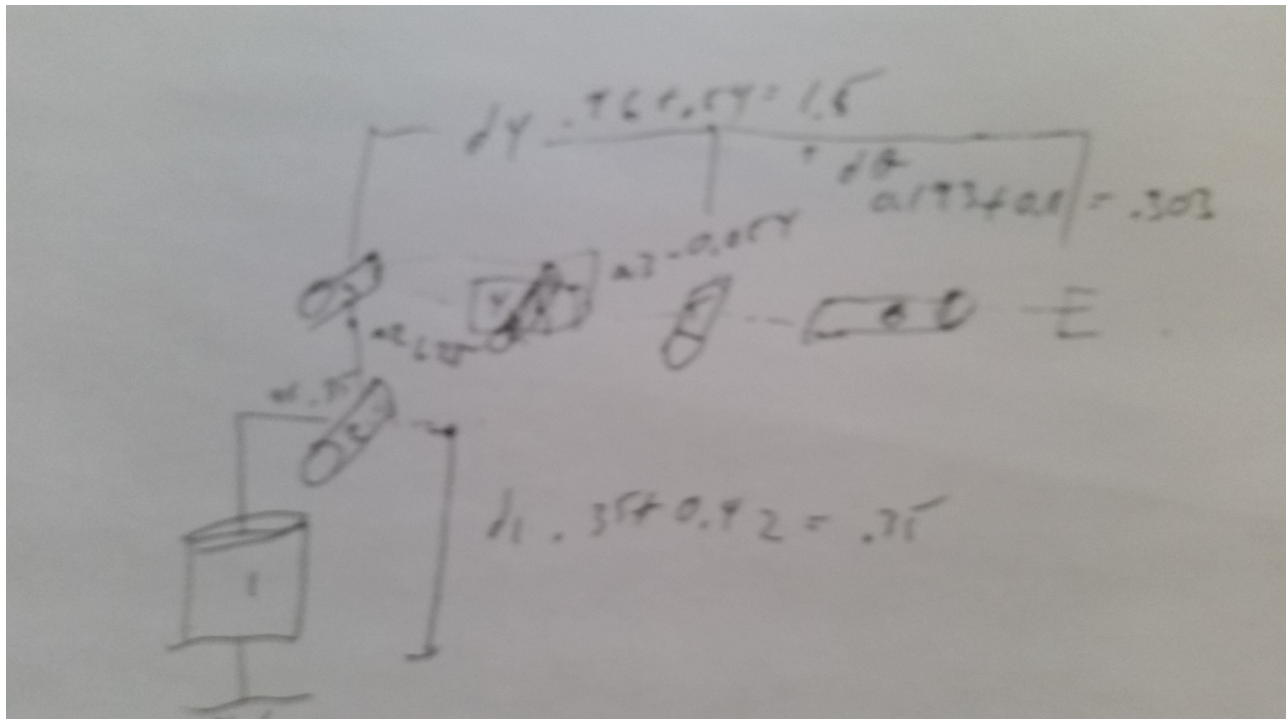


The joint values shown in the kr210.urdf.xacro are:

joint_1	0,	0,	0.33
joint_2	0.35,	0,	0.42
joint_3	0,	0,	1.25
joint_4	0.96,	0,	-0.054
joint_5	0.54,	0,	0
joint_6	0.193,	0,	0



```

# Create Modified DH parameters
DH_Table = { linkTwist0: 0,      linkLength0: 0,      linkOffset1: 0.75, joint1: joint1,
              linkTwist1: -pi/2., linkLength1: 0.35, linkOffset2: 0, joint2: -pi/2. + joint2,
              linkTwist2: 0,      linkLength2: 1.25, linkOffset3: 0, joint3: joint3,
              linkTwist3: -pi/2.0, linkLength3: -0.054, linkOffset4: 1.5, joint4: joint4,
              linkTwist4: pi/2.0, linkLength4: 0,      linkOffset5: 0, joint5: joint5,
              linkTwist5: -pi/2.0, linkLength5: 0,      linkOffset6: 0, joint6: joint6,
              linkTwist6: 0,      linkLength6: 0,      linkOffset7: 0.303, joint7: 0.0 }

```

```

def TF_Matrix(twist,length,Offset,q):
    TF = Matrix([[cos(q), -sin(q), 0.0, length],
                 [sin(q) * cos(twist),cos(q) * cos(twist), -sin(twist), -sin(twist) * Offset],
                 [sin(q) * sin(twist),cos(q) * sin(twist), cos(twist), cos(twist) * Offset],
                 [0.0, 0.0, 0.0, 1.0]])
    return TF

```

```

T0_1 = TF_Matrix(linkTwist0, linkLength0, linkOffset1,joint1).subs(DH_Table)
T1_2 = TF_Matrix(linkTwist1, linkLength1, linkOffset2,joint2).subs(DH_Table)
T2_3 = TF_Matrix(linkTwist2, linkLength2, linkOffset3,joint3).subs(DH_Table)
T3_4 = TF_Matrix(linkTwist3, linkLength3, linkOffset4,joint4).subs(DH_Table)
T4_5 = TF_Matrix(linkTwist4, linkLength4, linkOffset5,joint5).subs(DH_Table)
T5_6 = TF_Matrix(linkTwist5, linkLength5, linkOffset6,joint6).subs(DH_Table)
T6_EE =TF_Matrix(linkTwist6, linkLength6, linkOffset7,joint7).subs(DH_Table)
T0_EE = T0_1 * T1_2 * T2_3 * T3_4 * T4_5 * T5_6 * T6_EE;

```

$${}_0T^1 = \begin{bmatrix} \cos(joint\ 1) & -\sin(joint\ 1) & 0.0 & 0.0 \\ \sin(joint\ 1) & \cos(joint\ 1) & 0 & 0 \\ 0 & 0 & 1 & 0.7500000000000000 \\ 0.0 & 0.0 & 0.0 & 1.0000000000000000 \end{bmatrix}$$

$${}_1T^2 = \begin{bmatrix} \cos(joint\ 1) & -\sin(joint\ 1) & 0.0 & 0.0 \\ \sin(joint\ 1) & \cos(joint\ 1) & 0 & 0 \\ 0 & 0 & 1 & 0.7500000000000000 \\ 0.0 & 0.0 & 0.0 & 1.0000000000000000 \end{bmatrix}$$

$${}_2T^3 = \begin{bmatrix} \cos(joint\ 3) & -\sin(joint\ 3) & 0.0 & 1.250 \\ \sin(joint\ 3) & \cos(joint\ 3) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

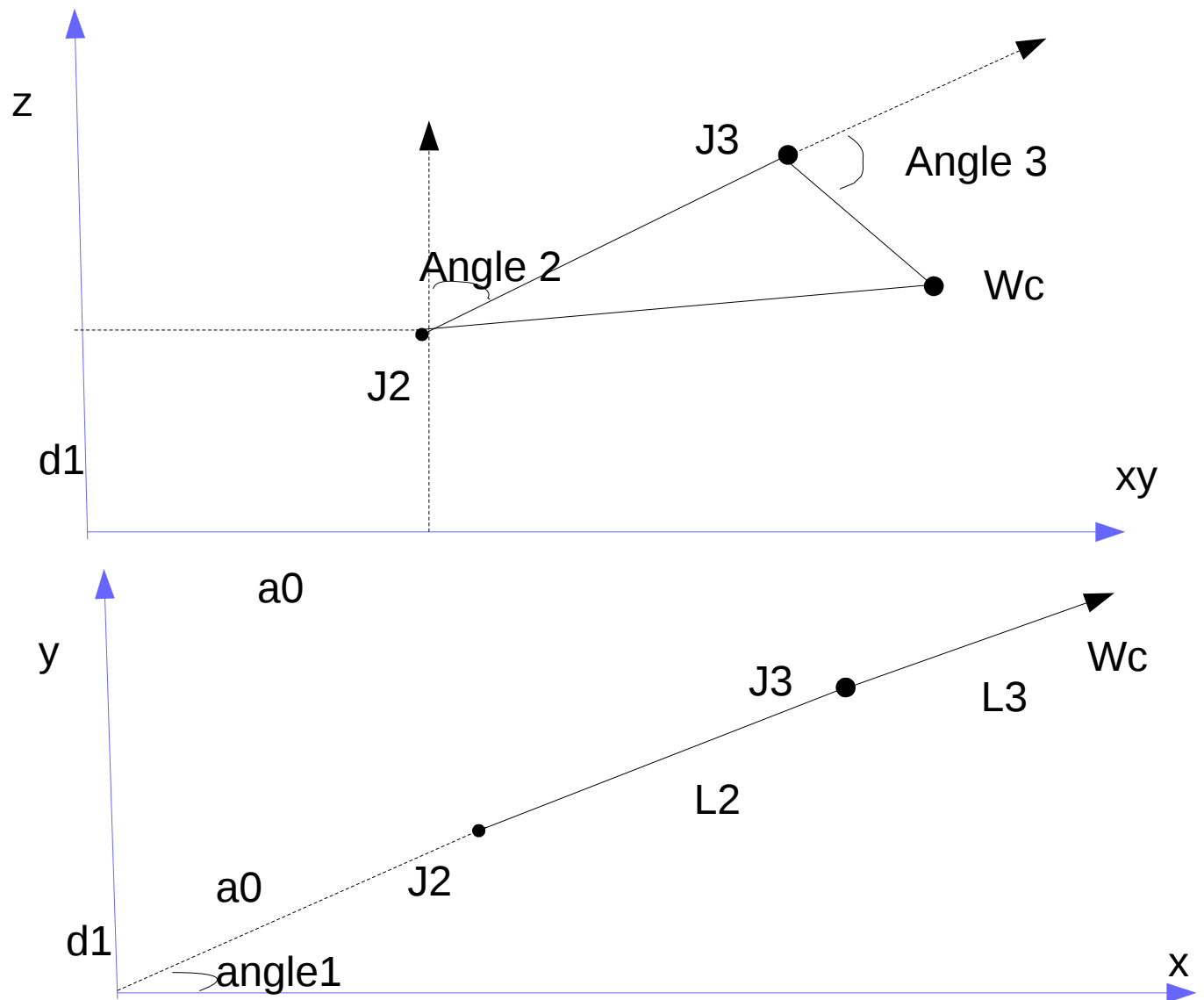
$${}_3T^4 = \begin{bmatrix} \cos(joint\ 4) & -\sin(joint\ 4) & 0.0 & -0.0540 \\ 0 & 0 & 1 & 1.50 \\ -\sin(joint\ 4) & -\cos(joint\ 4) & 0 & 0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}$$

$$\begin{aligned}
{}_4T^5 &= \begin{bmatrix} \cos(\text{joint } 5) & -\sin(\text{joint } 5) & 0.0 & 0.0 \\ 0 & 0 & -1 & 0 \\ \sin(\text{joint } 5) & \cos(\text{joint } 5) & 0 & 0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix} \\
{}_5T^6 &= \begin{bmatrix} \cos(\text{joint } 6) & -\sin(\text{joint } 6) & 0.0 & 0.0 \\ 0 & 0 & 1 & 0 \\ -\sin(\text{joint } 6) & -\cos(\text{joint } 6) & 0 & 0 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix} \\
{}_5T^6 &= \begin{bmatrix} 1 & 0 & 0.0 & 0.0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0.3030 \\ 0.0 & 0.0 & 0.0 & 1.0 \end{bmatrix}
\end{aligned}$$

Inverse Kinematics

The problem is broken into two parts, one for the revolute arm, another for a spherical arm. The revolute arm would return the joint angles for joints 1 through 3 and the spherical arm for joints 4 through 6.

So for the revolute part we would have:



For angle 1, it only lies on the xy plane so is $\text{atan2}(y,x)$

For the 2nd and 3rd angles, on the z plane use the cosine rule to obtain the angle first for angle 3, then calculating angle 2.

$L3 =$

$$\begin{aligned}
 l_2 &= \text{angle } 2 \\
 l_3 &= \sqrt{(a^2 + d^2)} \\
 \text{Angle } 3 &= 180 - \theta_3 \\
 xy &= \sqrt{(Wc_x^2 + Wc_y^2)} - a_0
 \end{aligned}$$

using the cosine rule

$$\begin{aligned}
 D^2 &= xy^2 + z^2 = l_2^2 + l_3^2 - 2(l_2)(l_3)\cos(\text{Angle } 3) \\
 \cos(\text{angle } 3) &= (xy^2 + z^2 - l_3^2 - l_2^2) / (2 * l_3 * l_2) = r \\
 \text{angle } 3 &= \text{atan2}(\sqrt{(1-r^2)}, r), \text{ where } \sqrt{(1-r^2)} = \sin(\text{angle } 3) \\
 \text{angle } 3 &= \text{atan2}(-\sqrt{(1-r^2)}, r) \\
 \text{angle } 2 = y - a_0 &= \text{atan2}(xy, z) - \text{atan2}(l_3 * \sin(\text{angle } 3), l_2 + l_3 * \cos(\text{angle } 3)) \quad \text{angle } 3 = \text{angle } 3 - \pi/2
 \end{aligned}$$

Angle 3 has to be -90 degrees as the starting point is horizontally right not vertically up.