

Dear Editor,

I have addressed all the queries raised by the referee. Detailed response to the queries of the referee has been provided below (Referee's comments are highlighted in blue). In view of these changes, I believe that the manuscript will now be judged suitable for publication.

Sincerely,

Ipsita Mandal

I. REVIEWER 2

1. I had asked if the author could provide physical insight for the approximate OMM-independence. An example of possible physical insight would be the ratio of certain energy scales being $\gg 1$. An argument of that type would be quite useful instead of one that depends on the behavior of certain integrals without addressing why the integrals behave that way.

The physical reason of the sub-dominance of the OMM can be seen as follows, which we have added in the manuscript: "The Boltzmann theory is valid in the semiclassical limit, which is valid in the limit of magnetic fields of a low magnitude. This condition ensures that the splitting between two subsequent Landau levels is small compared to the chemical potential, μ . One can show that the bound on B is captured by $B \ll (\mu/v_0)^2/e$, as demonstrated in the supplemental material of Ref. [1]. We will take this low- B -limit also to be such that it causes only a slight distortion of the Fermi surface [cf. Fig. 1(b)] via the effect of OMM. That is, we consider magnetic fields satisfying $e|\mathbf{B} \cdot \boldsymbol{\Omega}^s| \ll 1$, such that $|\varepsilon_{(m)}(\mathbf{k})|$ is small compared to $|\varepsilon^s(\mathbf{k})|$, i.e., $|\mathbf{B} \cdot \mathbf{m}^s(\mathbf{k})| \equiv e v_0 k |\mathbf{B} \cdot \boldsymbol{\Omega}^s| \ll |\varepsilon^s(\mathbf{k})|$. Now the OMM contributions appear in the conductivity via $\varepsilon_{(m)}$ and $\mathbf{v}_{(m)}$, which are naturally expected to be subleading compared to those coming from ε^s and $\mathbf{v}^s(\mathbf{k})$. This is reflected in Figs. 2 and 3, where the curves are not affected much when the OMM is not considered."

2. I'm happy with the response regarding numerical values. My only suggestion is to mention in the manuscript that trivial Fermi pockets without BC or OMM do not contribute intrinsically to LMC. A minor comment - I could not find the units of $\delta\sigma_z z$ in the plots or figure caption. That should be clarified.

I have added the statement that

"We would like to point out that the trivial Fermi pockets have zero BC or OMM vector fields and, hence, do not contribute intrinsically to longitudinal magnetoconductivity."

From the definition,

$$\delta\sigma_{zz}(s) \equiv \sigma_{zz}^s(B)/\sigma_{zz}^s(B=0) - 1,$$

that could be found in the manuscript, one can clearly see that $\delta\sigma_{zz}$ is unitless, as it is obtained from a ratio. So no further justification was required at any point for an observant reader.

3. I stand by my comment that the presentation is not suited for Scientific Reports. On one hand, considerable space has been devoted to trivial issues like transforming between Cartesian and polar coordinates, diagonalizing a 2×2 Hamiltonian, explaining the solution of linear equations etc. On the other hand, the manuscript contains unwieldy expressions, e.g. Eq 38, whose last equation alone occupies 3 lines. How do various terms in this expression rank in importance? Does the expression simplify in certain limits? Without further insight, equations like this generally belong to supplemental material/appendices even in specialized technical journals, and must be treated similarly for scientific reports as well.

It is extremely important to show the starting Hamiltonian and its eigenvalues, without which the system cannot be explained in the first place. It is also important to show the variables using the spherical polar coordinates as the collision integral contains terms using these coordinates, which reflects how a nontrivial angular dependence influences the conductivity. Finally, the parametrization of the eigenspinors feed into the expression for $\mathcal{T}_{s,\tilde{s}}(\theta, \theta')$ [cf. Eq. (21)] and it is important to show their expressions as well. However, I have moved some equations involving the BC and OMM to the supplementary material. The long expressions in Sec. IV [viz., "Comparison with results obtained from relaxation-time approximation"] have been moved to the supplementary material. Many equations from subsection "Kinetic equations driven by electromagnetic fields" have been moved to the supplementary material as well.

II. LIST OF CHANGES

The additions are marked in the marked-up version of the revised manuscript in the color red. Some materials have been relegated to the supplementary material.

- [1] A. Knoll, C. Timm, and T. Meng, Negative longitudinal magnetoconductance at weak fields in Weyl semimetals, *Phys. Rev. B* **101**, 201402 (2020).