Problem 1

$$\begin{aligned} \text{minimize} \quad & -e^{-w^T x} \\ \text{subject to} \quad & w^T A w - w^T A y - w^T x \leq -a \\ & y^T w - w^T x = b \end{aligned}$$

Problem 5

Let k be the quantity of knobs and m be the quantity of milk cartons. Then we have:

$$\begin{aligned} \text{minimize} \quad & -0.05k - 0.07m \\ \text{subject to} \quad & 3k + 4m \leq 240000 \\ & k + 2m \leq 6000 \end{aligned}$$

Problem 6

The partials are:

$$\frac{\partial f}{\partial x} = y(6x + 4y + 1) \qquad \frac{\partial f}{\partial y} = x(3x + 8y + 1)$$

So, clearly $(0, 0)$ is a critical point. Solving, we get that $(-\frac{1}{3}, 0)$, $(-\frac{1}{9}, -\frac{1}{12})$, and $(0, -\frac{1}{4})$ are also critical points. Calculating the Hessian determinant, we get:

$$H = -36x^2 - 12x(4y + 1) - (8y + 1)^2$$

So, we can see that $(0, 0)$, $(0, -\frac{1}{4})$, and $(-\frac{1}{3}, 0)$ are saddle points since at these points $H < 0$. Also, $H > 0$ and $f_{xx} < 0$ at $(-\frac{1}{9}, -\frac{1}{12})$, so it is a local maximum.

Problem 11

Proof. For some $x_0 \in \mathbb{R}$, consider:

$$x_1 = x_0 - \frac{f'(x_0)}{f''(x_0)}$$

Then, we have that:

$$\begin{aligned} x_1 &= x_0 - \frac{2ax_0 + b}{2a} \\ &= x_0 - x_0 - \frac{b}{2a} \\ &= -\frac{b}{2a} \end{aligned}$$

Since $a > 0$, we know that $-\frac{b}{2a}$ is the unique minimizer of f since $\frac{df}{dx} = 2ax + b = 0$ iff $x = -\frac{b}{2a}$ and $\frac{d^2 f}{dx^2} = 2a > 0$. $\square$