

# EE 338 - Digital Signal Processing

Filter Design Assignment

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## 1 Infinite Impulse Response Bandpass Filter

The filter number m = 44. Hence,

$$q(m) = \lfloor 0.1m \rfloor = 4$$
  
 $r(m) = m - 10 q(m) = 44 - 40 = 4$   
 $BL(m) = 25 + 1.7q(m) + 6.1r(m) = 55.8 \text{ kHz}$   
 $BH(m) = BL(m) + 20 = 75.8 \text{ kHz}$ 

## 1.1 Unnormalized Specifications

- Lower Passband Edge = 56.2 kHz
- Upper passband Edge = 76.2 kHz
- Transition band width = 4 kHz
- Passband tolerance = 0.15
- Stopband tolerance = 0.15
- Passband nature = monotonic
- Stopband nature = monotonic
- Sampling frequency = 330 kHz

## 1.2 Normalized Specifications

The sampling angular frequency  $2\pi f_s$  should map to  $2\pi$  after normalization. Hence the transformation used is

$$\omega = 2\pi \frac{f_s}{f}$$

The normalized specifications are as follows (following the notations used in the lecture)

• 
$$\omega_{p1} = 2\pi \, \frac{55.8}{330} = 0.3382 \, \pi$$

• 
$$\omega_{p2} = 2\pi \, \frac{75.8}{330} = 0.4594 \, \pi$$

- $\omega_{s1} = \omega_{p1} 0.0242 \,\pi = 0.314 \,\pi$
- $\omega_{s2} = \omega_{p2} + 0.0242 \,\pi = 0.4836 \,\pi$
- Passband and Stopband tolerance = 0.15
- Passband and Stopband nature = monotonic

## 1.3 Analog Filter Specifications using Bilinear Transform

We use the following transformation to move from the z domain to the s domain.

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

If we put  $z = j\omega$  and  $s = j\Omega$ , we get

$$\Omega = \tan \frac{\omega}{2}$$

The specifications now are

- Passband and Stopband nature = monotonic
- $\Omega_{s1} = 0.5375$
- $\Omega_{s2} = 0.9498$
- $\Omega_{p1} = 0.5875$
- $\Omega_{p2} = 0.8799$
- $\delta_1 = \delta_2 = 0.15$

## 1.4 Analog Frequency Transformation

Here we use the **Bandpass to Lowpass** analog frequency transformation, which is as follows:

$$\Omega_L = \frac{\Omega^2 - \Omega_0^2}{B\Omega}$$

where

$$\Omega_0 = \sqrt{\Omega_{p1} \, \Omega_{p2}} = 0.7189$$

and

$$B = \Omega_{p2} - \Omega_{p1} = 0.2924$$

Using the transformation mentioned above, we now have

- Passband and Stopband nature = monotonic
- $\Omega_{Ls1} = -1.4501$
- $\Omega_{Ls2} = 1.3874$
- $\Omega_{Lp1} = -1$
- $\Omega_{Lp2} = +1$
- $\delta_1 = \delta_2 = 0.15$

## 1.5 Equivalent Analog Lowpass Filter Specifications

Using the more stringent specifications from those obtained in section 1.4, we have

- Passband and Stopband nature = monotonic
- $\Omega_{Ls} = \min\{\Omega_{Ls2}, |\Omega_{Ls1}|\} = 1.3874$
- $\Omega_{Lp} = +1$
- $\delta_1 = \delta_2 = 0.15$
- $D_1 = \frac{1}{(1-\delta_1)^2} 1 = 0.384$
- $D_2 = \frac{1}{\delta_2^2} 1 = 43.44$

## 1.6 Analog Lowpass Magnitude Response

Since we have to make a Butterworth filter, the general response is given as

$$H_{LPF}(s_L) H_{LPF}(-s_L) = \frac{1}{1 + \left(\frac{s_L}{j\Omega_c}\right)^{2N}}$$

The parameters N and  $\Omega_c$  can be found as follows:

$$N = \left\lceil rac{\log(\sqrt{D_2/D_1})}{\log(\Omega_{Ls}/\Omega_{Lp})} 
ight
ceil$$

and

$$\frac{\Omega_{Lp}}{D_1^{\frac{1}{2N}}} \le \Omega_c \le \frac{\Omega_{Ls}}{D_2^{\frac{1}{2N}}}$$

Using the above relations, we get

$$N = 8$$

$$1.0616 \le \Omega_c \le 1.096$$

We choose  $\Omega_c=1.0788$ . Using these values, we have

$$H_{LPF}(s_L) H_{LPF}(-s_L) = \frac{1}{1 + \left(\frac{s_L}{j \cdot 1.0788}\right)^{16}}$$

After using MATLAB to find and plot the poles, we have

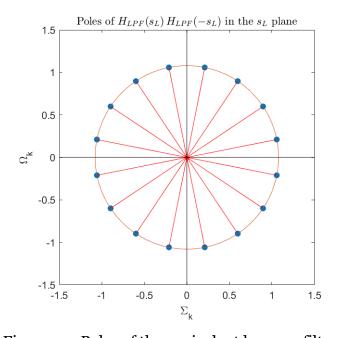


Figure 1.1: Poles of the equivalent lowpass filter

To have a realisable filter, we choose only those poles which lie on the left half of the complex  $s_L$  plane, which are as follows:

Pole	Value
$s_1$	-0.5993 + j  0.8970
$s_2$	-0.8970 + j  0.5993
$s_3$	-1.0581 + j  0.2105
$s_4$	-1.0581 - j  0.2105
$s_5$	-0.8970 - j  0.5993
$s_6$	-0.5993 - j  0.8970
$s_7$	-0.2105 - j  1.0581
$s_8$	-0.2105 + j  1.0581

Table 1.1: Poles on left half plane

Using the poles determined in table 1.1, the **equivalent analog lowpass transfer function** is given by

$$H_{LPF}(s_L) = \frac{\Omega_c^N}{\prod_{k=1}^N (s_L - s_k)} = \frac{1.8345}{\sum_{k=0}^8 a_k s_L^k}$$

where the coefficients of the denominator are

Coefficient	Value
$a_0$	1.8345
$a_1$	8.7167
$a_2$	20.708
$a_3$	31.921
$a_4$	34.794
$a_5$	27.428
$a_6$	15.289
$a_7$	5.5297
$a_8$	1

Table 1.2: Coefficients of Denominator

The magnitude and the phase responses of this equivalent Butterworth LPF are as follows:

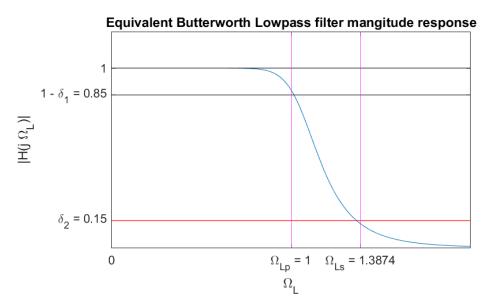


Figure 1.2: Magnitude Response

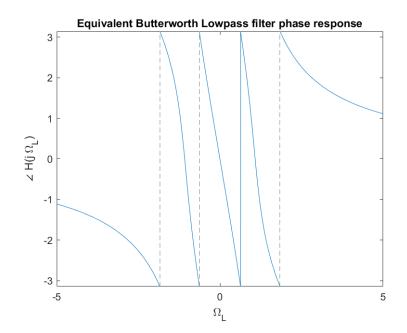


Figure 1.3: Phase Response

## 1.7 Analog Bandpass Transfer Function

We now use the inverse transformation to convert the equivalent Butterworth lowpass transfer function to get the Bandpass transfer function using the relation:

$$s_L \longleftarrow \frac{s^2 + \Omega_0^2}{B s}$$

where B and  $\Omega_0$  have the values as we found in section 1.4

$$H_{LPF}\left(\frac{s^2 + \Omega_0^2}{B \, s}\right) = H_{BPF}(s) = \frac{1.8345}{168879.9214 + \sum_{k=-8}^{8} a_k \, s^k}$$

The coefficients  $a_k$  are as follows:

Coefficient	Value
$a_1$	182483.5084
$a_{-1}$	94310.6177
$a_2$	252765.02515
$a_{-2}$	67513.5454
$a_3$	204166.44531
$a_{-3}$	28183.5223
$a_4$	220581.2811
$a_{-4}$	15736.8035
$a_5$	122303.3771
$a_{-5}$	4509.4469
$a_6$	101839.6751
$a_{-6}$	1940.6124
$a_7$	30259.6314
$a_{-7}$	298.0042
$a_8$	18714.6217
$a_{-8}$	95.2525

Table 1.3: Coefficients of Denominator in the Analog BP transfer function

The magnitude and frequency response are as follows:

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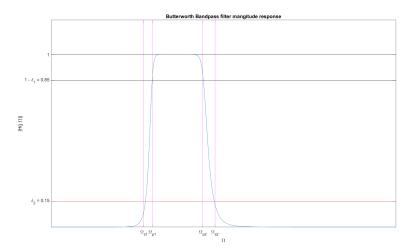


Figure 1.4: Magnitude response of the analog BPF

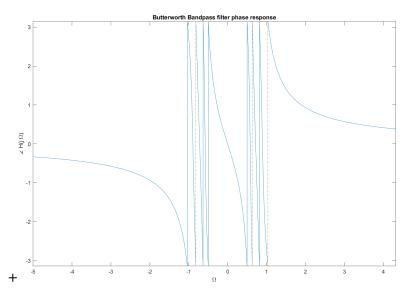


Figure 1.5: Phase response of the analog BPF

## 1.8 Discrete-Time Bandpass Transfer Function

We now make use of the Bilinear Transformation to convert the analog bandpass filter into a discrete bandpass filter in the normalized angular frequency domain. The transform is as follows:

$$s \longleftarrow \frac{1 - z^{-1}}{1 + z^{-1}}$$

The discrete time filter transfer function is

$$H_{BPF}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) = H(z) = \frac{\sum\limits_{k=0}^{16} b_k z^k}{\sum\limits_{k=0}^{16} a_k z^k}$$

## with coefficients

Coefficient	Value
$a_{16}$	0.0083
$a_{15}$	-0.0366
$a_{14}$	0.1197
$a_{13}$	-0.2688
$a_{12}$	0.5003
$a_{11}$	-0.7489
$a_{10}$	0.9662
$a_9$	-1.0495
$a_8$	0.9966
$a_7$	-0.8059
$a_6$	0.5697
$a_5$	-0.3389
$a_4$	0.1737
$a_3$	-0.0715
$a_2$	0.0244
$a_1$	-0.0057
$a_0$	0.0010

Coefficient	Value
$b_{16}$	1
$b_{15}$	0
$b_{14}$	-8
$b_{13}$	0
$b_{12}$	28
$b_{11}$	0
$b_{10}$	-56
$b_9$	0
$b_8$	70
$b_7$	0
$b_6$	-56
$b_5$	0
$b_4$	28
$b_3$	0
$b_2$	-8
$b_1$	0
$b_0$	1

Table 1.4: Denominator  $(\times 10^8)$ 

Table 1.5: Numerator

Please see the next page

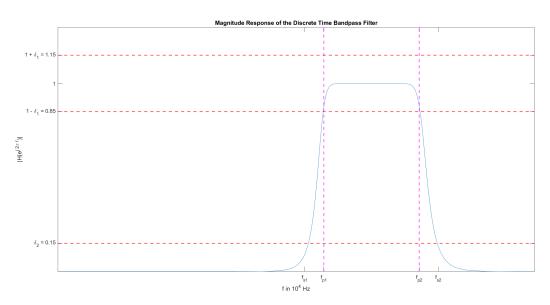


Figure 1.6: Magnitude Response of Discrete Time Bandpass Filter

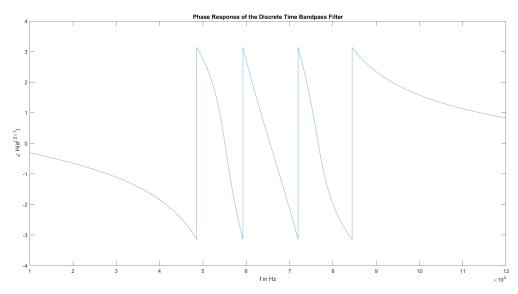


Figure 1.7: Phase Response of Discrete Time Bandpass Filter

## 2 Finite Impulse Response Bandpass Filter

In this part, we will use the windowing method to design a finite impulse response bandpass filter for the filter 44 using the same specifications as above. Recalling the specifications from section 1.2, we have the following normalized specifications:

• 
$$\omega_{p1} = 2\pi \frac{55.8}{330} = 0.3382 \,\pi$$

• 
$$\omega_{p2} = 2\pi \, \frac{75.8}{330} = 0.4594 \, \pi$$

- Transition bandwidth( $\Delta\omega_T$ ) =  $2\pi \frac{4}{330} = 0.0242 \,\pi$
- $\omega_{s1} = \omega_{p1} 0.0242 \,\pi = 0.314 \,\pi$
- $\omega_{s2} = \omega_{p2} + 0.0242 \,\pi = 0.4836 \,\pi$
- Passband and Stopband tolerance( $\delta$ ) = 0.15

We will use a time-shifted **Kaiser Window** and multiply it with the impulse response of an ideal bandpass filter to get a realizable FIR filter.

## 2.1 Implementation Technique

While calculating the ideal bandpass filter response, the mean of the lower passband and stopband edges are used as one of the lower passband edge and the mean of the upper passband and stopband edges are used to find the upper passband edge. The same is mentioned in the following figure:

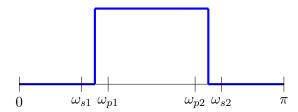


Figure 2.1: Ideal Bandpass Filter Magnitude Response

#### 2.2 Kaiser Window Parameters

We have

$$A = -20 \log_{10}(\delta) = 16.478$$

As A < 21, the shape parameter  $\beta = 0$  and the Kaiser window in this case is essentially a rectangular window. Also, the window width M is

$$M \ge \left[1 + \frac{A - 8}{2.285 \,\Delta\omega_T}\right] = 50$$

After tuning by hit and trail method so that the maximum error is within the tolerance, it has been found that a window width of M=66 satisfies all the required specifications.

#### 2.3 Discrete Time FIR Filter Transfer Function

The transfer function of the FIR filter obtained after adjusting for causality is as shown below. These are the coefficients of the Z transform of the impulse response of the FIR filter.

```
Columns 1 through 8

-0.0177 -0.0031 0.0115 0.0070 -0.0020 0.0001 0.0012 -0.0095

Columns 9 through 16

-0.0124 0.0084 0.0257 0.0067 -0.0264 -0.0241 0.0114 0.0274

Columns 17 through 24

0.0056 -0.0134 -0.0058 -0.0010 -0.0144 -0.0073 0.0341 0.0436

Columns 25 through 32

-0.0237 -0.0840 -0.0279 0.0904 0.0968 -0.0419 -0.1376 -0.0433

Columns 33 through 40

0.1175 0.1175 -0.0433 -0.1376 -0.0419 0.0968 0.0904 -0.0279

Columns 41 through 48

-0.0840 -0.0237 0.0436 0.0341 -0.0073 -0.0144 -0.0010 -0.0058

Columns 49 through 56

-0.0134 0.0056 0.0274 0.0114 -0.0241 -0.0264 0.0067 0.0257

Columns 57 through 64

0.0084 -0.0124 -0.0095 0.0012 0.0001 -0.0020 0.0070 0.0115

Columns 55 through 66

-0.0031 -0.0177
```

Figure 2.2: Bandpass FIR Transfer Function

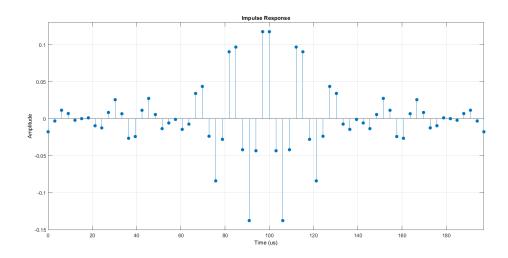


Figure 2.3: Bandpass FIR Impulse Response

# 2.4 Magnitude and Phase Response

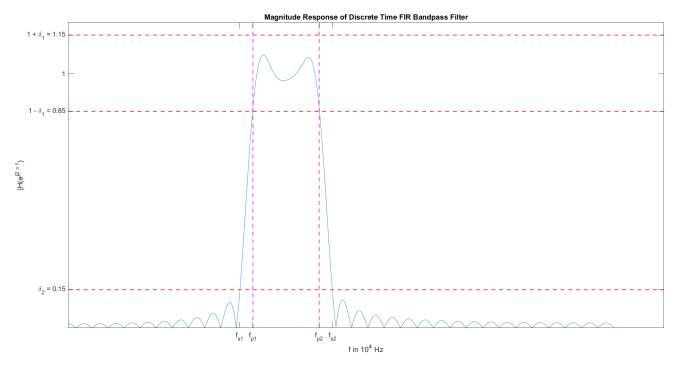


Figure 2.4: Magnitude Response of the FIR filter

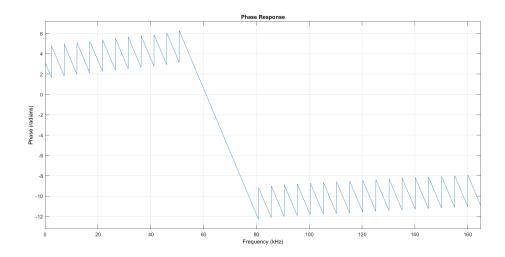


Figure 2.5: Phase Response of the FIR filter

## 2.5 Comparison with the IIR Filter

We can see that on comparing figure 1.7 and figure 2.5, we can see that the FIR filter gives a linear phase response for the frequencies in the passband range i.e from 55.8kHz to 75.8kHz.

Also, we can observe that the order of the IIR filter is 8 and the order of the FIR filter is 66 which is much higher. This once again proves the fact that *there* is no free lunch, i.e., we are getting a linear phase response at the expense of resources.

## 3 Infinite Impulse Response Bandstop Filter

The filter number m=44. Hence,

$$q(m) = \lfloor 0.1m \rfloor = 4$$
  
 $r(m) = m - 10 q(m) = 44 - 40 = 4$   
 $BL(m) = 25 + 1.9q(m) + 4.1r(m) = 49 \text{ kHz}$   
 $BH(m) = BL(m) + 20 = 69 \text{ kHz}$ 

## 3.1 Unnormalized Specifications

- Lower Passband Edge = 45 kHz
- Upper passband Edge = 73 kHz
- Transition band width = 4 kHz
- Lower Stopband Edge = 49 kHz
- Upper Stopband Edge = 69 kHz
- Passband tolerance = 0.15
- Stopband tolerance = 0.15
- Passband nature = equiripple
- Stopband nature = monotonic
- Sampling frequency = 260 kHz

## 3.2 Normalized Specifications

The sampling angular frequency  $2\pi f_s$  should map to  $2\pi$  after normalization. Hence the transformation used is

$$\omega = 2\pi \frac{f_s}{f}$$

The normalized specifications are as follows (following the notations used in the lecture)

• 
$$\omega_{p1} = 2\pi \, \frac{45}{260} = 0.3461 \, \pi$$

• 
$$\omega_{p2} = 2\pi \, \frac{73}{260} = 0.5615 \, \pi$$

• 
$$\omega_{s1} = 2\pi \frac{49}{260} = 0.3769 \,\pi$$

• 
$$\omega_{s2} = 2\pi \frac{69}{260} = 0.5307 \,\pi$$

- Transition bandwidth =  $2\pi\,\frac{4}{260}=0.0308\,\pi$
- Passband and Stopband tolerance = 0.15
- Passband nature = equiripple
- Stopband nature = monotonic

## 3.3 Analog Filter Specifications using Bilinear Transform

We use the following transformation to move from the z domain to the s domain.

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

If we put  $z = j\omega$  and  $s = j\Omega$ , we get

$$\Omega = \tan\frac{\omega}{2}$$

The specifications now are

- Passband nature = equiripple
- Stopband nature = monotonic
- $\Omega_{s1} = 0.6725$
- $\Omega_{s2} = 1.1014$
- $\Omega_{p1} = 0.6044$
- $\Omega_{p2} = 1.2146$
- $\delta_1 = \delta_2 = 0.15$

## 3.4 Analog Frequency Transformation

Here we use the **Bandstop to Lowpass** analog frequency transformation, which is as follows:

$$\Omega_L = \frac{B\,\Omega}{\Omega_0^2 - \Omega^2}$$

where

$$\Omega_0 = \sqrt{\Omega_{p1} \, \Omega_{p2}} = 0.8568$$

and

$$B = \Omega_{p2} - \Omega_{p1} = 0.6102$$

Using the transformation mentioned above, we now have

- Passband and Stopband nature = monotonic
- $\Omega_{Ls1} = 1.456$
- $\Omega_{Ls2} = -1.4031$
- $\Omega_{Lp1} = +1$
- $\Omega_{Lp2} = -1$
- $\delta_1 = \delta_2 = 0.15$

## 3.5 Equivalent Analog Lowpass Filter Specifications

Using the more stringent specifications from those obtained in section 3.4, we have

- Passband nature = equiripple
- Stopband nature = passband
- $\Omega_{Ls} = \min\{|\Omega_{Ls2}|, |\Omega_{Ls1}|\} = 1.4031$
- $\Omega_{Lp} = +1$
- $\delta_1 = \delta_2 = 0.15$
- $D_1 = \frac{1}{(1-\delta_1)^2} 1 = 0.384$
- $D_2 = \frac{1}{\delta_2^2} 1 = 43.44$

### 3.6 Analog Lowpass Magnitude Response

Since we have to make a Chebyshev filter, the general response is given as

$$H_{LPF}(s_L) H_{LPF}(-s_L) = \frac{1}{1 + \epsilon^2 C_N^2(s_L/j \Omega_{Lp})}$$

Where the parameters  $\epsilon$  and N are given by

$$\epsilon = \sqrt{D_1} = 0.6197$$

and

$$N = \left\lceil \frac{\cosh^{-1}(\sqrt{D_2/D_1})}{\cosh^{-1}(\Omega_{Ls}/\Omega_{Lp})} \right\rceil = 4$$

After using MATLAB to find and plot the poles, we have

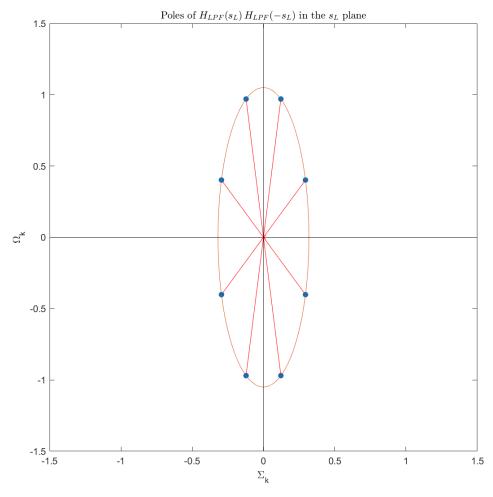


Figure 3.1: Poles of the equivalent lowpass filter

To have a realisable filter, we choose only those poles which lie on the left half of

the complex  $s_L$  plane, which are as follows:

Pole	Value
$s_1$	-0.1222 + j0.9698
$s_2$	-0.2949 + j  0.4017
$s_3$	-0.2949 - j  0.4017
$s_4$	-0.1222 - j  0.9698

Table 3.1: Poles on left half plane

Using the poles determined in table 3.1, the **equivalent analog lowpass transfer function** is given by

$$H_{LPF}(s_L) = \frac{A}{\prod\limits_{k=1}^{N} (s_L - s_k)}$$

Substituting the constraint that the value of the magnitude response at the passband edge is  $1 - \delta_1$ , we have

$$H_{LPF}(s_L) = \frac{\prod_{i=1}^{4} s_i}{\sqrt{1 + D_1} \left(\prod_{k=1}^{N} (s_L - s_k)\right)} = \frac{0.2373}{1.1764 \, s_L^4 + 0.9814 \, s_L^3 + 1.5858 \, s_L^2 + 0.7345 \, s_L + 0.2792}$$

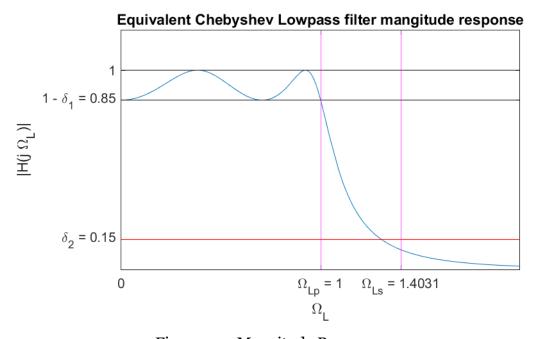


Figure 3.2: Magnitude Response

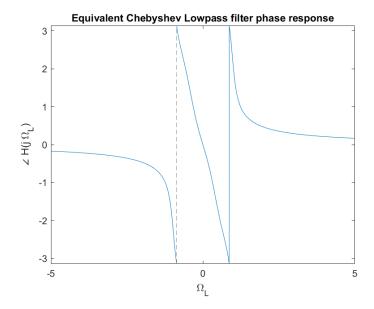


Figure 3.3: Phase Response

## 3.7 Analog Bandstop Transfer Function

We now use the inverse transformation to convert the equivalent Chebyshev lowpass transfer function to get the Bandstop transfer function using the relation:

$$s_L \longleftarrow \frac{B \, s}{s^2 + \Omega_0^2}$$

where B and  $\Omega_0$  have the values as we found in section 3.4

$$H_{LPF}\left(\frac{B\,s}{s^2 + \Omega_0^2}\right) = H_{BSF}(s) = \frac{1.3516\,s^8 + 3.9691\,s^6 + 4.3707\,s^4 + 2.1390\,s^2 + 3.9257}{4.6183 + \sum_{k=1}^8 b_k\,s^k}$$

Coefficient	Value
$b_8$	1.5902
$b_7$	2.5527
$b_6$	8.0327
$b_5$	6.8921
$b_4$	1.1001
$b_3$	5.0595
$b_2$	4.3289
$b_1$	1.0099

Table 3.2: Denominator

The magnitude and frequency response are as follows:

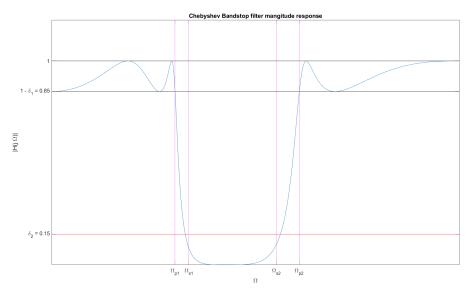


Figure 3.4: Magnitude Response of Chebyshev Bandstop filter

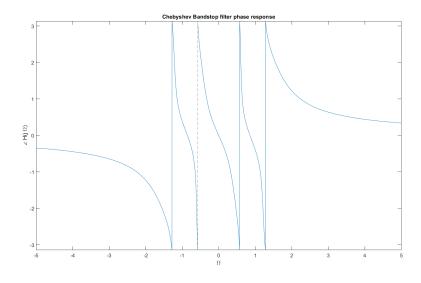


Figure 3.5: Phase Response of Chebyshev Bandstop filter

## 3.8 Discrete-Time Bandstop Transfer Function

We now make use of the Bilinear Transformation to convert the analog bandstop filter into a discrete bandstop filter in the normalized angular frequency domain. The transform is as follows:

$$s \longleftarrow \frac{1 - z^{-1}}{1 + z^{-1}}$$

The discrete time filter transfer function is

$$H_{BSF}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) = H(z) = \frac{\sum\limits_{k=0}^{8} b_k z^k}{\sum\limits_{k=0}^{8} a_k z^k}$$

Coefficient	Value
$a_8$	4.0936
$a_7$	-3.6764
$a_6$	8.8842
$a_5$	-5.8977
$a_4$	8.6077
$a_3$	-3.7768
$a_2$	3.6893
$a_1$	-1.0919
$a_0$	9.9081

Table 3.3: Denominator

Coefficient	Value
$b_8$	1.2223
$b_7$	-1.4993
$b_6$	5.5789
$b_5$	-4.6390
$b_4$	8.7240
$b_3$	-4.6390
$b_2$	5.5789
$b_1$	-1.4993
$b_0$	1.2223

Table 3.4: Numerator

*Please see the next page* 

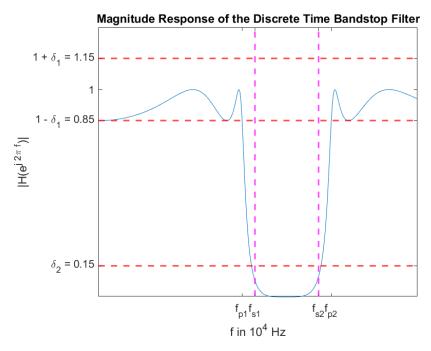


Figure 3.6: Magnitude Response of Discrete Time Bandstop Filter

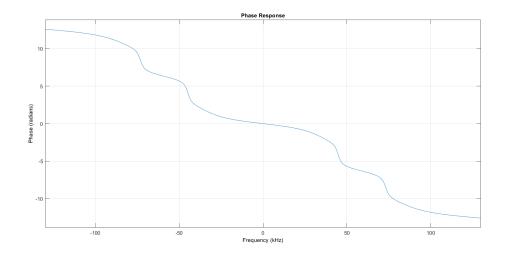


Figure 3.7: Phase Response of Discrete Time Bandstop Filter

## 4 Finite Impulse Response Bandstop Filter

In this part, we will use the windowing method to design a finite impulse response bandstop filter for the filter 44 using the same specifications as above. Recalling the specifications from section 3.2, we have the following normalized specifications:

• 
$$\omega_{p1} = 2\pi \, \frac{45}{260} = 0.3461 \, \pi$$

• 
$$\omega_{p2} = 2\pi \, \frac{73}{260} = 0.5615 \, \pi$$

• 
$$\omega_{s1} = 2\pi \frac{49}{260} = 0.3769 \,\pi$$

• 
$$\omega_{s2} = 2\pi \frac{69}{260} = 0.5307 \,\pi$$

- Transition bandwidth ( $\Delta\omega_T$ ) =  $2\pi\,\frac{4}{260}=0.0308\,\pi$
- Passband and Stopband tolerance = 0.15

We will use a time-shifted **Kaiser Window** and multiply it with the impulse response of an ideal bandstop filter to get a realizable FIR filter.

## 4.1 Implementation Technique

While calculating the ideal bandstop filter response, the mean of the lower passband and stopband edges are used as one of the lower stopband edge and the mean of the upper passband and stopband edges are used to find the upper stopband edge. The same is mentioned in the following figure:

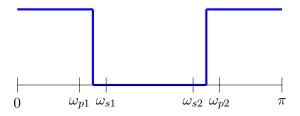


Figure 4.1: Ideal Bandstop Filter Magnitude Response

#### 4.2 Kaiser Window Parameters

We have

$$A = -20 \log_{10}(\delta) = 16.478$$

As A < 21, the shape parameter  $\beta = 0$  and the Kaiser window in this case is essentially a rectangular window. Also, the window width M is

$$M \ge \left[1 + \frac{A - 8}{2.285 \, \Delta \omega_T}\right] = 40$$

After tuning by hit and trail method so that the maximum error is within the tolerance, it has been found that a window width of M=53 satisfies all the required specifications.

#### 4.3 Discrete Time FIR Filter Transfer Function

The transfer function of the FIR filter obtained after adjusting for causality is as shown below. These are the coefficients of the Z transform of the impulse response of the FIR filter.

Columns 1 through 8							
-0.0188	0.0098	0.0156	-0.0020	-0.0028	0.0005	-0.0144	-0.0088
Columns 9	through 10	5					
0.0267	0.0228	-0.0271	-0.0326	0.0161	0.0273	-0.0030	-0.0028
Columns 17	through 2	24					
0.0018	-0.0346	-0.0232	0.0693	0.0674	-0.0839	-0.1220	0.0684
Columns 25	through 3	32					
0.1671	-0.0263	0.8154	-0.0263	0.1671	0.0684	-0.1220	-0.0839
Columns 33	Columns 33 through 40						
0.0674	0.0693	-0.0232	-0.0346	0.0018	-0.0028	-0.0030	0.0273
Columns 41	Columns 41 through 48						
0.0161	-0.0326	-0.0271	0.0228	0.0267	-0.0088	-0.0144	0.0005
Columns 49 through 53							
-0.0028	-0.0020	0.0156	0.0098	-0.0188			

Figure 4.2: Bandstop FIR Transfer Function

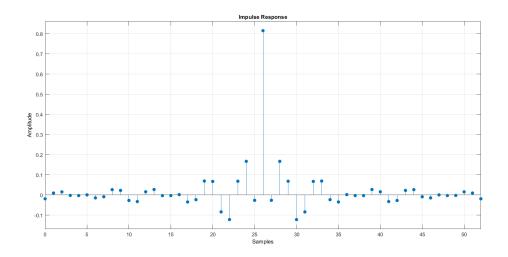


Figure 4.3: Bandstop FIR Impulse Response

# 4.4 Magnitude and Phase Response

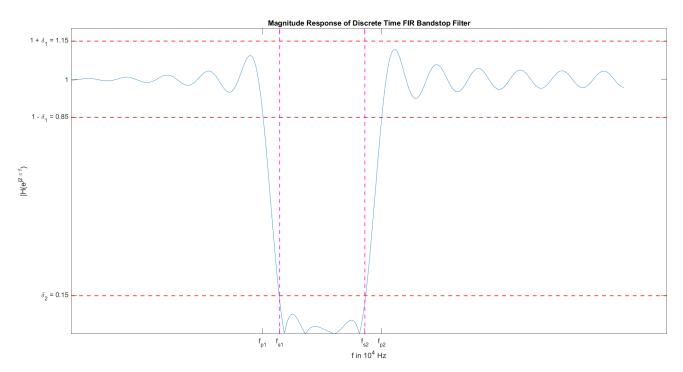


Figure 4.4: Magnitude Response of the FIR filter

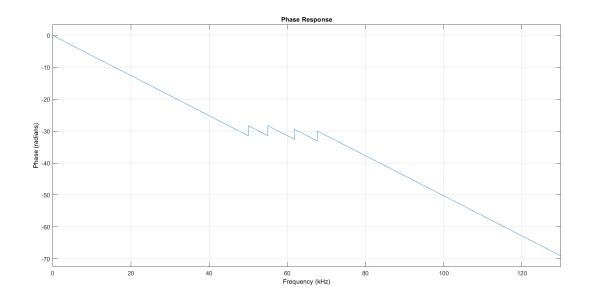


Figure 4.5: Phase Response of the FIR filter

## 4.5 Comparison with the IIR Filter

We can see that on comparing figure 3.7 and figure 4.5, we can see that the FIR filter gives a linear phase response for the frequencies in the passband range.

Also, we can observe that the order of the IIR filter is 4 and the order of the FIR filter is 53 which is much higher. This once again proves the fact that *there* is no free lunch, i.e., we are getting a linear phase response at the expense of resources.

## 5 Peer Review

#### 5.1 Review Received

Review by: Shlok Vaibhav Singh, 18D070064, Group 1

I have reviewed this report made by Mantri Krishna Sri Ipsit, I certify that he has completed all the steps for the design of a band-pass and a band-stop filter using IIR, FIR and elliptic filter design methods.

#### 5.2 Review Given

Review given to: Amol Girish Shah, 18D070005, Group 1

I have reviewed the report made by Amol Girish Shah and I certified that he has completed all the steps for the design of a band-pass and a band-stop filter using IIR, FIR and elliptic filter design methods.