

1 Problem 3.6:

Proof. $B_i \cap B_j = \emptyset \ \forall i \neq j$, so $(A \cap B_i) \cap (A \cap B_j) = \emptyset \ \forall i \neq j$, i.e. $\{A \cap B_i\}_{i \in I}$ is a collection of pairwise-disjoint events. Since it is indexed by a finite or countable set I , then, we have that:

$$\begin{aligned} \sum_{i \in I} P(A \cap B_i) &= P\left(\bigcup_{i \in I} A \cap B_i\right) \\ &= P\left(A \cap \left(\bigcup_{i \in I} B_i\right)\right) && \text{by the distributive property of } \cap \\ &= P(A \cap \Omega) && \text{by assumption} \\ &= P(A) && \text{since } A \subseteq \Omega \end{aligned}$$

□