Example Project | Simulating Portfolio Option VaR with EWMA and GARCH

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Introduction

This report aims to calculate the Value-at-Risk (VaR) for a portfolio consisting of Apple (AAPL) and Intel (INTC), both of which are traded on NASDAQ, along with two options. The data sample spans 19 years of daily observations, providing a robust basis for analysis. Two key risk forecasting methods—Exponentially Weighted Moving Average (EWMA) and Generalized Autoregressive Conditional Heteroskedasticity (GARCH)—have been employed to model volatility.

VaR is the focal point of this report as it quantifies potential portfolio losses at a given probability level, making it a crucial tool for risk measurement in financial markets. In particular, Simulated VaR has been calculated for the two assets and two options using various simulation sizes and compared against the True VaR, which is directly derived from the dataset.

During the analysis, the convergence behavior of Simulated VaR was observed; however, a notable difference between Simulated VaR and True VaR emerged. This discrepancy raises questions about the underlying assumptions and the influence of simulation size, which will be explored further in the next phase of this project.

Data Description and Preparation

The data has been downloaded from Jon Danielsson's R Notebook of Financial Risk Forecasting, with Apple (AAPL) and Intel (INTC) chosen for their shared characteristics as major tech institutions, both traded on the NASDAQ. The dataset spans a period of 19 years, from January 3, 2003, to December 30, 2022, offering a robust sample for analysis. To prepare the data, prices were first adjusted for stock splits and dividends to ensure accuracy. From the adjusted prices, log returns were extracted, as they provide a stationary measure suitable for Value-at-Risk (VaR) calculations. Summary statistics for the returns are presented below, showing comparable risk profiles for inclusion in the same portfolio.

Table 1: Summary Statistics for AAPL and INTC Returns

	AAPL	INTC
Mean Return	0.0012628	0.0001950
Std Dev	0.0212219	0.0197679
Max	0.1302421	0.1783246
Min	-0.1974579	-0.1989562
Median	0.0011121	0.0004712

Methodology

After obtaining Returns and Prices as two separate datasets, a general strategy has been devised for the analysis. Two different types of covariance structures have been generated for ease of use. The first approach calculates a combined portfolio risk using portfolio weights, which in this case are 0.5/0.5. Inspired by Jon Danielsson's Risk Forecasting Methods R Notebook, this method combines the two assets into a single risk measure and will be referred to as Sigma throughout the report. The second approach calculates a covariance matrix based on individual volatilities of the two assets. While Sigma is used as an input to the Black-Scholes model for option pricing, the covariance matrix is utilized for simulating prices.

The volatility estimation process involves two key models: EWMA and GARCH. Both models are implemented as functions, adapted from Jon Danielsson. The EWMA function estimates volatility recursively using the following formula:

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)y_{t-1}^2$$

where λ (lambda) is set to 0.94 as it provides a good balance between historical and recent information. For the GARCH function, a GARCH(1,1) model has been utilized, which can be expressed as:

$$\sigma_t^2 = \omega + \alpha y_{t-1}^2 + \beta\sigma_{t-1}^2$$

This model captures volatility clustering, a common characteristic of financial time series. The Black-Scholes model has been applied to analytically calculate option prices. The formula for the call option and the put option is given by:

$$\text{put}_t = Xe^{-r(\Upsilon-\tau)} - p_t + \text{call}_t$$

$$\text{call}_t = p_t\Phi(d_1) - Xe^{-r(\Upsilon-\tau)}\Phi(d_2)$$

where $\Phi()$ is the cumulative distribution function of the standard normal distribution, X is the strike price, r is the risk-free rate, and $\Upsilon - \tau$ is the time to maturity. The process can be summarized as follows. First, Simulated VaR is generated. In this step, today's prices are taken as the last observation in the sample, and option prices are calculated analytically using the Black-Scholes model with Sigma as an input. Next, tomorrow's prices are simulated using the covariance matrix and the rmvnorm function. Based on these simulated prices, option prices are recalculated using the Black-Scholes model. We chose put options with a strike price equal to 80% of the current price to reflect a conservative hedging strategy, where the options are out-of-the-money and provide protection against significant downside risk while limiting the cost of the options. Finally, the True VaR is calculated using the dataset. The difference between Simulated VaR and True VaR is then observed across various simulation sizes to analyze convergence behavior.

Results

First, we compare the volatilities between GARCH and EWMA:

Table 2: EWMA and GARCH Volatilities for AAPL and INTC

Asset	EWMA	GARCH
AAPL	0.022942	0.021759
INTC	0.021431	0.021921
Together	0.021494	0.019481

From the table, it is observed that GARCH volatility is consistently slightly lower than EWMA volatility for AAPL, indicating that recent volatility spikes are captured more strongly by EWMA due to its fixed decay factor, which places greater emphasis on recent data. In contrast, for INTC, GARCH volatility is slightly higher than EWMA volatility, suggesting that the dynamic modeling of GARCH captures additional persistent volatility not emphasized by EWMA.

Additionally, AAPL exhibits higher volatility compared to INTC under the EWMA model, indicating more pronounced short-term variability in AAPL's returns. However, under the GARCH model, AAPL volatility is slightly lower than INTC's, reflecting that GARCH smooths out the shocks in AAPL more effectively compared to INTC.

We continue to observe how the volatilities estimated by EWMA and GARCH impact the simulated VaR and true VaR. For the EWMA case, the final simulated VaR is 4.553, while the true VaR is 8.519. The simulated VaR appears noisy at the beginning due to the finite sample size and gradually converges to a stable value. However, it is important to note that this converged value does not align with the true VaR, which highlights a systematic underestimation of risk.

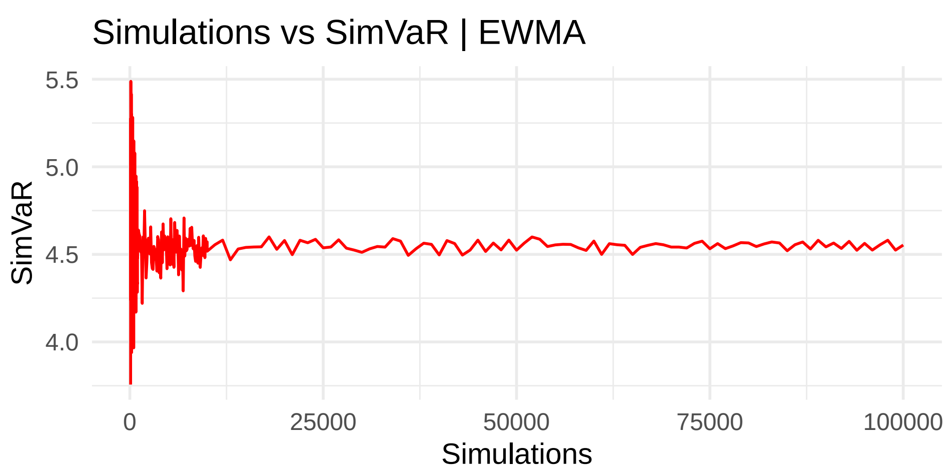
In the GARCH case, the final simulated VaR is 4.24, and the true VaR is 8.083. Similar to the EWMA model, the simulated VaR converges over time and aligns more closely with the true

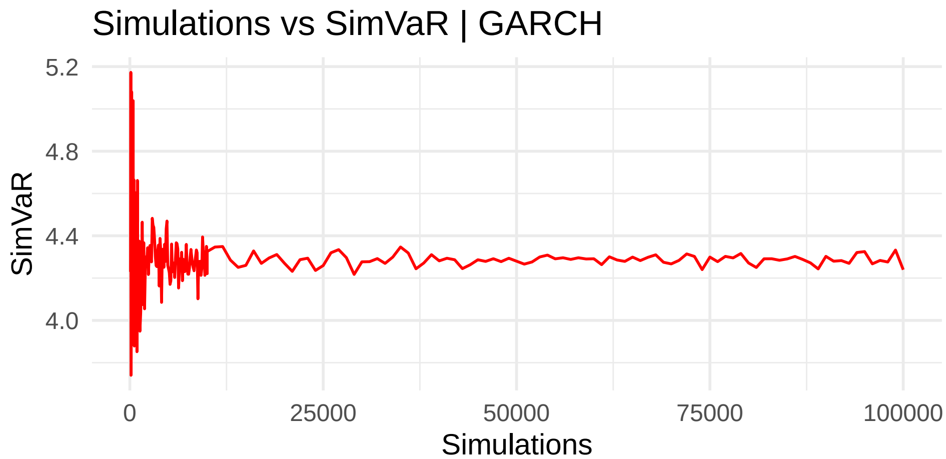
VaR. This improvement suggests that the GARCH model captures the persistence of volatility more effectively, reducing the discrepancies observed in earlier simulations.

However, slight deviations may still occur due to model assumptions and the nature of financial return distributions. These deviations highlight the importance of selecting appropriate models and parameters to balance responsiveness to short-term volatility and accuracy in capturing longer-term risk dynamics. Some factors to note are:

1. Parameter choices in the models: For the EWMA model, the fixed decay factor $\lambda = 0.94$ laces more weight on recent volatility while discounting older observations. While this makes EWMA responsive to sudden changes, it may fail to capture persistent volatility dynamics observed in long-term clustering.
2. GARCH(1,1) model limitations: The GARCH(1,1) model assumes conditional normality and smooths volatility over time. While it captures volatility clustering effectively, it still underestimates extreme risks because it does not account for fat tails or asymmetries in return distributions.

By underestimating risk relative to the true VaR, these models highlight the limitations of assuming normality in both volatility forecasts and VaR calculations. This reinforces the need for more flexible approaches, such as incorporating Student-t GARCH or non-parametric methods, to better model the extreme events that drive tail risk.





Conclusion and Recommendations

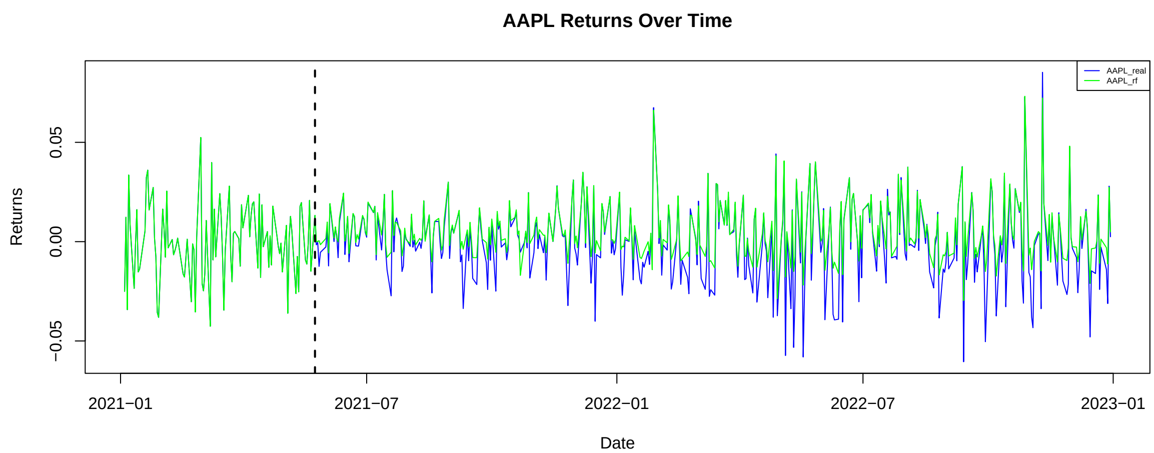
We prepared the data, applied volatility estimation models, and simulated VaR for our portfolio. However, we encountered issues with the converged simulated VaR consistently underestimating the true VaR. This highlights limitations in the current approach and areas for improvement.

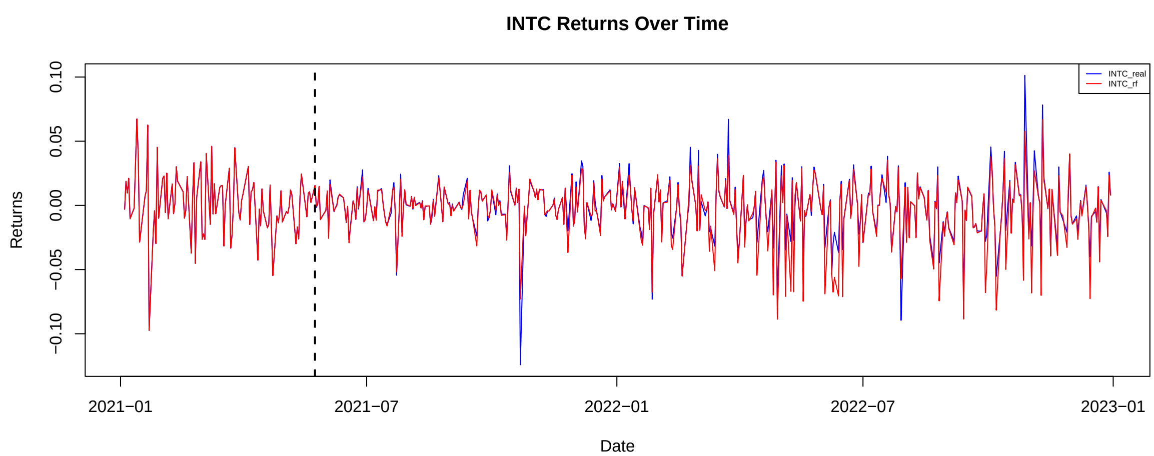
To address these issues, further actions can include exploring additional risk estimation methods such as t-GARCH to better capture fat-tailed behavior, incorporating DCC (Dynamic Conditional Correlation) or CCC (Constant Conditional Correlation) models for a more accurate covariance structure, and improving the price simulation strategy to reflect more realistic market dynamics. In the next stage of this project, I will explore these enhancements in greater detail.

Given these limitations, it's important to acknowledge that the current methodology underestimates risk significantly. Between AAPL and INTC, INTC appears to be a better choice for a risk-averse investor due to its relatively lower volatility under EWMA, offering more stability. On the other hand, a more risk-tolerant investor might prefer AAPL, which exhibits higher short-term volatility under EWMA but shows less persistence under GARCH, potentially providing greater opportunities for return.

Snapshot of a Possible Fix | Predicting Returns with Machine Learning

A potential improvement involves simulating returns using machine learning techniques, such as random forests. This approach introduces flexibility in capturing nonlinear patterns and dependencies in financial time series data. For instance, while the simulated returns for AAPL tend to underestimate the observed variability, the results for INTC align more closely with the actual return trends. This demonstrates the capacity of random forest models to adjust better to stable assets, though further refinement is needed to account for the more dynamic behavior observed in AAPL's returns.





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