Private Debt Control in Peru for the Period of 1991-2014

Iván Pérez Avellaneda.
University of Vermont

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Sección. 1

Problem Statement

Motivation

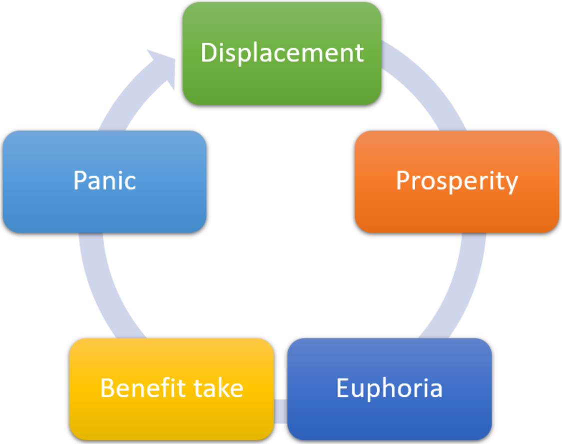
Reasons to control private debt in Peru:

- It represents a small open economy.
- It is vulnerable to changes in the price of commodities, specifically the price of copper.
- The appreciation of a sector can generate a stage of debt issuing euphoria. (Minsky, 1982)

- Peru is the second worldwide copper productor after Chile.*
- The total share of the exports of the mining and hydrocarbon sectors were 70% and the share of the exports of copper were 30%.**
- A big deposit of rock of Lithium was discovered in 2018 in Peru.***

* U.S Geological Survey, Mineral Commodity Summaries 2018
** BCRP Annual Report 2017, external sector.
*** Reuters.

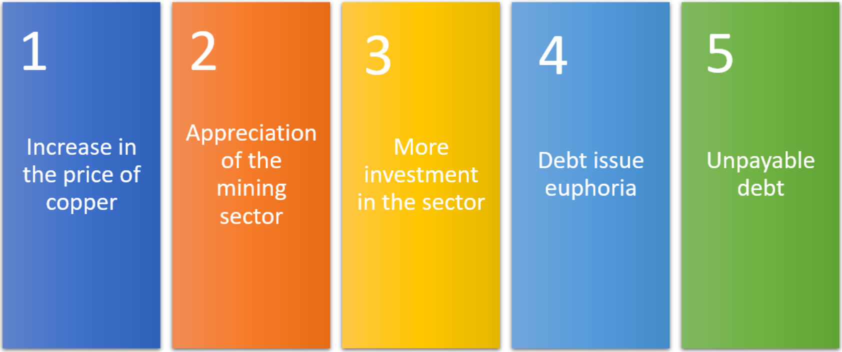
Minsky cicle of debt:



Minsky cicle of debt:

- Displacement: external event that increases income. Example: technology development. In particular: dotcom burst, appreciation of the house market (2008 financial crisis).
- Prosperity: the increasing of income generates more debt capacity.
- Euphoria: banks issue debt with little restrictions.
- Benefit take: markets insiders take advantage of the information and generate profit by selling their goods.
- Panic: the value of the goods decreases, but debt remains.

Transmission mechanism due to the mining sector:



Context


EKONOMIPRISET 2022
THE PRIZE IN ECONOMIC SCIENCES 2022
KUNGL. VETENSKAPS-AKADEMIEN
THE ROYAL SWEDISH ACADEMY OF SCIENCES
Ben S. Bernanke
The Brookings Institution, USA
Douglas W. Diamond
University of Chicago, USA
Philip H. Dybvig
Washington University in St. Louis, USA
"för forskning om banker och finanskriser"
"for research on banks and financial crises"
#nobelprize
THE NOBEL PRIZE

Sección. 2

Theory

Subsección. 1

Goodwin Model

- Non-linear.
- Endogenous cicles.
- Represents the interaction between households and firms.

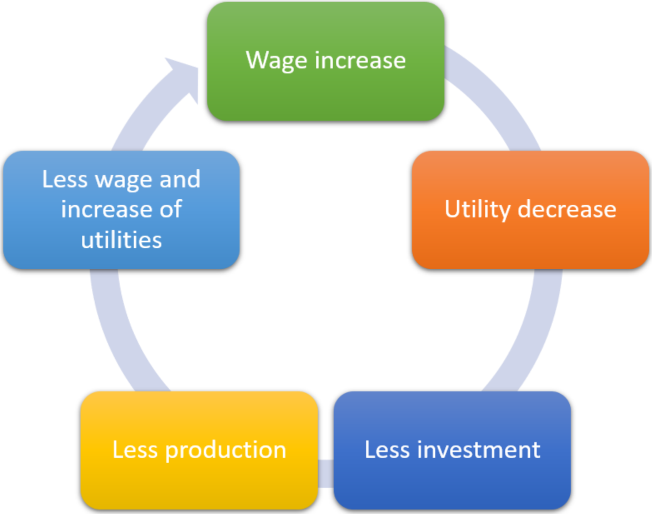


Figura.: Dynamic of the Goodwin model.

Goodwin Model

$$\begin{aligned} \dot{\omega}(t) &= \omega(t)\left[\Phi(\lambda(t)) - \alpha\right] \\ \dot{\lambda}(t) &= \lambda(t)\left[\frac{1-\omega(t)}{\nu} - \alpha - \beta - \delta\right] \end{aligned} \quad (1)$$

1. Variables:
   - $\omega(t)$: labor share (total wage income over GDP)
   - $\lambda(t)$: employment rate
2. Parameters:
   - α: productivity growth rate.
   - β: labor force growth rate.
   - δ: capital depreciation rate.
   - ν: capital-product constant rate.
3. Function: Φ: Phillipds curve.

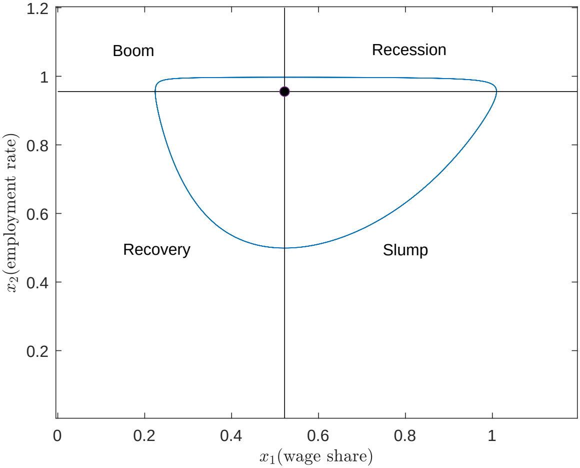


Figura.: Stages of the Goodwin cicle.

Subsección. 2

Keen Model

- Bigger non-linearity.
- It is based on the Goodwin model.
- Adds the financial sector.
- Also based on Minsky's Instability Hypothesis. (Minsky, 1982)

1. Variables
    - $\omega(t)$: laboral share
    - $\lambda(t)$: employment rate
    - $d(t)$: real debt of firms
2. Parámetros
    - α: productivity growth rate.
    - β: labor force growth rate.
    - δ: capital depreciation rate.
    - ν: capital-product constant rate.
    - r: interest rate of reference.
3. Funciones:
    - Φ: Phillips curve
    - κ: new investment rate
    - π: firms utility

Keen Model

$$\dot{\omega}(t) = \omega(t)\left[\Phi(\lambda) - \alpha\right]$$

$$\dot{\lambda}(t) = \lambda(t)\left[\frac{(\kappa \circ \pi)(t)}{\nu} - \alpha - \beta - \delta\right]$$

$$\dot{d}(t) = d(t)\left[r - \frac{(\kappa \circ \pi)(t)}{\nu} + \delta\right] + \left[(\kappa \circ \pi)(t) - (1 - \omega(t))\right]$$

where the new investment rate $\kappa(x)$ satisfies regularity conditions. The utility of the firms is defined as $\pi(t) := 1 - \omega(t) - rd(t)$.

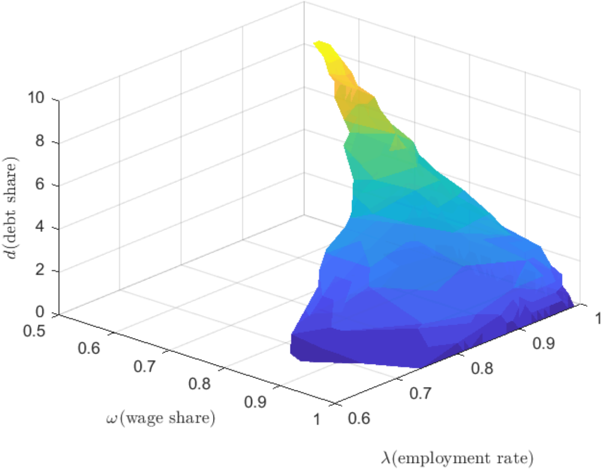


Figura.: Stability region.

Sección. 3

Empirical relevance

Subsección. 1

Stilyzed facts

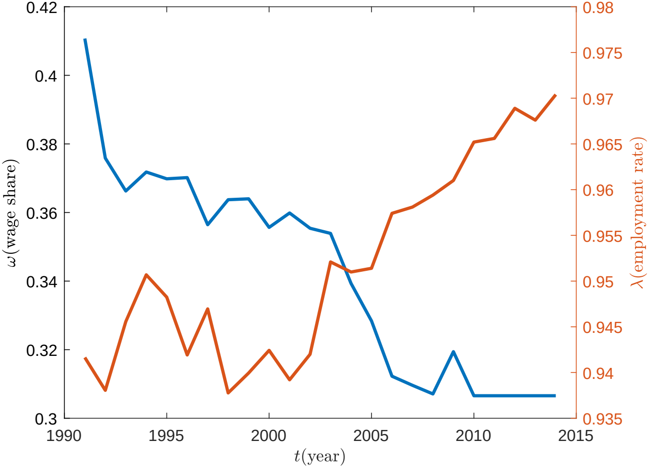


Figura.: Wage share and employment rate.

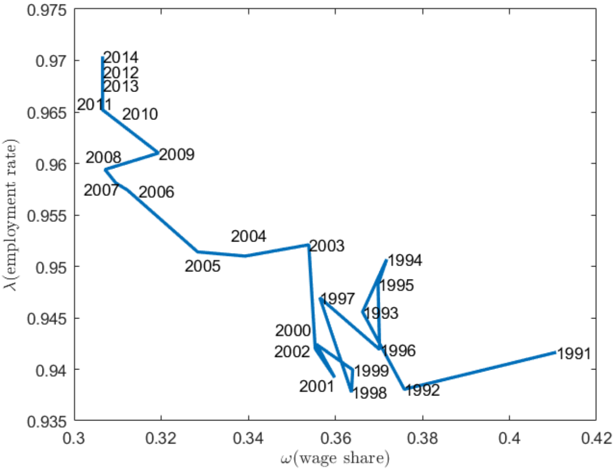


Figura.: Wage share and employment rate.

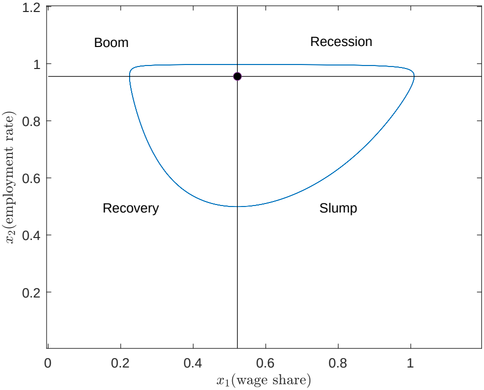


Figura.: Stages of the Goodwin model cicle.

Are we recovering from the Lost Decade? the price of copper decreased, El Niño phenomenon.

Subsección. 2

Estimation for the Peruvian case

Goodwin model calibration

- The Goodwin model is linear in the parameters.
- The model assumes constant growrth of labor productivity and total labor force.

$$\frac{\dot{a}(t)}{a(t)} = \alpha$$
$$\frac{\dot{N}(t)}{N(t)} = \beta.$$

then

$$\ln(a_t) = \ln a_0 + \alpha t + \varepsilon, \;\; \varepsilon \sim N(0, \sigma_1)$$
$$\ln(N_t) = \ln N_0 + \beta t + \varepsilon, \;\; \varepsilon \sim N(0, \sigma_2)$$

Goodwin model calibration

- Two types of Phillips curve are assumed for the estimation

$$\Phi = \phi_1 \lambda + \phi_0, \tag{2}$$
$$\Phi = \phi_1/(1-\lambda)^2 + \phi_0 \tag{3}$$

- Two ways to estimate the Phillips curve:

$$\frac{\omega[k+h] - \omega[k]}{\omega[k]} + \alpha = \Phi[k], \tag{4}$$
$$\nu \frac{\lambda[k+h]^2 - \lambda[k]\lambda[k+2h]}{\lambda[k+h]\lambda[k]} = \Phi[k] \tag{5}$$

Goodwin model calibration

Parameter	Value	Methodology
$\hat{\alpha}$	0.0129	Least square
$\hat{\beta}$	0.0337	Least square
$\overline{\nu}$	4.7055	Mean
$\overline{\delta}$	0.055	Mean

Table: Estimated parameters.

Goodwin model calibration

Varible	Description	Source
NGDP	GDP (current US$)	WDI
RGDP	GDP (constant 2010 US$)	WDI
CAPITAL	Capital Stock at Constant National Prices for Peru	FRED
EMPR	Employment to population ratio, 15+, total (%) (modeled ILO estimate)	WDI
LABFR	Labor force share rate, total (% of total population ages 15+) (modeled ILO estimate)	WDI
POP	Population, total	WDI
POP14	Population ages 0-14 (% of total)	WDI

Table: Variables usadas para la calibración

Goodwin model calibration

LABSH	Share of Labour Compensation in GDP at Current National Prices for Peru, Ratio, Annual, Not Seasonally Adjusted	FRED
DELTA	Constant of capital depreciation	PWT90
INV	Gross fixed capital formation, private sector (% of GDP)	WDI
RIR	Real interest rate	WDI
DEBT	Domestic credit to private sector by banks (% of GDP)	WDI
REV	Revenue, excluding grants (% of GDP)	WDI
TAXREV	Taxes on income, profits and capital gains (% of revenue)	WDI
TAXPROF	Profit tax (% of commercial profits)	WDI

Table: Used variables for the calibration

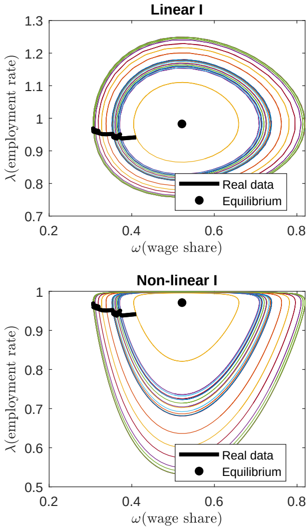


Figura.: Goodwin model for Peru.

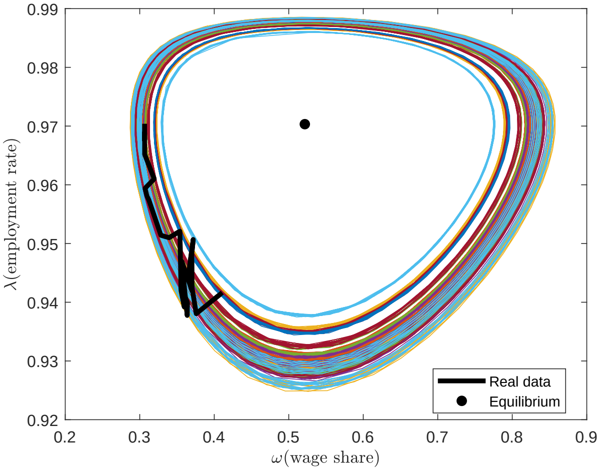


Figura.: Adjusted Goodwin model.

Approached $\Phi(.)$	$\hat{\phi}_1$	$\hat{\phi}_0$	$\overline{\lambda}$	Cycle length (years)
Linear I	0.4313	0.4238	0.9827	28.9881
Linear II	2.5588	2.4948	0.9716	11.9175
Non-linear I	$1.9286 \cdot 10^{-5}$	0.0231	0.09711	15.1248
Non-linear II	$9.7709 \cdot 10^{-5}$	0.1084	0.9801	11.9175
Non-linear II adjusted	$4.8855 \cdot 10^{-5}$	0.5548	0.9703	3.1333

Table: Summary of the estimation of parameters, cicle length.

Keen model calibration

Investment share:

$$\kappa(x) = \kappa_0 + \kappa_1 \arctan(\kappa_2 x + \kappa_3).$$

Regularity criterion of the stable equilibrium on the investment share function:

$$\kappa(-\infty) = 0, \tag{6}$$

$$\kappa(+\infty) = 1, \tag{7}$$

$$\kappa'(\overline{\pi}) = 1.5, \tag{8}$$

$$\kappa(\overline{\pi}) = \nu(\alpha + \beta + \delta), \tag{9}$$

$$\overline{\pi} = 0.46 \tag{10}$$

where $\kappa(.)$ y π represent the investment share and the utility respectively.

Keen model calibration

Parameter	value
$\hat{\kappa_0}$	0.5
$\hat{\kappa_1}$	0.31831
$\hat{\kappa_2}$	4.7341
$\hat{\kappa_3}$	-2.24556
$\bar{r}$	0.24592

Table: Estimated parameters.

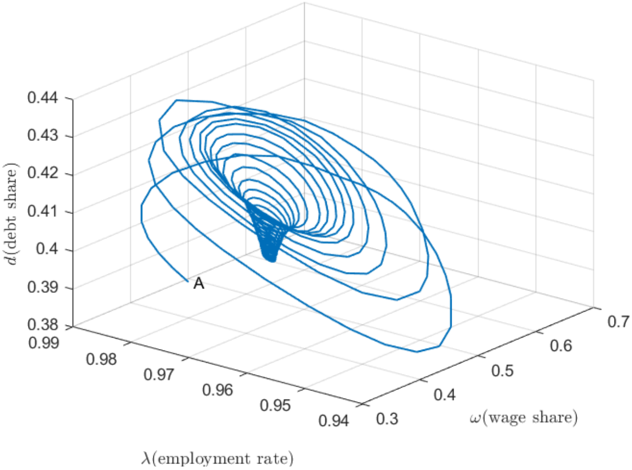


Figura.: Trajectory of the Keen model for Peru where A=2014.

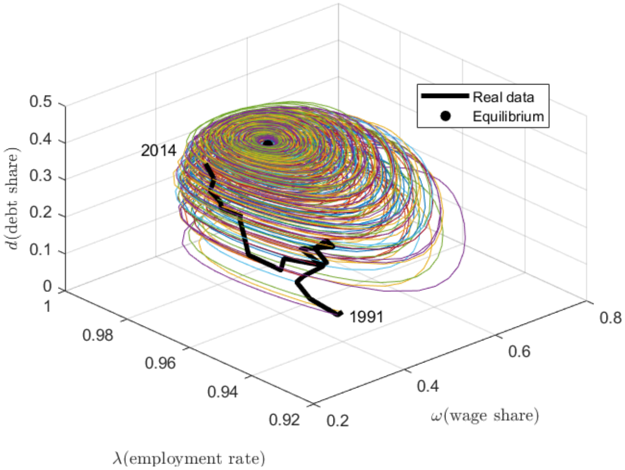


Figura.: Set of trajectories of the Keen model for Peru.

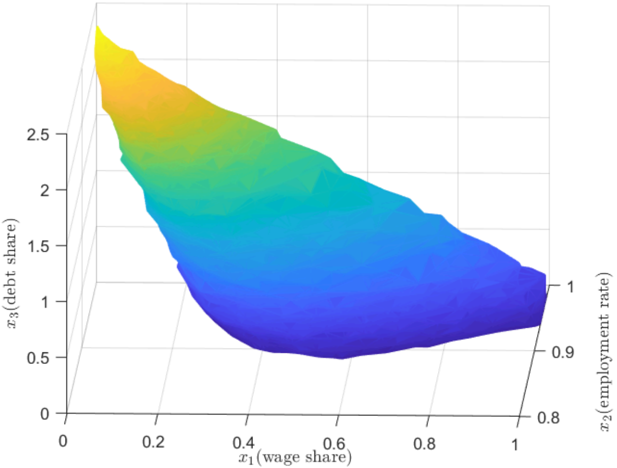


Figura.: Stability region of the Keen model for Peru.

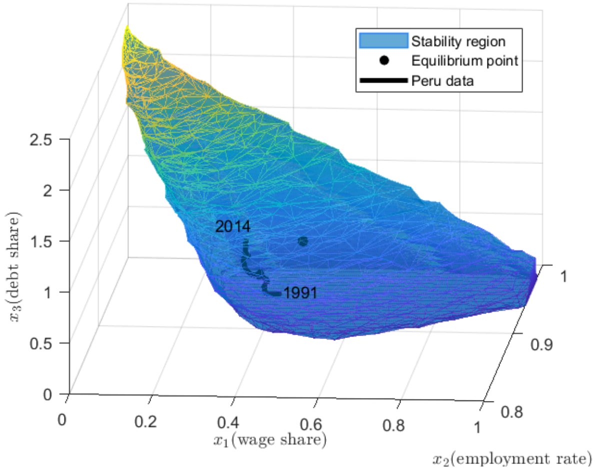


Figura.: Stability region of the Keen model for Peru and real trajectory.

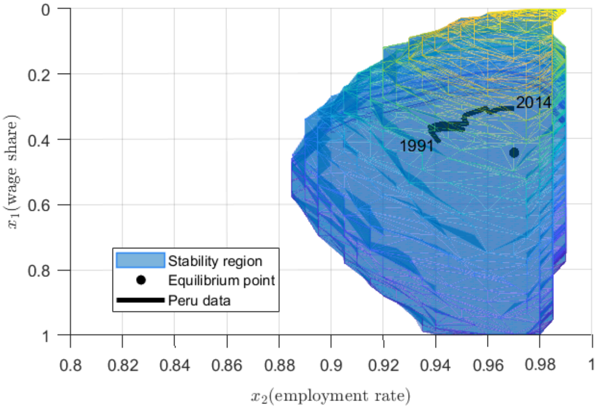


Figura.: Stability region of the Keen model for Peru and real trajectory.

Sección. 4

Revisited problem

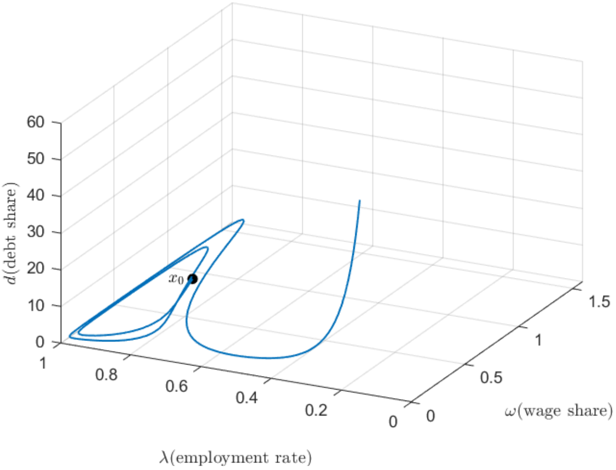


Figura.: The country moved to the instability region.

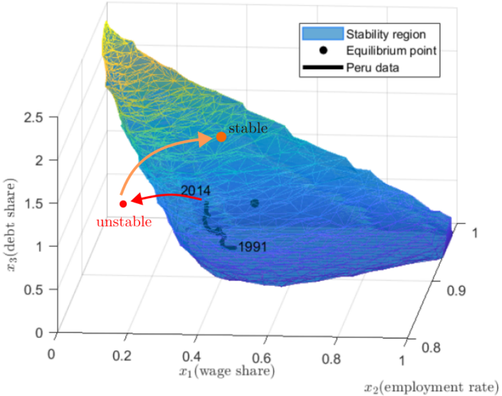


Figura.: Diagram of the stability problem.

Sección. 5

Technical aspect

Subsección. 1

Trajectory Planning

Gradient based planning

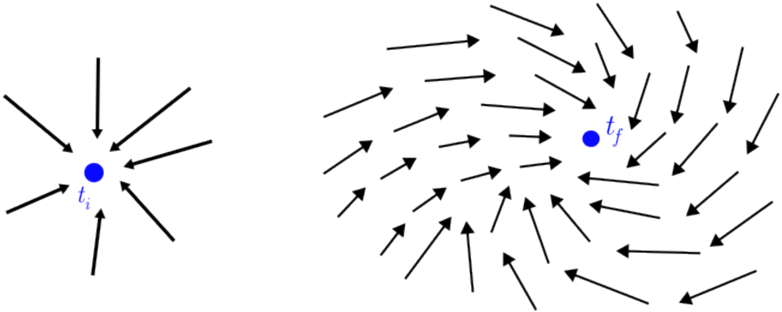


Figura.: On the left, the initial point is generated by an attractor field and on the right the vector field of the model.

Gradient based planning

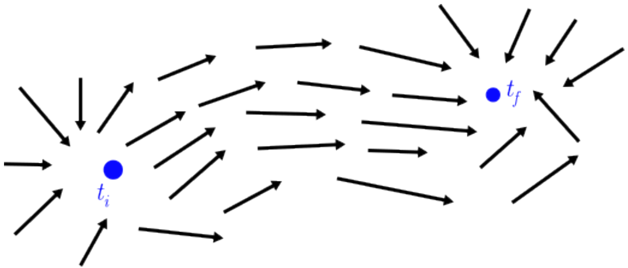


Figura.: Transposed vector fields.

Gradient based planning

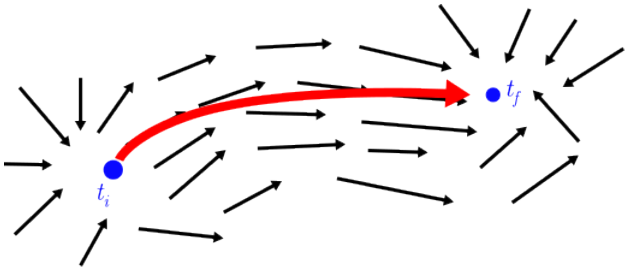


Figura.: Generated trajectory.

Algorithm 1 Gradient Based Planner

Input: $x_0, x_f, F_K, F_x, T, \delta$

Output: Path between x_0, x_f

Initialization : u_0

$F_s \leftarrow F_K + F_x$

while $i \neq T \vee ||x_f - \text{position}|| < \delta$ **do**

$\quad \text{position} \leftarrow x_0$

$\quad \text{Delta} \leftarrow \text{interpolate}(F_s, \text{position})$

$\quad \text{position} \leftarrow \text{position} + \text{Delta}/(T||\text{Delta}||)$

$\quad \text{route} \leftarrow \text{stack(route,position)}$

$\quad i \leftarrow i + 1$

end while

return *route*

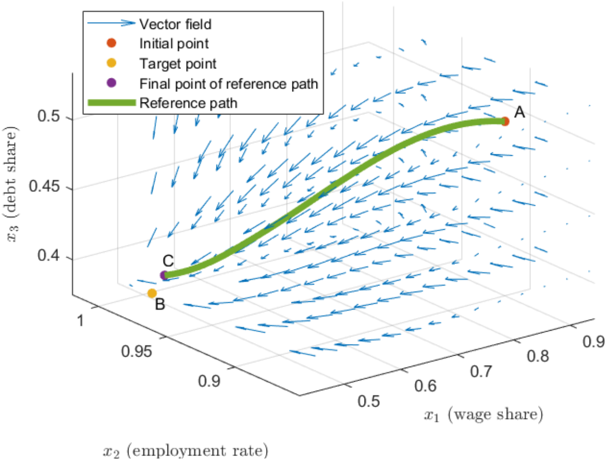


Figura.: Reference trajectory.

Subsección. 2

Linearization

Definition 5.1 (Lie Derivative).

Let f and h smooth functions. Denote

$$L_f h(x) = \frac{\partial h}{\partial x} \cdot f(x)$$

the *Lie derivative* of h with respect to f, and

$$L_f^k h(x) = L_f(L_f^{k-1} h(x)) = \frac{\partial L_f^{k-1} h(x)}{\partial x} \cdot f(x)$$

is the k-th Lie derivative of h with respect to f.

Feedback linearization

Definition 5.2 (Relative degrgee).

Consider the system

$$\dot{x} = f(x) + g(x)\,u, \qquad x(0) = x_0, \tag{11a}$$
$$y = h(x) \tag{11b}$$

with $x \in \mathbb{R}^n$ and f, g and h sufficiently smooth in $R \subset \mathbb{R}^n$. (11) have *relative degree* $0 \leq p \leq n$ in the region $R_{x_0} \in \mathbb{R}^n$ if for all $x_0 \in R_{x_0}$

$$L_g L_f^{i-1} h(x) = 0 \qquad \text{for} \quad i = 1, 2, \ldots, p-1,$$
$$L_g L_f^{p-1} h(x) \neq 0,$$

Feedback linearization

The relative degree p of a system is equal to the number of times the output is derivated until the input appears for the first time. The functions $h(x), L_f h(x), \ldots, L_f^{p-1} h(x)$ define a change of local coordinates of the system around x_0. To see this, define the new change of coordinates $z = T(x)$ where $T : \mathbb{R}^n \rightarrow \mathbb{R}^p$

$$T(x) = \begin{bmatrix} h(x) \\ L_f h(x) \\ \vdots \\ L_f^{p-1} h(x) \end{bmatrix} \tag{12}$$

Feedback linearization

After the change of coordinates, the system is the following:

$$\dot{h} = z_1 \tag{13}$$

$$\dot{z}_1 = z_2 \tag{14}$$

$$\vdots \tag{15}$$

$$\dot{z}_{p-2} = z_{p-1} \tag{16}$$

$$\dot{z}_{p-1} = L_f^p h(x_0) + L_g L_f^{p-1} h(x_0) u(t_0) \tag{17}$$

definamos $D := L_g L_f^{p-1} h(x_0)$ y hagamos

$$u = D^{-1}(v - L_f^p h(x_0)) \tag{18}$$

Feedback linearization

The following system is obtained:

$$\begin{bmatrix} \dot{h} \\ \dot{z}_1 \\ \vdots \\ \dot{z}_{p-2} \\ \dot{z}_{p-1} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \begin{bmatrix} h \\ z_1 \\ \vdots \\ z_{p-2} \\ z_{p-1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} v \tag{19}$$

Compared with the original system:

$$\dot{x} = f(x) + g(x)\,u, \qquad x(0) = x_0, \tag{20a}$$

$$y = h(x) \tag{20b}$$

Feedback linearization

For a system with ℓ outputs and m inputs, the relative degree can be computed for each output, say, p_i for $i = 1, \ldots, \ell$, just by computing the derivatives until the input appears. This defines a vector of relative degrees $(p_1, \ldots, p_\ell)$, that is well defined if

$$D(x) = \begin{bmatrix} L_{g_1} L_f^{p_1 - 1} h_1(x) & \cdots & L_{g_m} L_f^{p_1 - 1} h_1(x) \\ \vdots & \ddots & \vdots \\ L_{g_1} L_f^{p_\ell - 1} h_\ell(x) & \cdots & L_{g_m} L_f^{p_\ell - 1} h_\ell(x) \end{bmatrix} \tag{21}$$

is invertible.

Model 1

$$\dot{x}_1 = x_1[\Phi(x) - u_1]$$

$$\dot{x}_2 = x_2\left[\frac{\kappa(x)}{\nu} - u_1 - u_2 - \delta\right]$$

$$\dot{x}_3 = x_3[r - \frac{\kappa(x_1, x_3, x_4)}{\nu} + \delta] + \kappa(x) - (1 - x_1)$$

$$\dot{x}_4 = u_3$$

$$y = [x_1 \; x_2 \; x_3]^T = [\omega \; \lambda \; d]^T$$

with $u_1 = \alpha$ (productivity growth rate), $u_2 = \beta$ (labor force rate of growth) y $u_3 = \dot{r}$ (interest rate rate of growth)

$$\kappa(x_1, x_3, x_4) = \kappa_0 + \kappa_1 \arctan(\kappa_2(1 - x_1 - x_4 x_3)),$$

$$x(0) = (0.9, 0.9, 0.5), u(0) = (\alpha, \beta, 0)$$

Model 2

$$
\begin{aligned}
\dot{x}_1 &= x_1[\Phi(x) - \alpha_0 - x_5] \\
\dot{x}_2 &= x_2\left[\frac{\kappa(x)}{\nu} - \alpha_0 - x_5 - \beta_0 - x_6\right] \\
\dot{x}_3 &= x_3[x_4 - \frac{\kappa(x)}{\nu} + \delta] + \kappa(x) - (1 - x_1) \\
\dot{x}_4 &= u_3 \\
\dot{x}_5 &= -\tau_1(x_5 - u_1) \\
\dot{x}_6 &= -\tau_2(x_6 - u_1) \\
y &= [x_1 \ x_2 \ x_3]^T
\end{aligned}
$$

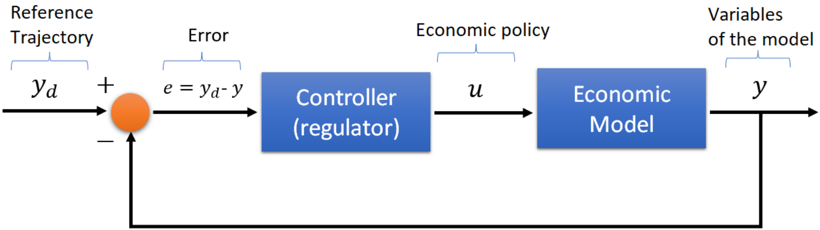
with

$\kappa(x_1, x_3, x_4) = \kappa_0 + \kappa_1 \arctan(\kappa_2(1 - x_1 - x_4x_3)), x(0) = (0.9, 0.9, 0.5), u$

Subsección. 3

Control

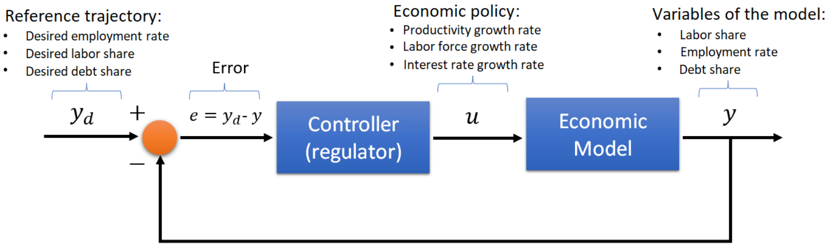
General control diagram



Type of controllers used:

- Pole placement
- Optimal control

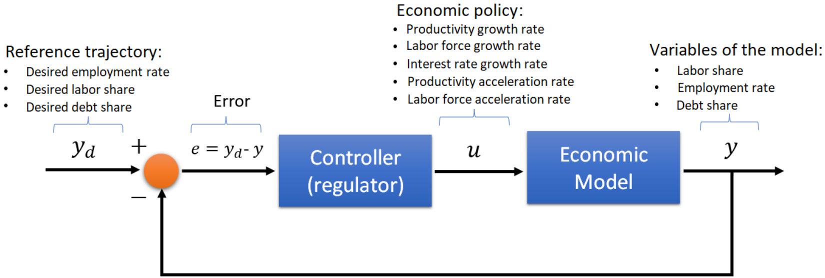
Model 1



Types of controllers used:

- Pole placement
- Optimal control

Model 2



Types of controllers used:

- Pole placement
- Optimal control

Pole Placement

Consider the following linear system

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

assign $u = y - ke$, where the error is $e = x - y_r$ and the reference trajectory is y_r.

Optimal control

Consider the discrete system

$$z[k+1] = M[k]z[k] + Nv[k] \tag{22a}$$
$$y[k] = Wz[k] \tag{22b}$$

the error to minimize is

$$e[k] = y[k+1] - y_r[k+\tau_S] \tag{23}$$

define the weight matrix Q for the error and the objective function is

$$J(v[k]) = e[k]^T Qe[k] \tag{24}$$

the objective function can be expressed in terms of the original input $u[k]$ in the following manner:

$$J(u[k]) = e_u[k]^T Q e_u[k],$$
$$e_u[k] = W(M[k]z[k] + N(D_k(z[k])u[k] + \alpha(z[k]))) - y_r[k + \tau_S]$$

bounds of 1 y -1 are imposed for $u(1)$, $u(2)$ y $u(3)$. This is,

$$L : \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} u[k] \leq \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{25}$$

In the models, $x(4)$ is the growth of the real interest rate. In order to make sure of its positivity, the following restrictions are imposed:

$$(r_{min} - x(4))/\tau_S \leq u(3) \leq (r_{max} - x(4))/\tau_S \tag{26}$$

in matrix terms, this is,

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \end{bmatrix} u[k] \leq \begin{bmatrix} (r_{max} - x(4))/\tau_S \\ (x(4) - r_{min})/\tau_S \end{bmatrix} \tag{27}$$

the optimal input is then defined as

$$u^*[k] := \arg\min J(u[k]) \tag{28}$$

$$\text{s.t} \quad L \tag{29}$$

Pole placement without saturation in model 1 (5 years)

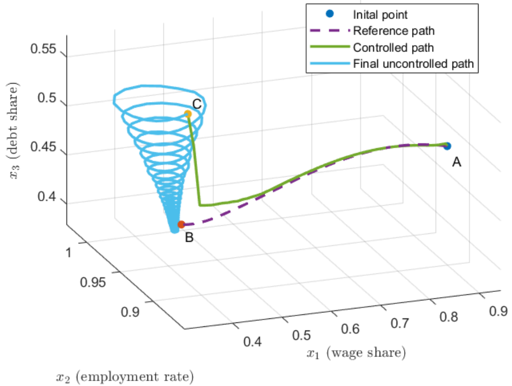


Figura.: Diagram of the control towards the stability region.

Pole placement without saturation in model 1 (5 years)

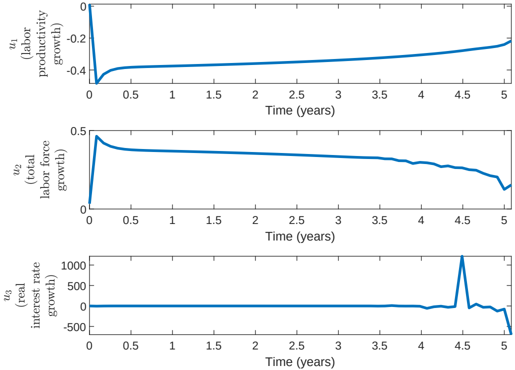


Figura.: Economic policies.

Pole placement without saturation in model 1 (5 years)

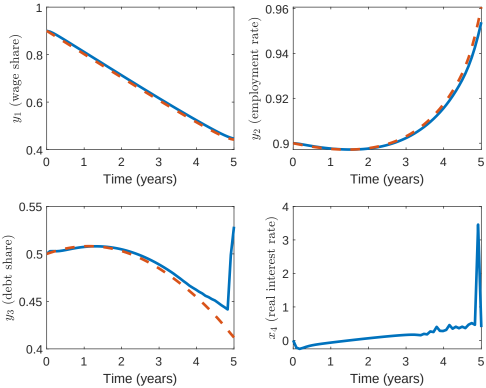


Figura.: Outputs.

Pole placement without saturation in model 1 (9 years)

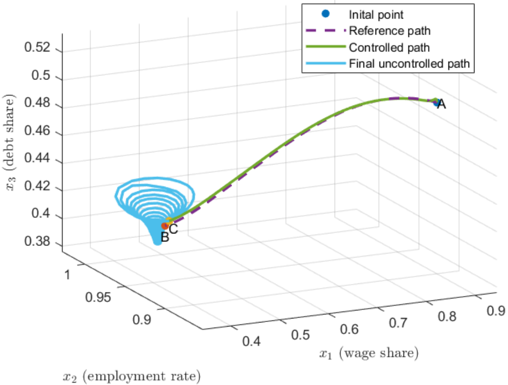


Figura.: Diagram of the control towards the stability region.

Pole placement without saturation in model 1 (9 years)

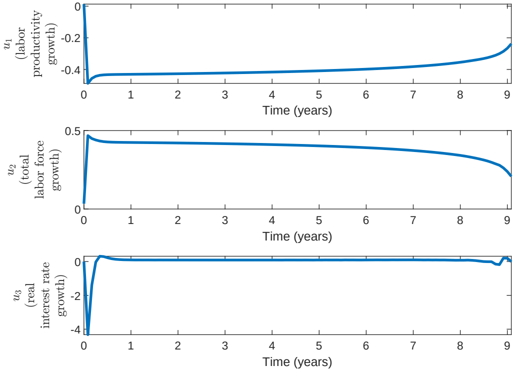


Figura.: Economic policies.

Pole placement without saturation in model 1 (9 years)

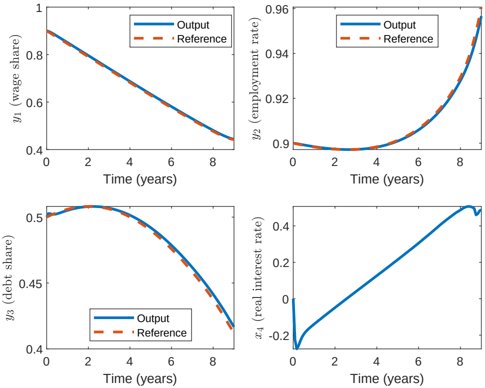


Figura.: Outputs.

Pole placement with saturation of 0.09 in model 1 (3 years)

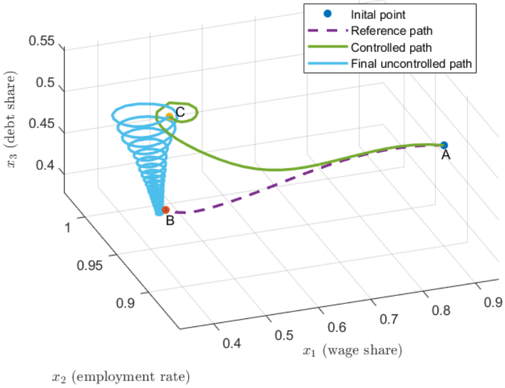


Figura.: Diagram of the control towards the stability region.

Pole placement with saturation of 0.09 in model 1 (3 years)

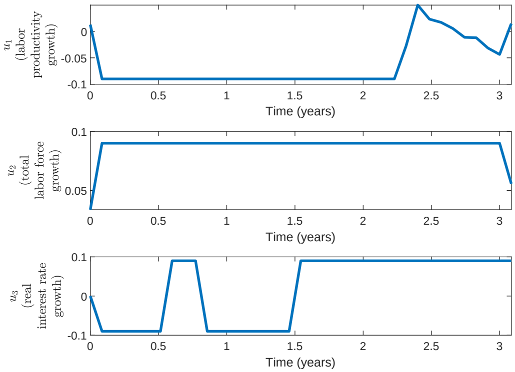


Figura.: Economic policies.

Optimal control with saturation of 1 in model 2

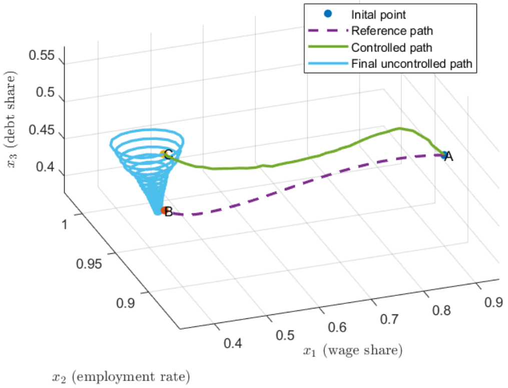


Figura.: Diagram of the control towards the stability region.

Optimal control with saturation of 1 in model 2

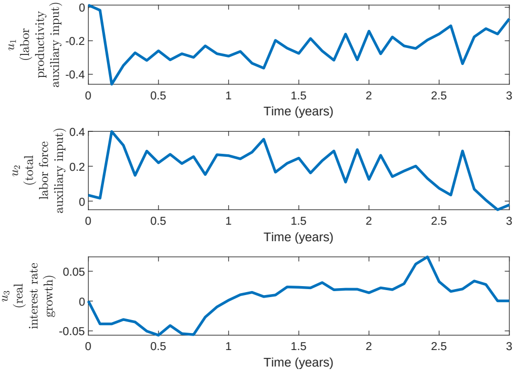


Figura.: Economic policies.

Optimal control with saturation of 1 in model 2

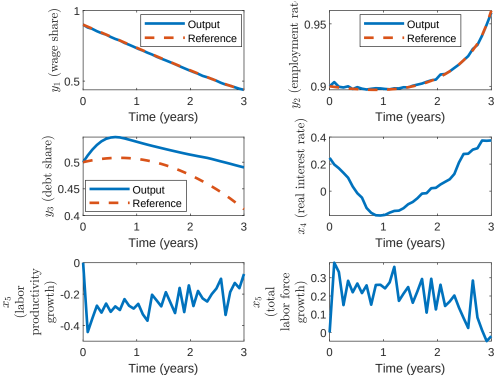


Figura.: Output.

Optimal control with restriction of 1 in the input and 0.01 in the growth rate of the real interest rate in model 2

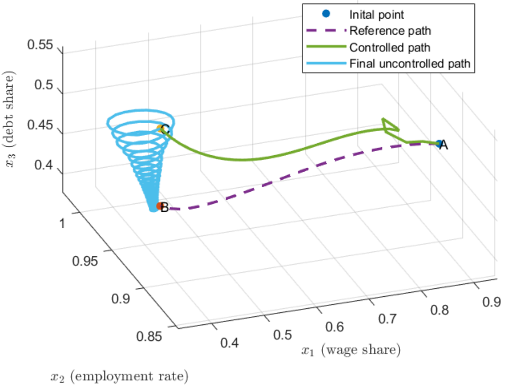


Figura.: Diagram of the control towards the stability region.

Optimal control with restriction of 1 in the input and 0.01 in the rate of growth of the real interest rate in model 2

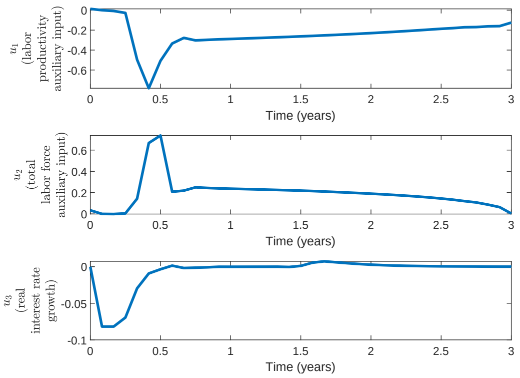


Figura.: Economic policies.

Optimal control with restriction of 1 in the input and 0.01 in the rate of growth of the real interest rate in model 2

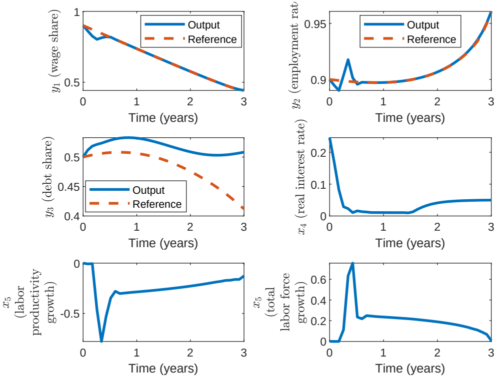


Figura.: Outputs.

Conclusions:

- Peru is inside the stability region for the studied period.
- The longer the control time, the better the performance of control policies.
- Control without restrictions does not guarantee realistic policies.
- It is possible to stabilize the economy with feedback control with saturation and with optimal control. The last one has less variation range.
- The optimal control with restriction and with delay in the controllers shows the smoothest economic policies.

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Thank you!
https://iperezav.github.io