

DEDUCTION OF THE EXPRESSION OF THE QUADRATIC EQUATION SOLUTION

Let $ax^2 - bx + c = 0$ be the quadratic equation for which you want to find the solution.

In order to have a simpler expression we are going to normalize by a and thus we have:

$$x^2 - \left(\frac{b}{a}\right)x + \frac{c}{a} = 0$$

If we define $b_1 = \frac{b}{a}$ and $c_1 = \frac{c}{a}$ for convenience and apply the changes, the quadratic equation would be:

$$x^2 - b_1x + c_1 = 0$$

If we take c_1 to the other side of equality, the equation is as follows:

$$x^2 - b_1x = -c_1$$

We know that $(x-p)^2 = x^2 - 2px + p^2$ We are looking for the form $x^2 - 2px + p^2$ to replace it with $(x-p)^2$ Therefore, if we add $\left(\frac{b_1}{2}\right)^2$ in both sides of the equation we will have the square we are looking for.

$$x^2 - b_1x + \left(\frac{b_1}{2}\right)^2 = -c_1 + \left(\frac{b_1}{2}\right)^2 \Rightarrow \left(x - \frac{b_1}{2}\right)^2 = -c_1 + \left(\frac{b_1}{2}\right)^2$$

Now the square can be removed and brought to the other member as a square root. This gives back TWO possible solutions that are represented by the \pm symbol.

$$x - \frac{b_1}{2} = \pm \sqrt{-c_1 + \left(\frac{b_1}{2}\right)^2}$$

Solving for x in the equation and rearranging it, we obtain:

$$x = \frac{b_1}{2} \pm \sqrt{-c_1 + \left(\frac{b_1}{2}\right)^2} \Rightarrow x = \frac{b_1}{2} \pm \sqrt{\frac{b_1^2 - 4c_1}{4}}$$

If we reduce we get the following expression:

$$x = \frac{b_1 \pm \sqrt{b_1^2 - 4c_1}}{2}$$

Substituting b_1 and c_1 for their corresponding values $b_1 = \frac{b}{a}$ and $c_1 = \frac{c}{a}$, the expression is as follows:

$$x = \frac{\frac{b}{a} \pm \sqrt{\left(\frac{b}{a}\right)^2 - 4\frac{c}{a}}}{2}$$

If we rationalize it to simplify it, we have:

$$\frac{\frac{b}{a} \pm \sqrt{\left(\frac{b}{a}\right)^2 - 4\frac{c}{a}}}{2} = \frac{\frac{b}{a} \pm \sqrt{\frac{b^2}{a^2} - 4\frac{c}{a}}}{2} = \frac{\frac{b}{a} \pm \sqrt{\frac{b^2}{a^2} - 4\frac{ac}{a^2}}}{2} = \frac{\frac{b}{a} \pm \sqrt{\frac{b^2 - 4ac}{a^2}}}{2} = \frac{\frac{b}{a} \pm \frac{\sqrt{b^2 - 4ac}}{a}}{2} = \frac{b \pm \sqrt{b^2 - 4ac}}{2a}$$

And so we get that the expressions for the 2 solutions of the quadratic equation are:

$$x_1 = \frac{b + \sqrt{b^2 - 4ac}}{2a} \quad \text{and} \quad x_2 = \frac{b - \sqrt{b^2 - 4ac}}{2a}$$