## Deep learning inference of the neutron star equation of state

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## Supplementary material

$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Classification NN								
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	n	N	Structure	batch size	epochs	binary accuracy	val. binary acc.		
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	100	15	$(30) \rightarrow 35$ , relu $\rightarrow 2$ , sigmoid	50	100	$0.754 \pm 0.015$	$0.753 \pm 0.016$		
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	100	15	$(30) \rightarrow 70$ , relu $\rightarrow 2$ , sigmoid	50	100	$0.802 \pm 0.008$	$0.802 \pm 0.009$		
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	100	15	$(30) \rightarrow 80$ , relu $\rightarrow 70$ , relu $\rightarrow 2$ ,	50	100	$0.817 \pm 0.002$	$0.809 \pm 0.009$		
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$			sigmoid						
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	200	15	$(30) \rightarrow 80$ , relu $\rightarrow 70$ , relu $\rightarrow 2$ ,	100	100	$0.823 \pm 0.002$	$0.818 \pm 0.008$		
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$			sigmoid						
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	100	20	$(40) \rightarrow 35$ , relu $\rightarrow 2$ , sigmoid	50	100	$0.759 \pm 0.013$	$0.753 \pm 0.015$		
$\begin{array}{ c c c c c c c c c }\hline 100 & 20 & (40) \rightarrow 80, \ relu \rightarrow 70, \ relu \rightarrow 2, \ 50 & 100 & 0.825 \pm 0.004 & 0.809 \pm 0.017\\ \hline sigmoid & 20 & (40) \rightarrow 120, \ relu \rightarrow 2, \ sigmoid & 100 & 100 & 0.844 \pm 0.003 & 0.842 \pm 0.007\\ \hline 200 & 20 & (40) \rightarrow 120, \ relu \rightarrow 120, \ relu \rightarrow & 100 & 100 & 0.862 \pm 0.001 & 0.845 \pm 0.004\\ \hline 110, \ relu \rightarrow 2, \ sigmoid & & & & & & & & \\ \hline 300 & 20 & (40) \rightarrow 120, \ relu \rightarrow & 120, \ relu \rightarrow & 100 & 100 & 0.862 \pm 0.002 & 0.849 \pm 0.005\\ \hline 110, \ relu \rightarrow 2, \ sigmoid & 50 & 100 & 0.804 \pm 0.008 & 0.791 \pm 0.014\\ \hline 100 & 30 & (60) \rightarrow & 120, \ relu \rightarrow & 2, \ sigmoid & 50 & 100 & 0.832 \pm 0.006 & 0.818 \pm 0.014\\ \hline 200 & 30 & (60) \rightarrow & 120, \ relu \rightarrow & 2, \ sigmoid & 100 & 100 & 0.843 \pm 0.007 & 0.830 \pm 0.009\\ \hline 200 & 30 & (60) \rightarrow & 200, \ relu \rightarrow & 2, \ sigmoid & 100 & 100 & 0.876 \pm 0.003 & 0.866 \pm 0.008\\ \hline 300 & 30 & (60) \rightarrow & 120, \ relu \rightarrow & 2, \ sigmoid & 100 & 100 & 0.854 \pm 0.007 & 0.846 \pm 0.012\\ \hline 300 & 30 & (60) \rightarrow & 180, \ relu \rightarrow & 2, \ sigmoid & 100 & 100 & 0.879 \pm 0.002 & 0.873 \pm 0.011\\ \hline 300 & 30 & (60) \rightarrow & 215, \ relu \rightarrow & 2, \ sigmoid & 100 & 100 & 0.884 \pm 0.002 & 0.878 \pm 0.007\\ \hline 300 & 30 & (60) \rightarrow & 215, \ relu \rightarrow & 2, \ sigmoid & 100 & 100 & 0.884 \pm 0.002 & 0.878 \pm 0.007\\ \hline 300 & 30 & (60) \rightarrow & 215, \ relu \rightarrow & 2, \ sigmoid & 100 & 100 & 0.889 \pm 0.001 & 0.865 \pm 0.016\\ \hline \end{array}$	100	20	$(40) \rightarrow 70$ , relu $\rightarrow 2$ , sigmoid	50	100	$0.804 \pm 0.007$	$0.798 \pm 0.010$		
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	100	20	$(40) \rightarrow 120$ , relu $\rightarrow 2$ , sigmoid	50	100	$0.833 \pm 0.005$	$0.829 \pm 0.008$		
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	100	20	$(40) \rightarrow 80$ , relu $\rightarrow 70$ , relu $\rightarrow 2$ ,	50	100	$0.825 \pm 0.004$	$0.809 \pm 0.017$		
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$			sigmoid						
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	200	20	$(40) \rightarrow 120$ , relu $\rightarrow 2$ , sigmoid	100	100	$0.844 \pm 0.003$	$0.842 \pm 0.007$		
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	200	20	$(40) \rightarrow 120$ , relu $\rightarrow 120$ , relu $\rightarrow$	100	100	$0.862 \pm 0.001$	$0.845 \pm 0.004$		
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$									
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	300	20		100	100	$0.862 \pm 0.002$	$0.849 \pm 0.005$		
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$			110, relu $\rightarrow$ 2, sigmoid						
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	100	30	$(60) \rightarrow 70$ , relu $\rightarrow 2$ , sigmoid	50	100	$0.804 \pm 0.008$	$0.791 \pm 0.014$		
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	100	30			100	$0.832 \pm 0.006$	$0.818 \pm 0.014$		
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	200	30	$(60) \rightarrow 120$ , relu $\rightarrow 2$ , sigmoid	100	100	$0.843 \pm 0.007$	$0.830 \pm 0.009$		
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	200	30	$(60) \rightarrow 200$ , relu $\rightarrow 2$ , sigmoid	100	100	$0.876 \pm 0.003$	$0.866 \pm 0.008$		
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	300	30	$(60) \rightarrow \overline{120}$ , relu $\rightarrow 2$ , sigmoid		100		$0.846 \pm 0.012$		
300 30 $(60) \rightarrow 215$ , relu $\rightarrow 200 \rightarrow 2$ , sig- 100 100 $0.889 \pm 0.001$ $0.865 \pm 0.016$	300	30	$(60) \rightarrow \overline{180}$ , relu $\rightarrow 2$ , sigmoid	100	100	$0.879 \pm 0.002$	$0.873 \pm 0.011$		
	300	30	$(60) \rightarrow 215$ , relu $\rightarrow 2$ , sigmoid	100	100	$0.884 \pm 0.002$	$0.878 \pm 0.007$		
Lieu Lieu Lieu Lieu Lieu Lieu Lieu Lieu	300	30	$(60) \rightarrow 215$ , relu $\rightarrow 200 \rightarrow 2$ , sig-	100	100	$0.889 \pm 0.001$	$0.8\overline{65} \pm 0.016$		
IIIOIG			moid						

Table 1: Summary of the most efficient models we trained for the classification network for the M-R input model. We vary the number (n) of noise injections we perform on each of the doublet, the size of the observation set (N), the number of layers and nodes. We trained each model for 100 epochs, and we found that a batch size of 100 is optimal for most configurations. However, for most of the cases where n = 100, a batch size of 50 was preferred. We trained each network 20 times, and report the mean binary accuracy and validation binary accuracy with their respective standard deviation.

		Classi	ification NN			
n	N	Structure	batch size	epochs	binary accuracy	val. binary ac- curacy
100	15	$(45) \rightarrow 35$ , relu $\rightarrow 2$ , sigmoid	50	100	$0.761 \pm 0.013$	$0.754 \pm 0.013$
100	15	$(45) \rightarrow 90$ , relu $\rightarrow 2$ , sigmoid	50	100	$0.808 \pm 0.003$	$0.803 \pm 0.007$
100	15	$(45) \rightarrow 90$ , relu $\rightarrow 70$ , relu $\rightarrow 2$ , sigmoid	50	100	$0.816 \pm 0.003$	$0.806 \pm 0.006$
200	15	$(45) \rightarrow 90$ , relu $\rightarrow 2$ , sigmoid	100	100	$0.821 \pm 0.002$	$0.813 \pm 0.008$
200	15	$(45) \rightarrow 90$ , relu $\rightarrow 70$ , relu $\rightarrow 2$ , sigmoid	100	100	$0.827 \pm 0.002$	$0.821 \pm 0.008$
100	20	$(60) \rightarrow 90$ , relu $\rightarrow 2$ , sigmoid	50	100	$0.808 \pm 0.006$	$0.800 \pm 0.011$
100	20	$(60) \rightarrow 120$ , relu $\rightarrow 2$ , sigmoid	50	100	$0.821 \pm 0.006$	$0.812 \pm 0.009$
100	20	$(60) \rightarrow 120$ , relu $\rightarrow 70$ , relu $\rightarrow 2$ , sigmoid	50	100	$0.831 \pm 0.003$	$0.817 \pm 0.006$
200	20	$(60) \rightarrow 120$ , relu $\rightarrow 2$ , sigmoid	100	100	$0.839 \pm 0.005$	$0.833 \pm 0.011$
200	20	$(60) \rightarrow 120$ , relu $\rightarrow 70$ , relu $\rightarrow 2$ , sigmoid	100	100	$0.845 \pm 0.003$	$0.835 \pm 0.012$
300	20	$(60) \rightarrow 120$ , relu $\rightarrow 2$ , sigmoid	100	100	$0.849 \pm 0.003$	$0.845 \pm 0.006$
300	20	$(60) \rightarrow 120$ , relu $\rightarrow 70$ , relu $\rightarrow 2$ , sigmoid	100	100	$0.853 \pm 0.003$	$0.848 \pm 0.005$
100	30	$(90) \rightarrow 120$ , relu $\rightarrow 2$ , sigmoid	50	100	$0.818 \pm 0.007$	$0.802 \pm 0.012$
100	30	$(90) \rightarrow 180$ , relu $\rightarrow 2$ , sigmoid	50	100	$0.830 \pm 0.004$	$0.819 \pm 0.012$
100	30	$ \begin{array}{c} (90) \rightarrow 180, \text{ relu } \rightarrow 120, \text{ relu } \rightarrow \\ 2, \text{ sigmoid} \end{array} $	50	100	$0.840 \pm 0.005$	$0.822 \pm 0.011$
200	30	$(90) \rightarrow 180$ , relu $\rightarrow 2$ , sigmoid	100	100	$0.850 \pm 0.005$	$0.844 \pm 0.008$
200	30	$(90) \rightarrow 180$ , relu $\rightarrow 120$ , relu $\rightarrow$ 2, sigmoid	100	100	$0.867 \pm 0.003$	$0.853 \pm 0.009$
300	30	$(90) \rightarrow 180$ , relu $\rightarrow 2$ , sigmoid	100	100	$0.862 \pm 0.006$	$0.857 \pm 0.008$
300	30	$(90) \rightarrow 180$ , relu $\rightarrow 120$ , relu $\rightarrow$ 2, sigmoid	100	100	$0.876 \pm 0.003$	$0.864 \pm 0.012$
300	30	$(90) \rightarrow 270$ , relu $\rightarrow 2$ , sigmoid	100	100	$0.878 \pm 0.003$	$0.872 \pm 0.011$

Table 2: Summary of the most efficient models we trained for the classification network for the M-R- $k_2$  input model. We vary the number (n) of noise injections we perform on each of the triplet, the size of the observation set (N), the number of layers and nodes. We trained each model for 100 epochs, and we found that a batch size of 100 is optimal for most configurations. However, for most of the cases where n = 100, a batch size of 50 was preferred. We trained each network 20 times, and report the mean binary accuracy and validation binary accuracy with their respective standard deviation.

		Regression N	NN					
n	N	Structure	batch size	epochs	msle train.	loss	msle val_los	s
100	15	$(30) \rightarrow 35$ , relu $\rightarrow 15$ , sigmoid	100	100	0.01819	±	0.01823	±
					0.00003		0.00003	
100	15	$(30) \rightarrow 45$ , relu $\rightarrow 15$ , sigmoid	100	100	0.01815	±	0.01814	±
					0.00003		0.00003	
200	15	$(30) \rightarrow 35$ , relu $\rightarrow 15$ , sigmoid	100	100	0.01809	±	0.01810	±
					0.00002		0.00003	
200	15	$(30) \rightarrow 45$ , relu $\rightarrow 15$ , sigmoid	100	100	0.01806	土	0.01807	±
					0.00002		0.00003	
100	20	$(40) \rightarrow 35$ , relu $\rightarrow 15$ , sigmoid	100	100	0.01814	土	0.01821	±
					0.00003		0.00004	
100	20	$(40) \rightarrow 45$ , relu $\rightarrow 15$ , sigmoid	100	100	0.01810	$\pm$	0.01818	$\pm$
					0.00004		0.00004	
200	20	$(40) \rightarrow 35$ , relu $\rightarrow 15$ , sigmoid	100	100	0.01807	$\pm$	0.01811	$\pm$
					0.00004		0.00004	
200	20	$(40) \rightarrow 45$ , relu $\rightarrow 15$ , sigmoid	100	100	0.01804	土	0.01808	±
					0.00002		0.00003	
100	30	$(60) \rightarrow 45$ , relu $\rightarrow 15$ , sigmoid	100	100	0.01809	$\pm$	0.01814	$\pm$
					0.00003		0.00005	
200	30	$(60) \rightarrow 60$ , relu $\rightarrow 15$ , sigmoid	100	100	0.01802	土	0.01807	±
					0.00002		0.00005	
300	30	$(60) \rightarrow 60$ , relu $\rightarrow 15$ , sigmoid	100	100	0.01800	±	0.01804	±
					0.00002		0.00004	
300	30	$(60) \rightarrow 70$ , relu $\rightarrow 15$ , sigmoid	100	100	0.01799	土	0.01802	±
					0.00002		0.00004	

Table 3: Summary of the most efficient models we trained for the regression network for the M-R input model. We vary the number (n) of noise injections we perform on each of the triplet, the size of the observation set (N), the number of layers and nodes. We trained each model for 100 epochs, and we found that a batch size of 100 is the most optimal choice. We trained each network 20 times, and report the mean training and validation losses with their respective standard deviation.

		Regression N	N					
n	N	Structure	batch size	epochs	msle train.	loss	msle val_loss	
100	15	$(45) \rightarrow 45$ , relu $\rightarrow 15$ , sigmoid	100	100	0.01815	±	0.01816	±
					0.00003		0.00006	
100	15	$(45) \rightarrow 65$ , relu $\rightarrow 15$ , sigmoid	100	100	0.01809	±	0.01810	±
					0.00001		0.00003	
200	15	$(45) \rightarrow 45$ , relu $\rightarrow 15$ , sigmoid	100	100	0.01810	土	0.01817	±
					0.00003		0.00005	
200	15	$(45) \rightarrow 65$ , relu $\rightarrow 15$ , sigmoid	100	100	0.01804	土	0.01811	±
					0.00002		0.00003	
200	15	$(45) \rightarrow 90$ , relu $\rightarrow 15$ , sigmoid	100	100	0.01813	±	0.01808	±
					0.00009		0.00003	
100	20	$(60) \rightarrow 65$ , relu $\rightarrow 15$ , sigmloid	100	100	0.01807	$\pm$	0.01813	±
					0.00002		0.00006	
100	20	$(60) \rightarrow 90$ , relu $\rightarrow 15$ , sigmoid	100	100	0.01803	±	0.01808	±
					0.00001		0.00002	
100	20	$(60) \rightarrow 120$ , relu $\rightarrow 15$ , sigmoid	100	100	0.01801	±	0.01806	±
					0.00001		0.00004	
200	20	$(60) \rightarrow 90$ , relu $\rightarrow 15$ , sigmoid	100	100	0.01798	±	0.01805	±
					0.00001		0.00002	
200	20	$(60) \rightarrow 120$ , relu $\rightarrow 15$ , sigmoid	100	100	0.01798	±	0.01799	±
					0.00001		0.00003	
300	20	$(60) \rightarrow 120$ , relu $\rightarrow 15$ , sigmoid	100	100	0.01795	土	0.01800	Ŧ
					0.00001		0.00002	
100	30	$(90) \rightarrow 90$ , relu $\rightarrow 15$ , sigmoid	100	100	0.01805	±	0.01808	±
					0.00001		0.00004	
100	30	$(90) \rightarrow 180$ , relu $\rightarrow 15$ , sigmoid	100	100	0.01799	±	0.01804	±
					0.00001		0.00003	
200	30	$(90) \rightarrow 90$ , relu $\rightarrow 15$ , sigmoid	100	100	0.01802	±	0.01803	±
					0.00001		0.00003	
200	30	$(90) \rightarrow 180$ , relu $\rightarrow 15$ , sigmoid	100	100	0.01796	±	0.01799	±
					0.00001		0.00004	
300	30	$(90) \rightarrow 90$ , relu $\rightarrow 15$ , sigmoid	100	100	0.01799	±	0.01800	±
					0.00002		0.00003	
300	30	$(90) \rightarrow 180$ , relu $\rightarrow 15$ , sigmoid	100	100	0.01794	±	0.01796	+
					0.00001		0.00006	

Table 4: Summary of the most efficient models we trained for the regression network for the M-R- $k_2$  input model. We vary the number (n) of noise injections we perform on each of the triplet, the size of the observation set (N), the number of layers and nodes. We trained each model for 100 epochs, and we found that a batch size of 100 is the most optimal choice. We trained each network 20 times, and report the mean training and validation losses with their respective standard deviation.