

# PETRI NET ENCODINGS FOR REGULAR OPEN MAS

## TEST CASES

### Test Case 1

Let  $\mathcal{O} = \langle A, E \rangle$  be a regular OMAS given as follows:

**Agent Template**  $A = (L, \iota, Act, P, tr, leave)$  with

- $L = \{l_0, l_1, l_2\}$
- $\iota = l_0$
- $Act = \{a_1, a_2\}$
- $P = \{l_0 : a_1, l_1 : a_2\}$
- $leave = l_2$
- $tr = \{\langle l_0, a_1, \{a_1\}, b, l_1 \rangle, \langle l_0, a_1, \emptyset, b, l_1 \rangle, \langle l_1, a_2, \{a_2\}, b, l_2 \rangle, \langle l_1, a_2, \emptyset, b, l_2 \rangle\}$

**Environment**  $E = (L_E, \iota_E, Act_E, P_E, tr_E)$  with

- $L_E = \{l_e\}$
- $\iota_E = l_e$
- $Act_E = \{b\}$
- $P_E = \{l_e : b\}$
- $tr_E = \{\langle l_e, b, \{a_1\}, l_e \rangle, \langle l_e, b, \{a_2\}, l_e \rangle\}$

The equivalent Petri net  $\langle P_{\mathcal{O}}, T_{\mathcal{O}}, F_{\mathcal{O}}, W_{\mathcal{O}} \rangle$  is defined as follows:

- $P_{\mathcal{O}} = L \cup L_E = \{l_0, l_1, l_2, l_e\}$ .
- Recall that the computation of  $T_{\mathcal{O}}$  depends, essentially, on the computation of  $\mathcal{TR}_{\mathcal{O}}$ , which is done iteratively over the transitions in  $tr_E$ . In the present case,  $\mathcal{TR}_{\mathcal{O}} = \mathcal{TR}_{\langle l_e, b, \{a_1\}, l_e \rangle} \cup \mathcal{TR}_{\langle l_e, b, \{a_2\}, l_e \rangle}$ .
  - $\mathcal{TR}_{\langle l_e, b, \{a_1\}, l_e \rangle}$  contains all those global joint transitions which are consistent with environment transition  $\langle l_e, b, \{a_1\}, l_e \rangle$  and crucially depends on  $(b, \{a_1\})$ . Hence,  $\mathcal{TR}_{\langle l_e, b, \{a_1\}, l_e \rangle} = \{(\langle l_0, a_1, \emptyset, b, l_1 \rangle, \langle l_e, b, \{a_1\}, l_e \rangle), (\langle l_0, a_1, \{a_1\}, b, l_1 \rangle, \langle l_0, a_1, \{a_1\}, b, l_1 \rangle, \langle l_e, b, \{a_1\}, l_e \rangle)\}$
  - Similarly,  $\mathcal{TR}_{\langle l_e, b, \{a_2\}, l_e \rangle}$  contains all those global joint transitions which are consistent with environment transition  $\langle l_e, b, \{a_2\}, l_e \rangle$  and crucially depends on  $(b, \{a_2\})$ . Hence,  $\mathcal{TR}_{\langle l_e, b, \{a_2\}, l_e \rangle} = \{(\langle l_1, a_2, \emptyset, b, l_2 \rangle, \langle l_e, b, \{a_2\}, l_e \rangle), (\langle l_1, a_2, \{a_2\}, b, l_2 \rangle, \langle l_1, a_2, \{a_2\}, b, l_2 \rangle, \langle l_e, b, \{a_2\}, l_e \rangle)\}$
- Let us denote the tuples in  $\mathcal{TR}_{\langle l_e, b, \{a_1\}, l_e \rangle}$  by  $\mathfrak{t}_1$  and  $\mathfrak{t}_2$  and  $\mathcal{TR}_{\langle l_e, b, \{a_2\}, l_e \rangle}$  by  $\mathfrak{t}_3$  and  $\mathfrak{t}_4$ , respectively. Then,  $T_{\mathcal{O}} = \{t_{aj}, t_{al}, t_{\mathfrak{t}_1}, t_{\mathfrak{t}_2}, t_{\mathfrak{t}_3}, t_{\mathfrak{t}_4}\}$ .
- The information computed in  $\mathcal{TR}$  helps us to augment  $F_{\mathcal{O}}$  as follows. Recall that  $F_{\mathcal{O}}$  is initialized to be  $\{\langle t_{aj}, l_0 \rangle, \langle l_2, t_{al} \rangle\}$ . We add the following edges to  $F_{\mathcal{O}}$ :  $\{\langle l_0, t_{\mathfrak{t}_1} \rangle, \langle l_e, t_{\mathfrak{t}_1} \rangle, \langle t_{\mathfrak{t}_1}, l_1 \rangle, \langle t_{\mathfrak{t}_1}, l_e \rangle\} \cup \{\langle l_0, t_{\mathfrak{t}_2} \rangle, \langle l_e, t_{\mathfrak{t}_2} \rangle, \langle t_{\mathfrak{t}_2}, l_1 \rangle, \langle t_{\mathfrak{t}_2}, l_e \rangle\} \cup \{\langle l_1, t_{\mathfrak{t}_3} \rangle, \langle l_e, t_{\mathfrak{t}_3} \rangle, \langle t_{\mathfrak{t}_3}, l_2 \rangle, \langle t_{\mathfrak{t}_3}, l_e \rangle\} \cup \{\langle l_1, t_{\mathfrak{t}_4} \rangle, \langle l_e, t_{\mathfrak{t}_4} \rangle, \langle t_{\mathfrak{t}_4}, l_2 \rangle, \langle t_{\mathfrak{t}_4}, l_e \rangle\}$ .
- Similarly, the initial content of  $W_{\mathcal{O}} = \{\langle t_{aj}, l_0 \rangle \mapsto 1, \langle l_2, t_{al} \rangle \mapsto 1\}$ . We augment  $W_{\mathcal{O}}$  with the following weights computed from  $\mathcal{TR}$ , corresponding to the edges in  $F_{\mathcal{O}}$ :  $\{\langle l_0, t_{\mathfrak{t}_1} \rangle \mapsto 1, \langle l_e, t_{\mathfrak{t}_1} \rangle \mapsto 1, \langle t_{\mathfrak{t}_1}, l_1 \rangle \mapsto 1, \langle t_{\mathfrak{t}_1}, l_e \rangle \mapsto 1\} \cup \{\langle l_0, t_{\mathfrak{t}_2} \rangle \mapsto 2, \langle l_e, t_{\mathfrak{t}_2} \rangle \mapsto 1, \langle t_{\mathfrak{t}_2}, l_1 \rangle \mapsto 2, \langle t_{\mathfrak{t}_2}, l_e \rangle \mapsto 1\} \cup \{\langle l_1, t_{\mathfrak{t}_3} \rangle \mapsto 1, \langle l_e, t_{\mathfrak{t}_3} \rangle \mapsto 1, \langle t_{\mathfrak{t}_3}, l_2 \rangle \mapsto 1, \langle t_{\mathfrak{t}_3}, l_e \rangle \mapsto 1\} \cup \{\langle l_1, t_{\mathfrak{t}_4} \rangle \mapsto 2, \langle l_e, t_{\mathfrak{t}_4} \rangle \mapsto 1, \langle t_{\mathfrak{t}_4}, l_2 \rangle \mapsto 2, \langle t_{\mathfrak{t}_4}, l_e \rangle \mapsto 1\}$ .

## Test Case 2

Let  $\mathcal{O} = \langle A, E \rangle$  be a regular OMAS given as follows:

**Agent Template**  $A = (L, \iota, Act, P, tr, leave)$  with

- $L = \{l_0, l_1, l_2\}$
- $\iota = l_0$
- $Act = \{a_1, a_2\}$
- $P = \{l_0 : a_1, l_1 : a_2\}$
- $leave = l_2$
- $tr = \{\langle l_0, a_1, \{a_1\}, b, l_1 \rangle, \langle l_1, a_2, \{a_2\}, b, l_2 \rangle\}$

**Environment**  $E = (L_E, \iota_E, Act_E, P_E, tr_E)$  with

- $L_E = \{l_e\}$
- $\iota_E = l_e$
- $Act_E = \{b\}$
- $P_E = \{l_e : b\}$
- $tr_E = \{\langle l_e, b, \{a_1\}, l_e \rangle, \langle l_e, b, \{a_2\}, l_e \rangle\}$

The equivalent Petri net  $\langle P_{\mathcal{O}}, T_{\mathcal{O}}, F_{\mathcal{O}}, W_{\mathcal{O}} \rangle$  is defined as follows:

- $P_{\mathcal{O}} = L \cup L_E = \{l_0, l_1, l_2, l_e\}$ .
- Recall that the computation of  $T_{\mathcal{O}}$  depends, essentially, on the computation of  $\mathcal{TR}_{\mathcal{O}}$ , which is done iteratively over the transitions in  $tr_E$ . In the present case,  $\mathcal{TR}_{\mathcal{O}} = \mathcal{TR}_{\langle l_e, b, \{a_1\}, l_e \rangle} \cup \mathcal{TR}_{\langle l_e, b, \{a_2\}, l_e \rangle}$ .
  - $\mathcal{TR}_{\langle l_e, b, \{a_1\}, l_e \rangle}$  contains all those global joint transitions which are consistent with environment transition  $\langle l_e, b, \{a_1\}, l_e \rangle$  and crucially depends on  $(b, \{a_1\})$ . Hence,  $\mathcal{TR}_{\langle l_e, b, \{a_1\}, l_e \rangle} = \{\langle l_0, a_1, \{a_1\}, b, l_1 \rangle, \langle l_0, a_1, \{a_1\}, b, l_1 \rangle, \langle l_e, b, \{a_1\}, l_e \rangle\}$
  - Similarly,  $\mathcal{TR}_{\langle l_e, b, \{a_2\}, l_e \rangle}$  contains all those global joint transitions which are consistent with environment transition  $\langle l_e, b, \{a_2\}, l_e \rangle$  and crucially depends on  $(b, \{a_2\})$ . Hence,  $\mathcal{TR}_{\langle l_e, b, \{a_2\}, l_e \rangle} = \{\langle l_1, a_2, \{a_2\}, b, l_2 \rangle, \langle l_1, a_2, \{a_2\}, b, l_2 \rangle, \langle l_e, b, \{a_2\}, l_e \rangle\}$
- Let us denote the tuple in  $\mathcal{TR}_{\langle l_e, b, \{a_1\}, l_e \rangle}$  by  $\mathfrak{t}_1$  and  $\mathcal{TR}_{\langle l_e, b, \{a_2\}, l_e \rangle}$  by  $\mathfrak{t}_2$ , respectively. Then,  $T_{\mathcal{O}} = \{t_{aj}, t_{al}, t_{\mathfrak{t}_1}, t_{\mathfrak{t}_2}\}$ .
- The information computed in  $\mathcal{TR}$  helps us to augment  $F_{\mathcal{O}}$  as follows. Recall that  $F_{\mathcal{O}}$  is initialized to be  $\{\langle t_{aj}, l_0 \rangle, \langle l_2, t_{al} \rangle\}$ . We add the following edges to  $F_{\mathcal{O}}$  :  $\{\langle l_0, t_{\mathfrak{t}_1} \rangle, \langle l_e, t_{\mathfrak{t}_1} \rangle, \langle t_{\mathfrak{t}_1}, l_1 \rangle, \langle t_{\mathfrak{t}_1}, l_e \rangle\} \cup \{\langle l_1, t_{\mathfrak{t}_2} \rangle, \langle l_e, t_{\mathfrak{t}_2} \rangle, \langle t_{\mathfrak{t}_2}, l_2 \rangle, \langle t_{\mathfrak{t}_2}, l_e \rangle\}$ .
- Similarly, the initial content of  $W_{\mathcal{O}} = \{\langle t_{aj}, l_0 \rangle \mapsto 1, \langle l_2, t_{al} \rangle \mapsto 1\}$ . We augment  $W_{\mathcal{O}}$  with the following weights computed from  $\mathcal{TR}$ , corresponding to the edges in  $F_{\mathcal{O}}$ :  $\{\langle l_0, t_{\mathfrak{t}_1} \rangle \mapsto 2, \langle l_e, t_{\mathfrak{t}_1} \rangle \mapsto 1, \langle t_{\mathfrak{t}_1}, l_1 \rangle \mapsto 2, \langle t_{\mathfrak{t}_1}, l_e \rangle \mapsto 1\} \cup \{\langle l_1, t_{\mathfrak{t}_2} \rangle \mapsto 2, \langle l_e, t_{\mathfrak{t}_2} \rangle \mapsto 1, \langle t_{\mathfrak{t}_2}, l_2 \rangle \mapsto 2, \langle t_{\mathfrak{t}_2}, l_e \rangle \mapsto 1\}$ .

## Test Case 3

Let  $\mathcal{O} = \langle A, E \rangle$  be a regular OMAS given as follows:

**Agent Template**  $A = (L, \iota, Act, P, tr, leave)$  with

- $L = \{l_0, l_1, l_2\}$
- $\iota = l_0$
- $Act = \{a_1, a_2\}$
- $P = \{l_0 : a_1, l_1 : a_2\}$
- $leave = l_2$
- $tr = \{\langle l_0, a_1, \emptyset, b, l_1 \rangle, \langle l_1, a_2, \emptyset, b, l_2 \rangle\}$

**Environment**  $E = (L_E, \iota_E, Act_E, P_E, tr_E)$  with

- $L_E = \{l_e\}$
- $\iota_E = l_e$
- $Act_E = \{b\}$
- $P_E = \{l_e : b\}$
- $tr_E = \{\langle l_e, b, \{a_1\}, l_e \rangle, \langle l_e, b, \{a_2\}, l_e \rangle\}$

The equivalent Petri net  $\langle P_{\mathcal{O}}, T_{\mathcal{O}}, F_{\mathcal{O}}, W_{\mathcal{O}} \rangle$  is defined as follows:

- $P_{\mathcal{O}} = L \cup L_E = \{l_0, l_1, l_2, l_e\}$ .
- Recall that the computation of  $T_{\mathcal{O}}$  depends, essentially, on the computation of  $\mathcal{TR}_{\mathcal{O}}$ , which is done iteratively over the transitions in  $tr_E$ . In the present case,  $\mathcal{TR}_{\mathcal{O}} = \mathcal{TR}_{\langle l_e, b, \{a_1\}, l_e \rangle} \cup \mathcal{TR}_{\langle l_e, b, \{a_2\}, l_e \rangle}$ .
  - $\mathcal{TR}_{\langle l_e, b, \{a_1\}, l_e \rangle}$  contains all those global joint transitions which are consistent with environment transition  $\langle l_e, b, \{a_1\}, l_e \rangle$  and crucially depends on  $(b, \{a_1\})$ . Hence,  $\mathcal{TR}_{\langle l_e, b, \{a_1\}, l_e \rangle} = \{\langle l_0, a_1, \emptyset, b, l_1 \rangle, \langle l_e, b, \{a_1\}, l_e \rangle\}$
  - Similarly,  $\mathcal{TR}_{\langle l_e, b, \{a_2\}, l_e \rangle}$  contains all those global joint transitions which are consistent with environment transition  $\langle l_e, b, \{a_2\}, l_e \rangle$  and crucially depends on  $(b, \{a_2\})$ . Hence,  $\mathcal{TR}_{\langle l_e, b, \{a_2\}, l_e \rangle} = \{\langle l_1, a_2, \emptyset, b, l_2 \rangle, \langle l_e, b, \{a_2\}, l_e \rangle\}$
- Let us denote the tuple in  $\mathcal{TR}_{\langle l_e, b, \{a_1\}, l_e \rangle}$  by  $\mathfrak{t}_1$  and  $\mathcal{TR}_{\langle l_e, b, \{a_2\}, l_e \rangle}$  by  $\mathfrak{t}_2$ , respectively. Then,  $T_{\mathcal{O}} = \{t_{aj}, t_{al}, t_{\mathfrak{t}_1}, t_{\mathfrak{t}_2}\}$ .
- The information computed in  $\mathcal{TR}$  helps us to augment  $F_{\mathcal{O}}$  as follows. Recall that  $F_{\mathcal{O}}$  is initialized to be  $\{\langle t_{aj}, l_0 \rangle, \langle l_2, t_{al} \rangle\}$ . We add the following edges to  $F_{\mathcal{O}}$  :  $\{\langle l_0, t_{\mathfrak{t}_1} \rangle, \langle l_e, t_{\mathfrak{t}_1} \rangle, \langle t_{\mathfrak{t}_1}, l_1 \rangle, \langle t_{\mathfrak{t}_1}, l_e \rangle\} \cup \{\langle l_1, t_{\mathfrak{t}_2} \rangle, \langle l_e, t_{\mathfrak{t}_2} \rangle, \langle t_{\mathfrak{t}_2}, l_2 \rangle, \langle t_{\mathfrak{t}_2}, l_e \rangle\}$ .
- Similarly, the initial content of  $W_{\mathcal{O}} = \{\langle t_{aj}, l_0 \rangle \mapsto 1, \langle l_2, t_{al} \rangle \mapsto 1\}$ . We augment  $W_{\mathcal{O}}$  with the following weights computed from  $\mathcal{TR}$ , corresponding to the edges in  $F_{\mathcal{O}}$ :  $\{\langle l_0, t_{\mathfrak{t}_1} \rangle \mapsto 1, \langle l_e, t_{\mathfrak{t}_1} \rangle \mapsto 1, \langle t_{\mathfrak{t}_1}, l_1 \rangle \mapsto 1, \langle t_{\mathfrak{t}_1}, l_e \rangle \mapsto 1\} \cup \{\langle l_1, t_{\mathfrak{t}_2} \rangle \mapsto 1, \langle l_e, t_{\mathfrak{t}_2} \rangle \mapsto 1, \langle t_{\mathfrak{t}_2}, l_2 \rangle \mapsto 1, \langle t_{\mathfrak{t}_2}, l_e \rangle \mapsto 1\}$ .