

# Gauges

## 1 Synchronous Gauge

The component  $g_{00}$  and  $g_{0i}$  of the metric tensor in the *synchronous gauge* are by definition unperturbed. The line element is given by

$$ds^2 = a^2(\tau) \{-d\tau^2 + (\gamma_{ij} + h_{ij})dx^i dx^j\}.$$

Since the choice of the initial hypersurface and its coordinate assignments are arbitrary, the synchronous gauge conditions do not fix the gauge degrees of freedom completely.

- Ma, Chung-Pei, and Edmund Bertschinger. 1995. “Cosmological Perturbation Theory in the Synchronous and Conformal Newtonian Gauges.” *arXiv [astro-Ph]*. arXiv. <http://arxiv.org/abs/astro-ph/9506072>.

## 2 Conformal Newtonian gauge (longitudinal gauge)

The *conformal Newtonian gauge* (also known as the *longitudinal gauge*) advocated by Mukhanov et al. (1992) is a particularly simple gauge to use for the scalar mode of metric perturbations. The perturbations are characterized by two scalar potentials  $\psi$  and  $\phi$  which appear in the line element as

$$ds^2 = a^2(\tau) \{-(1 + 2\psi)d\tau^2 + (1 - 2\phi)dx^i dx_i\}.$$

It should be emphasized that the conformal Newtonian gauge is a restricted gauge since the metric is applicable only for the scalar mode of the metric perturbations; the vector and the tensor degrees of freedom are eliminated from the beginning. Nonetheless, it can be easily generalized to include the vector and the tensor modes (Bertschinger 1995).

One advantage of working in this gauge is that the metric tensor  $g_{\mu\nu}$  is diagonal.

Another advantage is that  $\psi$  plays the role of the gravitational potential in the Newtonian limit and thus has a simple physical interpretation.

They are called Newtonian gauge because  $\phi$  is the Newtonian gravitational potential of classical Newtonian gravity, which satisfies the Poisson equation  $\nabla^2 \phi = 4\pi G \rho$  for non-relativistic matter and on scales where the expansion of the universe may be neglected.

- Ma, Chung-Pei, and Edmund Bertschinger. 1995. “Cosmological Perturbation Theory in the Synchronous and Conformal Newtonian Gauges.” *arXiv [astro-Ph]*. arXiv. <http://arxiv.org/abs/astro-ph/9506072>.
- [Newtonian gauge] [https://en.wikipedia.org/wiki/Newtonian\\_gauge](https://en.wikipedia.org/wiki/Newtonian_gauge)
- <http://www.damtp.cam.ac.uk/user/db275/Cosmology/Chapter4.pdf>

## 3 Spatially-flat gauge

A convenient gauge for computing inflationary perturbations is  $C = E = 0$ . (4.2.65) In this gauge, we will be able to focus most directly on the fluctuations in the inflaton field  $\phi$ , (see Chapter 6).

<http://www.damtp.cam.ac.uk/user/db275/Cosmology/Chapter4.pdf>

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## 4 Uniform density gauge

We can use the freedom in the time-slicing to set the total density perturbation to zero

$$\delta\rho=0.$$

→ <http://www.damtp.cam.ac.uk/user/db275/Cosmology/Chapter4.pdf>

## 5 Comoving gauge

Similarly, we can ask for the scalar momentum density to vanish,  $q = 0$ . (4.2.89) Fluctuations in comoving gauge are most naturally connected to the inflationary initial conditions. This will be explained in §4.3.1 and Chapter 6.

There are different versions of uniform density and comoving gauge depending on which of the metric fluctuations is set to zero. In these lectures, we will choose  $B = 0$ .

→ <http://www.damtp.cam.ac.uk/user/db275/Cosmology/Chapter4.pdf>