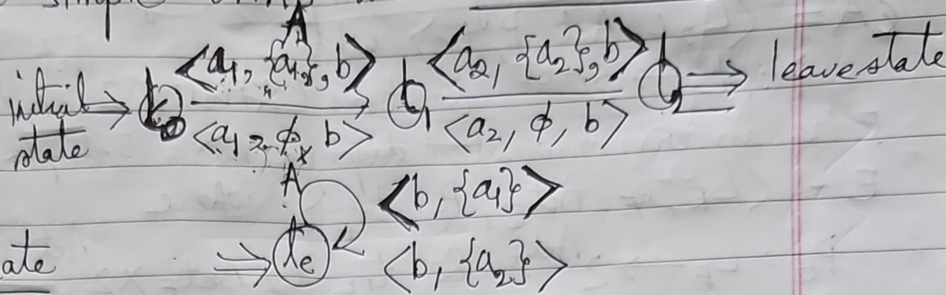
OMAS to Petrinet Encoding Algorithm

We consider a simple OMAS with a leave state

Agent Template



Environment Template

Agent Locations $\{l_0, l_1, l_2\}$

Agent Actions $\{a_1, a_2\}$

Agent Protocols
$P(l_0) = \{a_1\}$
$P(l_1) = \{a_2\}$
$P(l_2) = \phi$

Agent Transitions : $tr_1 = \boxed{\langle l_0, a_1, \{a_1\}, b, l_1\rangle}$
$tr_2 = \langle l_1, a_2, \{a_2\}, b, l_2\rangle$

Environment Locations : $\{l_e\}$ Environment actions $\{b\}$

Env. Protocols : $P(l_e) = \{b\}$

Env. Transitions : $tre_1 = \boxed{\langle l_e, b, \{a_1\}, l_e\rangle}$
$tre_2 = \langle l_e, b, \{a_2\}, l_e\rangle$

$A_1 = \{a_2, a_2, \_\_\_ a_n\}$

$A_2 = \{\}$

Given a regular system $\Theta = \langle A, E \rangle$ we define a Petri net $N_\Theta = \langle P_\Theta, T_\Theta, F_\Theta, W_\Theta \rangle$ which encodes the behaviour of Θ. A is an agent template defined as a 5-tuple.
$A = \langle L, \iota, Act, P, tr \rangle$ where
$P : L \rightarrow 2^{Act}$ & $tr : L \times Act \times 2^{Act} \times Act_E \rightarrow L$
E is an environment defined as a 5-tuple
$E = \langle L_E, \iota_E, Act_E, P_E, tr_E \rangle$ where
$P_E : L_E \rightarrow 2^{Act_E}$ & $tr_E : L_E \times Act_E \times 2^{Act} \rightarrow L_E$

The encoding N_Θ is defined as follows:
(*) $P_\Theta = L \uplus L_E$
(*) $T_\Theta, F_\Theta, W_\Theta$ are defined inductively as follows:

Base Case: $T_\Theta = \{t_{aj}, t_{al}\}$
$F_\Theta = \{\langle t_{aj}, \iota \rangle, \langle leave, t_{al} \rangle\}$
$W_\Theta = \{\langle t_{aj}, \iota \rangle \mapsto 1, \langle leave, t_{al} \rangle \mapsto 1\}$

Induction case: For every environment transition $et = \langle l_e, b, A, l_e' \rangle$ in tr_E, we augment T_Θ, ~~F_Θ, W_Θ~~ as follows:
Let $A = \{a_1, \ldots a_k\}$ i.e. $|A| = k$. For every $k \le n \le 2k$, ~~compute~~ consider unordered n-tuples from tr^n of the kind:
$\langle l_1, a_1, A_1, b, l_1' \rangle \langle l_2, a_2, A_2, b, l_2' \rangle \ldots \langle l_n, a_n, A_n, b, l_n' \rangle$ gt
If gt satisfies the following conditions then $gt \cdot \langle l_e, b, A, l_e' \rangle$ is added to TR_{et}

① $\{a_1, a_2, \ldots a_n\} = A$ ‖ For any $a \in A$, a occurs at most twice in the sequence $\langle a_1, a_2, \ldots, a_n \rangle$

② For every i; $1 \le i \le 2n$: $A_i = \{a_1, a_2, \ldots a_{i-1}, a_{i+1}, \ldots a_n\}$

$TR = \bigcup_{et \in tr_E} TR_{et}$ $T_\Theta = T_\Theta \uplus \{t_{tr} \mid tr \in TR\}$

After we have constructed T_Θ, from TR, we augment F_Θ & W_Θ as follows. For every $tr = \langle l_1, a_1, A_1, l_1' \rangle \ldots \langle l_n, a_n, A_n, l_n' \rangle \langle l_e, b, A, l_e' \rangle$ in TR, let pre(tr) & post(tr):
$pre(tr) = \{l_1, l_2, \ldots, l_n, l_e\}$
$post(tr) = \{l_1', l_2', \ldots l_n', l_e'\}$

We let f^{tr} & w^{tr} as follows:

$$f^{tr} = \{(l, t_{tr}) \mid l \in pre(tr)\} \cup \{(t_{tr}, l') \mid l' \in post(tr)\}$$

$$w^{tr} = \{(l, tr) \mapsto n_l^{tr} \mid l \in pre(tr)\} \cup \{(t_{tr}, l') \mapsto n_{l'}^{tr}\}$$

n_l^{tr} — no. of time l occurs in the tuple $\langle l_1, l_2, \dots, l_n \rangle$

$n_{l'}^{tr}$ — no. of times l' ———— $\langle l_1', l_2', \dots, l_n' \rangle$

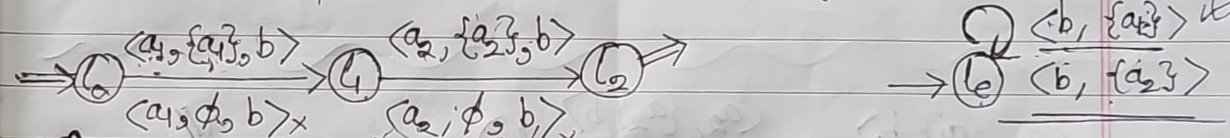
$$TR = \bigcup_{tr \in TR} f^{tr} \qquad WF = \bigcup_{tr \in TR} w^{tr}$$

$$F_\theta = F_\theta \cup TR \qquad W_\theta = W_\theta \cup WF$$

S n

Now, we will apply this construction to at least two inputs: Done

$Act = \{a_1, a_2\}$



tr $n = 1$ Agent l_e b, A, $|A| = 1$

TR $A \subseteq Act$ $1 \le n \le 2$

$\langle b, \{a_1\} \rangle$

$1 \le n \le 2$

$\langle l_0, \{a_1\}, b, l_1 \rangle \langle l_e, b, \{a_1\}, l_e \rangle \times$

$\boxed{\langle l_0, \{\}, b, l_1 \rangle \langle l_e, b, \{a_1\}, l_e \rangle}$ ✓

$\langle l_1, a_2, \{a_2\}, b, l_2 \rangle \langle l_e, b \{a_1\}, l_e \rangle \times$

$\langle l_1, a_2, \phi, b, l_2 \rangle \langle l_e, b, \{a_1\}, l_e \rangle \times$

$\boxed{\langle l_0, a_1, \{\}, b, l_1 \rangle \langle l_0, a_1, \{a_1\}, b, l_1 \rangle} \times$

$\langle l_0, a_1, \phi, b, l_1 \rangle \langle l_0, a_1, \{a_1\}, b, l_1 \rangle \times$

$\langle l_0, a_1, \{a_1\}, b, l_1 \rangle \langle l_0, a_1, \{a_1\}, b, l_1 \rangle$ ✓

$\langle l_0, a_1, \{a_1\}, b, l_1 \rangle$

$TR_{\langle b, \{a_2\} \rangle}$ $1 \le n \le 2$

So, $TR = \{ \overset{t_1}{(\langle l_0, a_1, \phi, b, l_1 \rangle \langle l_e, b, \{a_1\}, l_e' \rangle}$

$\overset{t_2}{(\langle l_0, a_2, \phi, b, l_1 \rangle \langle l_e, b, \{a_2\}, l_e' \rangle)}$

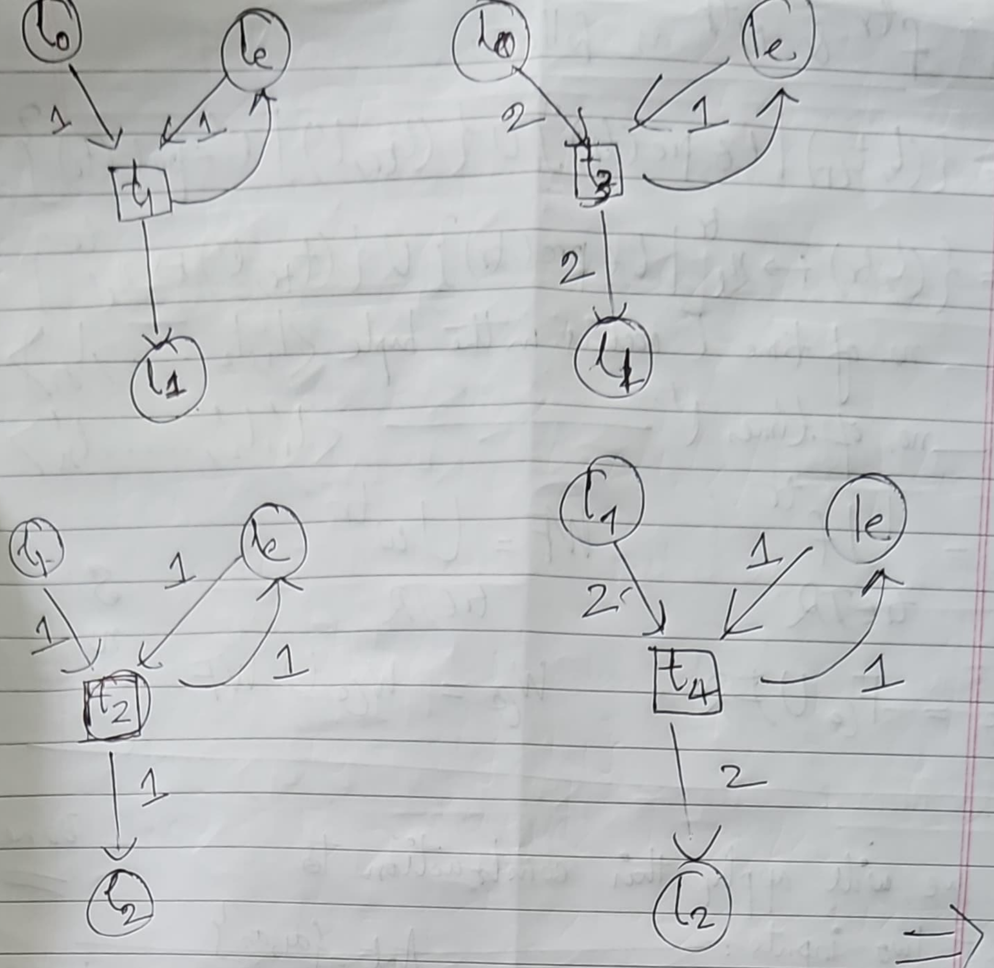
$(\langle l_0, a_1, \{a_1\}, b, l_1 \rangle \langle l_0, a_1, \{a_1\}, b, l_1 \rangle$
t_3 $\langle l_e, b, \{a_1\}, l_e' \rangle$

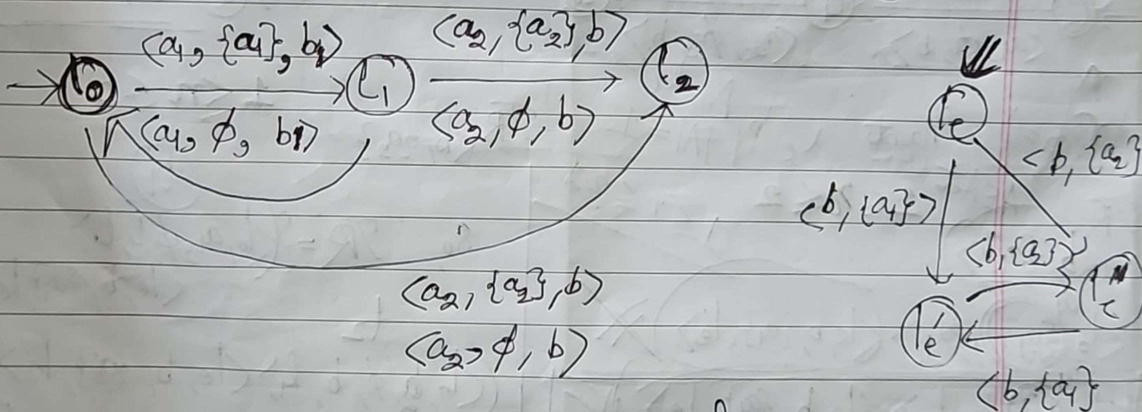
$(\langle l_1, a_2, \{a_2\}, b, l_2 \rangle \langle l_1, a_2, \{a_2\}, b, l_2 \rangle$
t_4 $\langle l_e, b, \{a_2\}, l_e' \rangle \}$

$\langle l_e, b, \{a_1\}, l_e \rangle$

$\ne \phi$

$A = \{a_2, \dots, a_n\}$





Doubling in the global joint transitions is happening due to ~~the~~ the presence of two different environment transitions each for $\langle b, \{a_1\}\rangle$ & $\langle b, \{a_2\}\rangle$

$\langle l_e, b, \{a_1\}, l_e' \rangle \oplus TR_{\langle l_e'', b, \{a_1\}, l_e' \rangle}$

$TR_{\langle l_e', b, \{a_2\}, l_e'' \rangle} \oplus TR_{\langle l_e, b, \{a_2\}, l_e'' \rangle}$

$$TR_{\langle b, \{a_1\} \rangle} = \Big\{ \underbrace{(\langle l_0, a_1, \phi, b, l_1 \rangle)}^{t_1},\ \underbrace{(\langle l_0, a_1, \{a_1\}, b, l_1 \rangle\ \langle l_0, a_1, \{a_1\}, b, l_1 \rangle)}^{t_3},$$

$$\underbrace{(\langle l_1, a_1, \phi, b, l_0 \rangle)}_{t_2},\ \underbrace{(\langle l_1, a_1, \{a_1\}, b, l_0 \rangle\ \langle l_1, a_1, \{a_1\}, b, l_0 \rangle)}^{t_4}$$

$$\underbrace{(\langle l_0, a_1, \{a_1\}, b, l_1 \rangle\ \langle l_1, a_1, \{a_1\}, b, l_0 \rangle)}^{t_5} \Big\}$$

$1 \leq n \leq 2$

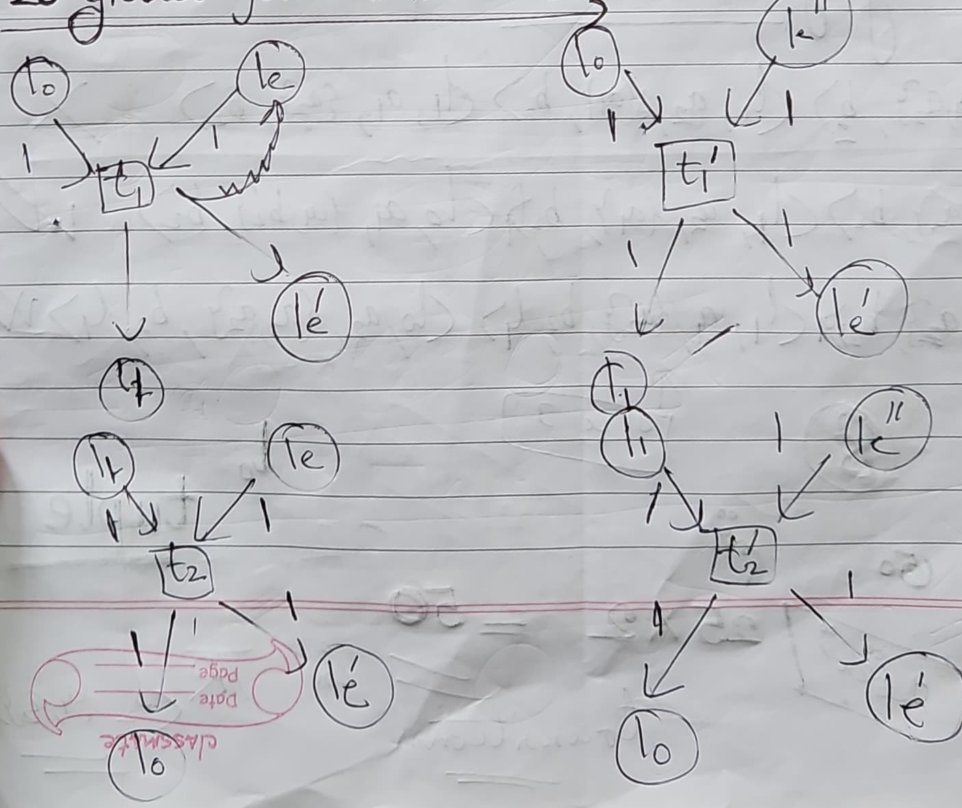
$$TR_{\langle b, \{a_2\} \rangle} = \Big\{ \underbrace{(\langle l_0, a_2, \phi, b, l_2 \rangle)}^{t_6},\ \underbrace{(\langle l_0, a_2, \{a_2\}, b, l_2 \rangle\ \langle l_0, a_2, \{a_2\}, b, l_2 \rangle)}^{t_8}$$

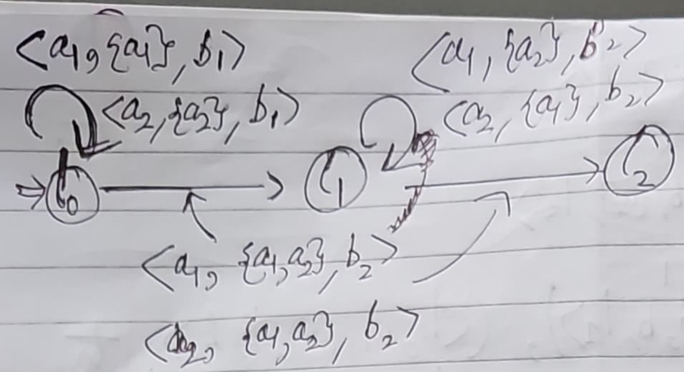
$$\underbrace{(\langle l_1, a_2, \phi, b, l_2 \rangle)}_{t_7},\ \underbrace{(\langle l_1, a_2, \{a_2\}, b, l_2 \rangle\ \langle l_1, a_2, \{a_2\}, b, l_2 \rangle)}^{t_9}$$

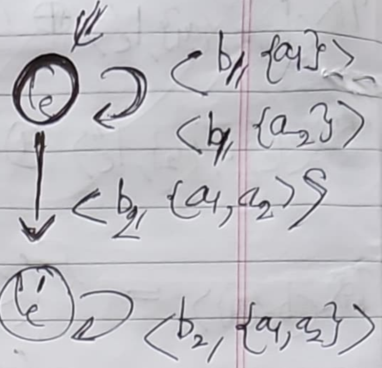
$$t_{10}\ (\langle l_0, a_2, \{a_2\}, b, l_2 \rangle\ \langle l_1, a_2, \{a_2\}, b, \{l_2\} \rangle) \Big\}$$

$1 \leq n \leq 2$

20 global joint transitions







$$TR_{\langle b_1, \{a_1\}\rangle} = \{(\langle l_0, a_1, \{a_1\}, b_1, l_0\rangle \langle l_0, a_1, \{a_1\}, b_1, l_0\rangle)\}$$

$$TR_{\langle b_1, \{a_2\}\rangle} = \{(\langle l_0, a_2, \{a_2\}, b_1, l_0\rangle \langle l_0, a_2, \{a_2\}, b_1, l_0\rangle)\}$$

16 $\boxed{\text{size } 4}$

$$TR_{\langle b_2, \{a_1, a_2\}\rangle} = \{(\langle l_0, a_1, \{a_1, a_2\}, b_2, l_1\rangle \langle l_0, a_1, \{a_1, a_2\}, b_2, l_1\rangle$$

$2 \leq n \leq 4$ $\quad \langle l_0, a_2, \{a_1, a_2\}, b_2, l_1\rangle \langle l_0, a_2, \{a_1, a_2\}, b_2, l_1\rangle)$

$\boxed{\text{size } 2} \leftarrow (\langle l_1, a_1, \{a_2\}, b_2, l_1\rangle \langle l_1, a_2, \{a_1\}, b_2, l_1\rangle)$ ✓

1

l_0

Is the following global transition possible

$\langle l_0, a_1, \{a_1, a_2\}, b_2\rangle \langle l_1, a_2, \{a_1\}, b_2\rangle \langle l_1, a_1, \{a_2\}, b_2\rangle$ ✗

$\langle l_0, a_1, \{a_1, a_2\}, b_2\rangle \langle l_1, a_2, \{a_1\}, b_2, l_1\rangle \langle l_0, a_1, \{a_1, b_2\}, b_2\rangle$ ✓

$\langle l_0, a_2, \{a_1, a_2\}, b_2, l_1\rangle \langle l_1, a_1, \{a_2\}, b_2, l_1\rangle \langle l_0, a_2, \{a_1, a_2\}, b_2, l_1\rangle$ ✓

8 $\qquad$ $\boxed{\text{size } 3}$

l_0 $\qquad$ tuple

50

$25 \times 2 = 50$ in $TR_{\langle b_2, \{a_1, a_2\}\rangle}$

transitions

what is the size of $\mathbb{T}_R$?

what is the size of $\mathbb{T}_G$?

what is the size of $\mathbb{F}_G$?

in terms of ① $|L|$ ② $|L_E|$ ③ $|Act|$ ④ $|Act_E|$

$|A| = 5K$

$1K$

$3 \leq a_1, a_2, a_3 \leq 6$

$a_1, a_1, a_2, a_2, a_3, a_3$

$a_1 a_2 a_3 \to b$

$a_1 a_1 a_2 a_3$

$a_1 a_1 a_2 a_2 a_3$

$a_1 a_1 a_2 a_2 a_3 a_3$

$a_1 a_1 a_2 a_2 a_3$

$a_1 a_2 a_2 a_3 a_3$

$a_1 a_2 a_3 a_3$

$a_1 a_1 a_2 a_3 a_3$

$a_1 a_2$

$a_1 a_1 a_2$

$a_1 a_1 a_2 a_2$

$a_1 a_2 a_2$

$\to a_1 a_1 a_2 a_3 a_3 a_2 a_3$

2^k

$(L_i, a, b, \{a_1, a_2, a_3\}, L_f)$

a_1

dictionary

$b + a + act \to L_1 \; L_2$

key

value