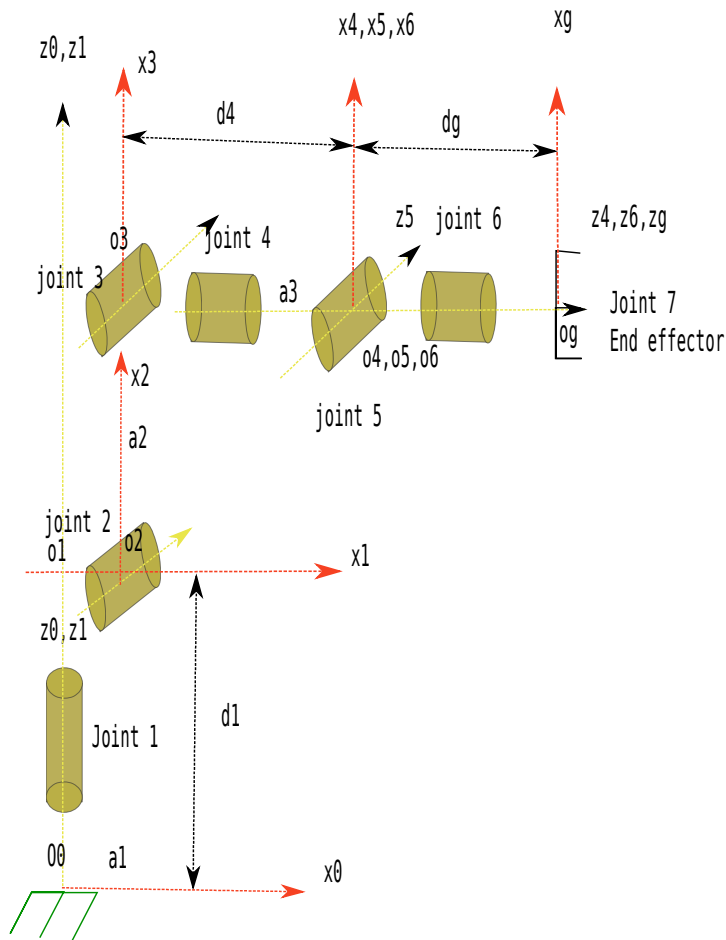


The joint values shown in the kr210.urdf.xacro are:



joint_1	0,	0,	0.33
joint_2	0.35,	0,	0.42
joint_3	0,	0,	1.25
joint_4	0.96,	0,	-0.054
joint_5	0.54,	0,	0
joint_6	0.193,	0,	0

Joint	a(joint-1)	a(joint-1)	d(joint)	theta(joint)
1	0	0	d1	theta1
2	-90	a1	0	Theta2 - 90
3	0	a2	0	Theta 3
4	-90	a3	d4	theta4
5	90	0	0	theta5
6	-90	0	0	theta6
7	0	0	dg	0

Joint	a(joint-1)	a(joint-1)	d(joint)	theta(joint)
1	0	0	.75	theta1
2	-90	.35	0	Theta2 - 90
3	0	1.25	0	Theta 3
4	-90	-0.054	1.5	theta4
5	90	0	0	theta5
6	-90	0	0	theta6
7	0	0	.303	0

Create Modified DH parameters

```
DH_Table = { linkTwist0: 0,      linkLength0: 0,   linkOffset1: 0.75, joint1: joint1,
              linkTwist1: -pi/2., linkLength1: 0.35, linkOffset2: 0,   joint2: -pi/2. + joint2,
              linkTwist2: 0,      linkLength2: 1.25, linkOffset3: 0,   joint3: joint3,
              linkTwist3: -pi/2.0, linkLength3: -0.054, linkOffset4: 1.5, joint4: joint4,
              linkTwist4: pi/2.0, linkLength4: 0,   linkOffset5: 0,   joint5: joint5,
              linkTwist5: -pi/2.0, linkLength5: 0,   linkOffset6: 0,   joint6: joint6,
              linkTwist6: 0,      linkLength6: 0,   linkOffset7: 0.303, joint7: 0.0 }
```

```
def TF_Matrix(twist,length,Offset,q):
    TF = Matrix([[cos(q), -sin(q), 0.0, length],
                 [sin(q) * cos(twist), cos(q) * cos(twist), -sin(twist), -sin(twist) * Offset],
```

```

        [sin(q) * sin(twist),cos(q) * sin(twist), cos(twist),   cos(twist) * Offset],
        [0.0,           0.0, 0.0,           1.0]])
return TF

```

```

T0_1 = TF_Matrix(linkTwist0, linkLength0, linkOffset1,joint1).subs(DH_Table)
T1_2 = TF_Matrix(linkTwist1, linkLength1, linkOffset2,joint2).subs(DH_Table)
T2_3 = TF_Matrix(linkTwist2, linkLength2, linkOffset3,joint3).subs(DH_Table)
T3_4 = TF_Matrix(linkTwist3, linkLength3, linkOffset4,joint4).subs(DH_Table)
T4_5 = TF_Matrix(linkTwist4, linkLength4, linkOffset5,joint5).subs(DH_Table)
T5_6 = TF_Matrix(linkTwist5, linkLength5, linkOffset6,joint6).subs(DH_Table)
T6_EE =TF_Matrix(linkTwist6, linkLength6, linkOffset7,joint7).subs(DH_Table)
T0_EE = T0_1 * T1_2 * T2_3 * T3_4 * T4_5 * T5_6 * T6_EE;

```

$$\begin{aligned}
{}_0T^1 &= \begin{bmatrix} \cos(joint\ 1) & -\sin(joint\ 1) & 0.0 & 0.0 \\ \sin(joint\ 1) & \cos(joint\ 1) & 0 & 0 \\ 0 & 0 & 1 & 0.7500000000000000 \\ 0.0 & 0.0 & 0.0 & 1.0000000000000000 \end{bmatrix} \\
{}_1T^2 &= \begin{bmatrix} \cos(joint\ 1) & -\sin(joint\ 1) & 0.0 & 0.0 \\ \sin(joint\ 1) & \cos(joint\ 1) & 0 & 0 \\ 0 & 0 & 1 & 0.7500000000000000 \\ 0.0 & 0.0 & 0.0 & 1.0000000000000000 \end{bmatrix} \\
{}_2T^3 &= \begin{bmatrix} \cos(joint\ 3) & -\sin(joint\ 3) & 0.0 & 1.250 \\ \sin(joint\ 3) & \cos(joint\ 3) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix} \\
{}_3T^4 &= \begin{bmatrix} \cos(joint\ 4) & -\sin(joint\ 4) & 0.0 & -0.0540 \\ 0 & 0 & 1 & 1.50 \\ -\sin(joint\ 4) & -\cos(joint\ 4) & 0 & 0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix} \\
{}_4T^5 &= \begin{bmatrix} \cos(joint\ 5) & -\sin(joint\ 5) & 0.0 & 0.0 \\ 0 & 0 & -1 & 0 \\ \sin(joint\ 5) & \cos(joint\ 5) & 0 & 0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix} \\
{}_5T^6 &= \begin{bmatrix} \cos(joint\ 6) & -\sin(joint\ 6) & 0.0 & 0.0 \\ 0 & 0 & 1 & 0 \\ -\sin(joint\ 6) & -\cos(joint\ 6) & 0 & 0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix} \\
{}_6T^7 &= \begin{bmatrix} 1 & 0 & 0.0 & 0.0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.3030 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}
\end{aligned}$$

Inverse Kinematics

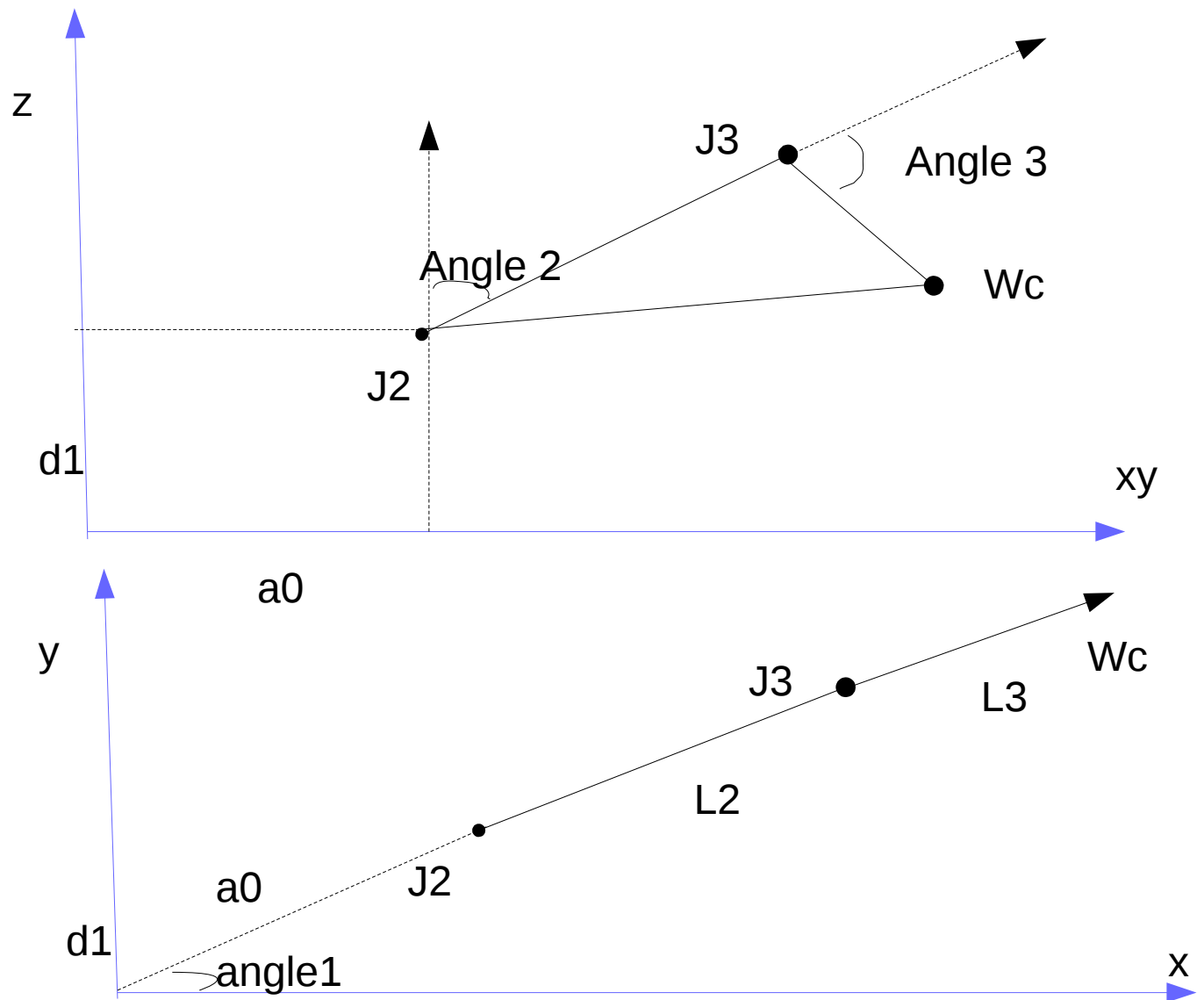
The problem is broken into two parts, one for the revolute arm joints, another for a spherical arm. The revolute arm would return the joint angles for joints 1 through 3 and the spherical arm for joints 4 through 6. The reason for doing this is because the Joint 7 end effector, has a common point of intersection (wrist center).

$$\theta_4 = \text{atan2}(r_{36}[2, 2], -r_{36}[0, 2])$$

$$\theta_5 = \text{atan2}(\sqrt{r_{36}[0, 2]^2 + r_{36}[2, 2]^2}, r_{36}[1, 2])$$

$$\theta_6 = \text{atan2}(-r_{36}[1, 1], r_{36}[1, 0])$$

So for the revolute part we would have:



For angle 1, it only lies on the xy plane so is $\text{atan2}(y,x)$

For the 2nd and 3rd angles, on the z plane use the cosine rule to obtain the angle first for angle 3, then calculating angle 2.

$$l2 = angle2$$

$$l3 = \sqrt{(a3^2 + d4^2)}$$

$$Angle3 = 180 - \theta3$$

$$xy = \sqrt{(Wc_x^2 + Wc_y^2)} - a0$$

using the cosine rule

$$D^2 = xy^2 + z^2 = l2^2 + l3^2 - 2(l2)(l3)\cos(\text{Angle } 3)$$

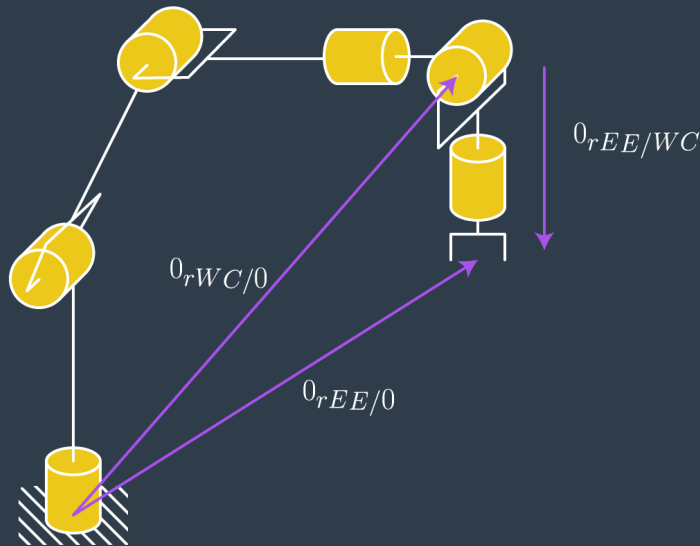
$$\cos(\text{angle } 3) = (xy^2 + z^2 - l3^2 - l2^2) / (2 * l3 * l2) = r$$

$$\text{angle } 3 = \text{atan2}(\sqrt{(1-r^2)}, r), \text{ where } \sqrt{(1-r^2)} = \sin(\text{angle } 3)$$

$$\text{angle } 3 = \text{atan2}(-\sqrt{(1-r^2)}, r)$$

$$\text{angle } 2 = y - a0 = \text{atan2}(xy, z) - \text{atan2}(l3 * \sin(\text{angle } 3), l2 + l3 * \cos(\text{angle } 3)) \quad \text{angle } 3 = \text{angle } 3 - \pi/2$$

Angle 3 has to be -90 degrees as the starting point is horizontally right not vertically up.



Angle for joint 4,5,6

The overall roll, pitch, yaw for the end effector relative to the base link is as follows

$$\frac{a}{b} R_{zyx} = R_z(\alpha) R_y(\beta) R_x(\gamma) = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

The complete transform is calculated by multiplying all the individual transforms

$$R0_6 = R0_1 * R1_2 * R2_3 * R3_4 * R4_5 * R5_6$$