DEDUCTION OF THE EXPRESSION OF THE QUADRATIC EQUATION SOLUTION

Let $ax^2+bx+c=0$ be the quadratic equation for which you want to find the solution.

In order to have a simpler expression we are going to normalize by a and thus we have:

$$x^2+\left(\frac{b}{a}\right)x+\frac{c}{a}=0$$

If we define $b_1=\frac{b}{a}$ and $c_1=\frac{c}{a}$ for convenience and apply the changes, the quadratic equation would be:

$$x^2+b_1x+c_1=0$$

If we take c_1 to the other side of equality, the equation is as follows:

$$x^2+b_1x=-c_1$$

We know that $(x+p)^2=x^2+2px+p^2$ We are looking for the form $x^2+2px+p^2$ to replace it with $(x+p)^2$ Therefore, if we add $\left(\frac{b_1}{2}\right)^2$ in both sides of the equation we will have the square we are looking for.

$$x^2+b_1x+\left(\frac{b_1}{2}\right)^2=-c_1+\left(\frac{b_1}{2}\right)^2\Rightarrow\left(x+\frac{b_1}{2}\right)^2=-c_1+\left(\frac{b_1}{2}\right)^2$$

Now the square can be removed and brought to the other member as a square root. This gives back TWO possible solutions that are represented by the ± symbol.

$$x+\frac{b_1}{2}=\pm\sqrt{-c_1+\left(\frac{b_1}{2}\right)^2}$$

Solving for x in the equation and rearranging it, we obtain:

$$x=-\frac{b_1}{2}\pm\sqrt{-c_1+\left(\frac{b_1}{2}\right)^2}\Rightarrow x=-\frac{b_1}{2}\pm\sqrt{\frac{b_1^2-4c_1}{4}}$$

If we reduce we get the following expression:

$$x=\frac{b_1\pm\sqrt{b_1^2-4c_1}}{2}$$

Substituting b_1 and c_1 for their corresponding values $b_1=\frac{b}{a}$ and $c_1=\frac{c}{a}$, the expression is as follows:

$$x=\frac{-\frac{b}{a}\pm\sqrt{\left(\frac{b}{a}\right)^2-4\frac{c}{a}}}{2}$$

If we rationalize it to simplify it, we have:

$$\frac{-\frac{b}{a}\pm\sqrt{\left(\frac{b}{a}\right)^2-4\frac{c}{a}}}{2}=\frac{-\frac{b}{a}\pm\sqrt{\frac{b^2}{a^2}-4\frac{c}{a}}}{2}=\frac{-\frac{b}{a}\pm\sqrt{\frac{b^2}{a^2}-4\frac{ac}{a^2}}}{2}=\frac{-\frac{b}{a}\pm\sqrt{\frac{b^2-4ac}{a^2}}}{2}=\frac{-\frac{b}{a}\pm\frac{\sqrt{b^2-4ac}}{a}}{2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

And so we get that the expressions for the 2 solutions of the quadratic equation are:

$$x_1=\frac{-b+\sqrt{b^2-4ac}}{2a}\quad\text{and}\quad x_2=\frac{-b-\sqrt{b^2-4ac}}{2a}$$