

Dear Editor,

I have addressed all the queries raised by the referee. Detailed response to the queries of the referee has been provided below (Referee's comments are highlighted in blue). In view of these changes, I believe that the manuscript will now be judged suitable for publication.

Sincerely,
Ipsita Mandal

I. REVIEWER 2

1. What is the "physical" reason for the approximate OMM-independence? The author has given explanations in terms of the behavior of certain integrals, but these are mathematical arguments, not physical.

The physical reason of the OMM can be seen as follows, which we have added in the manuscript:

"In the semiclassical picture, a Bloch electron is modelled by a wave-packet in a Bloch band, which is found to rotate about its center of mass in general, yielding an intrinsic magnetic moment dubbed as the OMM, which is topological in origin [1–5]. In the presence of a magnetic field, the electronic band-structure energy acquires a correction term from this intrinsic orbital moment. This invariably generates a modification of the Fermi surface, making it anisotropic, as shown in Fig. 1 (b). This brings about the possibility of the OMM generating nontrivial terms compared to the case when it is neglected. Now, whether it gives rise to a nonzero term or not depends on whether it can generate a nonzero intergral, similar to the BC part. This can be roughly determined by looking at whether the resulting integrand contains odd or even powers of the components of the momentum [see, for example, the discussions around Eq. (34)]."

Which terms exactly cancel or which term survive is precisely a mathematical analysis and the result of the mathematical analysis leads us to the physical answer. Without doing that analysis, one cannot arrive at the physical answer.

2. Besides listing several candidate materials, I suggest the author provide numerical estimates for the size of the effect in at least one material. Do none of these materials have trivial Fermi pockets? Most Weyl semimetals - including TaAs - have trivial Fermi pockets and they contribute significantly to transport.

I do not understand the numerical estimate of which effect the referee wants to know about. The entire paper deals with finding the numerical values of the effects of longitudinal magnetotransport, the representative numerical values of whose conductivity are provided in Figs. 2, 3 and 4. I would like to point out that it was already explained in the manuscript that

"Experimentally, the smoking-gun properties of KWNs have been observed in materials like β -Ag₂Se [6], which crystallises in a nonsymmorphic chiral form. In this paper, we will consider isotropic KWNs, which arises at a node harbouring the high-symmetry cubic point group, {T, O}. The candidate materials include K₂Sn₂O₃ [7], RhSi [8], CoSi [8], AlPt [9], and PtGa [10]." Following on that statement, the parameter values correspond to the material β -Ag₂Se have been used in making the plots. We had already mentioned this in our manuscript:

"Our choice of parameter regimes is dictated by the values mentioned in the literature [6]."

The explicit values of these parameters are provided in the captions and labels of the plots. Also, the referee seems to have confused TaAs, provided as an example of a conventional band-inversion Weyl semimetal, with the examples of *chiral crystals* provided for KWNs, which is the subject of study for the manuscript. The conductivity curves also describe the error one would make on neglecting the OMM part.

There can be many trivial (meaning, non-topological) Fermi pockets. But the crux of the matter is that these trivial Fermi pockets *do not* contribute to longitudinal magnetotransport (LMC) by the virtue of the very fact that their BC and OMM are zero. This is also the reason why LMC has emerged as a crucial signature to identify topologically-protected nodal points. Of course there can be other kinds of transport where they can contribute to (for example, ordinary Hall effect and the electric and magnetic fields are perpendicular to each other), but that is not what we are studying here. As such, there is no scope of the trivial Fermi pockets contributing to longitudinal magnetotransport.

3. I believe moving many details to a different part of the paper - methods or supplementary material - will increase the readability given the broad appeal of scientific reports. The current structure is acceptable for a more specialized journal for theoretical physics.

I do not agree with the belief of the referee on this point, the reasons being articulated already in the earlier response. I repeat those here so that the referee understands and appreciates the overarching reasons: I have kept the essential minimal Boltzmann equations in the main text which are required to set the notations of the final expressions to be solved. Without them, the readability and continuity of the manuscript will be completely lost. Furthermore, in Scientific Reports, appendices are not allowed — only supplementary information pdf is allowed. In

order not to sacrifice the readability of the manuscript, I disagree with the observation that “The current structure is acceptable for a more specialized journal for theoretical physics.” *On the contrary, these steps and explanations are quintessential for a broad non-specialized audience, uninitiated to Boltzmann formalism and other essential physical concepts.* In other words, I cannot comply with this suggestion without sacrificing the readability of the manuscript, which is the most important factor while thinking up the structure of the paper.

II. LIST OF CHANGES

The additions are marked in the marked-up version of the revised manuscript in the color red.

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 - [9] N. B. M. Schröter, D. Pei, M. G. Vergniory, Y. Sun, K. Manna, F. de Juan, J. A. Krieger, V. Süß, M. Schmidt, P. Dudin, B. Bradlyn, T. K. Kim, T. Schmitt, C. Cacho, C. Felser, V. N. Strocov, and Y. Chen, Chiral topological semimetal with multifold band crossings and long Fermi arcs, *Nature Physics* **15**, 759 (2019).
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