Analysis of Gravitational Interactions and Saturn V Performance for Apollo Missions

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Date: 03/09/2025

I. Introduction

The Apollo mission represents a pivotal endeavour in human space exploration, relying heavily on a fundamental understanding of gravitational interaction and rocket propulsion to complete their missions. This report aims to present a detailed analysis of the gravitational potential and forces within the Earth-Moon system, followed by a projection of the Staurn V Stage 1 rocket's performance. The findings presented will aid to refining mission trajectory planning, providing critical data to secure funding and support from Congress for NASA's Apollo missions.

II. The gravitational potential of the Earth-Moon system

The gravitational potential ϕ at a distance r from a mass M is described by $\phi(r) \; = -\frac{GM}{r}$, where G represents the universal gravitational constant. The Earth and Moon are modelled as point masses, positioned at the origin and at $\left(\frac{distance\ between\ Earth\ and\ Moon}{\sqrt{2}}\right)$, respectively.

A computational analysis was conducted to generate a two separate two-dimensional color-mesh representation of the gravitational potential of the Earth-Moon system. Figure 1 from the code, illustrates the gravitational potential with logarithmic scaling applied to enhance the interpretability. As expected the shown curve follows the inverse relationship between gravitational potential and distance from the mass. Additionally, Figure 2, from the code, shows the gravitational potential field exhibits a central concentration around the Earth, with a gradual decline in intensity as the distance increases.

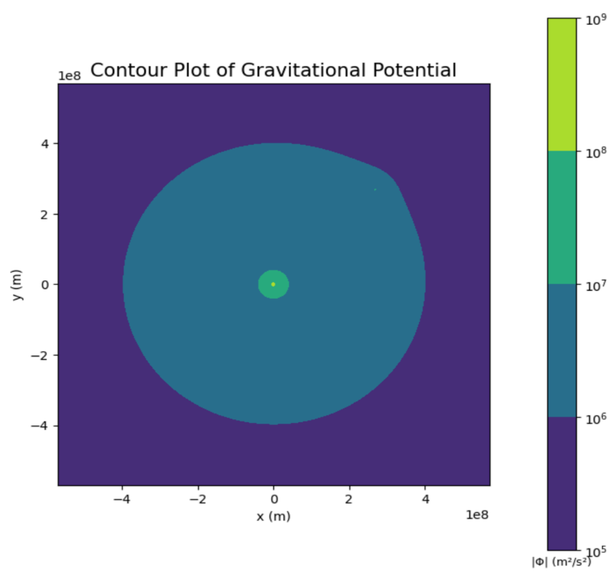

Figure 4. Contour Plot of Gravitational Potential for the Earth-Moon System

An important feature in this system are Lagrange points, where the gravitational and centrifugal forces balance, allowing objects to maintain a stable position, are identified and hold significant implications for mission design and orbital stability analysis. As shown in Figure 4, these areas pinpoint regions where gravitational influences from both the Moon and Earth begin to balance, aligning closely with L_1 and L_2. These points provide potential locations for station-keeping maneuvers and deep-space observational platforms, contributing to long-term mission sustainability which can be used in Apollo missions.

III. The gravitational force of the Earth-Moon system

The gravitational force F exerted by a mass M on a secondary mass m is determined by using:

$$F = \frac{GMm}{r^2}\widehat{r_{21}} ,$$

$F_{21} =- G\frac{M_1 m_2}{|r_{21}|}\widehat{r_{21}}$, where r_{21} denotes the displacement vector from M_1 to m_2. The plot visualizes the gravitational force field of the Earth-Moon system, as the arrows indicate the direction and relative magnitude of the force acting at various points in space.

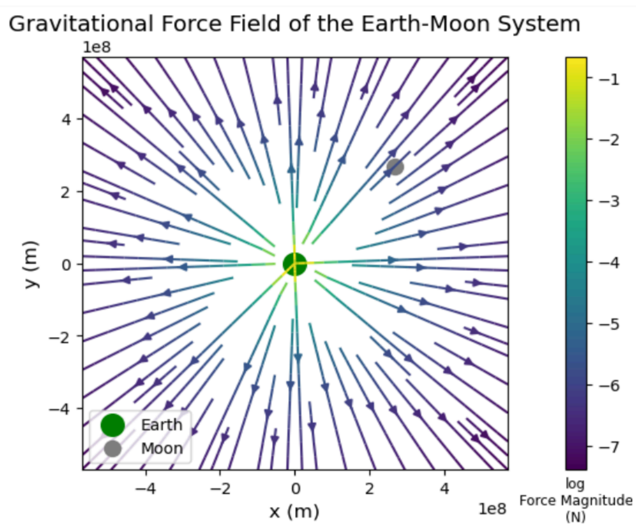

Figure 5: Streamplot of Gravitational Force Field of the Earth-Moon System

The visualization in Figure 5 allows for key observations regarding gravitational influences in the Earth-Moon system such as

- Earth's dominance seen as the force vectors are stronger near Earth, demonstrating its overwhelming gravitational influence in the region.
- Lunar influence is also present as a smaller, secondary region of attraction is observed around the Moon, where its gravitational force become significant.

- There is a shift in dominance between the Earth and the Moon, corresponding to Lagrange points, which is essential for understanding stable orbits and spacecraft planning as mentioned earlier.

These findings are essential for trajectory corrections, mission navigation strategies, and aid in optimizing lunar approach trajectories and ensuring the stability of spacecraft navigating between the Earth and the Moon.

IV. Projected performance of the Saturn V Stage 1

The performance of the Saturn V rocket is vital to the success of Apollo missions. The velocity change Δv of a rocky os governed by the Tsiolkovsky rocket equation,

$\Delta v(t) = v_e ln(\frac{m_0}{m(t)}) - gt$, where m_0 and m_f denote the initial (wet) and final(dry) masses, $\dot{m}$ is the rate of fuel consumption, v_e represents the exhaust velocity, and g is the acceleration due to gravity. The total burn time T is found by $T = \frac{m_0 - m_f}{\dot{m}}$.

These formulas were applied to find the altitude of h at burnout finding a burn duration of approximately 157.69 seconds at an altitude of 74.09 kilometers. Comparing this to recent test results from the first prototype of Saturn V indicates a burn duration of 160 seconds and a final altitude of 70 km, with a slight discrepancy between the two results. The difference can be attributed to unmodeled factors such as atmospheric drag, structural dynamics, and variations in fuel efficiency, which impact real-world performance.

V. Discussion and Future Work

During this study, there were multiple simplifying assumptions as the Earth and the Moon were modeled as point masses and neglected the shapes and gravitational variations of the Earth. Additionally, atmospheric drag effects were neglected when calculating the Saturn V’s performance. Refinements for the future will incorporate real-time propulsion modeling and engine efficiency, modelling to account for atmospheric drag, and an inhanced representation of Earth’s gravitational field incorporating oblateness.

All of these advancements will enhance the quality of mission simulations, supporting the continued success of the Apollo program. The findings presented in this report provide details to increase the feasibility of lunar missions and support for funding NASA’s lunar exploration initiatives.