Introduction to Trees

Contents

- Introduction: Trees and Binary Trees
- Representing Trees
    - Test Yourself #1
- Tree Traversals
    - Pre-order
    - Post-order
    - Level-order
    - Test Yourself #2
    - In-order
    - Test Yourself #3

Introduction

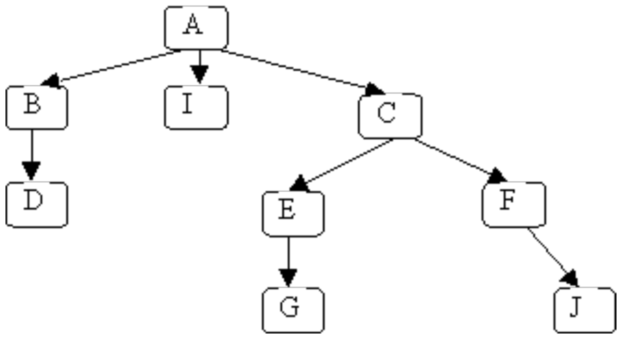
Lists, stacks, and queues are all **linear** structures: in all three data structures, one item follows another. Trees will be our first non-linear structure:

- More than one item can follow another.
- The number of items that follow can vary from one item to another.

Trees have many uses:

- representing family genealogies
- as the underlying structure in decision-making algorithms
- to represent priority queues (a special kind of tree called a **heap**)
- to provide fast access to information in a database (a special kind of tree called a **b-tree**)

Here is the conceptual picture of a tree (of letters):

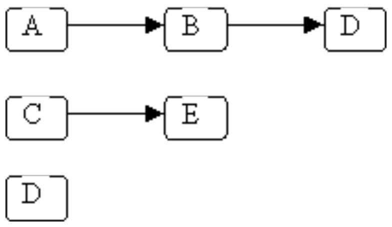


- each letter represents one **node**

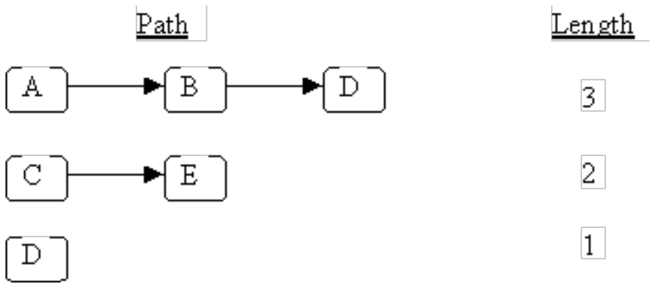
- the arrows from one node to another are called **edges**
- the topmost node (with no incoming edges) is the **root** (node A)
- the bottom nodes (with no outgoing edges) are the **leaves** (nodes D, I, G & J)

So a (computer science) tree is kind of like an upside-down real tree.

A **path** in a tree is a sequence of (zero or more) connected nodes; for example, here are 3 of the paths in the tree shown above:



The **length** of a path is the number of nodes in the path, e.g.:

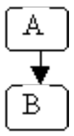


The **height** of a tree is the length of the longest path from the root to a leaf; for the above example, the height is 4 (because the longest path from the root to a leaf is A → C → E → G, or A → C → E → J). An empty tree has height = 0.

The **depth** of a node is the length of the path from the root to that node; for the above example:

- the depth of J is 4
- the depth of D is 3
- the depth of A is 1

Given two connected nodes like this:



Node A is called the **parent**, and node B is called the **child**.

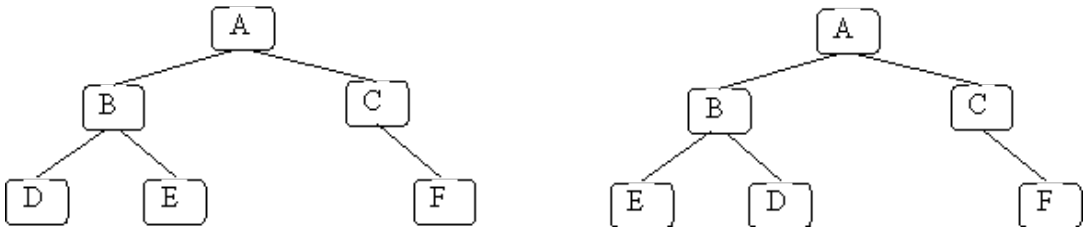
A **subtree** of a given node includes one of its children and all of that child's **descendants**. The descendants of a node N are all nodes reachable from N (N's children, its children's children, etc.). In the original example, node A has three subtrees:

1. B, D
2. I
3. C, E, F, G, J

An important special kind of tree is the **binary** tree. In a binary tree:

- Each node has 0, 1, or 2 children.
- Each child is either a left child or a right child.

Here are two examples of binary trees that are different:



The two trees are different because the children of node B are different: in the first tree, B's left child is D and its right child is E; in the second tree, B's left child is E and its right child is D. Also note that lines are used instead of arrows. We sometimes do this because it is clear that the edge goes from the higher node to the lower node.

Representing Trees

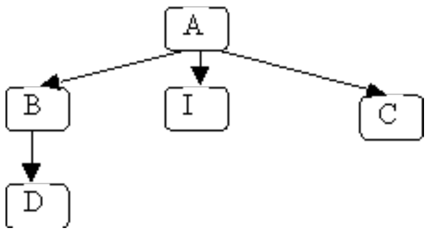
Since a binary-tree node never has more than two children, a node can be represented using a class with 3 fields: one for the data in the node plus two child pointers:

```
class BinaryTreenode<T> {
    // *** fields ***
    private T data;
    private BinaryTreenode<T> leftChild;
    private BinaryTreenode<T> rightChild;

    ...
}
```

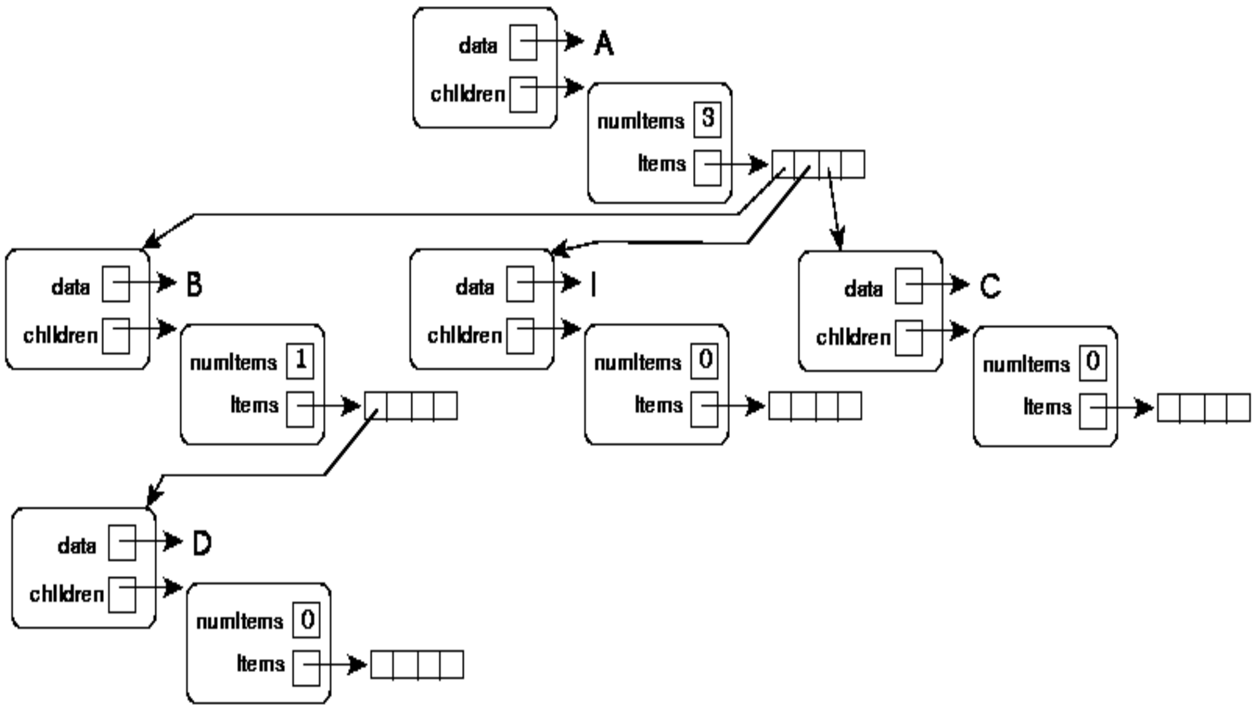
However, since a general-tree node can have an arbitrary number of children, a fixed number of child-pointer fields won't work. Instead, we can use a List to keep all of the child pointers:

```
class Treenode<T> {
    // *** fields ***
    private T data;
    private ListADT<Treenode<T>> children;

    ...
}
```

As we know, a list can be represented using either an array or a linked-list. For example, consider this general tree (a simplified version of the original example):



For the array representation of the List (where the array has an initial size of 4) we would have:



TEST YOURSELF #1

Draw a similar picture of the tree when the List fields are implemented using linked lists.

solution

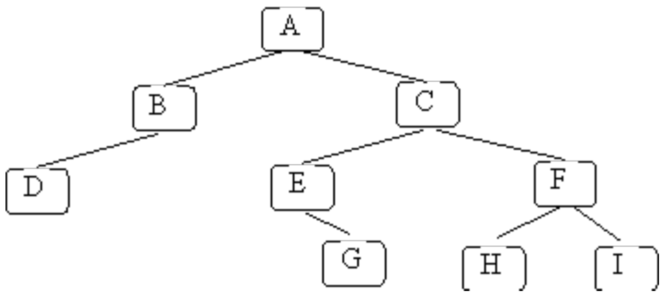
Tree Traversals

It is often useful to iterate through the nodes in a tree:

- to print all values
- to determine if there is a node with some property
- to make a copy

When we iterate through a List, we started with the first tree node and visited each node in turn. Since each node is visited, the best possible complexity is O(N) for a tree with N nodes. All of our traversal methods will achieve this complexity.

For trees, there are many different orders in which we might visit the nodes. There are three common traversal orders for general trees and one more for binary trees: pre-order, post-order, level-order, and in-order, all described below. We will use the following tree to illustrate each traversal:



Pre-order

A pre-order traversal can be defined (recursively) as follows:

1. ***visit the root***
2. perform a pre-order traversal of the first subtree of the root
3. perform a pre-order traversal of the second subtree of the root
4. etc., for all the subtrees of the root

If we use a pre-order traversal on the example tree given above and we print the letter in each node when we visit that node, the following will be printed: `A B D C E G F H I`.

Post-order

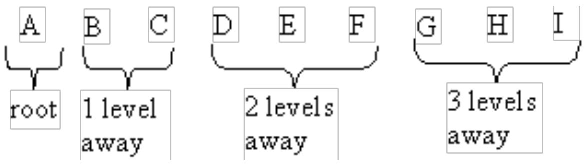
A post-order traversal is similar to a pre-order traversal, except that the root of each subtree is visited ***last*** rather than first:

1. perform a postorder traversal of the first subtree of the root
2. perform a postorder traversal of the second subtree of the root
3. etc., for all the subtrees of the root
4. ***visit the root***

If we use a post-order traversal on the example tree given above and we print the letter in each node when we visit that node, the following will be printed: `D B G E H I F C A`.

Level-order

The idea of a level-order traversal is to visit the root, then visit all nodes "1 level away" (depth 2) from the root (left to right), then all nodes "2 levels away" (depth 3) from the root, etc. For the example tree, the goal is to visit the nodes in the following order:



A level-order traversal requires using a queue (rather than a recursive algorithm, which implicitly uses a stack). Here's how to print the data in a tree in level order, using a queue `Q`, and using an iterator to access the children of each node (we assume that the root node is called `root` and that the `Treenode` class provides a `getChildren` method):

```
Q.enqueue(root)
while (!Q.empty()) {
    Treenode<T> n = Q.dequeue();
    System.out.print(n.getData());
    ListADT<Treenode<T>> kids = n.getChildren();
    Iterator<Treenode<T>> it = kids.iterator();
    while (it.hasNext()) {
        Q.enqueue(it.next());
    }
}
```

TEST YOURSELF #2

Draw pictures of `Q` as it would be each time around the outer while loop in the code given above for the example tree given above.

solution

In-order

An in-order traversal involves visiting the root "in between" visiting its left and right subtrees. Therefore, an in-order traversal only makes sense for binary trees. The (recursive) definition is:

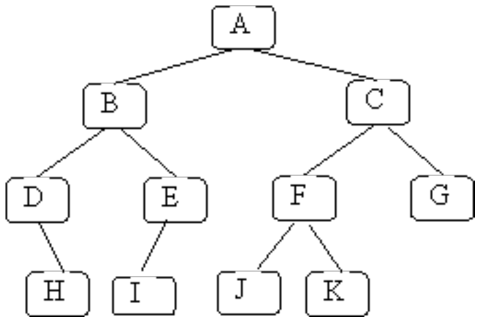
1. perform an in-order traversal of the left subtree of the root
2. ***visit the root***
3. perform an in-order traversal of the right subtree of the root

If we print the letters in the nodes of our example tree using an in-order traversal, the following will be printed: `D B A E G C H F I`

The primary difference between the pre-order, post-order, and in-order traversals is where the node is visited in relation to the recursive calls; i.e., before, after, or in-between.

TEST YOURSELF #3

What is printed when the following tree is visited using (a) a pre-order traversal, (b) a post-order traversal, (c) a level-order traversal, and (d) an in-order traversal?



solution

Back

Next: Continue with Binary Search Trees