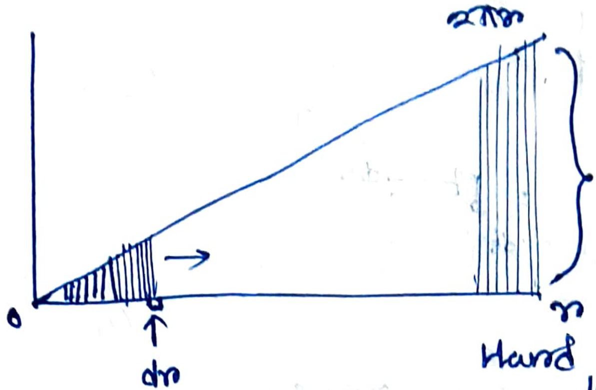
Essence of Calculus



$\frac{1}{2} r (r \cdot 2\pi)$

$= \pi R^2$

Hard problem

⇓

Sum of many small values

⇓

Area under a graph

Area $\approx 2\pi r \cdot dr$

$\approx 2\pi (0.0) \boxed{(0.1)}$

$\approx 2\pi (0.1) \boxed{(0.1)}$

$2\pi (0.2) \boxed{(0.1)}$

+

+

$\approx 2\pi (2.9) \boxed{(0.1)}$

$2\pi r \boxed{dr}$

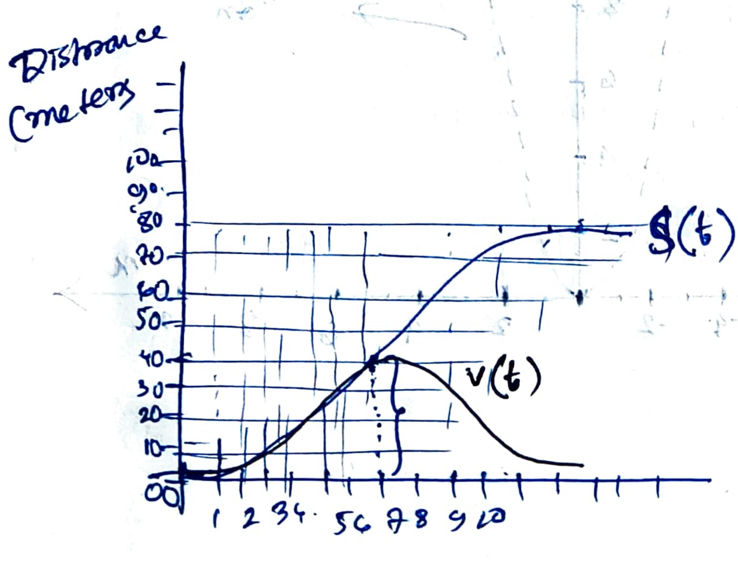
Derivative

Goal:- 1) Learn Derivatives 2) Avoid paradoxes

Instaneous rate of Changes

Instaneous → one point in time.

Change :→ Requires multiple point in time.



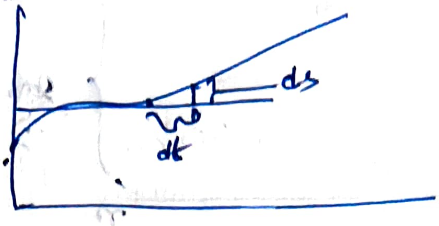
Velocity

↓

$\frac{\text{Change in Distance}}{\text{Change in time}}$

↓

$\frac{\text{(meters)}}{\text{(seconds)}}$

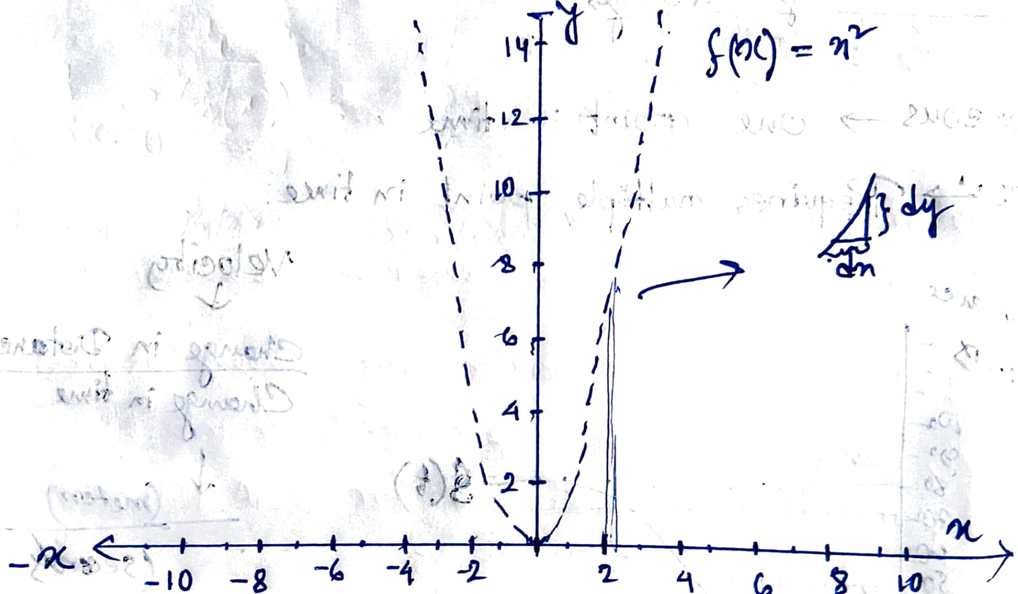


$$\frac{ds}{dt} = \frac{\text{rise}}{\text{run}}$$

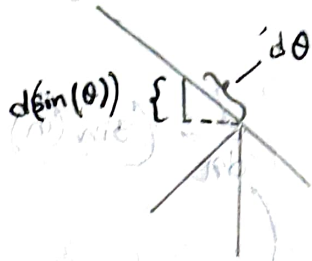
$$\frac{ds}{dt}(t)$$

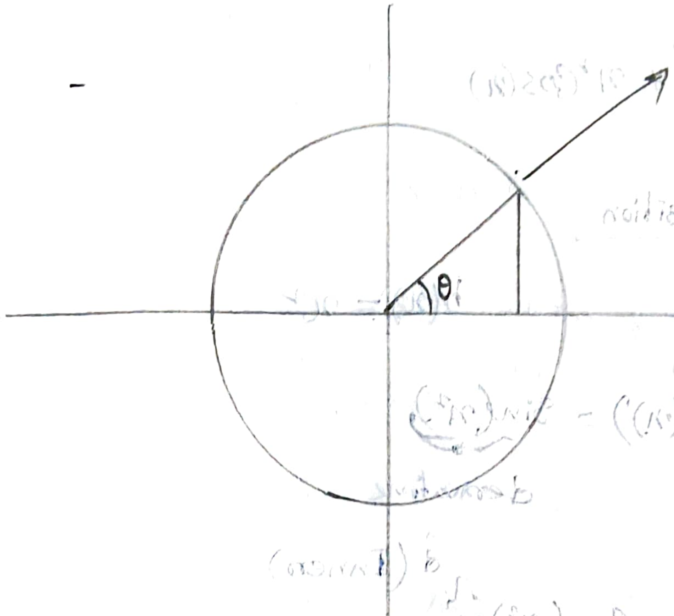
$$\frac{ds}{dt}(t) = \frac{s(t+dt) - s(t)}{dt}$$

Derivative formulas through geometry



$$\frac{d(x^n)}{dx} = nx^{n-1} \longrightarrow \text{Power rule}$$





Visulizing the chain Rule and product rule

"Using the chain rule is Like peeling an Onion: you have to deal with each layer at a time, and if it too big you will start Crying."

Sum Rule

$$\frac{d}{dx}(\sin(x) + x^2) = \cos(x) + 2x$$

$$\frac{d}{dx}(g(x) + h(x)) = \frac{dy}{dx} + \frac{dh}{dx}$$

$$df = g(x)\,dh + h(x)\,dy$$

$$\frac{df}{dx} = g(x)\frac{dh}{dx} + h(x)\frac{dg}{dx}$$

$$f(x) = \sin(x)\,x^2 = \text{Area}$$

$$df = \sin(x)\,d(x^2) + x^2\,d(\sin(x)) + \underset{\text{negligable}}{\dots}$$

$$\boxed{\frac{df}{dx} = \sin(x)2x + x^2\cos x}$$

" Left d (Right) + Right d (Left)"

$$\frac{d}{dx}\left(\sin(x)\, x^2\right)$$

$\rightarrow \sin(x)\,2x + x^2\cos(x)$

Function Composition

$g(x) = \sin(x)$ $\qquad$ $h(x) = x^2$

$$g(h(x)) = \sin(x^2)$$

Inner $\qquad$ derivative

$$\frac{d}{dx}\sin(x^2) = \cos(x^2)\,2x \qquad d\,(\text{Inner})$$

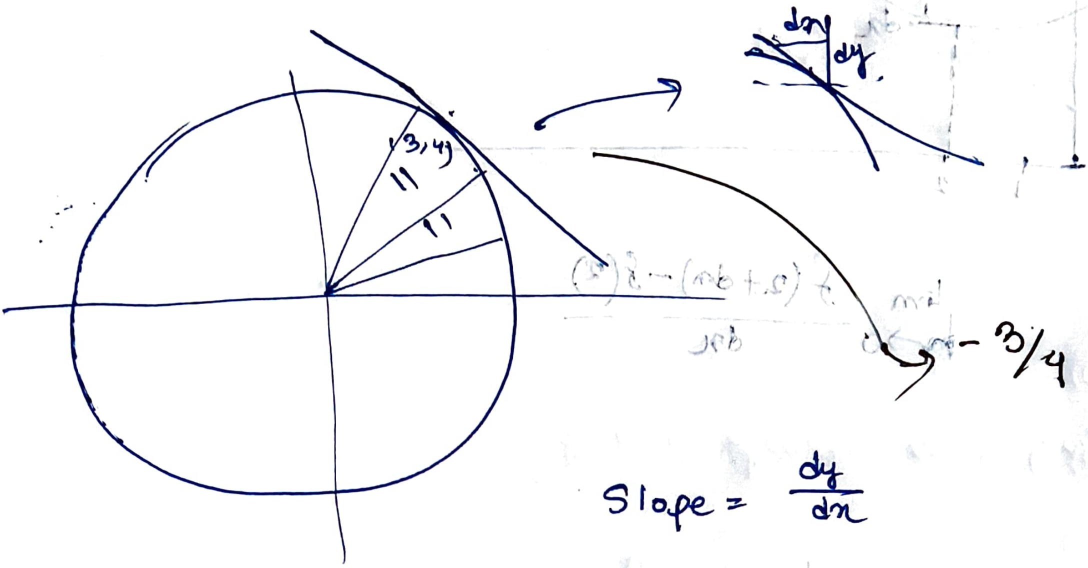
Outer $\qquad$ d (Outer)

$$\frac{d}{dx}\underbrace{g(\overbrace{h(x)}^{\text{Inner}})}_{\text{Outer}} = \underbrace{\frac{dg}{dh}(h(x))}_{d\,(\text{Outer})}\;\overbrace{\frac{dh}{dx}(x)}^{d\,(\text{Inner})}$$

Special about Euler's number 'e'?

$M(t) = 2^t$ $\qquad t = 3, 5 \to 6 \ldots$

$$\frac{dM}{dt} = \frac{2^{t+dt} - 2^t}{dt}$$



Slope = $\frac{dy}{dx}$

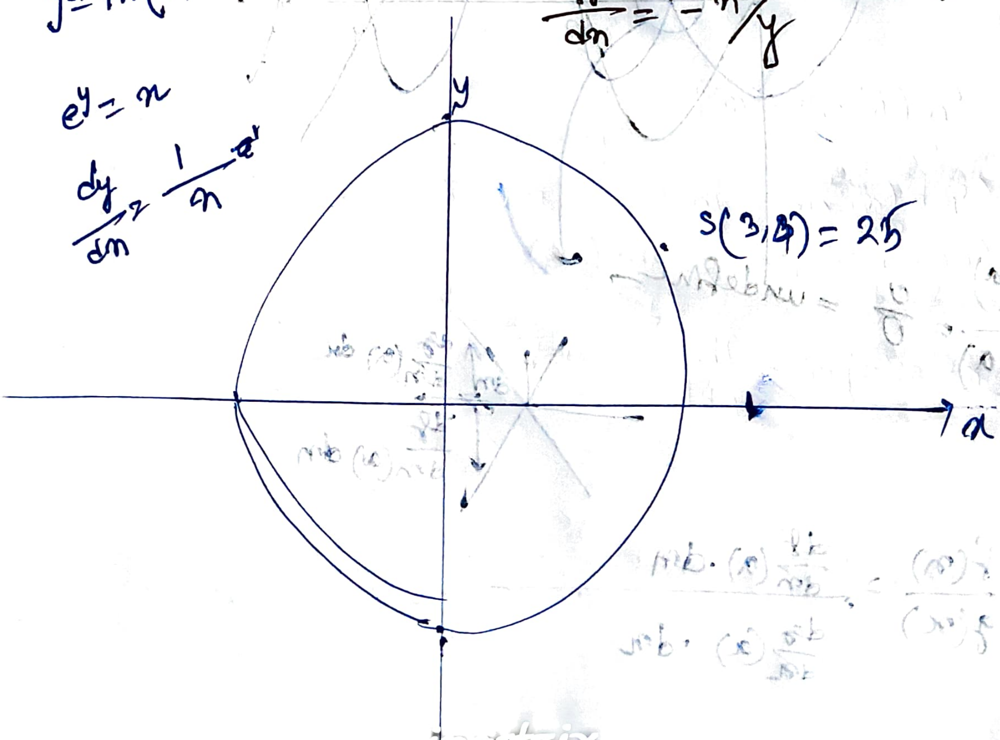
$x^2 + y^2 = 5^2$

$2x\,dx + 2y\,dy = 0$

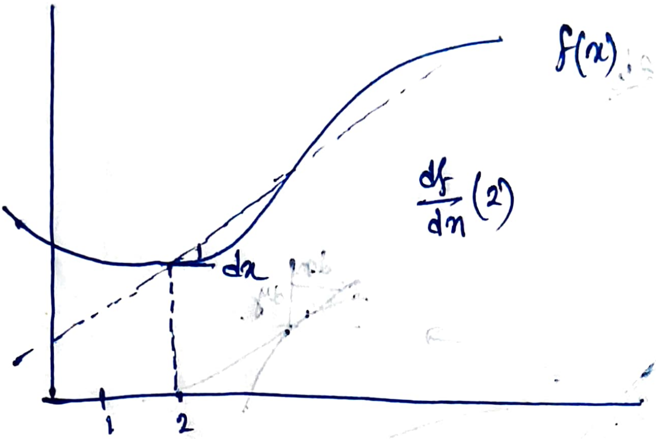
$y = \ln(x)$ $\qquad \frac{dy}{dx} = -x/y$

$e^y = x$

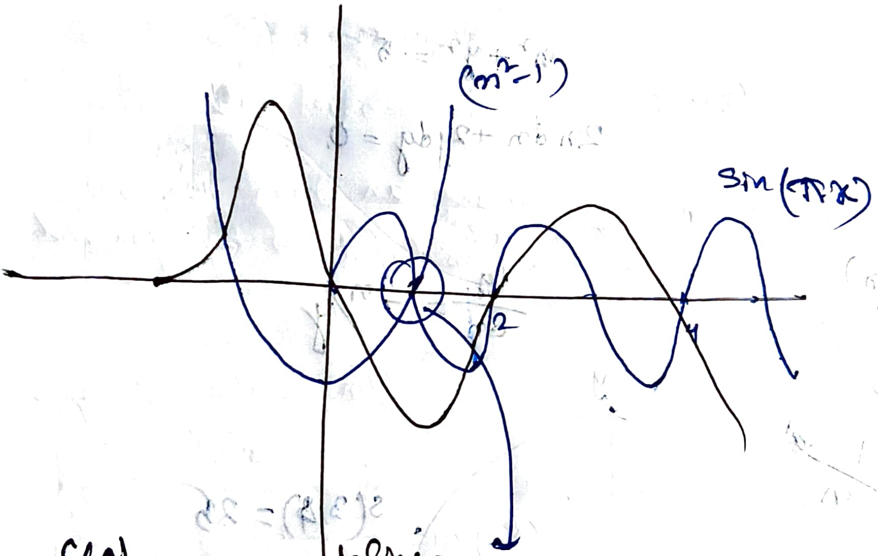
$\frac{dy}{dx} = \frac{1}{x}$



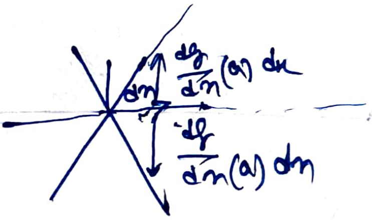
Limit , L- Hospital rule



$$\lim_{dx \to 0} \frac{f(2+dx)-f(2)}{dx}$$



$$\frac{f(a)}{g(a)} = \frac{0}{0} = \text{undefined}$$



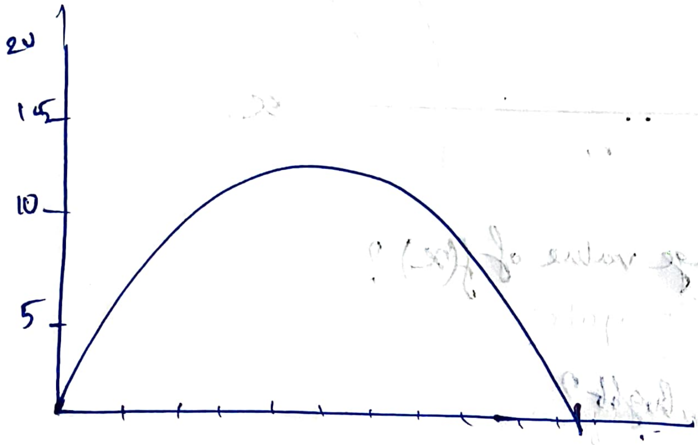
$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\frac{df}{dx}(a) \cdot dx}{\frac{dg}{dx}(a) \cdot dx}$$

$$\lim_{x \to 0} \frac{\sin(x)}{x} = \frac{\cos(0)}{1} = 1 \quad \text{(L'Hospital rule)}$$

Next

$$\frac{d(\sin x)}{dx}(x) = \lim_{h \to 0} \frac{\sin(x+h) - \sin(x)}{h}$$

Integration and fundamental theorem of Calculus

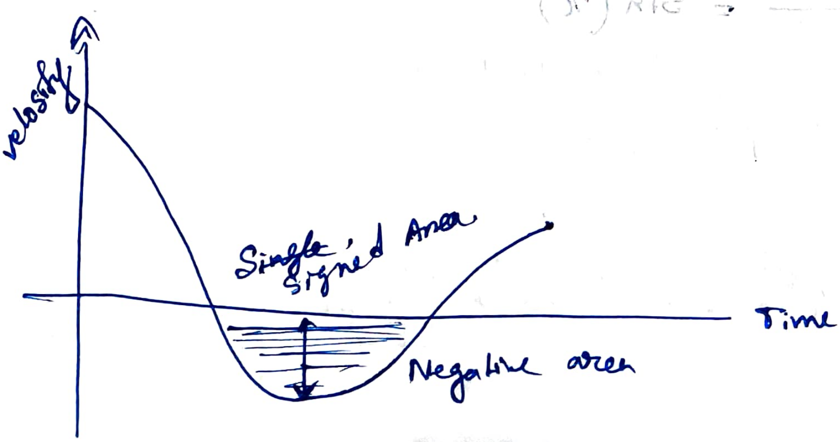


$$\int_0^T v(t)\, dt$$

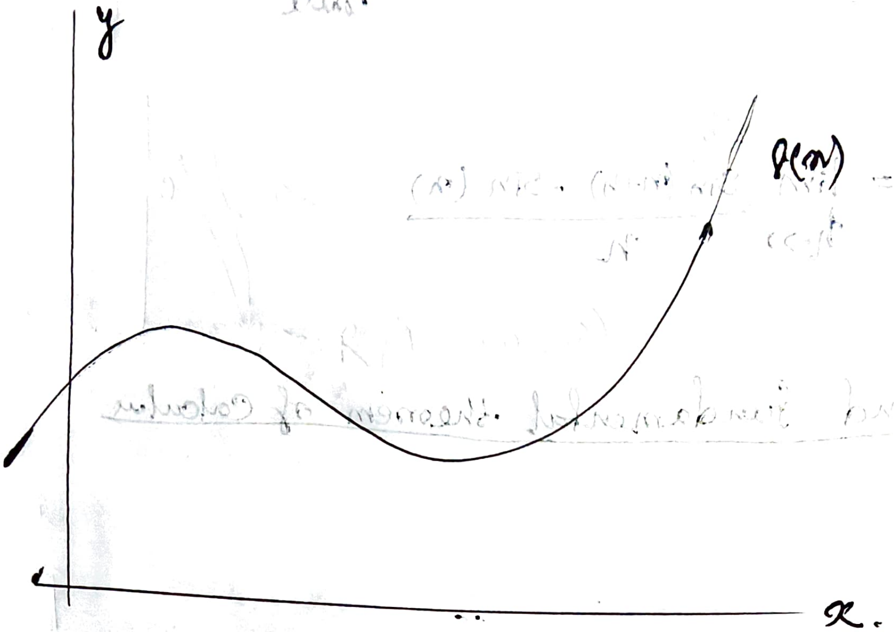
$$\frac{ds}{dt} = v(t)$$

$$v(t) = f(s \to t$$

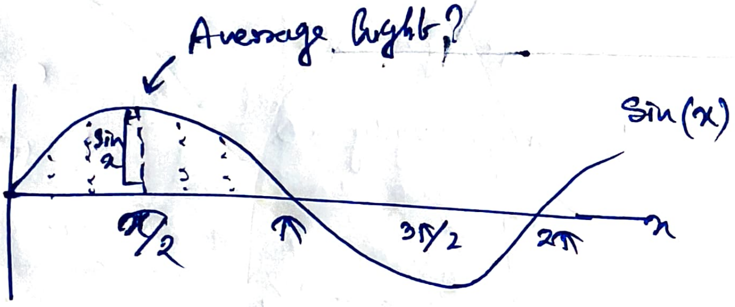
$$ds = v(t)\, dt$$



what does Area have to do with slope

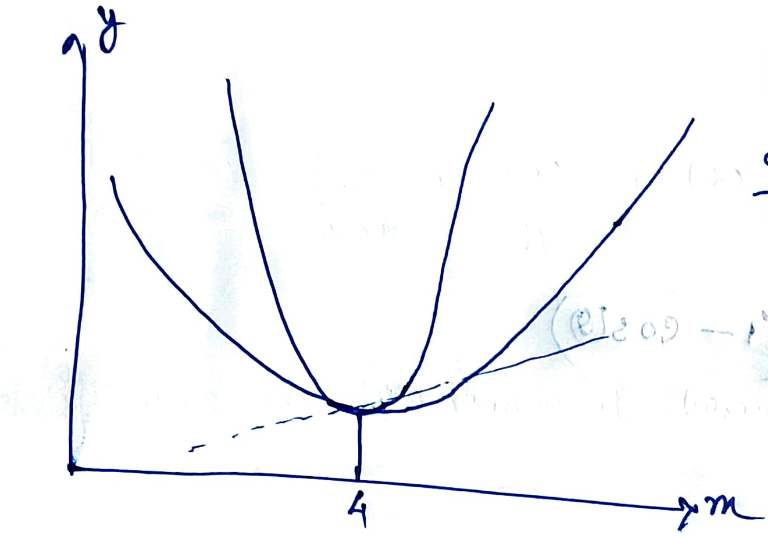


what is the average value of $f(x)$?



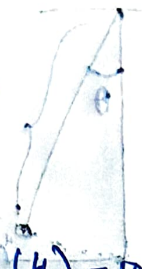
$$\frac{\int_0^{\pi} \sin(x)\,dx}{\pi} = \sin(x)$$

Higer Order Derivatives

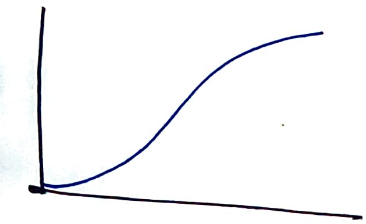


$f(x)$

$\frac{df}{dx}$

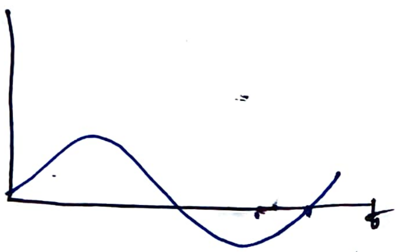


$\frac{d^2f}{dy^2}(4) = 10$

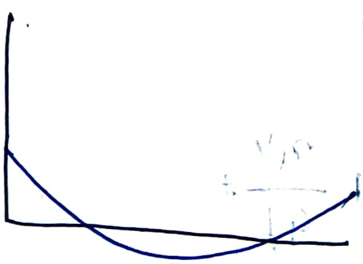


$s(t) \rightarrow$ Displacement

$\frac{ds}{dt}(t)$ = Velocity

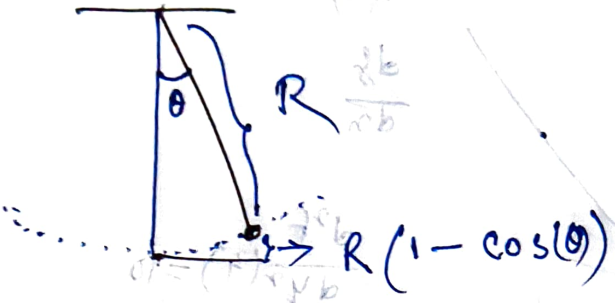


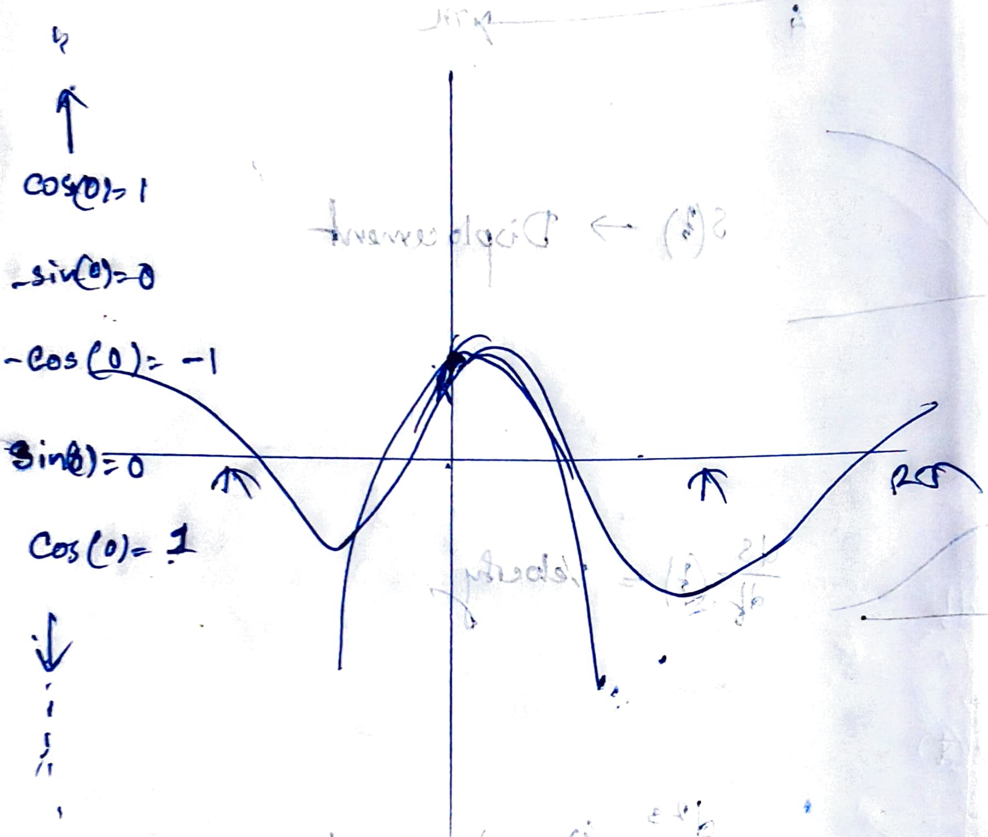
$\frac{d^2s}{dt^2}(t)$ = Acceleration.



$\frac{d^3s}{ds^3}(t) \rightarrow$ Jerk

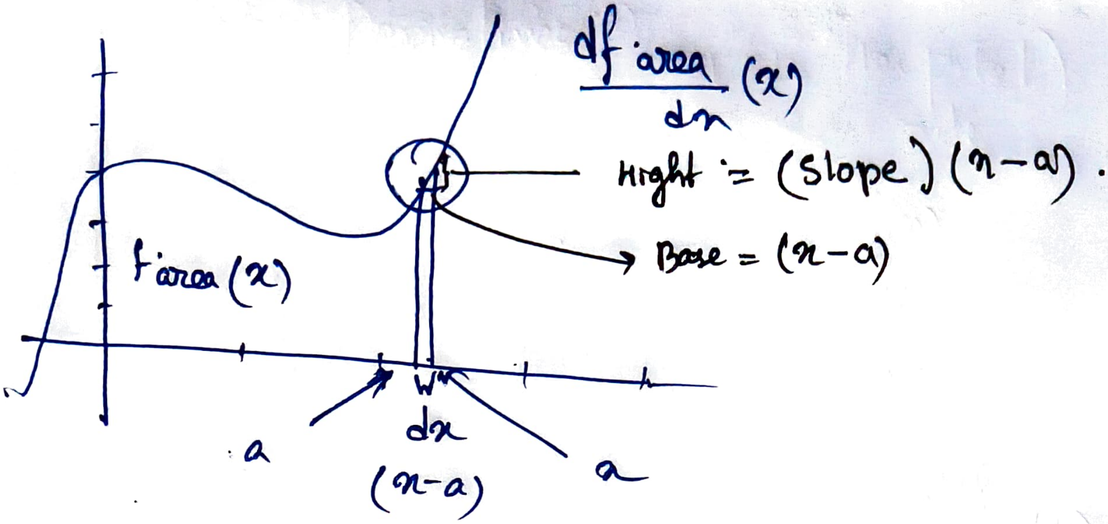
Taylor's Series





Derivative information at a point → Output information near the point

$$P(x) = 1 + 0\frac{x^1}{1!} + -1\frac{x^2}{2!} + 0\frac{x^3}{3!} + 1\frac{x^4}{4!} + \dots$$



$$\text{Hight} = \frac{d^2 f\,area}{dx^2}(a)\,(x-a)$$

In ~~tatlor~~ taylor series. It is use the derivative at a point and approximate the calculation of a function