Control Systems

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Contents

***Abstract*—The objective of this manual is to introduce control system design at an elementary level.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/ketan/codes
```

1 M and N circles

1.1. What are Constant M and N circles and how can we determine closed loop frequency response using M and N circles?

Solution: M circles are called constant magnitude Loci and N circles are called as constant phase angle Loci.These are helpful in determining the closed-loop frequency response of unity negative feedback systems.

Constant-Magnitude Loci(Mcircle): Let $G(\jmath\omega)$ be complex quantity it can be written as

$$G(\jmath\omega) = X + \jmath Y \tag{1.1.1}$$

where X,Y are real quantities. Let M be magnitude of closed loop transfer function.

$$M = \left|\frac{X + \jmath Y}{1 + X + \jmath Y}\right| \tag{1.1.2}$$

$$M^2 = \frac{X^2 + Y^2}{(1 + X)^2 + Y^2} \tag{1.1.3}$$

Hence,

$$X^2\left(1 - M^2\right) - 2M^2X - M^2 + \left(1 - M^2\right)Y^2 = 0 \tag{1.1.4}$$

If $M = 1$, then from Equation (**??**), we obtain $X = \frac{-1}{2}$ This is the equation of a straight line parallel to the Y axis and passing through the point $\left(\frac{-1}{2}, 0\right)$.

If $M \neq 1$ Equation (1.1.4) can be written as

$$X^2 + \frac{2M^2}{M^2 - 1}X + \frac{M^2}{M^2 - 1} + Y^2 = 0 \tag{1.1.5}$$

Simplifying,

$$\left(X + \frac{M^2}{M^2 - 1}\right)^2 + Y^2 = \frac{M^2}{\left(M^2 - 1\right)^2} \tag{1.1.6}$$

Equation (**??**) is the equation of a circle with center $\left(-\frac{M^2}{M^2-1}, 0\right)$ and radius $\left|\frac{M}{M^2-1}\right|$

Thus the intersection of Nquist plot with M circle at a frequency(ω) results as the magnitude of closed loop transfer function as M at frequency (ω)

Constant-Phase-Angle Loci (N Circles): Finding Phase angle α from (**??**) we get,

$$\alpha = \tan^{-1}\left(\frac{Y}{X}\right) - \tan^{-1}\left(\frac{Y}{1 + X}\right) \tag{1.1.7}$$

$$\text{Let } tan\alpha = N \tag{1.1.8}$$

$$N = tan\left(\tan^{-1}\left(\frac{Y}{X}\right) - \tan^{-1}\left(\frac{Y}{1 + X}\right)\right) \tag{1.1.9}$$

Simplifying,

$$N = \frac{Y}{X^2 + X + Y^2} \tag{1.1.10}$$

Further Simplifying..

$$\left(X + \frac{1}{2}\right)^2 + \left(Y - \frac{1}{2N}\right)^2 = \frac{1}{4} + \frac{1}{(2N)^2} \tag{1.1.11}$$

Equation (**??**) is the equation of a circle with center at $\left(\frac{-1}{2}, \frac{1}{2N}\right)$ and radius $\sqrt{\frac{1}{4} + \frac{1}{(2N)^2}}$

Thus the intersection of Nquist plot with N circle at a frequency(ω) results as the phase of closed loop transfer function as $tan^{-1}(N)$ at frequency (ω)

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1.2. For unity Feedback system given below,obtain closed loop frequency response using constant M and N circles.

$$G(s) = \frac{10}{s(s+1)(s+2)} \tag{1.2.1}$$

Solution: The following code plots Fig. **??**

```
codes/ee18btech11017_code1.py
```

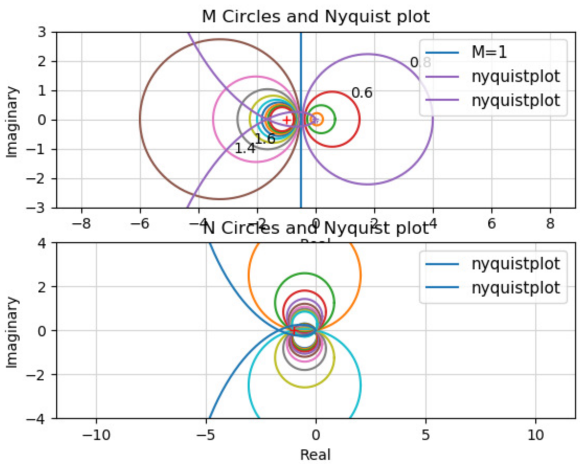

Fig. 1.2

1.3. Find the intersection of M and N circles with Nyquist plot at different frequencies.
Solution: The following code finds intersection of M and N circles with Nyquist plot at different frequencies

```
codes/ee18btech11020/
    ee18btech11020_code2.py
```

The points M and frequencies are listed in Table **??**

M in dB	**M**	ω
13.08	4.51	1.8
7.640	2.41	1.98
5.153	1.81	2.07
-0.81	0.91	2.37
-10.17	0.31	3.17
-40	0.01	8.47

TABLE 1.3

The points M and frequencies are listed in Table**??**

α	**N**	ω
113.49	-2.3	6.684
111.03	-2.6	7.67
105.94	-3.5	10.011
101.30	-5	14.09
102.80	-4.4	24.77
77.73	4.6	34.13

TABLE 1.3

The constant N locus for given value of α is not the entire circle but only an arc.This is beacuse tangent of angle remains same if $+180°$ or $-180°$ is added to the angle.

1.4. Plot Magnitude and Phase plot from the values obtained above.
Solution: The following code plots Fig. 1.4

```
codes/ee18btech11020/
    ee18btech11020_code3.py
```

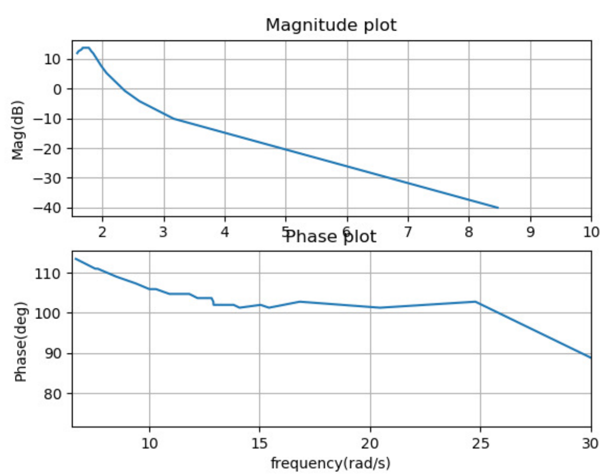

Fig. 1.4

1.5. Compare the above plot with bode plot of closed loop transfer function.
Solution: The following code plots Fig. 1.5

```
codes/ee18btech11020_code4.py
```

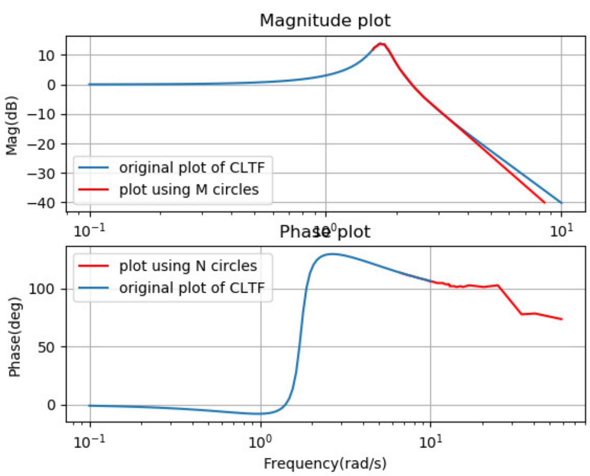
Magnitude plot
Mag(dB)
original plot of CLTF
plot using M circles
Phase plot
Phase(deg)
plot using N circles
original plot of CLTF
Frequency(rad/s)

Fig. 1.5