Analysis of Mining on the Moon and Earth

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I. Introduction

The current mining project involves a 4 km deep mine shaft located at the equator. Objects within this shaft experience the force of gravity, which will act differently depending on the depth of the object within the mine. The gravitational force can be assumed to be constant throughout the shaft, or it may vary, either linearly with depth or according to more complex functions based on the specific characteristics of the Earth's gravity at different altitudes. In addition to gravitational forces, the Coriolis effect, due to the Earth's rotation, will also influence the trajectory of objects falling within the shaft. The time it takes for an object to fall through the mine shaft has been proposed as a method for determining the shaft's depth. Under ideal conditions, the fall time can be predicted by considering the gravitational acceleration as constant, but a number of factors may affect the actual time observed in practice. These factors include air drag, the Coriolis force, and the potential variability of the gravitational force with depth. This report examines these physical effects and their influence on the accuracy of the time-based depth measurement. To analyze the system, Newton's Second Law was used to derive relevant differential equations and solved them numerically using the solve_ivp integrator in Python. This report analyzes the behavior of a test mass in free fall through different models of gravitational force.

II. Fall Time

We began by modeling the simplest case: an object falling under constant gravitational acceleration with no drag. Using the standard kinematic equation $t = \sqrt{2d/g}$, we calculated the fall time for a 4 km shaft to be approximately 28.6 seconds. This was verified through numerical integration of the differential equation $\frac{d^2y}{dt^2} = -g$, which produced consistent results. To account for the variation of gravity with height, we modeled gravity as $g(r) = g_0(r/R_{earth})$. This more accurate model resulted in a slightly increased fall time of approximately 28.9 seconds. The minimal change confirms that variable gravity has only a small effect over the 4 km shaft length. We then included air drag by modeling it as a force proportional to the square of the velocity (). The modified differential equation showed that the fall time increased to about 35.1 seconds to be around 83.5 seconds fall time, demonstrating the significant effect of air resistance during descent.

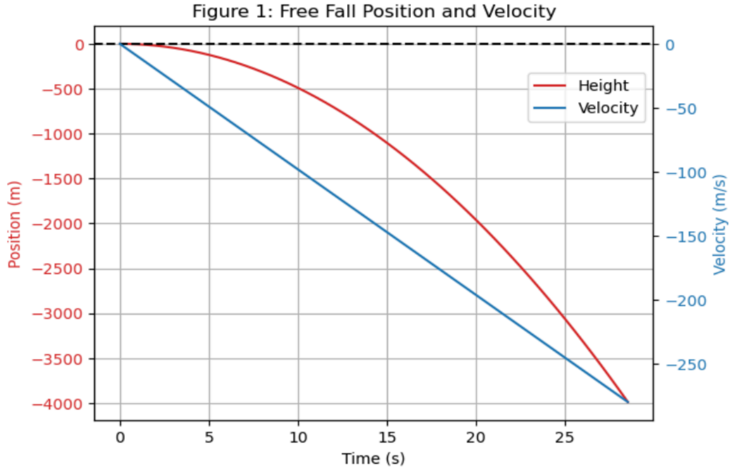


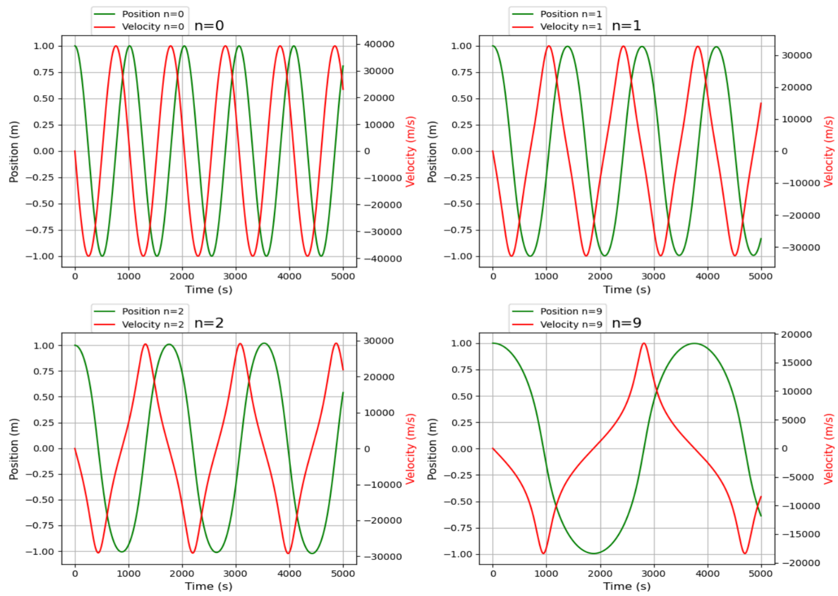
Figure 1: Free Fall Position and Velocity

III. Feasibility of Depth Measurement Approach

The Coriolis force is cause by Earth's rotation which can interfere with the falling mass by deflecting it t the side. In a coordinate system where $\hat{x}$is east, is down the shaft, and $\hat{z}$ is north, the Coriolis force introduces an acceleration $a_c \ - 2(\Omega \times \ v)$. We modeled the resulting equations of motion in two dimensions to account for eastward drift. Without drag, the object drifted eastward. Thus, the test mass would hit the wall before reaching the bottom at around 21.91 seconds. Including drag reduced this so the mass reach the bottom of the mine shaft in 28.5 seconds. This allowed the object to stay within the shaft and safely reach the bottom. The difference between the two fall times demonstrates how drag extends both the fall time and the descent distance.

IV. Crossing Times for Homogenegous and Non-Homogeneous Earth

To further explore the physics of th falling mass in gravitational fields, using simulation motion through a hyoptheical tunnel through the Earth provided great detail. All of the previous models were based on a constant density for Earth, but the density of Earth does vary with distance in real life. In the case of a homogeneous Earth (), gravity varies linearly with radius, producing simple harmonic motion. The time to reach the Earth's center for n = 0 was approximately 1266.6 seconds, and the full traversal time was 5069.37 seconds. In order to simulate a more realistic Earth using a non-uniform density model of the for $p(r) \ = p_n(1 - r^2/R^2)^n$, utilizing an extreme case n=9 would ensure proper measurements as it concentrates more mass near the center. There was a faster traversal time found for n =9, which reflects the increased gravitational acceleration in the inner regions of the Earth caused by a more centralized mass distribution.



This confirms the theoretical relationship between time and p, the average planetary density, as t is proportional to $1/\sqrt{p}$. Since the Moon's density is lower than the Earth's the fall time is correspondingly longer. For the Earth and the Moon, their average density ratio of p_{Earth}/p_{moon}, aligns well with the fall time ratio of t_{earth}/t_{moon}, which confirms the inverse square root relationship.

V. Discussion and Future Work

Our simulations show that while fall time measurements provide a useful theoretical model for estimating depth, real-world factors significantly complicate this approach. The Coriolis force alone can prevent a test mass from reaching the bottom unless drag is accounted for. Even then, accurate calibration of drag is necessary. For these reasons, using fall time alone is not a reliable method for depth measurement in deep equatorial shafts.

To enhance the realism of this study, future work could include modeling the Earth's actual density profile based on seismic data, incorporating variable atmospheric pressure and turbulent drag, and accounting for shaft imperfections or tilt. Additionally, the effect of performing the same experiment at different latitudes could be explored to assess the role of rotational forces.