

## The role of tilting rate and wear of surfaces on basic friction angle testing

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**ABSTRACT:** Basic friction angle is an essential parameter when estimating shear strength of rock joints, according to Barton's equation and other approaches. Nevertheless, there is still no standard procedure to estimate it by means of laboratory testing. This study investigates the influence of tilting rate on basic friction angle results, as well as the impact of previous wear on the surfaces when carrying out various tilt tests on the same surfaces. An experimental program including 5 tilting rates, 5 joints for every rate and 5 tests for every joint was carried out in rock samples of moderately weathered granite. Interpretation of results suggest that whereas tilt rate does not significantly affect results of basic friction angle, surface wear and the number of test performed on every sample may play a meaningful role. These observations are finally discussed with the aim of suggesting a guideline for testing procedures focusing on rock engineering purposes.

### 1 INTRODUCTION

#### 1.1 Background

Rock joints and other discontinuities control the behaviour of rock masses being, therefore, essential to understand how they behave. One of the most relevant parameters to assess the stability of natural or engineered rock slopes and underground excavations is the peak shear strength of discontinuities, which has commonly been estimated during last years by Barton and co-workers' expression, as shown by Equation 1 (Barton & Choubey 1977).

$$\tau = \sigma_n \times \tan \left[ \phi_r + \text{JRC} \times \log_{10} \left( \frac{\text{JCS}}{\sigma_n} \right) \right] \quad (1)$$

This equation takes into account the roughness of the discontinuity (JRC), the compressive strength of the rock (JCS) and the normal stress  $\sigma_n$  to which the discontinuity is subjected, to estimate the peak shear strength  $\tau$  of the rock joint.

The residual friction angle  $\phi_r$ , that considers the effect of joint weathering, can be estimated with equation (2):

$$\phi_r = (\phi_b - 20^\circ) + 20 \frac{r}{R} \quad (2)$$

where  $r$  and  $R$  correspond, respectively, to the number of Schmidt hammer rebounds for a weathered discontinuity ( $r$ ) – as commonly found *in situ* – and for a non-weathered one ( $R$ ).

Parameter  $\phi_b$  is the basic friction angle of a dry/wet saw-cut rock surface, determined respectively for dry or saturated discontinuities in laboratory by means of tilt testing, pull testing or shear testing on planar discontinuities.

#### 1.2 Laboratory testing procedure for estimating the basic friction angle

Over the last years, many authors (Stimpson 1981, Cruden & Hu 1988, Alejano et al. 2012, Pérez-Rey et al. 2015) delved into the study of the basic friction angle nature and the laboratory procedures for its estimation. Nevertheless, there is still no standard procedure to obtain this parameter by means of laboratory testing.

Tilt tests carried out for this study were developed by following guidelines suggested by Alejano et al. (2012) and Hencher (2012), which seem an appropriate starting point to obtain the basic friction angle from laboratory testing of dry, saw-cut rock surfaces.

The resorted guidelines comprise:

- Using rock slabs with at least 50 cm<sup>2</sup> surfaces and a length to height ratio of at least 2, in order to assure that stresses are compressive over the whole contact surface when sliding occurs.
- Wiping the surface of sliding with a dry cloth between each repetition to avoid flour effect.
- Placing specimens in horizontal initial positions.
- Repeating tilt tests at least three times; a large number of repetitions is not encouraged to avoid wear and polishing effects, and an odd number of repetitions is recommended.

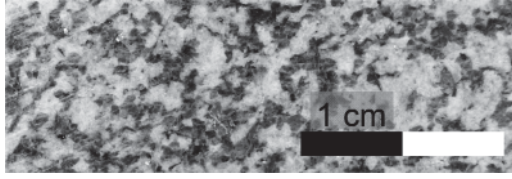


Figure 1. Detailed view of the *Amarelo País* granite texture.

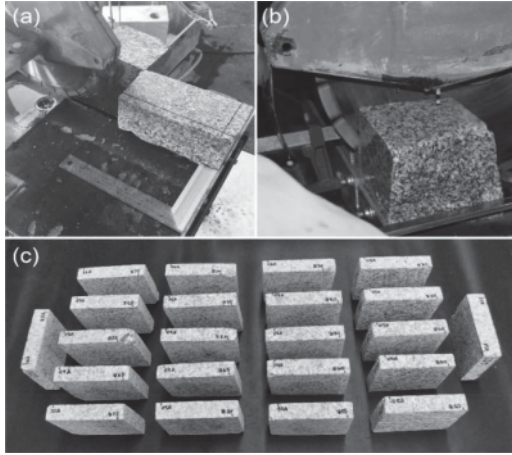


Figure 2. (a) Original block as received from the quarry; (b) cutting process and (c) rock samples prepared to be tested.

Since one of the main topics of this study is to evaluate the possible effect of the tilting rate on basic friction angle results, five tests were considered for each series, in order to provide enough data for subsequent and more advanced and reliable statistical analyses.

## 2 LABORATORY WORK

To develop this particular study, a moderately weathered, medium grain size granite (*Amarelo País*) was selected. It is a quite common rock in our surrounding region and it has been already used in our lab for other studies on the same topic (González et al. 2014; Pérez-Rey et al. 2015). A photograph of its texture can be seen in Figure 1.

This rock presents a density of  $2.61 \text{ g/cm}^3$ , a UCS mean value of 75 MPa and a tensile strength of 6.65 MPa.

Rock samples used in this study were cut from two  $300 \times 120 \times 120 \text{ mm}^3$  prismatic rock blocks, which were divided into 22 slab specimens by using a circular diamond saw-blade, as can be seen in the process shown in Figure 2.

The saw-blade used for the whole cutting process is 350 mm in diameter and 3.4 mm thick (Fig. 3). It has 23 teeth made up of a bronze alloy matrix with diamond concretion ( $\approx 0.6 \text{ ct/cm}^3$ ) and the teeth grit is approximately 50–60 *US Mesh*.

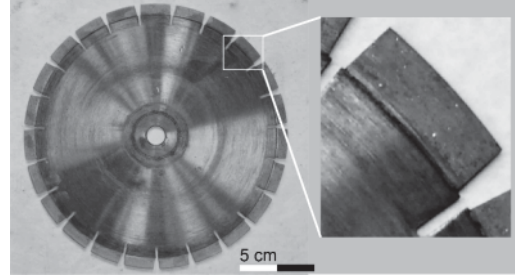


Figure 3. Saw blade used to obtain specimens for tilt-testing showing a detailed view of a tooth.

Final dimensions of rock samples were adjusted to fulfil those suggestions made by Alejano et al. (2012), resulting in 22 rectangular prism-shaped slabs of  $115 \times 100 \times 25 \text{ mm}$ . These samples were tested on both sides in order to take maximum advantage of the available rock surfaces.

The laboratory program comprised five testing series – including twenty tilt tests per series – each one carried out at a different tilting rate with the motorized tilting table already described in a previous paper (Alejano et al. 2012).

Modifying the input frequency of the induction motor – by means of a *Siemens Sinamics G110* frequency inverter – the machine platform was allowed to move at predefined tilting rates.

## 3 THE EFFECT OF TILTING RATE AND WEAR ON THE BASIC FRICTION ANGLE

The effect of the tilting rate on the basic friction angle results could be a matter of concern, since different laboratories performs tests at different velocities ranging from 2 to  $25^\circ/\text{minute}$ , depending on their own experience, technical capabilities or available equipment, and since there is still neither standard procedure to perform these tests nor recommendations for the tilting rates.

Aiming to reasonably cover tilt testing velocities reported in the literature, which are  $2.5^\circ/\text{min}$  (USBR 2009),  $8^\circ/\text{min}$  (Bruce et al. 1989),  $23^\circ/\text{min}$  (Ruiz & Li 2014) or  $24^\circ/\text{min}$  (Alejano et al. 2012), five series of twenty tilt tests each, run at 2, 4, 7.5, 12.5 and  $25^\circ/\text{min}$ , were performed following the above mentioned recommendations.

These tilting rates correspond to motor input frequencies of 4, 8, 15, 25 and 50 Hz, respectively. All results are presented in Table 1.

A simple look at these values indicates certain variability, as otherwise pointed out in previous studies (Nicholson 1994; Alejano et al. 2012; Hencher 2012). It seems that there is no clear correspondence between basic friction angle results and different angular velocities although the effect of wear begins to arise even for just five repetitions on the same surface, in line

Table 1. Basic friction angles (in degrees) performed at different tilting rates.

Series	$\omega$ (°/min)	Repetitions				
		$\phi_{b1}$ (°)	$\phi_{b2}$ (°)	$\phi_{b3}$ (°)	$\phi_{b4}$ (°)	$\phi_{b5}$ (°)
TR-1	25	32.5	30.9	30.2	29.8	27.6
TR-2	25	29.0	26.3	25.4	25.3	25.4
TR-3	25	31.3	32.7	31.7	31.5	31.2
TR-4	25	30.9	31.1	30.7	30.8	29.5
TR-5	12.5	29.2	29.2	31.0	28.9	28.9
TR-6	12.5	27.5	26.8	27.3	25.4	29.1
TR-7	12.5	27.1	27.2	26.4	26.7	27.2
TR-8	12.5	29.1	31.3	27.5	26.1	26.1
TR-9	7.5	30.7	30.8	29.4	30.0	29.6
TR-10	7.5	29.2	29.4	27.6	25.6	28.6
TR-11	7.5	32.2	29.6	30.7	29.8	28.0
TR-12	7.5	29.3	28.0	27.9	29.1	30.8
TR-13	4	31.1	32.2	31.0	30.4	30.2
TR-14	4	30.6	30.6	28.0	26.7	26.6
TR-15	4	28.9	28.7	30.8	29.6	30.2
TR-16	4	30.1	30.3	30.7	29.8	28.9
TR-17	2	32.2	30.0	28.6	29.9	26.6
TR-18	2	30.9	27.9	27.8	26.3	27.7
TR-19	2	30.2	25.6	32.1	24.1	23.4
TR-20	2	30.3	30.7	28.5	27.6	27.8

with previously reported studies on this topic by the authors (González et al. 2014; Pérez-Rey et al. 2015).

As a first attempt to study the possible influence of tilting rate on basic friction angle, raw test results were plotted against the angular tilting velocity of the rotating table (Fig. 4) and also against test number or repetition – up to five in the present work (Fig. 5).

In order to study if any kind of relationship exists between obtained results and test parameters, some procedures were utilized, starting from the simplest one – a line of best fit – and subsequently using more sophisticated descriptions and statistical methods.

If test results are plotted against different angular tilting velocities (Fig. 4), it can be observed that – disregarding natural variability – there is no correlation between basic friction angles and tilting rate, a conclusion that may be mathematically drawn from the low coefficient of determination of average values ( $R^2 = 0.0517$ ) and, moreover, the one of all value ( $R^2 = 0.007$ ). Besides that, even taken into consideration a linear fit, a negligible difference of about  $0.5^\circ$  can be observed between the extreme velocities of 2 and  $25^\circ/\text{min}$ . Additionally average values of  $\phi_b$  for different tilting rates, represented as large circles in Figure 4, do not show any clear trend.

In the case of plotting basic friction angle results against test repetitions on the same sliding surface, a clear decaying trend can be easily observed, which is in accordance with previous studies. Applying to the data the same procedure of Figure 5 – also ignoring dispersion associated to natural variability by estimating average values for each data group – a line of best fit with a quite larger coefficient of determination ( $R^2 = 0.939$ ) is obtained, implying a decay rate of  $0.5^\circ$

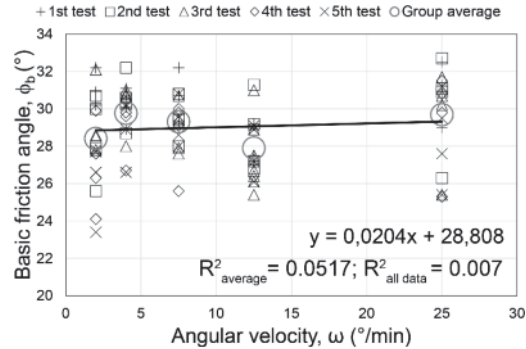


Figure 4. Basic friction angle versus angular velocity. Circles represent average values for data grouped by tilting rate.

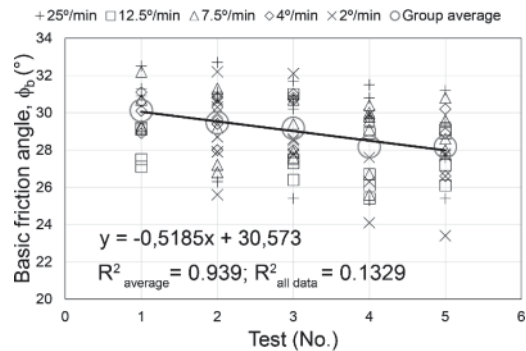


Figure 5. Basic friction angle versus each test repetition. Circles represent average values for data grouped by tilting rate.

per repetition: around  $2.1^\circ$  between first and fifth test. This fact puts forward that, for this moderately weathered granite, the effect of wear should not be neglected when estimating the basic friction angle. The same decreasing trend is also observed when considering different velocities for each group, supporting the fact that results do not display any velocity-dependence.

#### 4 STATISTICAL ANALYSIS

After performing some preliminary graph-based analyses and with the aim of further investigating the possible effect of tilting rate on basic friction angle results, some different graphs were plotted and statistical tests were carried out with the experimental data.

##### 4.1 Box plots

First of all, data were represented by box plots on Cartesian coordinated axes using *MatLab* software. This kind of plots provides information related to data variability and distribution in an easy-to-understand and summarized way. They include several descriptive parameters such as median (50th percentile), 25th

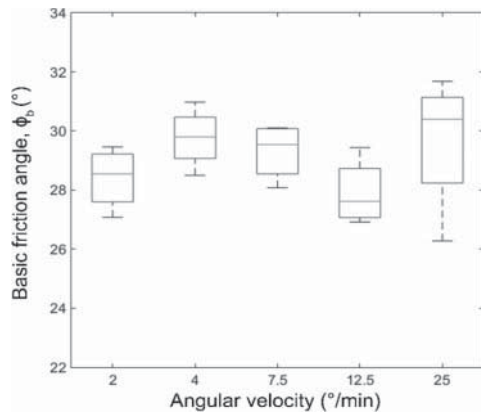


Figure 6. MatLab box-plots showing median, 25th, 75th percentiles and maximum/minimum values for data against different angular velocities.

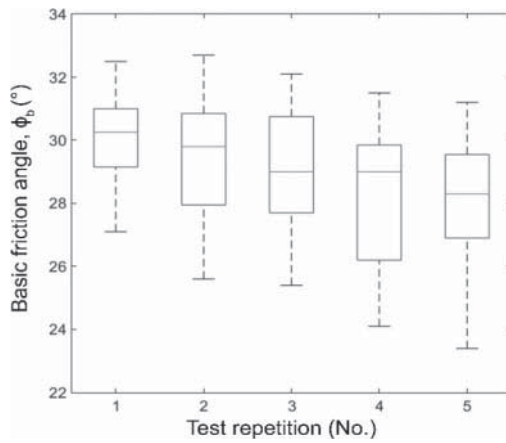


Figure 7. MatLab box-plots showing median, 25th, 75th percentiles and maximum/minimum values for data against different test repetitions.

percentile, 75th percentile and maximum/minimum represented by the line in the box, the top and bottom sides of the box and the edges of a dashed segment, respectively. Box plots also allow a first notion of the level of symmetry of the data set, which is related to the distribution normality, since for a normal distribution, segment edges, upper, lower and middle sides of the box are equally spaced.

Figure 6 shows box plot-type representations of the basic friction angle data series for different angular velocities. As can be seen, data do not follow any clear trend.

Representation of basic friction angles against the five repetitions can be seen in Figure 7. The effect of wear is revealed on the graph as a decreasing trend of the values, whose percentiles are roughly equally spaced indicating that data sets are close to a normal distribution.

#### 4.2 Analysis of variance (ANOVA)

A more rigorous way to study a possible effect of the tilting rate on basic friction angle is to compare whether test results are similar or belong to the same original population, by analysing data with certain statistical tests providing information regarding data origins.

An easy-to-apply and sufficiently powerful test is the *one-way analysis of variance (ANOVA)*. This test compares the means of different data sets and determines if they are statistically different from each other. Specifically, it tests the existence of the null hypothesis. This hypothesis or assumption implies that data belong to the same original population and, therefore, they are comparable among them.

To assert the null hypothesis, the output '*p-value*' – obtained after applying *ANOVA* to a number of data sets – has to be greater than a critical pre-defined value  $\alpha = 0.05$ , the considered significance level.

To study if the tilting rate may affect the friction angle the *one-way analysis of variance* was applied to the 20 groups of data presented in the rows of Table 1. The tool of *MS Excel 2013, Data Analysis*, was used to carry out this task. Results of this *ANOVA* test reflect that series are not comparable. Considering that *F* is the value of the statistic for the considered degrees of freedom (4 between groups and 95 within groups, for the first case) the output of the test becomes: ( $F(4, 95) = 3.916$ ,  $p = 0.05$ ). This means that it cannot be stated that they belong to the same population. It does not mean that they are not.

Student t-tests are the equivalent of *ANOVA* for two groups of tests. These tests have been applied for pairs of groups of data as in rows in Table 1, corresponding to tests with the same angular velocities. Results indicate that whether in the majority of comparisons results belong to the same population, in seven cases associated to series TR2, TR7 and TR19, this cannot be ensured. This means that groups of the original population were moderately similar, but not enough to fulfil the *ANOVA* criterion.

This fact can be attributed to the data large variability, associated to the natural variability of basic friction angle, but also to the variability induced by surface wear. In other words, the effect of wear is thought to be the reason for the negative result of the *ANOVA* analysis in this case. Recall that in average, each test performed on the same rock surface causes a decrease of the basic friction angle of about  $0.5^\circ$ .

In order to reduce the effect of wear and to make data presumably depending on a single parameter, the *ANOVA* analysis was thus applied to a new data set made up by the average results of the four series for each angular velocity. So we have now 5 groups (one for each tilting rate) of four data (average friction angle for the five repetitions  $\phi_b^{av}$ ) as shown in Table 2.

The same procedure carried out for the previous analysis with *ANOVA* was applied again to the averaged data series. In this case, the obtained output was, for the significance level  $\alpha = 0.05$ : ( $F(4, 15) = 1.377$ ,



Table 2. Averages of five repetitions friction angle results for every surface for different tilting rates.

Series	$\omega$ (°/min)	$1-\phi_b^{av}$	$2-\phi_b^{av}$	$3-\phi_b^{av}$	$4-\phi_b^{av}$
1	25	30.2	26.3	31.7	30.6
2	12.5	29.4	27.2	26.9	28.0
3	7.5	30.1	28.1	30.1	29.0
4	4	31.0	28.5	29.6	30.0
5	2	29.5	28.1	27.1	29.0

Table 3. Basic friction angle results performed at different tilting rates subtracting the effect of wear.

Series	$\omega$ (°/min)	$\phi_{b1}$ (°)	$\phi_{b2}$ (°)	$\phi_{b3}$ (°)	$\phi_{b4}$ (°)	$\phi_{b5}$ (°)
TR1	25	32.50	31.42	31.24	31.36	29.67
TR2	25	29.00	26.82	26.44	26.86	27.47
TR3	25	31.30	33.22	32.74	33.06	33.27
TR4	25	30.90	31.62	31.74	32.36	31.57
TR5	12.5	29.20	29.72	32.04	30.46	30.97
TR6	12.5	27.50	27.32	28.34	26.96	31.17
TR7	12.5	27.10	27.72	27.44	28.26	29.27
TR8	12.5	29.10	31.82	28.54	27.66	28.17
TR9	7.5	30.70	31.32	30.44	31.56	31.67
TR10	7.5	29.20	29.92	28.64	27.16	30.67
TR11	7.5	32.20	30.12	31.74	31.36	30.07
TR12	7.5	29.30	28.52	28.94	30.66	32.87
TR13	4	31.10	32.72	32.04	31.96	32.27
TR14	4	30.60	31.12	29.04	28.26	28.67
TR15	4	28.90	29.22	31.84	31.16	32.27
TR16	4	30.10	30.82	31.74	31.36	30.96
TR17	2	32.20	30.52	29.64	31.46	28.67
TR18	2	30.90	28.42	28.84	27.86	29.77
TR19	2	30.20	26.12	33.14	25.66	25.47
TR20	2	30.30	31.22	26.54	29.16	29.87

$p = 0.289$ ). Since  $p > 0.05$ , this result demonstrates in a statistically rigorous way that all results belong to the same population, that is, that tilting rate does not significantly affect basic friction angle results calculated as the average of five tests.

In a certain sense, this is also a confirmation that the effect of wear (as a decaying tendency on basic friction angle results for the same tested surface) should be accounted for, at least for the case of the rock type studied in this research (*Amarelo Pais*).

#### 4.3 Understanding results disregarding the effect of wear

With the aim of disregarding the effect of wear, the original database shown in Table 1 was remade, removing the effect due to wear as derived from the average fit of all values in relation with test number (Fig. 5): for test nr. 1 the values are the same; for test nr. 2 the values are decreased by 0.5185; for test nr. 3 the values are decreased by  $2 \times 0.5185$ , and so on.

The new set of data is presented in Table 3, and with these new values, the relative frequencies histogram of

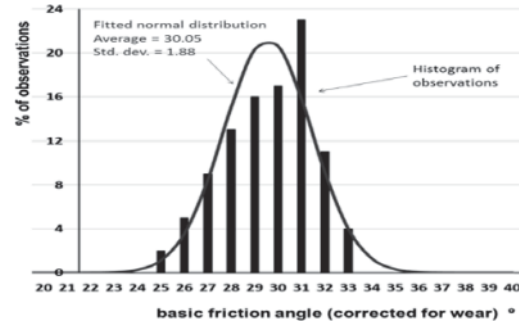


Figure 8. Histogram of observations of corrected basic friction angle estimates of planar rock joints in moderately weathered granite together with the fitted normal distribution function.

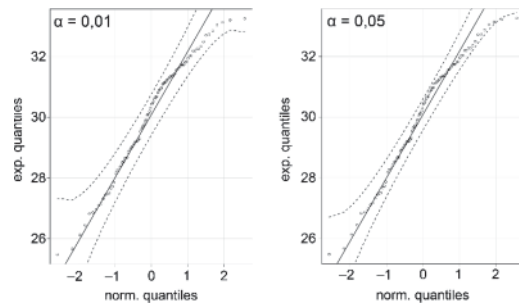


Figure 9. Q-Q plots of data after subtracting wear effect, for two different levels of significance ( $\alpha = 0.01$  and  $\alpha = 0.05$ ).

all data is plotted in Figure 8, where the average friction angle obtained is  $30.05^\circ$  and the standard deviation is  $1.88^\circ$ . This reduces to few degrees the standard deviation of the uncorrected data and, as shown in Figure 8, data seems to be reasonably close to the typical shape of a normal distribution.

Furthermore, the standard deviation seems to be rather small in comparison to standard deviation of other typical geomechanical parameters such as uniaxial compressive strength or elastic moduli of rock samples tested in the lab.

In order to assess the normality of the data set after subtracting the wear effect, a Kolmogorov-Smirnov test (K-S test) and Q-Q diagrams were considered.

To fulfil K-S conditions of normality, the corresponding statistic  $D$  of this test needs to be less than a critical value  $D_n$ , equal to 0.1031 in the present case, for the data to be considered normally distributed. It should be remarked that a lower significance level,  $\alpha = 0.01$ , was considered but it does not mean that normality conditions are not achieved. The obtained statistic for the corrected basic friction angle data is  $D = 0.094$ , less than the critical value  $D_n$ . Therefore, data can be considered, in general terms, normally distributed.

Q-Q plots can be also analysed for different levels of significance, as shown in Figure 9. These type of plots are graph-based representations intended to

strictly compare the equality, in terms of normality between a theoretical normal distribution (represented by a straight line) and the actual data set.

According to Figure 9, normality conditions are fulfilled for  $\alpha = 0.01$ . In the case of  $\alpha = 0.05$ , only two points are out of the corresponding significance area, enclosed by two dashed lines. Taking into account that the majority of data – especially those points located at the center of the plot – lie very close to the straight line in both assumptions, regarding to the performed histogram and to the K-S test, it may be considered in a reasonably rigorous way that representative values of basic friction angle, once disregarded the effect of wear, may be represented by a normally distributed data set.

## 5 CONCLUSIONS

No methodology is so far suggested to determine the basic friction angle of rock joints. Previous studies carried out by the authors have suggested some guidelines which should be followed in order to obtain reasonably reliable results (Alejano et al. 2012).

In this study, an experimental program was prepared to assess the influence of tilting rate on basic friction angle results by means of various tests on various rock joint surfaces at various velocities.

A rigorous statistical interpretation of results has demonstrated that the tilt rate in the range between 2 to 25°/min does not significantly affect results of the basic friction angle.

However, in accordance with previous studies, it was observed that previous wear of the surfaces caused by the number of tests performed with every sample, plays a non-irrelevant role, and so it should be somehow contemplated when analysing tilt test results on rock joints. This conclusion was clearer in this case probably due to the moderately weathered granite that was tested.

It is finally pertinent to remark that, once removed the wear effect, tilt test results provide basic friction angle values which can be fitted by normal distribution functions with standard deviations in the range of those typically encountered in other rock engineering parameters.

Further studies are needed to analyse and understand the effects of the rock cutting procedure on tests results, including the type of saw-blade used to prepare the samples. These studies could be helpful to provide a convenient testing methodology to estimate the basic friction angle of rock planar joints.

## ACKNOWLEDGEMENTS

The authors thank the Spanish Ministry of Economy and Competitiveness for partial financial support of

this study, awarded under Contract Reference No. BIA2014-53368P. This contract is partially financed by means of ERDF funds of the EU. Dr. Ángeles Saavedra is kindly acknowledged for her help in performing and understanding statistical tests.

## REFERENCES

- Alejano, L.R., González, J. & Muralha, J. 2012. Comparison of different techniques of tilt testing and basic friction angle variability assessment. *Rock Mechanics & Rock Engineering* 45(6): 1023–1035.
- Barton, N. & Choubey, V. 1977. The shear strength of rock joints in theory and practice. *Rock Mechanics* 10(1): 1–54.
- Bruce, I.G., Cruden, D.M. & Eaton, T.M. 1989. Use of a tilting table to determine the basic friction angle of hard rock samples. *Canadian Geotechnical Journal*, 26(3): 474–479.
- Cruden, D.M. & Hu, X.Q. 1988. Basic friction angles of carbonate rocks from Kananaskis country. *Bulletin of the International Association of Engineering Geology*, 38(1): 55–59.
- Hencher, S.R. 2012. Discussion of Alejano, Gonzalez and Muralha 2012. *Rock Mechanics & Rock Engineering*. 45(6): 1141–1143.
- González, J., González-Pastoriza, N., Castro, U., Alejano, L.R. & Muralha, J. 2014. Considerations on the laboratory estimate of the basic friction angle of rock joints. In L.R. Alejano, A. Perucho, C. Olalla and R. Jiménez (eds.). *Rock Engineering and Rock Mechanics: Structures in and on Rock Masses; Proc. EUROCK 2014, ISRM European Regional Symposium, Vigo, 26–28 May 2014*. Rotterdam: Balkema.
- Nicholson, G.A. 1994. A test is worth a thousand guesses—a paradox. In: P.P. Nelson & S.E. Laubach (eds.) *Proc. 1st NARMS Symposium, Austin, 1–3 June 1994*. Rotterdam: Balkema.
- Pérez-Rey, I., Alejano, L.R., González-Pastoriza, N., González, J. & Arzúa, J. 2015. Effect of time and wear on the basic friction angle of rock discontinuities. In W. Schubert & A. Kluckner (eds.) *Proc. ISRM European Regional Symposium EUROCK 2015 & 64th Geomechanics Colloquium, Salzburg, 7–9 October 2015*: 1115–1120. Salzburg: OGG.
- Ruiz, J. & Li, C.C. 2014. Measurement of the basic friction angle of rock by three different tilt test methods. In L.R. Alejano, A. Perucho, C. Olalla and R. Jiménez (eds.). *Rock Engineering and Rock Mechanics: Structures in and on Rock Masses; Proc. EUROCK 2014, ISRM European Regional Symposium, Vigo, 26–28 May 2014*: 261–266. Rotterdam: Balkema.
- Stimpson, B. 1981. A suggested technique for determining the basic friction angle of rock surfaces using core. *International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts*, 18(1): 63–65.
- USBR (2009) Procedure for determining the angle of basic friction (static) using a tilting table test (Designation USBR 6258-09). <http://www.usbr.gov/>