JSTAT-026P-1117: Reply to Referees

(Dated: February 18, 2018)

I. RESPONSES TO REFEREE COMMENTS

We thank the referees for their critical reading, valuable comments and suggestions on our manuscript. We have modified the manuscript and the following is our detailed response to the comments:

Referee comment: 1) My main concern is that it is unclear to me how much this work adds to the work reported in references 11-19. I suggest the authors clarify this.

Our response: The work in Ref. 11-19 do not consider disorder. It was not clear a priori whether the results derived there would hold for disordered systems, given the fact that there are considerable error bars for disorder realizations. The analytical expressions and scaling ansatz used for the clean cases need to be rederived in the presence of disorder. However, our numerical simulations indicate that the analytical arguments should also hold in presence of disorder.

Referee comment: 2) I agree with the authors that the unsuccessful study of the RCF system in Eq 5 is important to include. However, it would be illuminating if the authors added some comments on the reason of this failing. Why do they think the Monte-Carlo optimization fails here but not for the other models?

Our response: We have used a cluster update algorithm for the Monte Carlo updates for interactions involving nearest neighbors. It is nontrivial to design such an algorithm for next nearest neighbour interaction, and hence it is beyond the scope of the current work. Without such cluster updates, the efficiency of the Monte Carlo simulations is order of magnitude less, and with the computation power available to us, this did not lead to convergent results. We have added the above explanation in the manuscript.

Referee comment: 3) I recommend the authors add a few sentences on the background of RMI and phase transitions, explaining why a crossing of the RMI curves indicates a phase transition. This would help bring the communities of information theorists and physicists closer together.

Our response: The RMI scales as (see Ref. 9):

$$\frac{I_2(A,B)}{L} = a_2(\beta) + \frac{d_2(\beta)}{L} + \mathcal{O}\left(L^{-2}\right), \tag{1}$$

where the term $d_2(\beta)$ is related to the symmetry breaking of the lattice. When the symmetry breaking causes $d_2(\beta)$ to change sign, it passes through zero and the function $\frac{I_2(A,B)}{L}$ is then independent of system size up to order $\mathcal{O}\left(L^{-2}\right)$. One can show that $d_2(\beta) < 0$ for $T_c < T < 2T_c$, and positive elsewhere. This leads to the crossings in $\frac{I_2(L/2,L/2)}{L}$ at the temperatures T_c and $2T_c$ for different system sizes. We have added the above explanation in the manuscript.

Referee comment: 4) In the introduction, a classification of universality classes is promised but this is not taken up later in the paper. Unless I have missed something, the authors should either take out the promise or add this to the paper.

Our response: The universality class can be identified if we extract the central charge c by using geometric mutual information. In the current work, we have not tried to extract the value of c for each case. This can be done in a future work.

Referee comment: 5) Is it worth explaining what a 'glassy phase' is? (page 2)

Our response: Although we did not extensively define a 'glassy phase', we have mentioned its behaviour in the introduction of the article. Since we do not dwell on the glassy phase much throughout the paper, we feel it would not be worthwhile to further explain the phase at this stage. We have added a line in Sec. V pointing this out.

Referee comment: 6) Why is J set to 1 in Eq (4) but not in Eq (6)?

Our response: We thank the referee for pointing this out. We have now corrected the equation appropriately.

Referee comment: 7) Should it be $\beta = 1/(k_B T)$ rather than $\beta = 1/T$ on page 3?

Our response: We thank the referee for pointing this out. We have set $k_B = 1$ throughout. We have added this explanation in the manuscript.

Referee comment: 8) 3rd paragraph in Results: point to Fig 2.

Our response: We thank the referee for pointing this out. We have added the reference to the Figure.

Referee comment: 9) Last paragraph of Results: Give exact results from Eq 7 for comparison.

Our response: We have now added a couple of lines mentioning the exact results.

Referee comment: 10) The lower-value crossing points are never clearly visible. Maybe an inset with a zoom? I suspect this will show that they are hidden in the noise.

Our response: We thank the referee for the comment. We have now modified Fig. 2 to show zoomed regions.

Referee comment: 11) Why is Renyi entropy of order 2 used and not any other order? (Why is it called second Renyi entropy rather than Renyi entropy of order 2?)

Our response: For n-th order Renyi entropy, we need to use n copies/replicas for simulations. For second order Renyi entropy, we just need two copies. This is what we used in our simulations. The computational power needed for higher order Renyi entropies is more, while the second order Renyi entropy is enough to give us the values of T_c. Hence, higher order Renyi entropies are not needed for our purpose. The terminology "second Renyi entropy" is commonly used to indicate Renyi entropy of order 2.

II. LIST OF CHANGES

The following are the changes made in the manuscript based on the comments from the referees:

1. In the results section paragraph 2 we have added a couple of sentences as to why the Monte Carlo did not give good results for the Edwards-Anderson model. (Referee comment 2).

2. In the section “Methods”, in the paragraph below Eq. 9, we have added a couple of sentences on the relation between the Mutual information and the crossings at finite temperature. (Referee comment 3).

3. In the section “DISCUSSION AND CONCLUDING REMARKS”, we have added a line in the end. (Referee comment 4).

4. We have set $J = 1$ consistently throughout our article (Referee comment 6).

5. We have set $k_B = 1$ consistently throughout our article (Referee comment 7).

6. In the 3rd paragraph of the section “ Results”, we have added a reference to Fig 2 (Referee comment 8).

7. In the last paragraph of the section “Results”, we have added a couple of sentences mentioning the exact results for the random-bond Ising model at the self-dual point (Referee comment 9).

8. We have modified Figure 2 to include in the insets zoomed versions of the crossings (Referee comment 10).

We trust the manuscript will now be judged suitable for publication.

Sincerely,
Ipsita Mandal and P. V. Srilucksmy