PETRI NET ENCODINGS FOR REGULAR OPEN MAS

TEST CASES

Test Case 1

Let $\mathcal{O} = \langle A, E\rangle$ be a regular OMAS given as follows:

Agent Template $A = (L, \iota, Act, P, tr, leave)$ with

- $L = \{l_0, l_1, l_2\}$
- $\iota = l_0$
- $Act = \{a_1, a_2\}$
- $P = \{l_0 : a_1, l_1 : a_2\}$
- $leave = l_2$
- $tr = \{\langle l_0, a_1, \{a_1\}, b, l_1\rangle, \langle l_0, a_1, \emptyset, b, l_1\rangle, \langle l_1, a_2, \{a_2\}, b, l_2\rangle, \langle l_1, a_2, \emptyset, b, l_2\rangle\}$

Environment $E = (L_E, \iota_E, Act_E, P_E, tr_E)$ with

- $L_E = \{l_e\}$
- $\iota_E = l_e$
- $Act_E = \{b\}$
- $P_E = \{l_e : b\}$
- $tr_E = \{\langle l_e, b, \{a_1\}, l_e\rangle, \langle l_e, b, \{a_2\}, l_e\rangle\}$

The equivalent Petri net $\langle P_{\mathcal{O}}, T_{\mathcal{O}}, F_{\mathcal{O}}, W_{\mathcal{O}}\rangle$ is defined as follows:

- $P_{\mathcal{O}} = L \cup L_E = \{l_0, l_1, l_2, l_e\}$.
- Recall that the computation of $T_{\mathcal{O}}$ depends, essentially, on the computation of $\mathcal{TR}_{\mathcal{O}}$, which is done iteratively over the transitions in tr_E. In the present case, $\mathcal{TR}_{\mathcal{O}} = \mathcal{TR}_{\langle l_e,b,\{a_1\},l_e\rangle} \cup \mathcal{TR}_{\langle l_e,b,\{a_2\},l_e\rangle}$.
  - $\mathcal{TR}_{\langle l_e,b,\{a_1\},l_e\rangle}$ contains all those global joint transitions which are consistent with environment transition $\langle l_e, b, \{a_1\}, l_e\rangle$ and crucially depends on $(b, \{a_1\})$. Hence, $\mathcal{TR}_{\langle l_e,b,\{a_1\},l_e\rangle} = \{(\langle l_0, a_1, \emptyset, b, l_1\rangle, \langle l_e, b, \{a_1\}, l_e\rangle), (\langle l_0, a_1, \{a_1\}, b, l_1\rangle, \langle l_0, a_1, \{a_1\}, b, l_1\rangle, \langle l_e, b, \{a_1\}, l_e\rangle)\}$
  - Similarly, $\mathcal{TR}_{\langle l_e,b,\{a_2\},l_e\rangle}$ contains all those global joint transitions which are consistent with environment transition $\langle l_e, b, \{a_2\}, l_e\rangle$ and crucially depends on $(b, \{a_2\})$. Hence, $\mathcal{TR}_{\langle l_e,b,\{a_1\},l_e\rangle} = \{(\langle l_1, a_2, \emptyset, b, l_2\rangle, \langle l_e, b, \{a_2\}, l_e\rangle), (\langle l_1, a_2, \{a_2\}, b, l_2\rangle, \langle l_1, a_2, \{a_2\}, b, l_2\rangle, \langle l_e, b, \{a_2\}, l_e\rangle)\}$
- Let us denote the tuples in $\mathcal{TR}_{\langle l_e,b,\{a_1\},l_e\rangle}$ by $\mathbb{t}_1$ and $\mathbb{t}_2$ and $\mathcal{TR}_{\langle l_e,b,\{a_2\},l_e\rangle}$ by $\mathbb{t}_3$ and $\mathbb{t}_4$, respectively. Then, $T_{\mathcal{O}} = \{t_{aj}, t_{al}, t_{\mathbb{t}_1}, t_{\mathbb{t}_2}, t_{\mathbb{t}_3}, t_{\mathbb{t}_4}\}$.
- The information computed in $\mathcal{TR}$ helps us to augment $F_{\mathcal{O}}$ as follows. Recall that $F_{\mathcal{O}}$ is initialized to be $\{\langle t_{aj}, l_0\rangle, \langle l_2, t_{al}\rangle\}$. We add the following edges to $F_{\mathcal{O}}$: $\{\langle l_0, t_{\mathbb{t}_1}\rangle, \langle l_e, t_{\mathbb{t}_1}\rangle, \langle t_{\mathbb{t}_1}, l_1\rangle, \langle t_{\mathbb{t}_1}, l_e\rangle\}$ $\cup\{\langle l_0, t_{\mathbb{t}_2}\rangle, \langle l_e, t_{\mathbb{t}_2}\rangle, \langle t_{\mathbb{t}_2}, l_1\rangle, \langle t_{\mathbb{t}_2}, l_e\rangle\} \cup\{\langle l_1, t_{\mathbb{t}_3}\rangle, \langle l_e, t_{\mathbb{t}_3}\rangle, \langle t_{\mathbb{t}_3}, l_2\rangle, \langle t_{\mathbb{t}_3}, l_e\rangle\} \cup\{\langle l_1, t_{\mathbb{t}_4}\rangle, \langle l_e, t_{\mathbb{t}_4}\rangle, \langle t_{\mathbb{t}_4}, l_2\rangle, \langle t_{\mathbb{t}_4}, l_e\rangle\}$.
- Similarly, the initial content of $W_{\mathcal{O}} = \{\langle t_{aj}, l_0\rangle \mapsto 1, \langle l_2, t_{al}\rangle \mapsto 1\}$. We augment $W_{\mathcal{O}}$ with the following weights computed from $\mathcal{TR}$, corresponding to the edges in $F_{\mathcal{O}}$: $\{\langle l_0, t_{\mathbb{t}_1}\rangle \mapsto 1, \langle l_e, t_{\mathbb{t}_1}\rangle \mapsto 1, \langle t_{\mathbb{t}_1}, l_1\rangle \mapsto 1, \langle t_{\mathbb{t}_1}, l_e\rangle \mapsto 1\}\cup\{\langle l_0, t_{\mathbb{t}_2}\rangle \mapsto 2, \langle l_e, t_{\mathbb{t}_2}\rangle \mapsto 1, \langle t_{\mathbb{t}_2}, l_1\rangle \mapsto 2, \langle t_{\mathbb{t}_2}, l_e\rangle \mapsto 1\} \cup\{\langle l_1, t_{\mathbb{t}_3}\rangle \mapsto 1, \langle l_e, t_{\mathbb{t}_3}\rangle \mapsto 1, \langle t_{\mathbb{t}_3}, l_2\rangle \mapsto 1, \langle t_{\mathbb{t}_3}, l_e\rangle \mapsto 1\} \cup\{\langle l_1, t_{\mathbb{t}_4}\rangle \mapsto 2, \langle l_e, t_{\mathbb{t}_4}\rangle \mapsto 1, \langle t_{\mathbb{t}_4}, l_2\rangle \mapsto 2, \langle t_{\mathbb{t}_4}, l_e\rangle \mapsto 1\}$.

Test Case 2

Let $\mathcal{O} = \langle A, E\rangle$ be a regular OMAS given as follows:

Agent Template $A = (L, \iota, Act, P, tr, leave)$ with

- $L = \{l_0, l_1, l_2\}$
- $\iota = l_0$
- $Act = \{a_1, a_2\}$
- $P = \{l_0 : a_1, l_1 : a_2\}$
- $leave = l_2$
- $tr = \{\langle l_0, a_1, \{a_1\}, b, l_1\rangle, \langle l_1, a_2, \{a_2\}, b, l_2\rangle\}$

Environment $E = (L_E, \iota_E, Act_E, P_E, tr_E)$ with

- $L_E = \{l_e\}$
- $\iota_E = l_e$
- $Act_E = \{b\}$
- $P_E = \{l_e : b\}$
- $tr_E = \{\langle l_e, b, \{a_1\}, l_e\rangle, \langle l_e, b, \{a_2\}, l_e\rangle\}$

The equivalent Petri net $\langle P_\mathcal{O}, T_\mathcal{O}, F_\mathcal{O}, W_\mathcal{O}\rangle$ is defined as follows:

- $P_\mathcal{O} = L \cup L_E = \{l_0, l_1, l_2, l_e\}$.
- Recall that the computation of $T_\mathcal{O}$ depends, essentially, on the computation of $\mathcal{TR}_\mathcal{O}$, which is done iteratively over the transitions in tr_E. In the present case, $\mathcal{TR}_\mathcal{O} = \mathcal{TR}_{\langle l_e, b, \{a_1\}, l_e\rangle} \cup \mathcal{TR}_{\langle l_e, b, \{a_2\}, l_e\rangle}$.
  - $\mathcal{TR}_{\langle l_e, b, \{a_1\}, l_e\rangle}$ contains all those global joint transitions which are consistent with environment transition $\langle l_e, b, \{a_1\}, l_e\rangle$ and crucially depends on $(b, \{a_1\})$. Hence, $\mathcal{TR}_{\langle l_e, b, \{a_1\}, l_e\rangle} = (\langle l_0, a_1, \{a_1\}, b, l_1\rangle, \langle l_0, a_1, \{a_1\}, b, l_1\rangle, \langle l_e, b, \{a_1\}, l_e\rangle)\}$
  - Similarly, $\mathcal{TR}_{\langle l_e, b, \{a_2\}, l_e\rangle}$ contains all those global joint transitions which are consistent with environment transition $\langle l_e, b, \{a_2\}, l_e\rangle$ and crucially depends on $(b, \{a_2\})$. Hence, $\mathcal{TR}_{\langle l_e, b, \{a_1\}, l_e\rangle} = (\langle l_1, a_2, \{a_2\}, b, l_2\rangle, \langle l_1, a_2, \{a_2\}, b, l_2\rangle, \langle l_e, b, \{a_2\}, l_e\rangle)\}$
- Let us denote the tuple in $\mathcal{TR}_{\langle l_e, b, \{a_1\}, l_e\rangle}$ by $\mathbb{t}_1$ and $\mathcal{TR}_{\langle l_e, b, \{a_2\}, l_e\rangle}$ by $\mathbb{t}_2$, respectively. Then, $T_\mathcal{O} = \{t_{aj}, t_{al}, t_{\mathbb{t}_1}, t_{\mathbb{t}_2}\}$.
- The information computed in $\mathcal{TR}$ helps us to augment $F_\mathcal{O}$ as follows. Recall that $F_\mathcal{O}$ is initialized to be $\{\langle t_{aj}, l_0\rangle, \langle l_2, t_{al}\rangle\}$. We add the following edges to $F_\mathcal{O}$: $\{\langle l_0, t_{\mathbb{t}_1}\rangle, \langle l_e, t_{\mathbb{t}_1}\rangle, \langle t_{\mathbb{t}_1}, l_1\rangle, \langle t_{\mathbb{t}_1}, l_e\rangle\} \cup \{\langle l_1, t_{\mathbb{t}_2}\rangle, \langle l_e, t_{\mathbb{t}_2}\rangle, \langle t_{\mathbb{t}_2}, l_2\rangle, \langle t_{\mathbb{t}_2}, l_e\rangle\}$.
- Similarly, the initial content of $W_\mathcal{O} = \{\langle t_{aj}, l_0\rangle \mapsto 1, \langle l_2, t_{al}\rangle \mapsto 1\}$. We augment $W_\mathcal{O}$ with the following weights computed from $\mathcal{TR}$, corresponding to the edges in $F_\mathcal{O}$: $\{\langle l_0, t_{\mathbb{t}_1}\rangle \mapsto 2, \langle l_e, t_{\mathbb{t}_1}\rangle \mapsto 1, \langle t_{\mathbb{t}_1}, l_1\rangle \mapsto 2, \langle t_{\mathbb{t}_1}, l_e\rangle \mapsto 1\} \cup \{\langle l_1, t_{\mathbb{t}_2}\rangle \mapsto 2, \langle l_e, t_{\mathbb{t}_2}\rangle \mapsto 1, \langle t_{\mathbb{t}_2}, l_2\rangle \mapsto 2, \langle t_{\mathbb{t}_2}, l_e\rangle \mapsto 1\}$.

Test Case 3

Let $\mathcal{O} = \langle A, E \rangle$ be a regular OMAS given as follows:

Agent Template $A = (L, \iota, Act, P, tr, leave)$ with

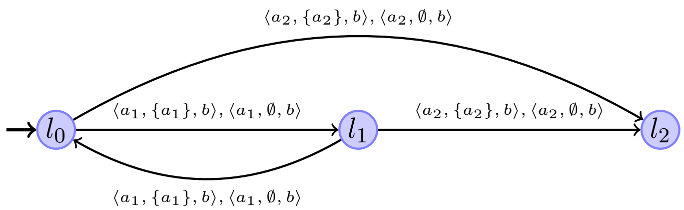
- $L = \{l_0, l_1, l_2\}$
- $\iota = l_0$
- $Act = \{a_1, a_2\}$
- $P = \{l_0 : a_1, l_1 : a_2\}$
- $leave = l_2$
- $tr = \{\langle l_0, a_1, \emptyset, b, l_1 \rangle, \langle l_1, a_2, \emptyset, b, l_2 \rangle\}$

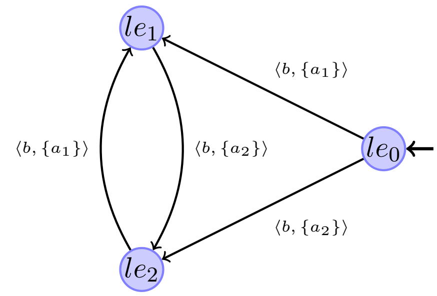
Environment $E = (L_E, \iota_E, Act_E, P_E, tr_E)$ with

- $L_E = \{l_e\}$
- $\iota_E = l_e$
- $Act_E = \{b\}$
- $P_E = \{l_e : b\}$
- $tr_E = \{\langle l_e, b, \{a_1\}, l_e \rangle, \langle l_e, b, \{a_2\}, l_e \rangle\}$

The equivalent Petri net $\langle P_{\mathcal{O}}, T_{\mathcal{O}}, F_{\mathcal{O}}, W_{\mathcal{O}} \rangle$ is defined as follows:

- $P_{\mathcal{O}} = L \cup L_E = \{l_0, l_1, l_2, l_e\}$.
- Recall that the computation of $T_{\mathcal{O}}$ depends, essentially, on the computation of $\mathcal{TR}_{\mathcal{O}}$, which is done iteratively over the transitions in tr_E. In the present case, $\mathcal{TR}_{\mathcal{O}} = \mathcal{TR}_{\langle l_e, b, \{a_1\}, l_e \rangle} \cup \mathcal{TR}_{\langle l_e, b, \{a_2\}, l_e \rangle}$.
  - $\mathcal{TR}_{\langle l_e, b, \{a_1\}, l_e \rangle}$ contains all those global joint transitions which are consistent with environment transition $\langle l_e, b, \{a_1\}, l_e \rangle$ and crucially depends on $(b, \{a_1\})$. Hence, $\mathcal{TR}_{\langle l_e, b, \{a_1\}, l_e \rangle} = \{(\langle l_0, a_1, \emptyset, b, l_1 \rangle, \langle l_e, b, \{a_1\}, l_e \rangle)\}$
  - Similarly, $\mathcal{TR}_{\langle l_e, b, \{a_2\}, l_e \rangle}$ contains all those global joint transitions which are consistent with environment transition $\langle l_e, b, \{a_2\}, l_e \rangle$ and crucially depends on $(b, \{a_2\})$. Hence, $\mathcal{TR}_{\langle l_e, b, \{a_1\}, l_e \rangle} = \{(\langle l_1, a_2, \emptyset, b, l_2 \rangle, \langle l_e, b, \{a_2\}, l_e \rangle)\}$
- Let us denote the tuple in $\mathcal{TR}_{\langle l_e, b, \{a_1\}, l_e \rangle}$ by $\mathbb{t}_1$ and $\mathcal{TR}_{\langle l_e, b, \{a_2\}, l_e \rangle}$ by $\mathbb{t}_2$, respectively. Then, $T_{\mathcal{O}} = \{t_{aj}, t_{al}, t_{\mathbb{t}_1}, t_{\mathbb{t}_2}\}$.
- The information computed in $\mathcal{TR}$ helps us to augment $F_{\mathcal{O}}$ as follows. Recall that $F_{\mathcal{O}}$ is initialized to be $\{\langle t_{aj}, l_0 \rangle, \langle l_2, t_{al} \rangle\}$. We add the following edges to $F_{\mathcal{O}}$: $\{\langle l_0, t_{\mathbb{t}_1} \rangle, \langle l_e, t_{\mathbb{t}_1} \rangle, \langle t_{\mathbb{t}_1}, l_1 \rangle, \langle t_{\mathbb{t}_1}, l_e \rangle\} \cup \{\langle l_1, t_{\mathbb{t}_2} \rangle, \langle l_e, t_{\mathbb{t}_2} \rangle, \langle t_{\mathbb{t}_2}, l_2 \rangle, \langle t_{\mathbb{t}_2}, l_e \rangle\}$.
- Similarly, the initial content of $W_{\mathcal{O}} = \{\langle t_{aj}, l_0 \rangle \mapsto 1, \langle l_2, t_{al} \rangle \mapsto 1\}$. We augment $W_{\mathcal{O}}$ with the following weights computed from $\mathcal{TR}$, corresponding to the edges in $F_{\mathcal{O}}$: $\{\langle l_0, t_{\mathbb{t}_1} \rangle \mapsto 1, \langle l_e, t_{\mathbb{t}_1} \rangle \mapsto 1, \langle t_{\mathbb{t}_1}, l_1 \rangle \mapsto 1, \langle t_{\mathbb{t}_1}, l_e \rangle \mapsto 1\} \cup \{\langle l_1, t_{\mathbb{t}_2} \rangle \mapsto 1, \langle l_e, t_{\mathbb{t}_2} \rangle \mapsto 1, \langle t_{\mathbb{t}_2}, l_2 \rangle \mapsto 1, \langle t_{\mathbb{t}_2}, l_e \rangle \mapsto 1\}$.





Test Case 4

Let $\mathcal{O} = \langle A, E\rangle$ be a regular OMAS given as follows:

Agent Template $A = (L, \iota, \mathit{Act}, P, \mathit{tr}, \mathit{leave})$ with

- $L = \{l_0, l_1, l_2\}$
- $\iota = l_0$
- $\mathit{Act} = \{a_1, a_2\}$
- $P = \{l_0 : \{a_1, a_2\}, l_1 : \{a_1, a_2\}\}$
- $\mathit{leave} = l_2$
- $\mathit{tr} = \{\langle l_0, a_1, \{a_1\}, b, l_1\rangle, \langle l_0, a_1, \emptyset, b, l_1\rangle, \langle l_0, a_2, \{a_2\}, b, l_2\rangle, \langle l_0, a_2, \emptyset, b, l_2\rangle,$ $\langle l_1, a_2, \{a_2\}, b, l_2\rangle, \langle l_1, a_2, \emptyset, b, l_2\rangle, \langle l_1, a_1, \{a_1\}, b, l_0\rangle, \langle l_1, a_1, \emptyset, b, l_0\rangle\}$

Environment $E = (L_E, \iota_E, \mathit{Act}_E, P_E, \mathit{tr}_E)$ with

- $L_E = \{le_0, le_1, le_2\}$
- $\iota_E = le_0$
- $\mathit{Act}_E = \{b\}$
- $P_E = \{le_0 : b, le_1 : b, le_2 : b\}$
- $\mathit{tr}_E = \{\langle le_0, b, \{a_1\}, le_1\rangle, \langle le_e, b, \{a_2\}, le_2\rangle, \langle le_2, b, \{a_1\}, le_1\rangle, \langle le_1, b, \{a_2\}, le_2\rangle\}$

The equivalent Petri net $\langle P_{\mathcal{O}}, T_{\mathcal{O}}, F_{\mathcal{O}}, W_{\mathcal{O}}\rangle$ is defined as follows:

$P_{\mathcal{O}}$**:** As usual $P_{\mathcal{O}} = L \cup L_E = \{l_0, l_1, l_2, le_0, le_1, le_2\}$

$T_{\mathcal{O}}$**:** In order to define $T_{\mathcal{O}}$, we need to compute $\mathcal{TR}_{\mathcal{O}}$ which is equal to $\mathcal{TR}_{\langle le_0, b, \{a_1\}, le_1\rangle} \cup \mathcal{TR}_{\langle le_e, b, \{a_2\}, le_2\rangle} \cup \mathcal{TR}_{\langle le_2, b, \{a_1\}, le_1\rangle} \cup \mathcal{TR}_{\langle le_1, b, \{a_2\}, le_2\rangle}$. These sets are, in turn, computed from $\mathcal{TR}_{\langle b, \{a_1\}\rangle}$ and $\mathcal{TR}_{\langle b, \{a_2\}\rangle}$. So, we proceed to compute them first.

$\mathcal{TR}_{\langle b, \{a_1\}\rangle}$ contains two (2) tuples of size one (1): $\langle l_0, a_1, \emptyset, b, l_1\rangle$ and $\langle l_1, a_1, \emptyset, b, l_0\rangle$.

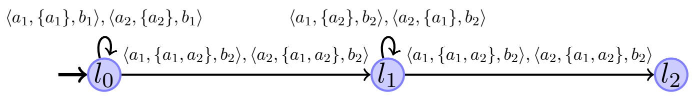
$\mathcal{TR}_{\langle b, \{a_1\}\rangle}$ contains three (3) tuples of size one (2): $(\langle l_0, a_1, \{a_1\}, b, l_1\rangle, \langle l_0, a_1, \{a_1\}, b, l_1\rangle)$, $(\langle l_1, a_1, \{a_1\}, b, l_0\rangle, \langle l_1, a_1, \{a_1\}, b, l_0\rangle)$ and $(\langle l_0, a_1, \{a_1\}, b, l_1\rangle, \langle l_1, a_1, \{a_1\}, b, l_0\rangle)$.

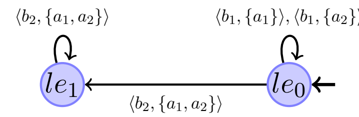
Now, $\mathcal{TR}_{\langle le_0, b, \{a_1\}, le_1\rangle} = \mathcal{TR}_{\langle b, \{a_1\}\rangle} \times \{\langle le_0, b, \{a_1\}, le_1\rangle\}$ and $\mathcal{TR}_{\langle le_2, b, \{a_1\}, le_1\rangle} = \mathcal{TR}_{\langle b, \{a_1\}\rangle} \times \{\langle le_2, b, \{a_1\}, le_1\rangle\}$.

$\mathcal{TR}_{\langle b, \{a_2\}\rangle}$ contains two (2) tuples of size one (1): $\langle l_0, a_2, \emptyset, b, l_2\rangle$ and $\langle l_1, a_2, \emptyset, b, l_2\rangle$.

$\mathcal{TR}_{\langle b, \{a_2\}\rangle}$ contains three (3) tuples of size one (2): $(\langle l_0, a_2, \{a_2\}, b, l_2\rangle, \langle l_0, a_2, \{a_2\}, b, l_2\rangle)$, $(\langle l_1, a_2, \{a_2\}, b, l_2\rangle, \langle l_1, a_2, \{a_2\}, b, l_2\rangle)$ and $(\langle l_0, a_2, \{a_2\}, b, l_2\rangle, \langle l_1, a_2, \{a_2\}, b, l_2\rangle)$

Now, $\mathcal{TR}_{\langle le_1, b, \{a_2\}, le_2\rangle} = \mathcal{TR}_{\langle b, \{a_2\}\rangle} \times \{\langle le_1, b, \{a_2\}, le_2\rangle\}$ and $\mathcal{TR}_{\langle le_0, b, \{a_2\}, le_2\rangle} = \mathcal{TR}_{\langle b, \{a_2\}\rangle} \times \{\langle le_0, b, \{a_2\}, le_2\rangle\}$.
$T_{\mathcal{O}} = \{t_{aj}, t_{al}\} \cup \{t_{\mathbb{tr}} \mid \mathbb{tr} \in \mathcal{TR}_{\mathcal{O}}\}$.





Test Case 5

Let $\mathcal{O} = \langle A, E\rangle$ be a regular OMAS given as follows:

Agent Template $A = (L, \iota, Act, P, tr, leave)$ with

- $L = \{l_0, l_1, l_2\}$
- $\iota = l_0$
- $Act = \{a_1, a_2\}$
- $P = \{l_0 : \{a_1, a_2\}, l_1 : \{a_1, a_2\}\}$
- $leave = l_2$
- $tr = \{\langle l_0, a_1, \{a_1\}, b_1, l_0\rangle, \langle l_0, a_2, \{a_2\}, b_1, l_0\rangle, \langle l_0, a_1, \{a_1, a_2\}, b_2, l_1\rangle, \langle l_0, a_2, \{a_1, a_2\}, b_2, l_1\rangle,$
  $\langle l_1, a_1, \{a_2\}, b_2, l_1\rangle, \langle l_1, a_2, \{a_1\}, b_2, l_1\rangle, \langle l_1, a_1, \{a_1, a_2\}, b_2, l_2\rangle, \langle l_1, a_2, \{a_1, a_2\}, b_2, l_2\rangle\}$

Environment $E = (L_E, \iota_E, Act_E, P_E, tr_E)$ with

- $L_E = \{le_0, le_1\}$
- $\iota_E = le_0$
- $Act_E = \{b_1, b_2\}$
- $P_E = \{le_0 : \{b_1, b_2\}, le_1 : b_2\}$
- $tr_E = \{\langle le_0, b_1, \{a_1\}, le_0\rangle, \langle le_0, b_1, \{a_2\}, le_0\rangle, \langle le_0, b_2, \{a_1, a_2\}, le_1\rangle, \langle le_1, b_2, \{a_1, a_2\}, le_1\rangle\}$

The equivalent Petri net $\langle P_{\mathcal{O}}, T_{\mathcal{O}}, F_{\mathcal{O}}, W_{\mathcal{O}}\rangle$ is defined as follows:

- $P_{\mathcal{O}} = L \cup L_E = \{l_0, l_1, l_2, le_0, le_1\}$.
- Recall that the computation of $T_{\mathcal{O}}$ depends, essentially, on the computation of $\mathcal{TR}_{\mathcal{O}}$, which is done iteratively over the transitions in tr_E. In the present case,
  $\mathcal{TR}_{\mathcal{O}} = \mathcal{TR}_{\langle le_0, b_1, \{a_1\}, le_0\rangle} \cup \mathcal{TR}_{\langle le_0, b_1, \{a_2\}, le_0\rangle} \cup \mathcal{TR}_{\langle le_0, b_2, \{a_1, a_2\}, le_1\rangle} \cup \mathcal{TR}_{\langle le_1, b_2, \{a_1, a_2\}, le_1\rangle}$.
  - $\mathcal{TR}_{\langle le_0, b_1, \{a_1\}, le_0\rangle} = \{(\langle l_1, a_1, \{a_1\}, b_2, l_1\rangle, \langle l_1, a_1, \{a_1\}, b_2, l_1\rangle, \langle le_0, b_1, \{a_1\}, le_0\rangle)\}$.
  - $\mathcal{TR}_{\langle le_0, b_1, \{a_2\}, le_0\rangle} = \{(\langle l_0, a_2, \{a_2\}, b_1, l_0\rangle, \langle l_0, a_2, \{a_2\}, b_1, l_0\rangle, \langle le_0, b_1, \{a_2\}, le_0\rangle)\}$.
  - $\mathcal{TR}_{\langle le_0, b_2, \{a_1, a_2\}, le_1\rangle}$ and $\mathcal{TR}_{\langle le_1, b_2, \{a_1, a_2\}, le_1\rangle}$ are computed from $\mathcal{TR}_{\langle b_2, \{a_1, a_2\}\rangle}$. $\mathcal{TR}_{\langle b_2, \{a_1, a_2\}\rangle}$ contains tuples of sizes 2, 3 and 4 from tr.
    - $\mathcal{TR}_{\langle b_2, \{a_1, a_2\}\rangle}$ contains one (1) tuple of size 2 – $(\langle l_1, a_1, \{a_2\}, b_2, l_1\rangle, \langle l_1, a_2, \{a_1\}, b_2, l_1\rangle)$.
    - $\mathcal{TR}_{\langle b_2, \{a_1, a_2\}\rangle}$ contains eight (8) tuples of size 3 – These tuples are obtained from the following pair of tuple by replacing (l, l') and $(l^\dagger, l^\ddagger)$ by the pairs from the set $\{(l_0, l_1), (l_1, l_2)\}$:
      $(\langle l, a_1, \{a_1, a_2\}, b_2, l'\rangle, \langle l_1, a_2, \{a_1\}, b_2, l_1\rangle, \langle l^\dagger, a_1, \{a_1, a_2\}, b_2, l^\ddagger\rangle)$ and
      $(\langle l, a_2, \{a_1, a_2\}, b_2, l'\rangle, \langle l_1, a_1, \{a_2\}, b_2, l_1\rangle, \langle l^\dagger, a_2, \{a_1, a_2\}, b_2, l^\ddagger\rangle)$.
    - $\mathcal{TR}_{\langle b_2, \{a_1, a_2\}\rangle}$ contains sixteen (16) tuples of size 4 – These tuples are obtained from the following tuple by replacing (l, l') by the pairs from the set $\{(l_0, l_1), (l_1, l_2)\}$:
      $(\langle l, a_1, \{a_1, a_2\}, b_2, l'\rangle, \langle l, a_1, \{a_1, a_2\}, b_2, l'\rangle, \langle l, a_2, \{a_1, a_2\}, b_2, l'\rangle, \langle l, a_2, \{a_1, a_2\}, b_2, l'\rangle)$.
      Note that in each sub-tuple $\langle l,, \{a_1, a_2\}, b_2, l'\rangle$, (l, l') may be replaced by an arbitrary pair from the set $\{(l_0, l_1), (l_1, l_2)\}$.

  After we have computed the set $\mathcal{TR}_{\langle b_2, \{a_1, a_2\}\rangle}$, we define the sets $\mathcal{TR}_{\langle le_0, b_2, \{a_1, a_2\}, le_1\rangle}$ and $\mathcal{TR}_{\langle le_1, b_2, \{a_1, a_2\}, le_1\rangle}$ as follows:

$$\mathcal{TR}_{\langle le_0, b_2, \{a_1, a_2\}, le_1\rangle} = \mathcal{TR}_{\langle b_2, \{a_1, a_2\}\rangle} \times \{\langle le_0, b_2, \{a_1, a_2\}, le_1\rangle\}$$
$$\mathcal{TR}_{\langle le_1, b_2, \{a_1, a_2\}, le_1\rangle} = \mathcal{TR}_{\langle b_2, \{a_1, a_2\}\rangle} \times \{\langle le_1, b_2, \{a_1, a_2\}, le_1\rangle\}$$

- We omit further details regarding $T_{\mathcal{O}}$, $F_{\mathcal{O}}$ and $W_{\mathcal{O}}$ as they are self evident.