Cheatsheet: 2D-3D Geometry and Camera Projection

1. Camera Projection Model

Projection Equation:

$$\mathbf{x} = P\mathbf{X}, \quad P = K[R|t]$$

Where:

$$K = \begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}, \quad P \in \mathbb{R}^{3\times 4}, \quad \mathbf{X} \in \mathbb{R}^{4\times 1}$$

Transformation Matrix:

$$T_{w_c} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix}, \quad T_{c_w} = \begin{bmatrix} R^\top & -R^\top t \\ 0 & 1 \end{bmatrix}$$

2. Fundamental Matrix ($F \in \mathbb{R}^{3\times 3}$)

Epipolar Constraint:

$$\mathbf{x'}^\top F\mathbf{x} = 0$$

8-Point Algorithm:

$$A\mathbf{f} = 0, \quad A \in \mathbb{R}^{N\times 9}, \quad \mathbf{f} = \text{flatten}(F)$$

SVD for rank-2 enforcement:

$$U, S, Vt = \text{SVD}(F), \quad S[-1] = 0, \quad F = USVt$$

Epipolar Line:

$$l_2 = F\mathbf{x}, \quad l_2 = [a, b, c]^\top, \quad ax + by + c = 0$$

Epipoles:

$$e_2 = \text{null}(F), \quad e_1 = \text{null}(F^\top)$$

3. Essential Matrix ($E \in \mathbb{R}^{3\times 3}$)

Essential Matrix from F:

$$E = K_2^\top F K_1$$

Decomposition of E:

$$E = U\Sigma V^\top, \quad R_1 = UWV^\top, \quad R_2 = UW^\top V^\top, \quad t = U[:, 2]$$

$$W = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

4. Triangulation ($X \in \mathbb{R}^{4\times 1}$)

Triangulation from Two Views:

$$A\mathbf{X} = 0, \quad A \in \mathbb{R}^{4\times 4}$$

$$A = \begin{bmatrix} x_1 P_3^1 - P_1^1 \\ y_1 P_3^1 - P_2^1 \\ x_2 P_3^2 - P_1^2 \\ y_2 P_3^2 - P_2^2 \end{bmatrix}$$

Solution using SVD:

$$\mathbf{X} = Vt[-1], \quad \mathbf{X}/ = \mathbf{X}[-1]$$

5. Homography ($H \in \mathbb{R}^{3\times 3}$)

DLT (Direct Linear Transform):

$$A\mathbf{h} = 0, \quad A \in \mathbb{R}^{2N\times 9}, \quad \mathbf{h} = \text{flatten}(H)$$

$$H = Vt[-1].reshape(3, 3), \quad H/ = H[2, 2]$$

Inverse Homography:

$$H_{12} = H_{21}^{-1}$$

6. RMSE for Homography

RMSE Formula:

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N} ||\mathbf{x'}_i - H_{21}\mathbf{x}_i||^2}$$

7. Other Useful Formulas

Projection Matrix:

$$P = K[R|t]$$

Where $K \in \mathbb{R}^{3\times 3}$ is the intrinsic matrix, $R \in \mathbb{R}^{3\times 3}$ is the rotation matrix, $t \in \mathbb{R}^{3\times 1}$ is the translation vector.

Transformation of Points Between Frames:

$$\mathbf{X'} = T\mathbf{X}, \quad T \in \mathbb{R}^{4\times 4}$$

Compute Essential Matrix from R and t:

$$E = [t]_\times R, \quad [t]_\times = \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix}$$