Estimated normals

Estimated normals (needle diagram)

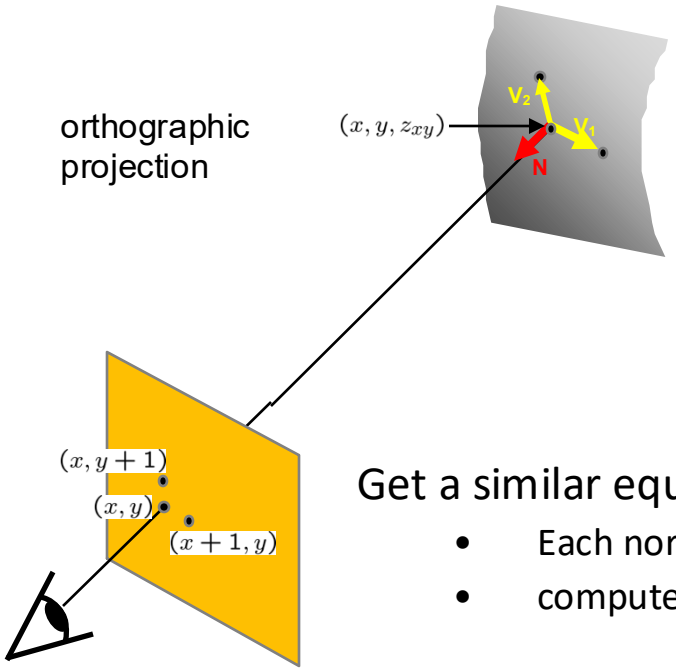
Normal Integration

- Integrating a set of derivatives is easy in 1D
  - (similar to Euler's method from diff. eq. class)
- Could integrate normals in each column / row separately
  - Wouldn't give a good surface
- Instead, we formulate as a linear system and solve for depths that *best agree with the surface normals*



Depth Map from Normal Map

- Given the surface normals we can iteratively compute depth?



Assume a smooth surface

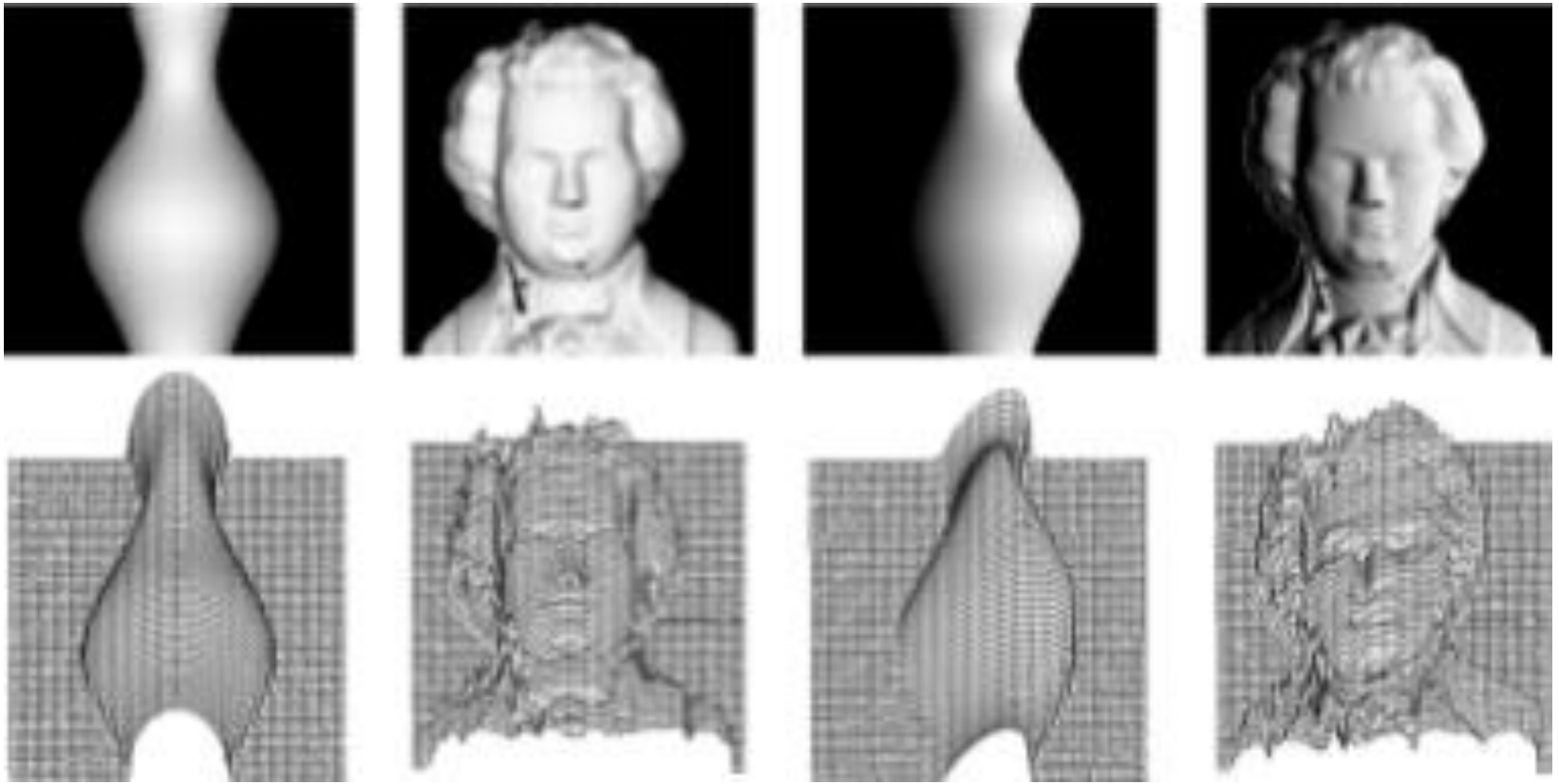
$$\begin{aligned} V_1 &= (x+1, y, z_{x+1,y}) - (x, y, z_{xy}) \\ &= (1, 0, z_{x+1,y} - z_{xy}) \end{aligned}$$

$$\begin{aligned} 0 &= N \cdot V_1 \\ &= (n_x, n_y, n_z) \cdot (1, 0, z_{x+1,y} - z_{xy}) \\ &= n_x + n_z(z_{x+1,y} - z_{xy}) \end{aligned}$$

Get a similar equation for $\mathbf{V_2}$

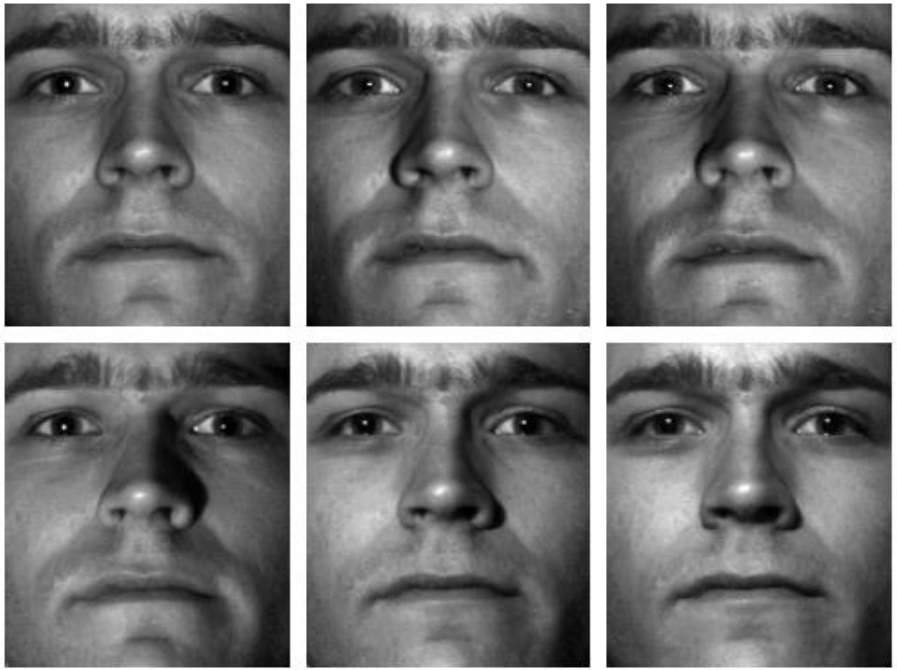
- Each normal gives us two linear constraints on z
- compute z values by solving a matrix equation

Results



[Slide credit: Andreas Geiger] [Frankot and Chellappa: A Method for enforcing integrability in shape from shading algorithms. TPAMI, 1988]

Results



[from Athos Georghiades]

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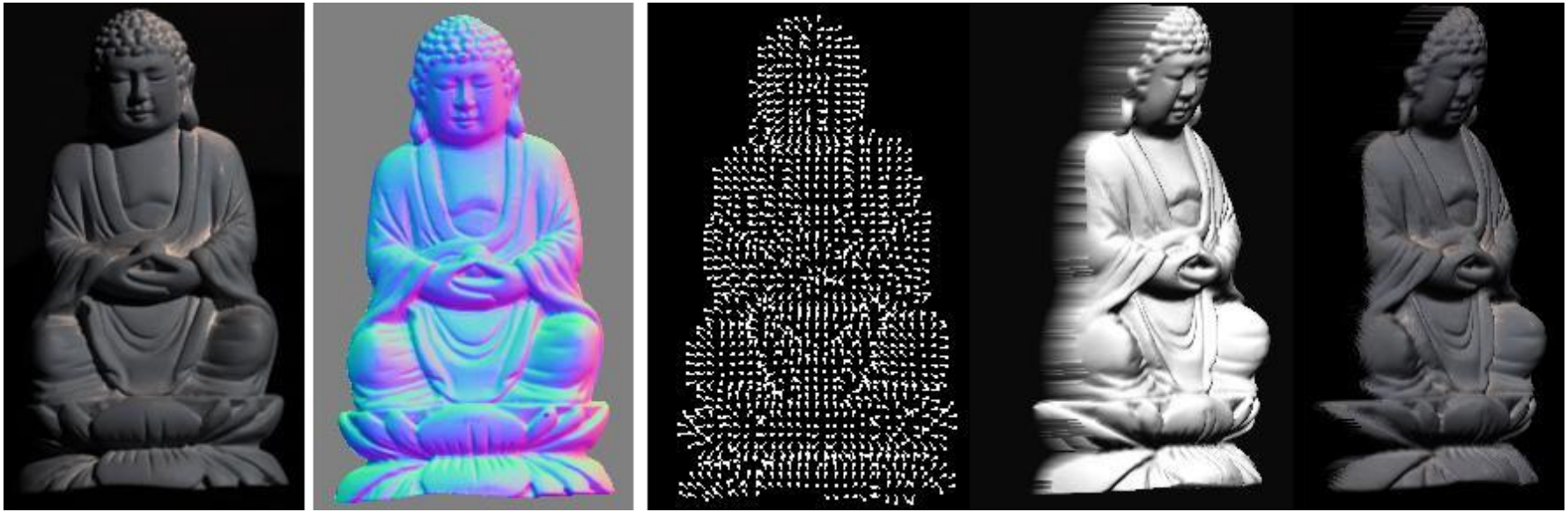
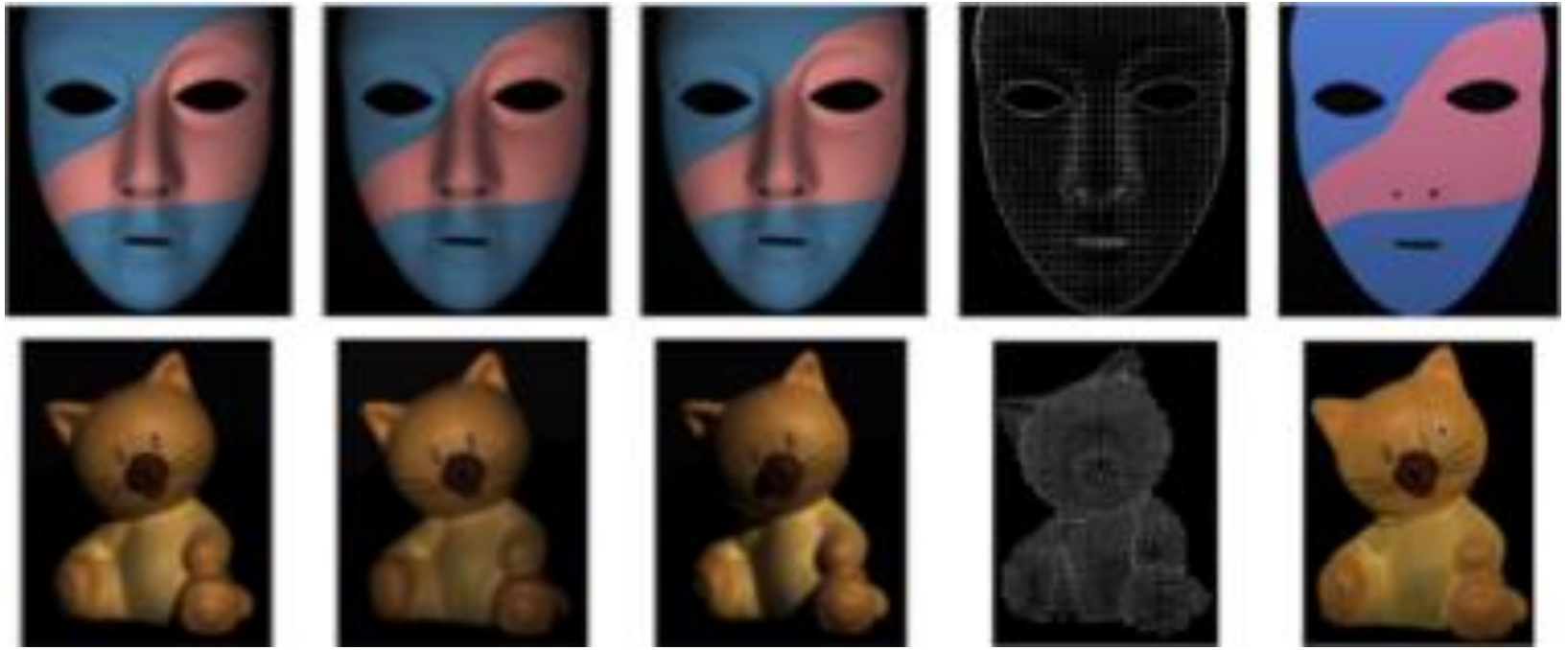
Results



[Woodham: Analysing images of curved surfaces. Artificial Intelligence, 1981]

Results

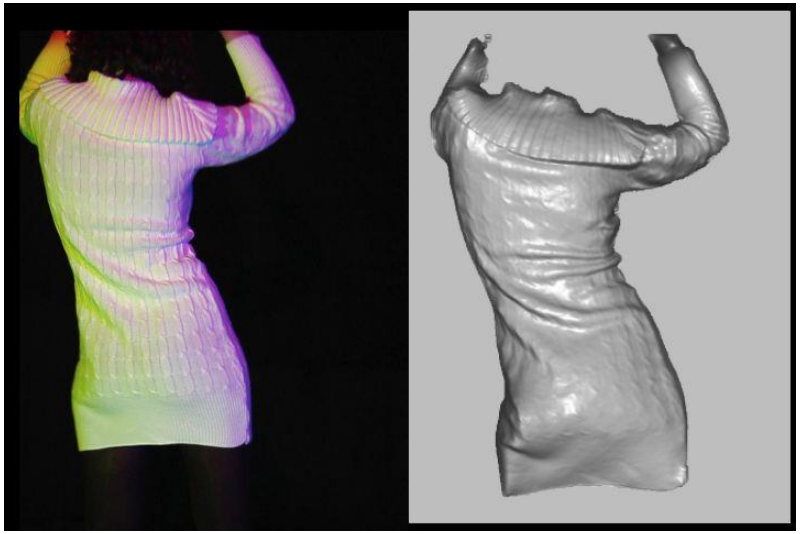


- For color images, apply PS to each channel separately to obtain color albedo.
- Deviations from Lambertian assumption and global illumination cause errors.

[slide credit: Andreas Geiger]

Extension

- Photometric Stereo from Colored Lighting



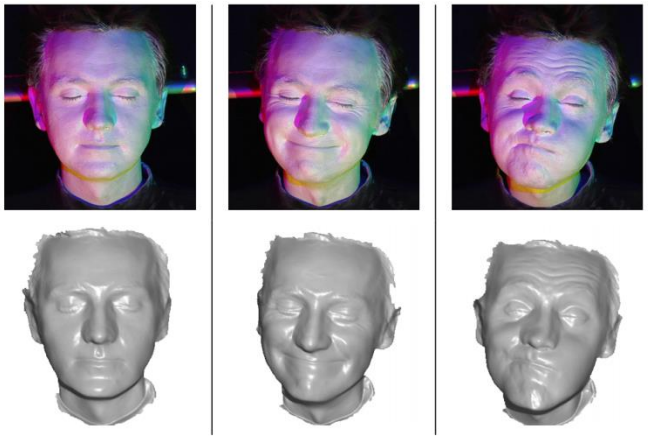
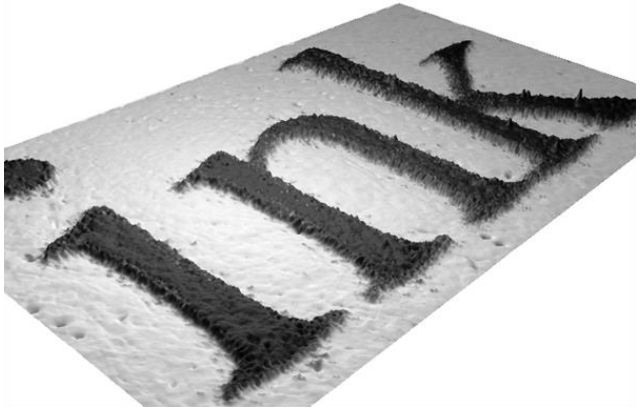


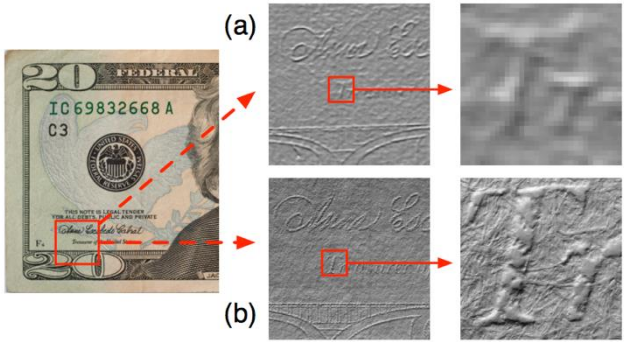
Fig. 2. **Applying the original algorithm to a face with white makeup.** Top: example input frames from video of an actor smiling and grimacing. Bottom: the resulting integrated surfaces.

Video Normals from Colored Lights
Gabriel J. Brostow, Carlos Hernández, George Vogiatzis, Björn Stenger, Roberto Cipolla
IEEE TPAMI, 2011.

Specialized High-Resolution Scanners

- Recovers extremely subtle details with high accuracy
- Photometric stereo yields unmatched performance



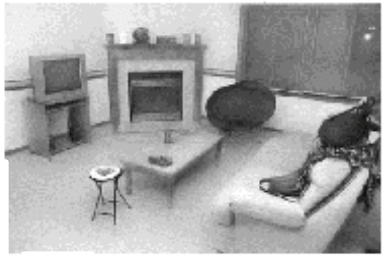


[Sensing Surfaces with GelSight, 2011]

Intrinsic Image Decomposition

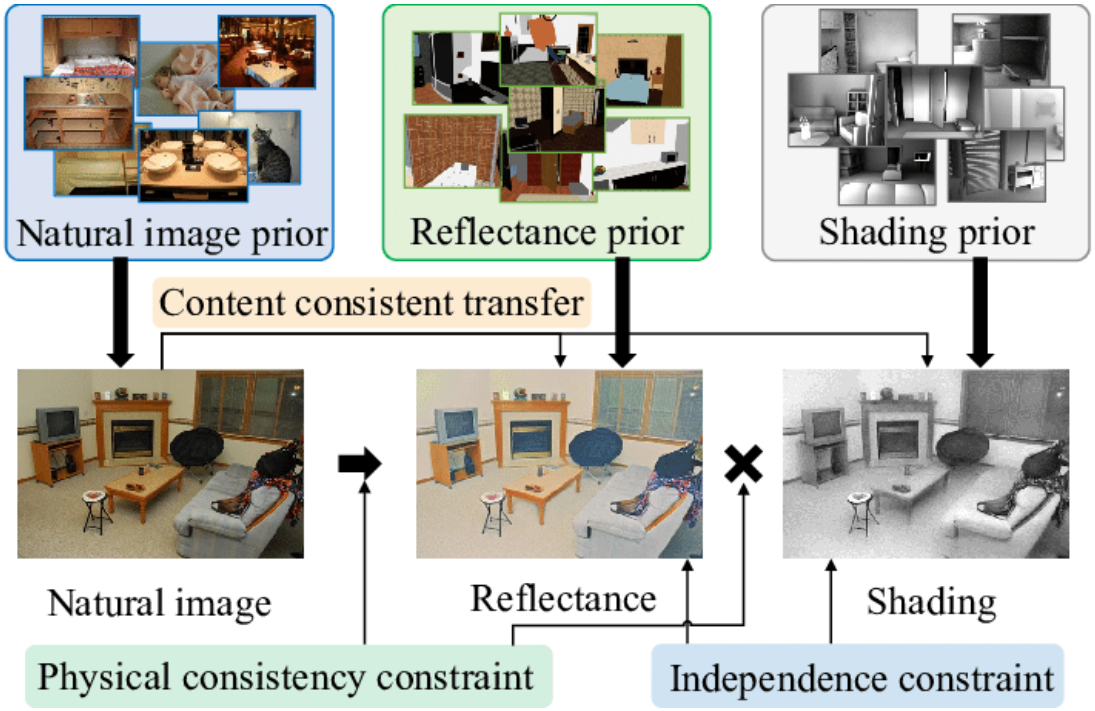
Natural image

Reflectance



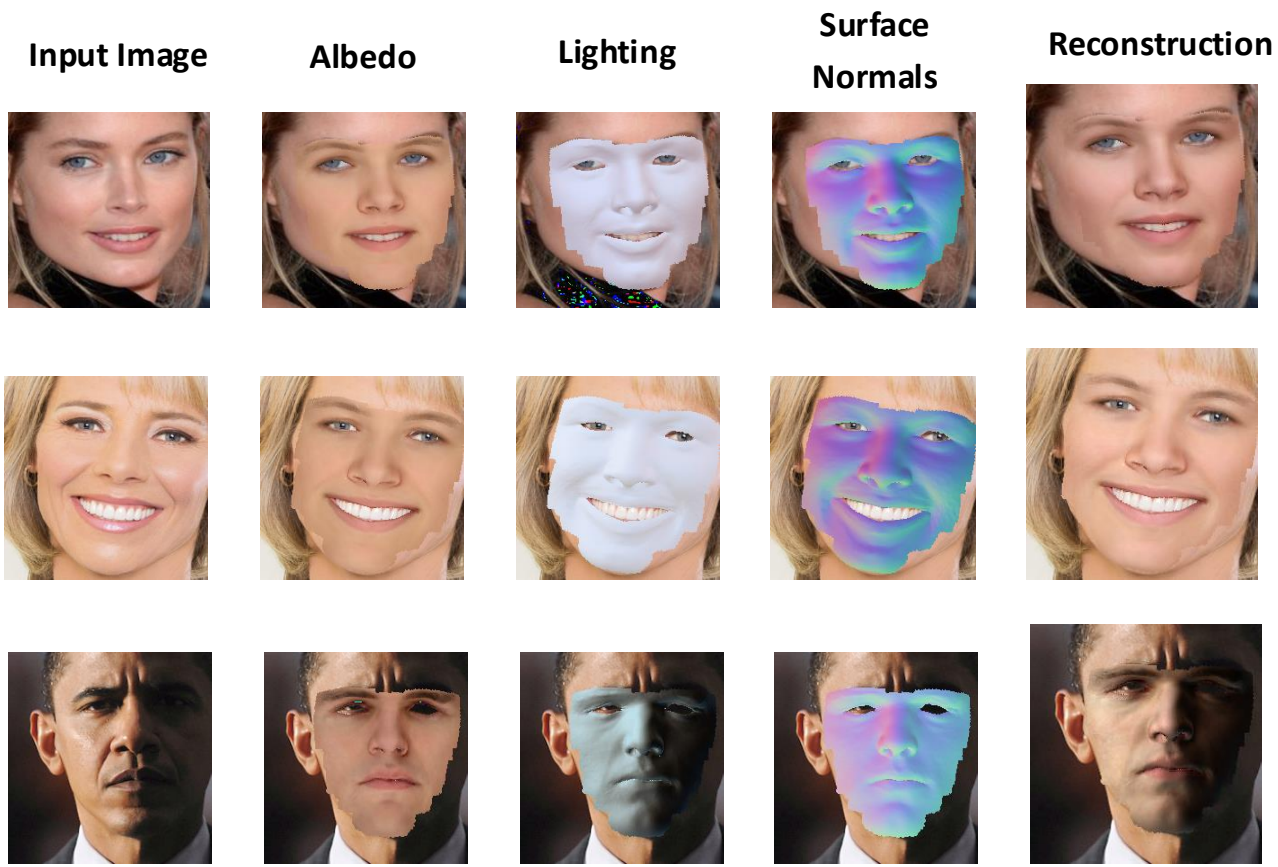
Shading

Intrinsic Image Decomposition



Unsupervised Learning for Intrinsic Image Decomposition from a Single Image
Yunfei Liu, Yu Li, Shaodi You and Feng Lu, CVPR2020

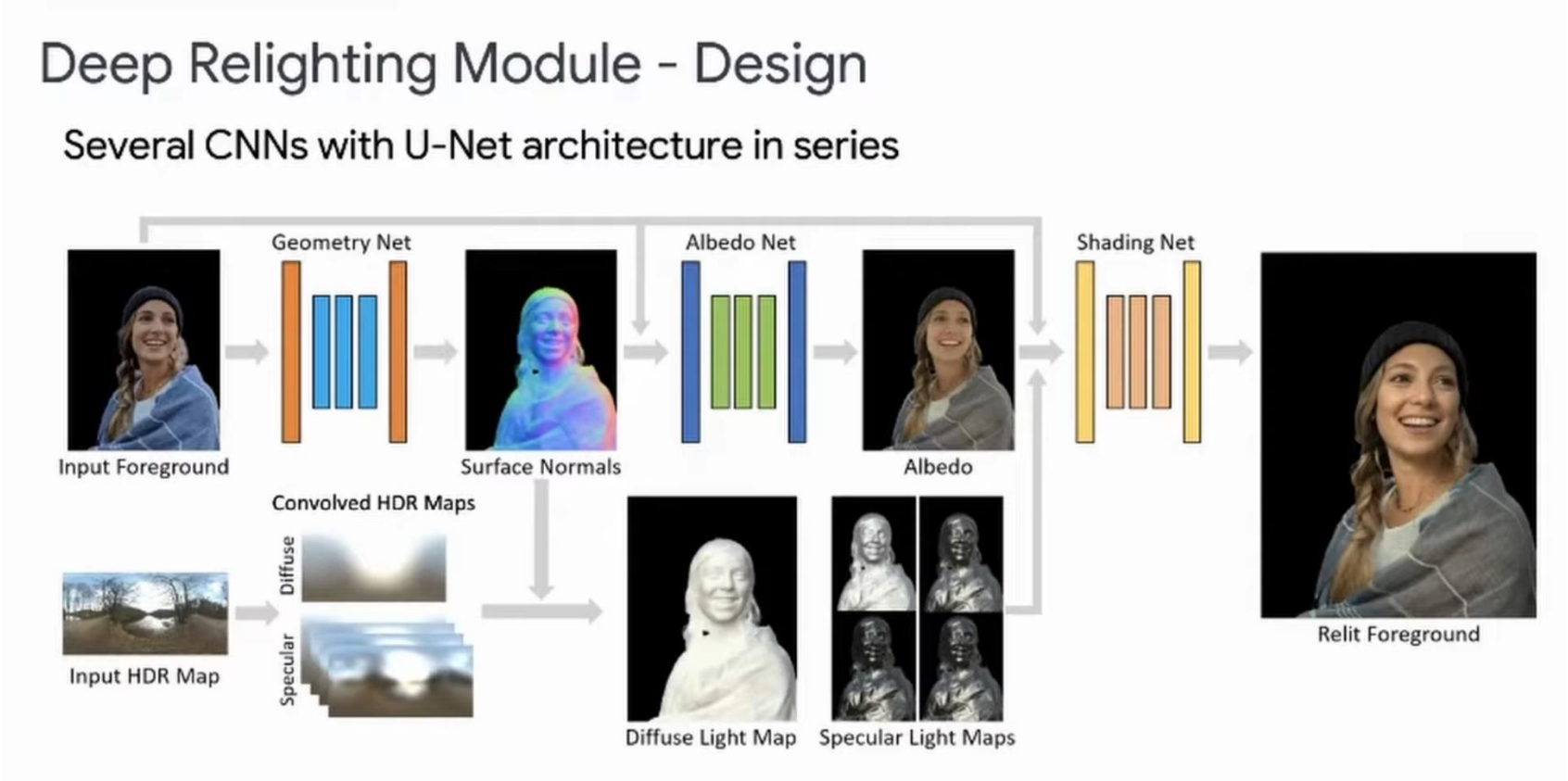
Diffuse Reflection



Total Relighting

[Pandey et al, SIGGRAPH'21, YouTube]

Total Relighting



[Pandey et al, SIGGRAPH'21, YouTube]

Image processing operations

- Point-wise operations
  - histogram operation
- Neighborhood operations
  - Spatial domain (image convolution)
  - Frequency domain (Fourier transform)
  - Binary image analysis
- Geometric operations
  - Rotation, translation, affine…

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Histograms

How do we ‘summarize’ images?

← https://rvdboomgaard.github.io/ComputerVision_LectureNotes/LectureNotes/IP/Images/ImageHistograms.htm

https://rvdboomgaard.github.io/ComputerVision_LectureNotes/LectureNotes/IP/PointOperators/index.html →

Image Histograms

1D / Univariate Histogram

Summarize image f via ***Histogram*** of its values
(over all possible values)

$$h_f[i] = \sum_{\substack{x \in E \\ \text{for } i = 0, \dots, k-1}} [e_i \leq f(x) < e_{i+1}]$$

Iverson Bracket

$$[c] = \begin{cases} 1 & \text{for } c = \text{true} \\ 0 & \text{for } c = \text{false} \end{cases}$$

Is pixel value inside ***bin***?

$$f(x) \in [e_i, e_{i+1})$$

Bin edges

e_i for $i = 0, \dots, k$

NumPy: `histogram()`

https://numpy.org/doc/stable/reference/generated/numpy.histogram.html

How choose ***bin size/number***?

- No clear answer
- Empirically → More art than science
- Sturges' rule (assumes *Gaussian* data):

$$k = \lceil \log_2 n \rceil + 1$$

bins number

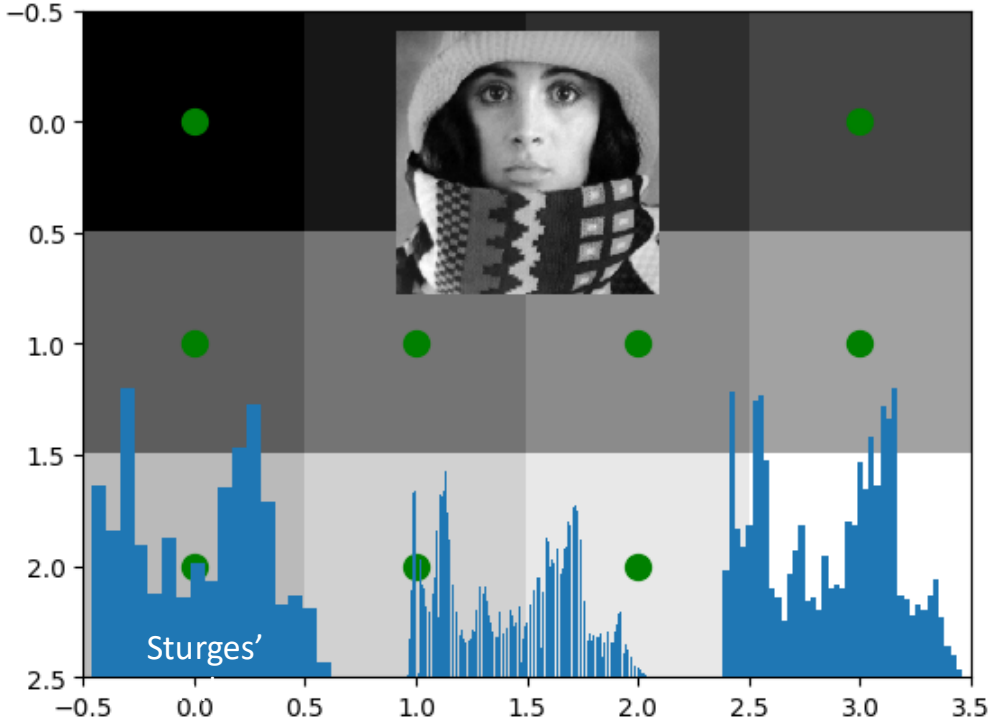
pixels number

`histogram_bin_edges()`

https://numpy.org/doc/stable/reference/generated/numpy.histogram_bin_edges.html

Image Histograms

1D / Univariate Histogram



Too many (and too small) bins?

→ Many bins will be empty

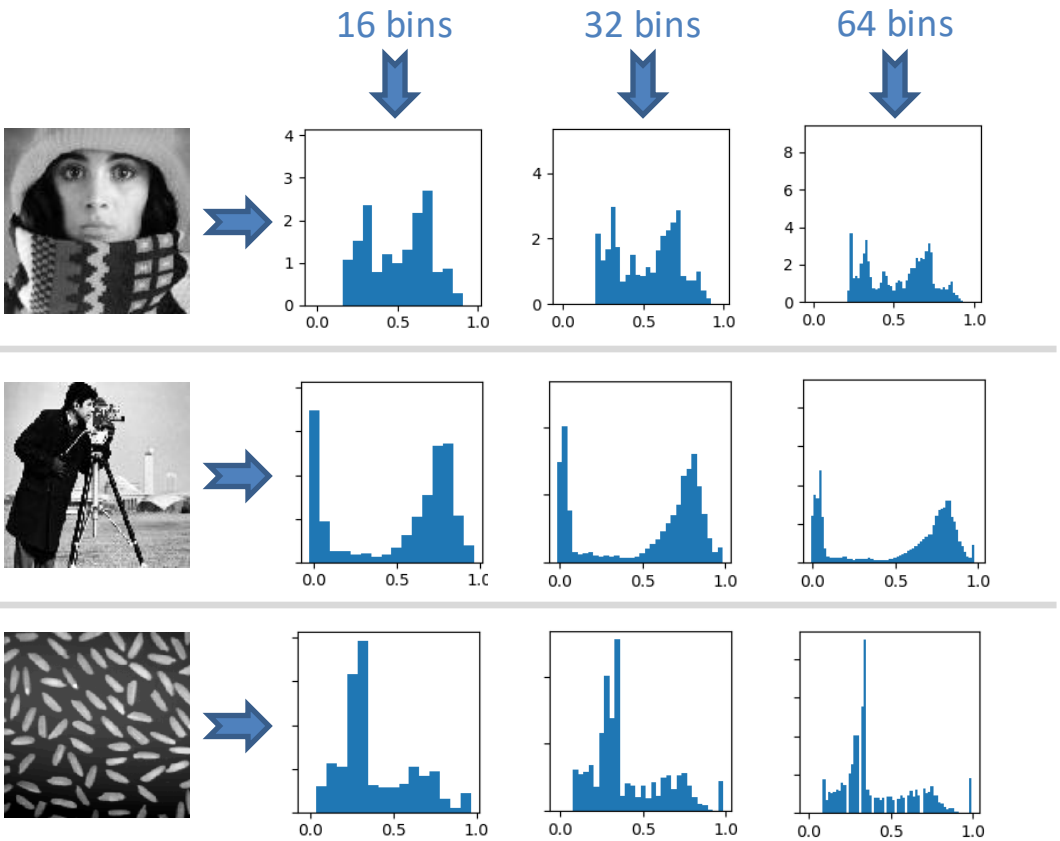
→'High-frequency' info

Too few (and too big) bins?

→ Will not encode 'well enough'

the distribution of values

Image Histograms



Too many (and too small) bins?

→ Many bins will be empty

→'High-frequency' info

Too few (and too big) bins?

→ Will not encode 'well enough'

the distribution of values

Image Histograms

Histogram → $h_f(u)$ counts pixels with value $f(x) = u$ — ***PDF*** – Probability-density function, normalized such that: $\sum h_f(u) = 1$

Cumulative Histogram → $H_f(u)$ counts pixels with value $f(x) \leq u$ — ***Cumulative distribution*** function

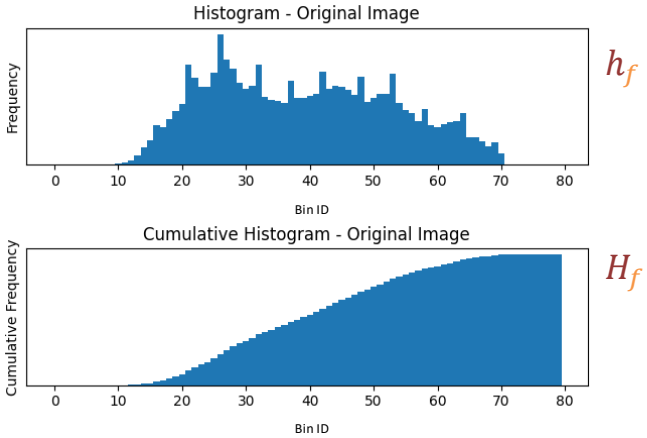
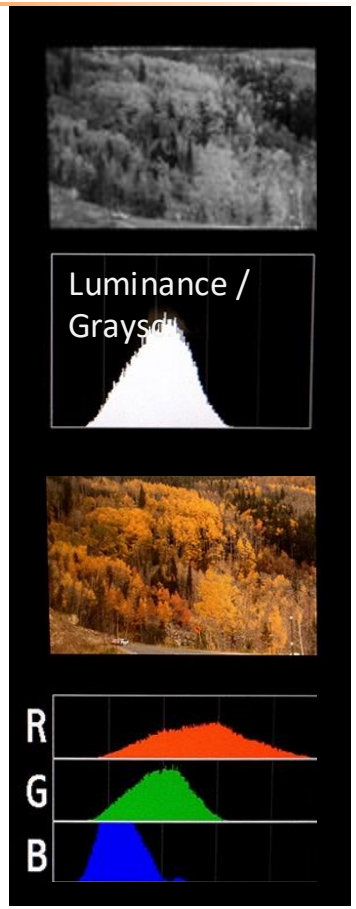


Image Histograms

3D / Multivariate Histogram

Grayscale images → ***1D Histogram***

(each pixel → Scalar value) (Univariate)



What about ***Color*** images?

(each pixel → Triplet of BGR value)

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{bmatrix} f_1(\boldsymbol{x}) \\ f_2(\boldsymbol{x}) \\ f_3(\boldsymbol{x}) \end{bmatrix} = \begin{bmatrix} b(\boldsymbol{x}) \\ g(\boldsymbol{x}) \\ r(\boldsymbol{x}) \end{bmatrix}$$

Ignores color **Correlations**!

Trivial solution: ***3x separate*** 1D histograms per channel

Image Histograms

3D / Multivariate Histogram

Better solution → ***3D Histogram***

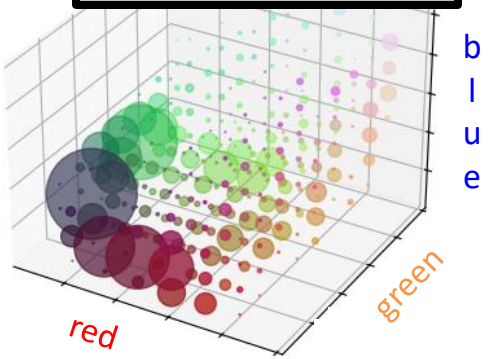
$$h_f[i,j,k] = \sum_{x \in E} [e_{1,i} \le f_1(x) < e_{1,i+1}] \; [e_{2,j} \le f_2(x) < e_{2,j+1}] \; [e_{3,k} \le f_3(x) < e_{3,k+1}]$$

Is pixel value inside

3D bin

with indices (i, j, k) ?

3D bins: 8x8x8 cubes



Bin centered @ RGB point

Sphere size == Hist. count

`histogramdd()`

https://numpy.org/doc/stable/reference/generated/numpy.histogramdd.html

Image Histograms

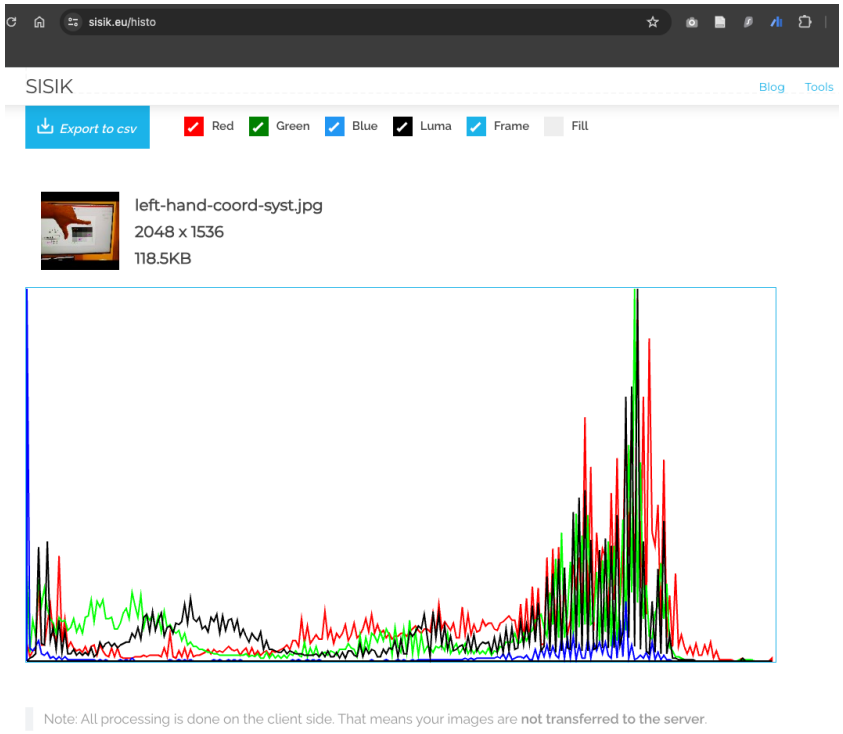
Online Demo - 1D Histograms

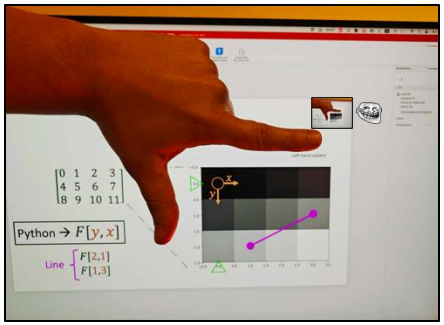
*'This tool basically creates **256 bins** for each color (red, green, blue) and greyscale (luma) intensity. The number of bins is shown on the **horizontal** axis.'*

*'The tool then loops through every image pixel and counts the **occurrence of each intensity**. The counts of occurrences in each bin are then displayed on **vertical** axis.'*

*'Counts for each pixel-intensity are **normalized** to range 0 to 255 before they are displayed on the graph.'*

'Used JavaScript in combination with WebAssmebly to create this tool.'





https://sisik.eu/histo

Point Operators

Histograms

f: $\mathcal{D} \rightarrow ([0,1] \in \mathbb{R})$

φ: $([0,1] \in \mathbb{R}) \rightarrow ([0,1] \in \mathbb{R})$

$$(\varphi f)(x) = \varphi\big(f(x)\big)$$

Monadic
'in-place'
Point Operator

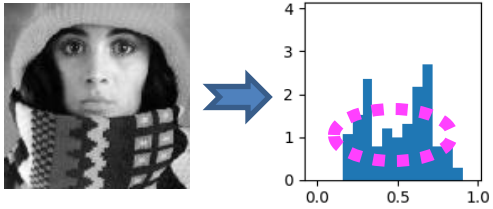
Replaces pixel value $f(x)$ ***independent*** of:

- Position x
- All other pixel values in neighborhood

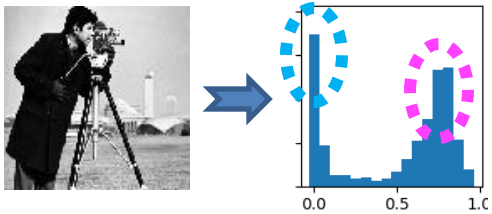
How can we 'manipulate' histograms with Monadic Point Operators?

Point Operators

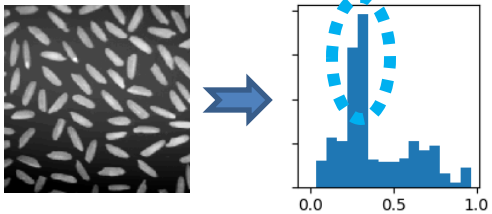
Histograms (issues)



Only *mid-range* gray
No extremes



Spikes for *bright* and *very-dark* areas



Mostly *dark-ish* areas

Not using full range
of luminance
might be sub-optimal

Image might be … :

Missing *large (bright)* values? → … too *dark*

Missing *low (dark)* values? → … too *bright*

Changing illumination conditions
for same scene → very different histograms!

Point Operators

Histogram Contrast Stretching

$f(x)$: $\mathbb{R}^2 \rightarrow [a, b] \subset [0,1] \in \mathbb{R}$

Contrast Stretching { Construct new image using full range of luminance

Monadic Operator

$$g(x) = \varphi(f(x)) = \frac{f(x) - a}{b - a}$$

'Distance' from min luminance in f

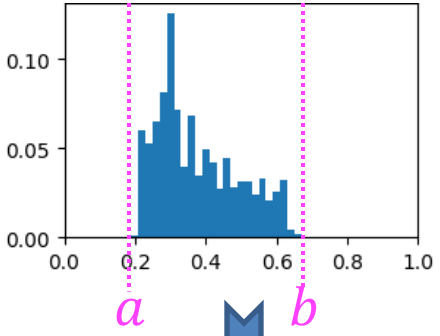
Stretches luminance range to [0,1]

$$g = \frac{f - f_{min}}{f_{max} - f_{min}}$$

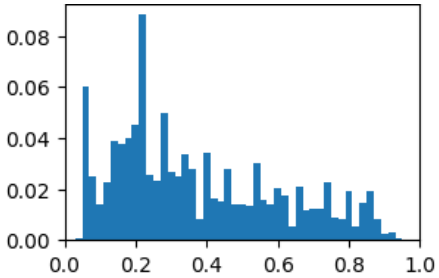
Lifted arithmetical operators

X axis: luminance values (bins)
Y axis: frequency for each value

Input: **Low Contrast**

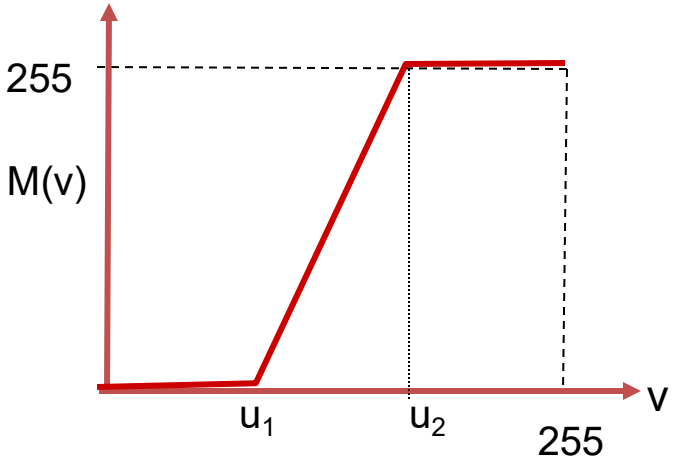


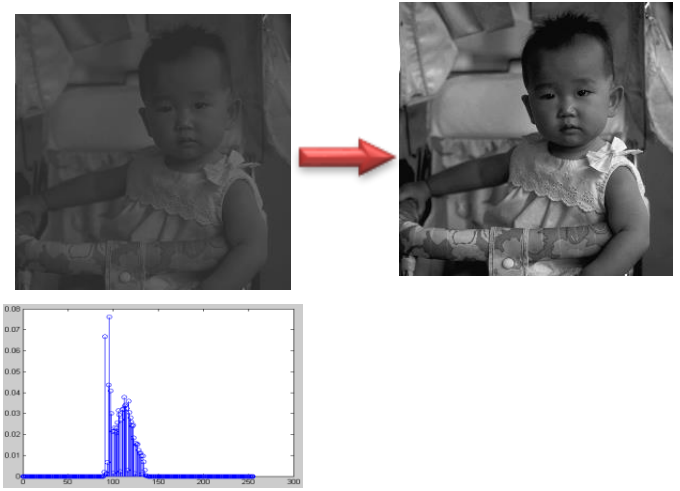
Output: **Contrast Stretched**



Contrast Enhancement

- If most of the gray-levels in the image are in $[u_1\ u_2]$, the following mapping increases the image contrast.
- The values u_1 and u_2 can be found by using the image's accumulated histogram.





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Gamma Correction

Non-linear scaling (power-law transformation):

Output gray level $s(r) = cr^{\gamma}$

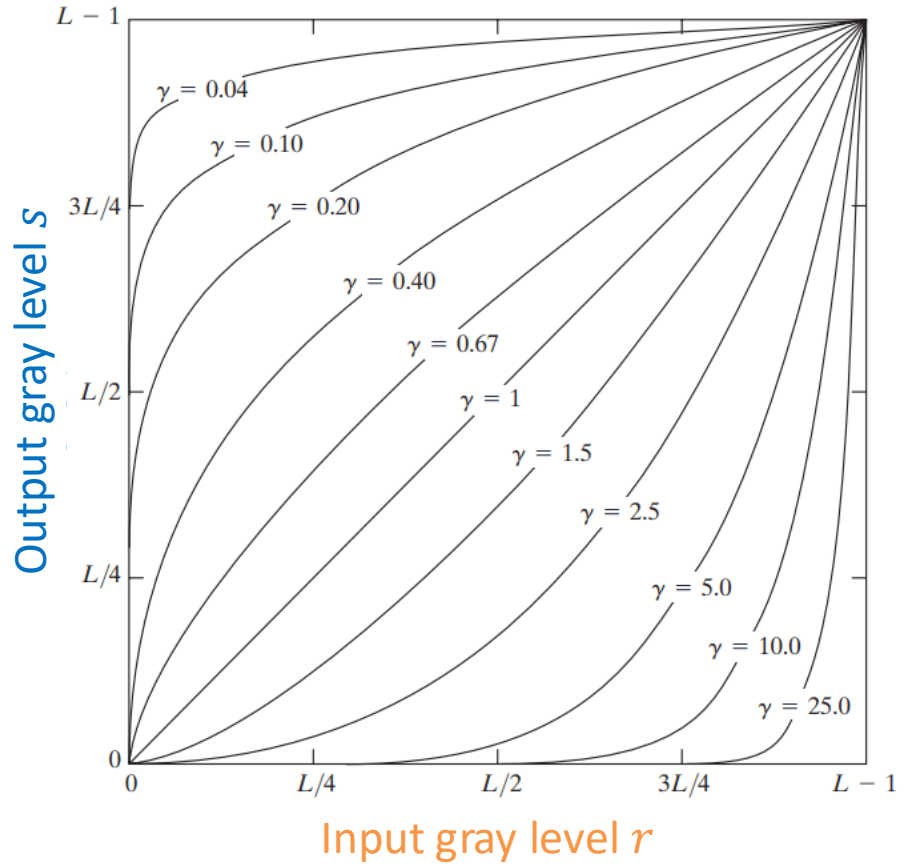
Normalization constant

Input gray level

- Note: $s(r)$ is linear for $\gamma = 1$.

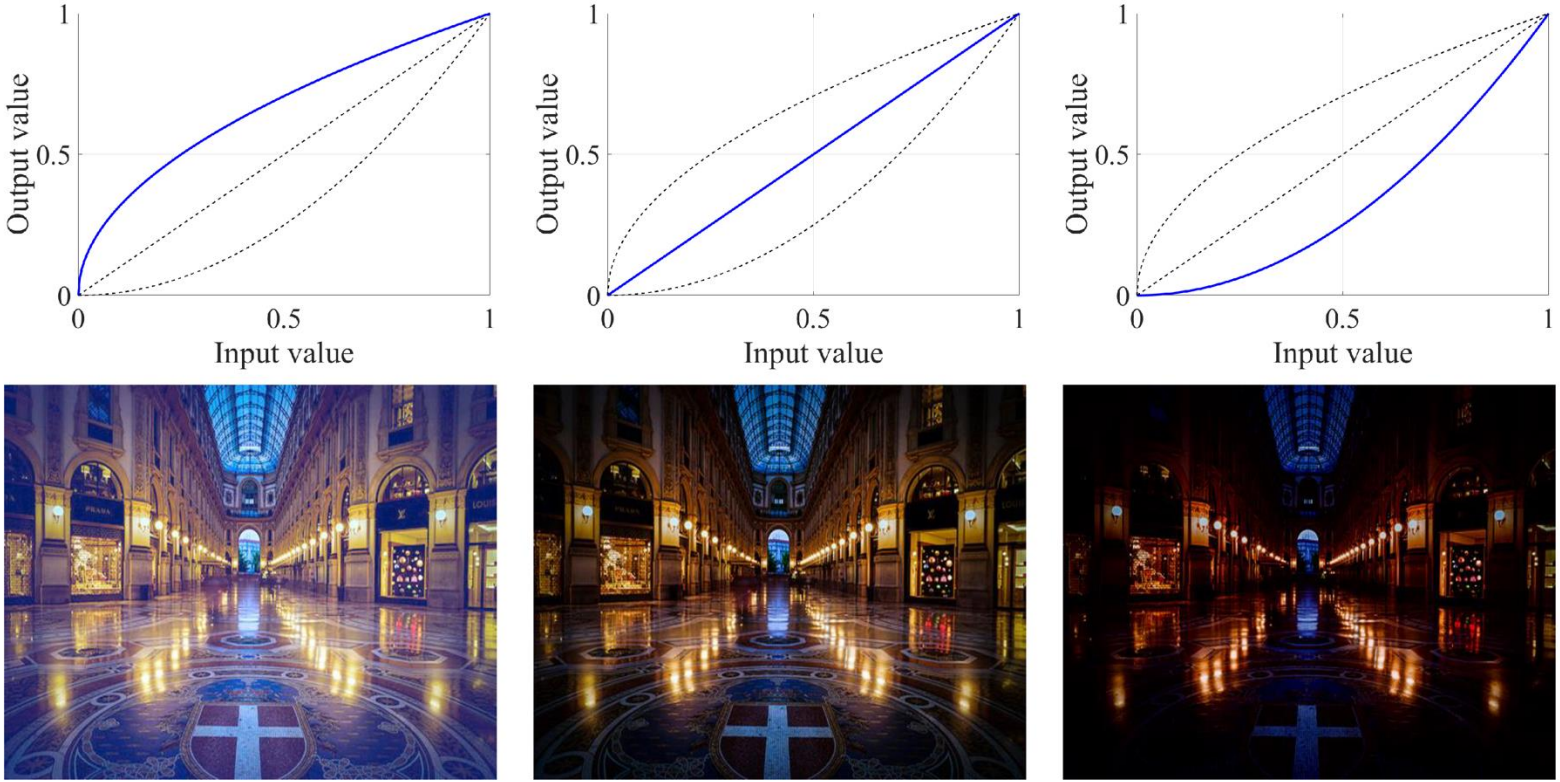
gamma=0.5 gamma=1.0 gamma=1.8 gamma=2.0 gamma=2.5

[Artlets.com]



[Yin at al., 2022]

Gamma Correction



[Ngo and Kang, Electronics, 2021]

Gamma Correction

gamma=1.0

gamma=3.0

gamma=5.0

gamma=5.0

Point Operators

Histogram Equalization (idealized)

Produce image $g = \varphi f$ with ***'constant' (flat) histogram*** h_g

All luminance values should be 'equally' probable
(as 'equal' as possible for finite number of pixels)

Useful consequence: ***cumulative histogram*** $H_g(u) = u$

Why? Think: integrate a constant

Luminance value: $u = f(x)$

Preserve ***order***: $u_1 \leq u_2 \Rightarrow \varphi(u_1) \leq \varphi(u_2)$

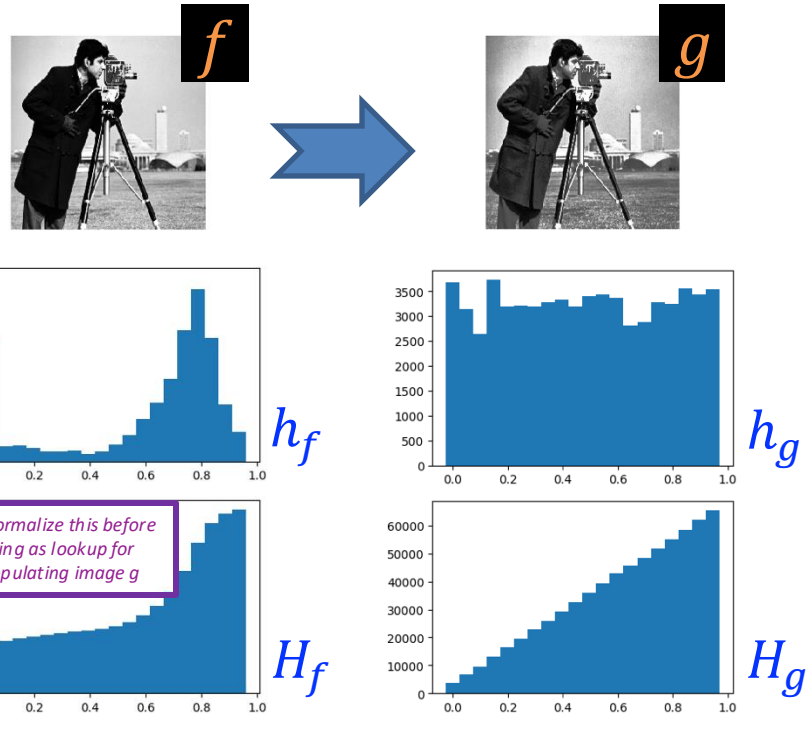
So, the **pth percentile** of f & g should be the same:

$$H_f(u) = H_g(\varphi(u)) = \varphi(u)$$

Visit each pixel of f
replace its value u with:

$$\varphi(u) = H_f^{norm}(u)$$

To populate a pixel of g:
- visit the corresponding pixel location of f
- read its value u (i.e., $f(x)$)
- use u as key to 'query' (the *normalized*) H_f as "lookup table"
- Copy value from "lookup table" as value of pixel of g



Point Operators

Histogram Thresholding

Segment foreground (objects of interests) from background

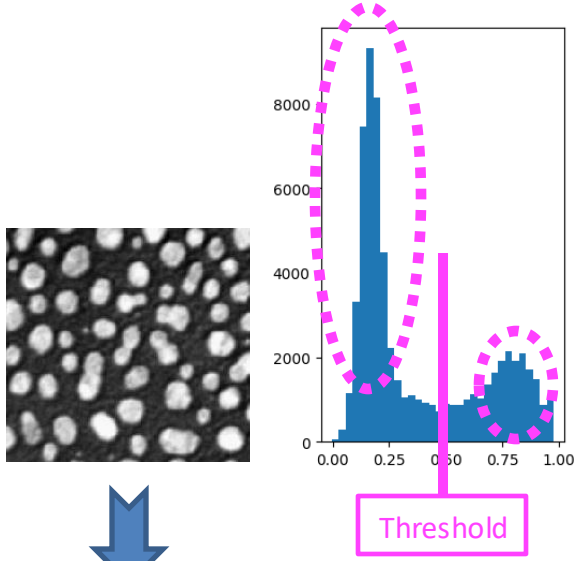
- Manually ☹ choose threshold t → looking at histogram (bimodal distribution)
- ***Automatically*** choose threshold t → ***IsoData*** thresholding

$m_L(t)$ → Mean of pixels with value: $\leq t$

$m_H(t)$ → Mean of pixels with value: $> t$

$$t = \frac{m_L(t) + m_H(t)}{2} = m(t)$$

Function of itself!



Iterative Solution

- Start with initial guess t_i for $i = 0$
- Compute $t_{i+1} = m(t_i)$
- Repeat until convergence
  (i.e. repeat as long as t_{i+1} and t_i are not too similar)

Resources for Histogram Slides

https://rvdboomgaard.github.io/ComputerVision_LectureNotes/LectureNotes/IP/Images/index.html

https://rvdboomgaard.github.io/ComputerVision_LectureNotes/LectureNotes/IP/PointOperators/index.html

Additional Resources

- Photometric Stereo
  - Shree Nayar: YouTube
  - Andreas Geiger: YouTube
- Specular and Diffuse Reflection
  - Steve Seitz: YouTube / YouTube

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