Laboratory Session 1: 2D-3D Geometry in Homogeneous Coordinates and Camera Projection

In this laboratory session we are going to introduce homogeneous coordinates in order to understand the projection of 3D points on 2D camera planes and basic geometric 2D and 3D operations.

Goals of the assignment:

1. Practice with homogeneous coordinates to code 2D and 3D geometrical entities.
2. Practice with perspective projection of basic geometrical entities.
3. Understanding SVD for solving homogeneous systems of linear equations.

image 1

image 2

We provide the camera pose R_wc1, R_wc1, t_wc1, and calibration parameters f_x, f_y,x_0, $y_0$of two RGB views from a sequence in the Scannet dataset, with partial common field of view and 3D coordinates of some selected points.

<table>
<tr><td colspan="2">Calibration parameters:

f_x = 1165.723022 px
f_y = 1165.738037 px
x_0 = 649.094971 px
y_0 = 484.765015 px</td><td colspan="3">R_w_c1 = [-0.107841 0.3888 -0.91495
0.99396 0.023439 -0.1072
-0.02024 -0.921 -0.3891]
t_w_c1 = [6.1377 1.3981 1.4762]’ m
R_w_c2 = [0.107366 0.306900 -0.945666
0.993780 -0.061402 0.092902
-0.029555 -0.949759 -0.311584]
t_w_c2 = [4.4462 1.41698 1.5726]’ m</td></tr>
<tr><td>X_A = [3.44,
0.80,
0.82]</td><td>X_B = [4,20,
0.80,
0.82]</td><td>X_C = [4.20,
0.60,
0.82]</td><td>X_D = [3.55,
0.60,
0.82]</td><td>X_E = [-0.01,
2.6,
1.21]</td></tr>
</table>

From this data for both images:

1. Projection matrices

1.1. Compute the projection matrices P1 and P2.

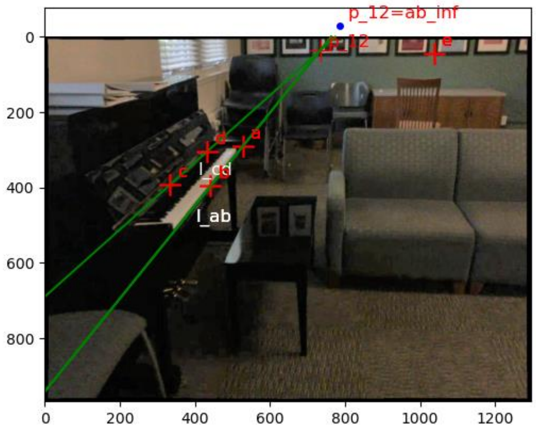
1.2. Compute and plot all the projections of the 3D points A,B,C,D,E in both images.

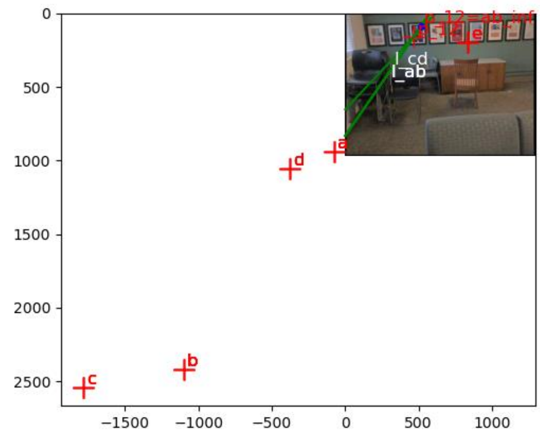
Solution of the projected points:

Coordinates of 2D points on image 1	Coordinates of 2D points on image 2
a b c d e x: 527.7, 441.7, 334.1, 433.5, 1036.9 y: 292.9, 393.8, 392.1, 303.7, 43.9	a b c d e x: -72.7, -1106.1, -1781.3, -377.3, 835.0 y: 943.2, 2410.0, 2538.0, 1052.4, 196.8

2. 2D Lines and vanishing points

2.1. Compute and plot in each image the line l_ab defined by a, and b (projections on A and B).
2.2. Compute and plot in the image the line l_cd, defined by c and d (projections on C and D) in the image.
2.3. Compute p_12 the intersection point of l_ab and l_cd
2.4. Compute the 3D infinite point corresponding to the 3D direction defined by points A and B, AB_inf.
2.5. Project the point AB_inf with matrix P to obtain the corresponding vanishing point ab_inf.
What





3. Understanding SVD with 2D lines fitting

The file `line2DFittingSVD.py` includes an example of 2D line fitting using SVD. First, a set of 5 points `xGT` perfectly lying on a line `l_GT` are computed. Then, Gaussian noise is added to the points such that they are not perfectly lying on the line anymore `x` (actually both sets of points are loaded from external files to assure the repeatability of the results). Finally, an estimation of the line is computed using SVD.

3.1. Modify the provided code to fit the line with SVD using only the 2 extreme points from `x`.
3.2. Modify the provided code to fit the line with SVD using the original 5 "perfect points" `xGT`.

What are the sizes of U, S and V matrices in each case?

3.3. Interpret the resulting singular values in each case.
3.4. At the end of the provided code a singular value is modified and a variation of the original input matrix is re-composed. What represent the data of this matrix? Why?

4. Planes in homogeneous coordinates

4.1. Compute the equation of the 3D plane π defined by the points A, B, C, D. You can check the provided file `line2DFittingSVD.py` for a SVD programming example. *Solution:* [-0.-0. 0.7733 -0.6341]

4.2. Compute the distance of each of the 3D points A,B,C,D to the plane π

Solution:

d_A = 0. m   d_B = 0. m   d_C = 0. m   d_D = 0. m   d_E = 0.39 m