Assignment 1

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1 Problem

1.1. Let

$$x(n) = \left\{\underset{\uparrow}{1}, 2, 3, 4, 2, 1\right\} \tag{1.1.1}$$

$$y(n) + \frac{1}{2}y(n-1) = x(n) + x(n-2) \tag{1.1.2}$$

1.2. Compute

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \dots, N-1 \tag{1.2.1}$$

and $H(k)$ using h(n).

1.3. Compute

$$Y(k) = X(k)H(k) \tag{1.3.1}$$

2 Solution

2.1. We know that,the Impulse Response of the LTI system is the output of the system when Unit Impulse Signal is given as input to the system. So, using Eq (1.1.2) the Impulse Response of the System can be found as,

$$h(n) + \frac{1}{2}h(n-1) = \delta(n) + \delta(n-2) \tag{2.1.1}$$

where $h(n)$ is an IIR Filter.

2.2. DFT of a Input Signal $x(n)$ is

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \dots, N-1 \tag{2.2.1}$$

For N = 6, The above expression can be written in matrix form as below:

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \end{bmatrix} = \begin{bmatrix} W_6^0 W_6^0 W_6^0 & W_6^0 & W_6^0 & W_6^0 \\ W_6^0 W_6^1 W_6^2 & W_6^3 & W_6^4 & W_6^5 \\ W_6^0 W_6^2 W_6^4 & W_6^6 & W_6^8 & W_6^{10} \\ W_6^0 W_6^3 W_6^6 & W_6^9 & W_6^{12} & W_6^{15} \\ W_6^0 W_6^4 W_6^8 & W_6^{12} & W_6^{16} & W_6^{20} \\ W_6^0 W_6^5 W_6^{10} & W_6^{15} & W_6^{20} & W_6^{25} \end{bmatrix} \begin{bmatrix} x(1) \\ x(2) \\ x(3) \\ x(4) \\ x(5) \\ x(6) \end{bmatrix} \tag{2.2.2}$$

Where $W_N = e^{-j2\pi/N}$

$$\implies W_6 = e^{-j2\pi/6} = \frac{1 - j\sqrt{3}}{2} \tag{2.2.3}$$

Using $x(n)$ from Eq(1.1.1), we get

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \end{bmatrix} = \begin{bmatrix} W_6^0 W_6^0 W_6^0 & W_6^0 & W_6^0 & W_6^0 \\ W_6^0 W_6^1 W_6^2 & W_6^3 & W_6^4 & W_6^5 \\ W_6^0 W_6^2 W_6^4 & W_6^6 & W_6^8 & W_6^{10} \\ W_6^0 W_6^3 W_6^6 & W_6^9 & W_6^{12} & W_6^{15} \\ W_6^0 W_6^4 W_6^8 & W_6^{12} & W_6^{16} & W_6^{20} \\ W_6^0 W_6^5 W_6^{10} & W_6^{15} & W_6^{20} & W_6^{25} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 2 \\ 1 \end{bmatrix} \tag{2.2.4}$$

On simplifying we get,

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \end{bmatrix} = \begin{bmatrix} 1+2+3+4+2+1 \\ 1+(2)e^{-j\pi/3} + \dots + (1)e^{-j5\pi/3} \\ 1+(2)e^{-2j\pi/3} + \dots + (1)(e^{-2j5\pi/3} \\ 1+(2)e^{-3j\pi/3} + \dots + (1)e^{-3j5\pi/3} \\ 1+(2)e^{-4j\pi/3} + \dots + (1)e^{-4j5\pi/3} \\ 1+(2)e^{-5j\pi/3} + \dots + (1)e^{-5j5\pi/3} \end{bmatrix} \tag{2.2.5}$$

Finally,

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \end{bmatrix} = \begin{bmatrix} 13+0j \\ -4-1.732j \\ 1+0j \\ -1+0j \\ 1+0j \\ -4+1.732j \end{bmatrix} \tag{2.2.6}$$

2.3. DFT of a Impulse Response $h(n)$ is

$$H(k) \triangleq \sum_{n=0}^{N-1} h(n) e^{-j2\pi kn/N}, \quad k = 0, 1, \dots, N-1 \tag{2.3.1}$$

Now to find $H(k)$ we need to know h(n) first. So we will first calculate h(n). For that we need to first find the H(z) by applying Z-transform on equation (**??**) i.e.,

$$H(z) = \frac{2(z^2+1)}{z(2z+1)} \tag{2.3.2}$$

$$H(z) = \frac{1+z^{-2}}{1+\frac{1}{2}z^{-1}} \tag{2.3.3}$$

From this we can say that h(n) is,

$$h(n) = Z^{-1}\left[\frac{1}{1+\frac{1}{2}z^{-1}} + \frac{z^{-2}}{1+\frac{1}{2}z^{-1}}\right] \tag{2.3.4}$$

$$h(n) = \left[\frac{-1}{2}\right]^n u(n) + \left[\frac{-1}{2}\right]^{n-2} u(n-2) \tag{2.3.5}$$

Similarly, converting the above expression in matrix form to find $H(k)$

$$\begin{bmatrix} H(0) \\ H(1) \\ H(2) \\ H(3) \\ H(4) \\ H(5) \end{bmatrix} = \begin{bmatrix} W_6^0 W_6^0 W_6^0 W_6^0 W_6^0 W_6^0 \\ W_6^0 W_6^1 W_6^2 W_6^3 W_6^4 W_6^5 \\ W_6^0 W_6^2 W_6^4 W_6^6 W_6^8 W_6^{10} \\ W_6^0 W_6^3 W_6^6 W_6^9 W_6^{12} W_6^{15} \\ W_6^0 W_6^4 W_6^8 W_6^{12} W_6^{16} W_6^{20} \\ W_6^0 W_6^5 W_6^{10} W_6^{15} W_6^{20} W_6^{25} \end{bmatrix} \begin{bmatrix} h(1) \\ h(2) \\ h(3) \\ h(4) \\ h(5) \\ h(6) \end{bmatrix} \tag{2.3.6}$$

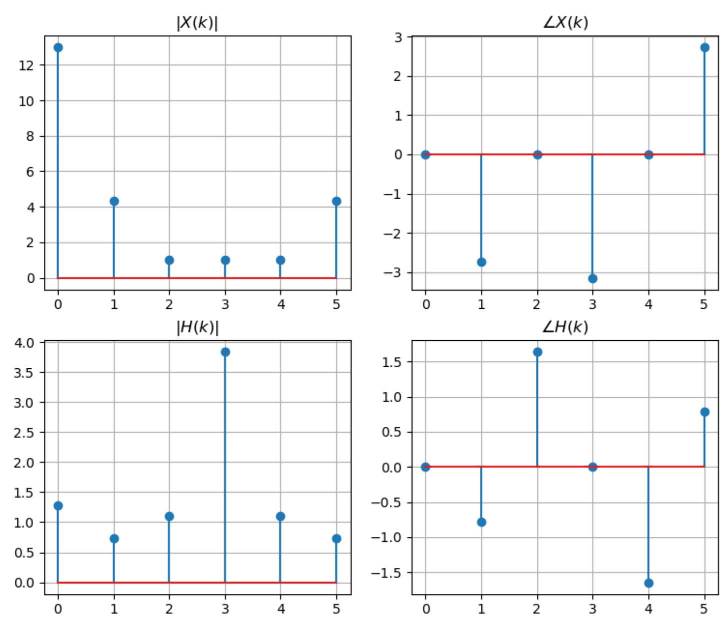
Further Simplification we get,

$$\begin{bmatrix} H(0) \\ H(1) \\ H(2) \\ H(3) \\ H(4) \\ H(5) \end{bmatrix} = \begin{bmatrix} h(0)+h(1)+h(2)+h(3)+h(4)+h(5) \\ h(0)+h(1)e^{-j\pi/3}+\dots+h(5)e^{-j5\pi/3} \\ h(0)+h(1)e^{-2j\pi/3}+\dots+h(5)e^{-2j5\pi/3} \\ h(0)+h(1)e^{-3j\pi/3}+\dots+h(5)e^{-3j5\pi/3} \\ h(0)+h(1)e^{-4j\pi/3}+\dots+h(5)e^{-4j5\pi/3} \\ h(0)+h(1)e^{-5j\pi/3}+\dots+h(5)e^{-5j5\pi/3} \end{bmatrix} \tag{2.3.7}$$

Finally we get,

$$\begin{bmatrix} H(0) \\ H(1) \\ H(2) \\ H(3) \\ H(4) \\ H(5) \end{bmatrix} = \begin{bmatrix} 1.28125+0j \\ 0.51625-0.51418j \\ -0.07812+1.10956j \\ 3.84375+0j \\ -0.07182-1.10956j \\ 0.51625+0.51418j \end{bmatrix} \tag{2.3.8}$$

2.4. The magnitude and phase plots of $X(k)$ and $H(k)$



2.5. We can now compute $Y(k)$ using Eq (2.5.1)

$$Y(k) = X(k)H(k) \tag{2.5.1}$$

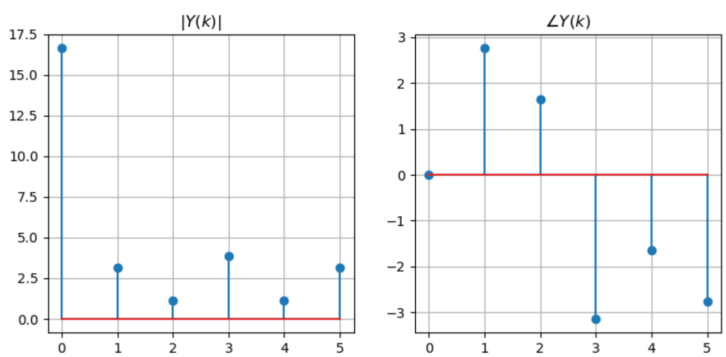
So, $Y(k)$ is obtained element wise multiplication of $X(k)$ and $H(k)$

$$\begin{bmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \\ Y(4) \\ Y(5) \end{bmatrix} = \begin{bmatrix} X(0)\cdot H(0) \\ X(1)\cdot H(1) \\ X(2)\cdot H(2) \\ X(3)\cdot H(3) \\ X(4)\cdot H(4) \\ X(5)\cdot H(5) \end{bmatrix} \tag{2.5.2}$$

Computing the above expression we get,

$$\begin{bmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \\ Y(4) \\ Y(5) \end{bmatrix} = \begin{bmatrix} 16.6562+0j \\ -2.95312+1.16372j \\ -0.07812+1.10959j \\ -3.84375-9.27556j \\ -0.07812-1.10959j \\ -2.95312-1.16372j \end{bmatrix} \tag{2.5.3}$$

The magnitude and phase plots of $Y(k)$ are



2.6. The following code plots all the above figures.

https://github.com/ipsingh85/EE3025_IDP/tree/main/Assingment_1/codes