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BENCHMARKS FOR CURRENT LINEAR AND MIXED INTEGER OPTIMIZATION SOLVERS

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Abstract

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Linear programming (LP) and mixed integer linear programming (MILP) problems belong among very important class of problems that find their applications in various managerial consequences. The aim of the paper is to discuss computational performance of current optimization packages for solving large scale LP and MILP optimization problems. Current market with LP and MILP solvers is quite extensive. Probably among the most powerful solvers GUROBI 6.0, IBM ILOG CPLEX 12.6.1, and XPRESS Optimizer 27.01 belong. Their attractiveness for academic research is given, except their computational performance, by their free availability for academic purposes. The solvers are tested on the set of selected problems from MIPLIB 2010 library that contains 361 test instances of different hardness (easy, hard, and not solved).

Keywords: benchmark, linear programming, mixed integer linear programming, optimization, solver

INTRODUCTION

Solving linear and mixed integer linear optimization problems (LP and MILP) that belong to one of the most often modelling tools, is unthinkable without high-quality software. Optimization problems of this nature find their applications in analysis of wide variety of decision making tasks. Among traditional applications of LP and MILP problems product mix problems, blending problems, transportation and vehicle routing problems, cutting stock problems, and many more belong. LP and MILP problems are often applied in not so traditional fields – e.g. efficiency evaluation of decision making problems using data envelopment analysis models may be a good example of untraditional applications of LP problems.

Optimization problems with continuous variables and with only several tens or few hundreds of variables and constraints only can be successfully solved using less-quality optimization tools as e.g. MS Excel Solver is. Its advantage consists in its availability for almost all users, disadvantage consists in its limits (around 200 variables only) and in its performance characteristics. Problems with integer variables need not be solved even in case of a very small size of the given problem. Real-world optimization problems have usually many thousands of variables and/or constraints. Problems with hundreds of thousands of variables (constraints) are not an exception. These problems must be solved using professional high-quality solvers. Among the top LP and MILP solvers on the marker belong IBM ILOG CPLEX, GUROBI and FICO XPRESS Optimizer. More information about them can be found in IBM (2015), GUROBI (2013) and FICO (2011).

Optimization solvers can be used either as a stand-alone software tools, i.e. users must prepare input data set in a required format and run the solver with the data set as a batch. This way can be used e.g. with CPLEX and GUROBI solvers but it is not usually convenient for users. Much more convenient is to use complex systems for modeling support, often denoted as modelling languages. They offer more or less user-friendly interface for creating user's own models and presentation of given results. IBM ILOG CPLEX Optimization Studio, FICO XPRESS-IVE, MPL for Windows, AIMMS, GAMS, and AMPL belong among the most widespread systems of this nature.

The aim of the paper is to compare computational performance of three top LP and MILP solvers in their latest versions, i.e. IBM ILOG CPLEX 12.6.1 (released in December 2014), GUROBI 6.0 (October 2014), and FICO XPRESS Optimizer 27.01.02 (December 2014). The first two mentioned systems are available for academic purposes free in their full professional versions. For the test purposes MIPLIB 2010 library that contains 361 MILP test instances was used.

The next section of the paper contains little more detailed information about solvers included in the study, informs about MIPLIB 2010 library and about data set for benchmarking. Section 3 presents benchmarks results and the final section discusses them and concludes the paper.

MATERIALS AND METHODS

Discrete optimization problems are of a great attention of researchers and practitioners. Almost any of real problems cannot be modeled without integer (binary) variables. It is possible to mention many examples of models with discrete variables (traveling salesman problem, covering problem, vehicle routing problems, cutting problems, etc.) but the aim of the paper is not to discuss possible applications of discrete models but their computational aspects.

Optimal solution of discrete models can be often only hardly computed. Even though a great progress in solution of such class of problems has been recognized in the last years the large-scale (not only) problems are only hardly solvable. It is of a great importance to test and benchmark capabilities of LP and MILP professional solvers because it helps users to choose the best option for their purposes. There are available various data sets (libraries of test problems) for benchmarking of different problem classes (not only LP and MILP problems).

One of the most important and most cited library is MIPLIB (Mixed Integer Programming LIBrary). The current version of this library is MIPLIB 2010

I: *Benchmark data set*

Instance	Rows	Columns	Integer	Binary	Cont.
30n20b8	0	18380	7344	11036	0
aflow40b	1442	2728	0	1364	1364
air04	823	8904	0	8904	0
bab5	4964	21600	0	21600	0
beasleyC3	1750	2500	0	1250	1250
cov1075	637	120	0	120	0
csched010	351	1758	0	1457	301
danoint	664	521	0	56	465
eil33-2	32	4516	0	4516	0
gmu-35-40	424	1205	0	1200	5
map20	328818	164547	0	146	164401
mcsched	2107	1747	14	1731	2
mzzv11	9499	10240	251	9989	0
n3seq24	6044	119856	0	119856	0
n4-3	1236	3596	174	0	3422
neos13	20852	1827	0	1815	12
neos18	11402	3312	0	3312	0
net12	14021	14115	0	1603	12512
newdano	576	505	0	56	449
noswot	182	128	25	75	28
ns1758913	624166	17956	0	17822	134
pg5_34	225	2600	0	100	2500
qiu	1192	840	0	48	792
ran16x16	288	512	0	256	256
reblock67	2523	670	0	670	0
rmine6	7078	1096	0	1096	0
roll3000	2295	1166	492	246	428
sp98ir	1531	1680	809	871	0
tanglegram2	8980	4714	0	4714	0
vpphard	47280	51471	0	51471	0

and it fifth version of this library since its creation in 1996. It contains 361 test problems of different nature, size, and computational complexity. The test instances of the library are divided into three main groups:

- *Easy* – instances that are solved using professional commercial solvers quite easily (within one hour); This group contains currently 215 instances;
- *Hard* – the set of instances that are already solved using specialized software tools and their optimum solution is known (64 instances);
- *Open* (not solved) – the instances where the optimal solution is not known and are not solved up to now (82 instances).

Assignment to the mentioned three groups is not fixed. As the progress in codes is very significant factor during the time some of the hard or open problems are moved into higher categories and it is difficult to estimate the numbers of the problems in particular categories after several years.

More information about MIPLIB 2010 library can be found in Koch *et al.* (2011). The previous version of this library MIPLIB 2003 is described in detail in Achtenberg *et al.* (2006).

Selected test problems of MIPLIB 2010 are included into the benchmark test set. This set contains 87 problems, all of them are in the "easy" group. In our study we work with the subset of the benchmark set that has 30 problems. The main reason for this reduction is given by presentation limits given by the given space for the paper. Our data set is presented in Tab. I. The first column of this table contains identification of the instance as it is denoted in the MIPLIB library. The next five columns informs about the number of constraints (Rows), total number of variables (Columns), and the number of integer, binary and continuous variables of the instance.

The reduced data set was tested using three most powerful MILP solvers: IBM ILOG CPLEX 12.6.1, GUROBI 6.0 and FICO XPRESS Optimizer 27.01. The following paragraphs contain brief characteristics of these three solvers:

IBM ILOG CPLEX was originally developed in the 80s of the last century by CPLEX Optimization that was founded by R.E. Bixby. This firm was sold to ILOG, Inc. in 1997 and in 2007 to IBM. During the last 25 years CPLEX become one of the most powerful LP and MILP solvers at all. The newest version of this software 12.6.1 was released in December 2014. It is interesting that this solver (together with modeling environment IBM ILOG CPLEX Optimization Studio) is available to academic researchers and students for free in its full professional version. CPLEX solver itself is MS DOS application that is controlled by commands from DOS prompt. Much more convenient for decision makers is to use it as a solver within a modeling language (MPL for Windows, GAMS, AIMMS, etc.).

GUROBI is quite new solver. It is product of Gurobi, Inc. that was founded in 2008 by Z. Gu, E. Rothberg and R. E. Bixby. This system contains top LP and MIP solver that reaches often better results than CPLEX. Similarly as CPLEX, GUROBI is free in its full professional version for academic purposes. It is a set of libraries that is controlled from DOS prompt using its own commands. An advantage is that GUROBI can be easily used within professional modelling languages which allows its application in academic environment for solving student test cases and in commercial environment for complex optimization tasks.

XPRESS Optimizer is originally a product of British firm Dash Optimization, Inc. It was sold to FICO, Inc. in 2008. XPRESS Optimizer is now a part of a large optimization and modeling system FICO XPRESS Optimization Suite. The current version of XPRESS Optimizer is 27.01.02. FICO XPRESS Optimization Suite is not generally available for academic purposes but one can ask for free licenses under FICO Academic Partner Program. XPRESS Optimizer contains three main powerful solvers: simplex, barrier and integer. The system selects the most appropriate solver itself based on an analysis of the data set.

RESULTS

Benchmarks for optimization software are subject to professional interest of many researchers. The selected results for MIPLIB 2010 library are available e.g. in Mittelmann (2015). Information about benchmarks with the previous version of the MIPLIB library (MIPLIB 2003) is presented in Jablonský (2008). Tab. II presents information about solving the test problems using the three above mentioned MILP solvers. The upper time limit for all solvers was set to 3600 seconds (1 hour). All solvers use a modification of branch and bound algorithm. The relative gap for stopping the solver was 0.0001 (i.e. 0.01%). This means that the calculation stops when the current best integer solution found by the solver differs from the lower bound less than 0.01%. All experiments were performed with Lenovo Yoga ultrabook with Intel Core i7 and 8GB RAM.

The first column in Tab. II contains optimum objective function values of the problems. The next three columns present either numerical values that express time of calculating the optimum solutions using given solvers, or symbols "N" or "O/G" followed by numerical values. "N" means that the optimum solution of the problem was not found using the solver within the given time (3600 seconds). "O/G x.x" indicates that the solver found the optimum solution but the calculation does not stop because of the positive gap. In this case the numerical value indicates the value of the gap in %. The best results for particular instances are bolded.

The problem is how to explain the results of benchmarks. Decision makers are interested in recommendation which solver is the best for their purposes but the numbers can be explained in many ways. The companies (IBM, GUROBI, and FICO)

II: *Time of calculating the optimum solution [sec]*

Instance	Objective	CPLEX	GUROBI	XPRESS
30n20b8	302	7.23	**2.29**	112.00
aflow40b	1168	**169.23**	390.42	489.90
air04	56137	17.74	**10.11**	11.00
bab5	–106411.84	1086.11	**448.26**	O/G 0.6
beasleyC3	754	**11.34**	12.21	14.60
cov1075	20	7.32	**4.06**	104.80
csched010	408	**992.52**	1266.42	3452.10
danoint	65.6667	**1344.28**	2876.83	4188.20
eil33-2	934.008	47.41	**43.75**	72.00
gmu-35-40	–2406733.369	**562.88**	958.03	N
map20	–922	102.69	**74.11**	203.30
mcsched	211913	366.08	44.34	**30.90**
mzzv11	–21718	13.09	15.31	**9.30**
n3seq24	52200	**99.83**	228.15	121.20
n4-3	8993	**24.59**	1010.50	1641.90
neos13	–95.4748	89.94	20.09	**13.50**
neos18	16	8.66	**6.31**	51.20
net12	214	138.32	77.21	**77.10**
newdano	65.6667	N	**365.08**	O/G 4.5
noswot	–41	114.05	**18.43**	317.70
ns1758913	–1454.67	202.55	**11.92**	41.90
pg5_34	–14339.4	**282.72**	287.97	515.70
qiu	–132.873	**7.21**	18.98	61.30
ran16x16	3823	134.11	**26.56**	537.40
reblock67	–34630648.44	571.68	**275.07**	N
rmine6	–457.186	2013.66	**524.49**	N
roll3000	12890	37.01	27.98	**16.90**
sp98ir	219676790.4	43.51	**23.28**	446.80
tanglegram2	443	1.29	0.51	**0.43**
vpphard	5	1283.27	403.45	**175.10**

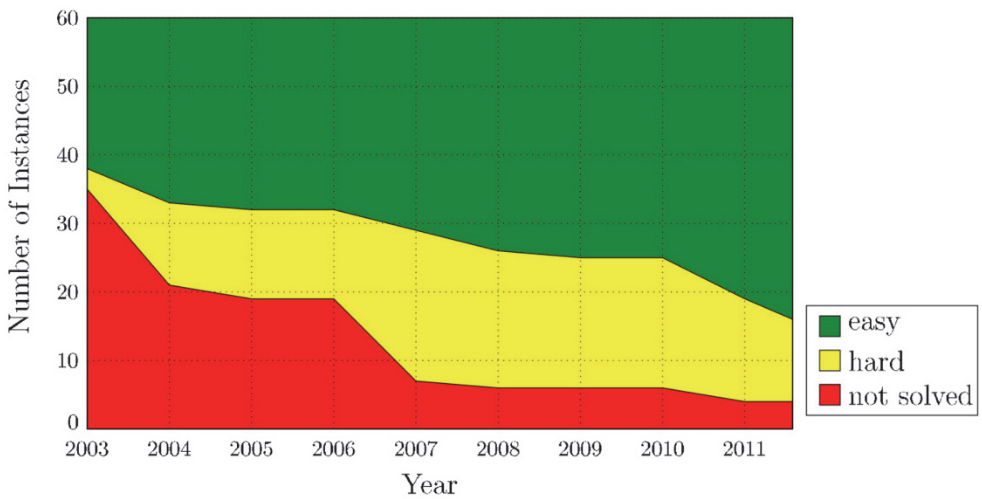
presents their own benchmarks that usually lead to a conclusion that their own solver is the fastest and overcomes the remaining ones. Of course this study is not complete and does not cover all MIPLIB 2010 instances but it is independent and is not burdened by any prejudices.

Everyone understands that it is not possible to rank the solvers according to the simple sum or average of computing times. The first orientation in selection of solvers can be given by the number of the "best" results. In our study, among 30 test problems GUROBI solver was the best (fastest) in 14 instances, IBM ILOG CPLEX in 9 instances and FICO XPRESS in 7 instances. It is interesting that FICO XPRESS is the fastest only in instances that were solved quite easily at least by one of the other two solvers (perhaps except *vpphard* instance). On the other hand FICO XPRESS is not able to reach optimum solution within one hour in three (four) instances, and probably is not able to reach the optimum solution of these instances at all. GUROBI has found optimum solution for all instances even it was not always the fastest solver. According to this criterion GUROBI is rated as the best, CPLEX the second and XPRESS as the worse among these three solvers.

We offer the following simple procedure for evaluation of solvers. The solution times are normalized and the value 1 is assigned to the slowest solver and the remaining ones have value lower than one (fraction of time needed to solve the instance comparing to the highest value). The cases where the solver is not able to find optimum solution was penalized by one (optimum solution was found but the solver does not stop within the given time) or two units (optimum solution was not found). The normalized values are given in Tab. III. The sum of normalized values is presented in the last row of Tab. III. The best solver according to this simple procedure is again GUROBI followed by IBM ILOG CPLEX and FICO XPRESS.

III: *Evaluation of solvers*

Instance	CPLEX	GUROBI	XPRESS
30n20b8	0.065	0.020	1.000
aflow40b	0.345	0.797	1.000
air04	1.000	0.570	0.620
bab5	1.000	0.413	2.000
beasleyC3	0.777	0.836	1.000
cov1075	0.070	0.039	1.000
csched010	0.288	0.367	1.000
danoint	0.321	0.687	1.000
eil33-2	0.658	0.608	1.000
gmu-35-40	0.588	1.000	3.000
map20	0.505	0.365	1.000
mcsched	1.000	0.121	0.084
mzzv11	0.855	1.000	0.607
n3seq24	0.438	1.000	0.531
n4-3	0.015	0.615	1.000
neos13	1.000	0.223	0.150
neos18	0.169	0.123	1.000
net12	1.000	0.558	0.557
newdano	3.000	1.000	2.000
noswot	0.359	0.058	1.000
ns1758913	1.000	0.059	0.207
pg5_34	0.548	0.558	1.000
qiu	0.118	0.310	1.000
ran16x16	0.250	0.049	1.000
reblock67	1.000	0.481	3.000
rmine6	1.000	0.260	3.000
roll3000	1.000	0.756	0.457
sp98ir	0.097	0.052	1.000
tanglegram2	1.000	0.395	0.333
vpphard	1.000	0.314	0.136
Sum	**20.465**	**13.636**	**31.684**



1: *Progress in MILP solvers – MIPLIB 2003 instances*

DISCUSSION AND CONCLUSION

Progress in optimization software is very significant and MILP problems that were unsolvable still few years ago can be solved now quite easily. This progress is very well illustrated on Fig. 1 – source Achtenberg *et al.* (2006). This figure presents the number of instances of MIPLIB 2003 library in three categories – easy, hard and not solved – within 8 years from 2003. It is evident that more than 50% of instances were unsolvable in 2003. After 8 years only 4 instances of 60 together were not solved. A similar progress can be expected with the new version(s) of MIPLIB library in the future.

Discrete optimization is a very complex task and the improvement and future development of MILP solvers is of a high importance due to increasing applications of discrete models. The main aim of the paper was to test and compare current top MILP solvers on the test problems taken as a subset of the benchmark set of MIPLIB 2010 library. Even our benchmarks do not include the complete benchmark set, the conclusion is almost clear. According to the proposed methodology GUROBI was identified as the best MILP solver. Its advantage is that can be incorporated into user's own applications or as a powerful solver in modeling systems. The other two tested solvers are very powerful and if possible they can be used as an alternative to GUROBI because it is not possible to estimate which of the solvers will be the best for a given MILP problem. An advantage of GUROBI and IBM ILOG CPLEX solvers is a possibility to get them free under academic programs of both firms.

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REFERENCES

ACHTENBERG, T., KOCH, T. and MARTIN A. 2006. MIPLIB 2003. *Operations Research Letters,* 34(4): 361–372.

FICO ©2015. *Solving Real World Problems with FICO Xpress Optimization Suite 7.2.* [Online]. Available at: http://www.fico.com/en/wp-content/secure_upload/Xpress_7.2_Benchmarking_2773FS.pdf. [Accessed: 15 February 2015].

GUROBI ©2015. *Benchmarks and Case Studies, 2013.* [Online]. Available at: http://www.gurobi.com/resources/benchmarks-and-case-studies. [Accessed: 10 February 2015].

IBM ©2015. *IBM ILOG CPLEX Optimizer Performance Benchmarks.* [Online]. Available at: http://www-01.ibm.com/software/commerce/optimization/cplex-performance/. [Accessed: 15 February 2015].

JABLONSKÝ, J. 2008. Advance in Optimization Software for LP and MIP Problems. In: *Quantitative Methods in Economics (Multiple Criteria Decision Making XIV).* High Tatras, 5–7 June. Bratislava, Iura Edition, 97–104.

KOCH, T., ACHTENBERG, T., ANDERSEN, E., BASTERT, O., BERTHOLD, T., BIXBY, R. E., DANNA, E., GAMRATH, G., GLEIXNER, A. M., HEINZ, S., LODI, A., MITTELMANN, H., RALPHS, T., SALVAGNIN, D., STEFFI, D. E. and WOLTER, K. 2011. MIPLIB 2010 – Mixed Integer Programming Library version 5. *Mathematical Programming Computation,* 3(2): 103–163.

MITTELMANN, H. 2015. *Mixed Integer Linear Programming Benchmark (MIPLIB 2010).* [Online]. Available at: http://plato.asu.edu/ftp/milpc.html. [Accessed: 20 January 2015].

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