EE 338 - Digital Signal Processing

Filter Design Assignment

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Abstract

This report is made as a part of the Evaluation Component - 4 of the Digital Signal Processing happenning in Autumn 2020 at IIT Bombay. This report summarizes the filter design techniques for designing a bandpass and a bandstop filter of Butterworth and Chebyshev nature.

Acknowledgements

I would like to thank and I am grateful to Professor V.M.Gadre for introducing and teaching me this fundamental and important course of Digital Signal Processing. The tasks that were a part of this course helped a lot in learning and understanding different aspects in this field, which even the ongoing pandemic couldn't stop. I would also like to thank my group, Mr. Shlok Vaibhav Singh and Mr. Amol Girish Shah for helping me and collaborating with me in various aspects of this course.

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1 Infinite Impulse Response Bandpass Filter

The filter number $m = 44$. Hence,

$$
\begin{aligned}
q(m) &= \lfloor 0.1m \rfloor = 4 \\
r(m) &= m - 10\, q(m) = 44 - 40 = 4 \\
BL(m) &= 25 + 1.7q(m) + 6.1r(m) = 55.8\,\text{kHz} \\
BH(m) &= BL(m) + 20 = 75.8\,\text{kHz}
\end{aligned}
$$

1.1 Unnormalized Specifications

- Lower Passband Edge = 55.8 kHz
- Upper passband Edge = 75.8 kHz
- Transition band width = 4 kHz
- Passband tolerance = 0.15
- Stopband tolerance = 0.15
- Passband nature = monotonic
- Stopband nature = monotonic
- Sampling frequency = 330 kHz

1.2 Normalized Specifications

The sampling angular frequency $2\pi f_s$ should map to 2π after normalization. Hence the transformation used is

$$\omega = 2\pi \frac{f_s}{f}$$

The normalized specifications are as follows (following the notations used in the lecture)

- $\omega_{p1} = 2\pi\, \frac{55.8}{330} = 0.3382\,\pi$
- $\omega_{p2} = 2\pi\, \frac{75.8}{330} = 0.4594\,\pi$

- Transition bandwidth = $2\pi \frac{4}{330} = 0.0242\,\pi$
- $\omega_{s1} = \omega_{p1} - 0.0242\,\pi = 0.314\,\pi$
- $\omega_{s2} = \omega_{p2} + 0.0242\,\pi = 0.4836\,\pi$
- Passband and Stopband tolerance = 0.15
- Passband and Stopband nature = monotonic

1.3 Analog Filter Specifications using Bilinear Transform

We use the following transformation to move from the z domain to the s domain.

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

If we put $z = j\omega$ and $s = j\Omega$, we get

$$\Omega = \tan\frac{\omega}{2}$$

The specifications now are

- Passband and Stopband nature = monotonic
- $\Omega_{s1} = 0.5375$
- $\Omega_{s2} = 0.9498$
- $\Omega_{p1} = 0.5875$
- $\Omega_{p2} = 0.8799$
- $\delta_1 = \delta_2 = 0.15$

1.4 Analog Frequency Transformation

Here we use the **Bandpass to Lowpass** analog frequency transformation, which is as follows:

$$\Omega_L = \frac{\Omega^2 - \Omega_0^2}{B\Omega}$$

where

$$\Omega_0 = \sqrt{\Omega_{p1}\,\Omega_{p2}} = 0.7189$$

and

$$B = \Omega_{p2} - \Omega_{p1} = 0.2924$$

Using the transformation mentioned above, we now have

- Passband and Stopband nature = monotonic
- $\Omega_{Ls1} = -1.4501$
- $\Omega_{Ls2} = 1.3874$
- $\Omega_{Lp1} = -1$
- $\Omega_{Lp2} = +1$
- $\delta_1 = \delta_2 = 0.15$

1.5 Equivalent Analog Lowpass Filter Specifications

Using the more stringent specifications from those obtained in section 1.4, we have

- Passband and Stopband nature = monotonic
- $\Omega_{Ls} = \text{min}\{\Omega_{Ls2}, |\Omega_{Ls1}|\} = 1.3874$
- $\Omega_{Lp} = +1$
- $\delta_1 = \delta_2 = 0.15$
- $D_1 = \frac{1}{(1-\delta_1)^2} - 1 = 0.384$
- $D_2 = \frac{1}{\delta_2^2} - 1 = 43.44$

1.6 Analog Lowpass Magnitude Response

Since we have to make a Butterworth filter, the general response is given as

$$H_{LPF}(s_L)\, H_{LPF}(-s_L) = \frac{1}{1 + \left(\frac{s_L}{j\,\Omega_c}\right)^{2N}}$$

The parameters N and Ω_c can be found as follows:

$$N = \left\lceil \frac{\log(\sqrt{D_2/D_1})}{\log(\Omega_{Ls}/\Omega_{Lp})} \right\rceil$$

and

$$\frac{\Omega_{Lp}}{D_1^{\frac{1}{2N}}} \leq \Omega_c \leq \frac{\Omega_{Ls}}{D_2^{\frac{1}{2N}}}$$

Using the above relations, we get

$$\boxed{N = 8}$$

$$\boxed{1.0616 \leq \Omega_c \leq 1.096}$$

We choose $\Omega_c = 1.0788$. Using these values, we have

$$H_{LPF}(s_L)\,H_{LPF}(-s_L) = \frac{1}{1 + \left(\frac{s_L}{j\,1.0788}\right)^{16}}$$

After using MATLAB to find and plot the poles, we have

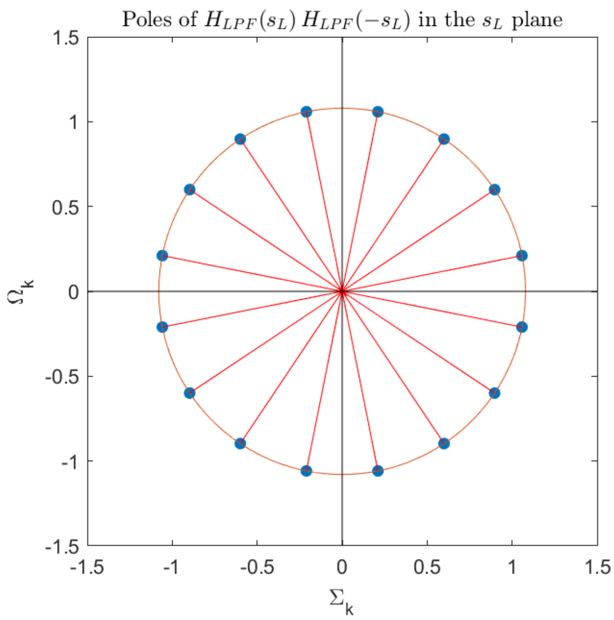

Figure 1.1: Poles of the equivalent lowpass filter

To have a realisable filter, we choose only those poles which lie on the left half of the complex s_L plane, which are as follows:

Pole	Value
s_1	$-0.5993 + j\,0.8970$
s_2	$-0.8970 + j\,0.5993$
s_3	$-1.0581 + j\,0.2105$
s_4	$-1.0581 - j\,0.2105$
s_5	$-0.8970 - j\,0.5993$
s_6	$-0.5993 - j\,0.8970$
s_7	$-0.2105 - j\,1.0581$
s_8	$-0.2105 + j\,1.0581$

Table 1.1: Poles on left half plane

Using the poles determined in table 1.1, the **equivalent analog lowpass transfer function** is given by

$$H_{LPF}(s_L) = \frac{\Omega_c^N}{\prod\limits_{k=1}^{N}(s_L - s_k)} = \frac{1.8345}{\sum\limits_{k=0}^{8} a_k\, s_L^k}$$

where the coefficients of the denominator are

Coefficient	Value
a_0	1.8345
a_1	8.7167
a_2	20.708
a_3	31.921
a_4	34.794
a_5	27.428
a_6	15.289
a_7	5.5297
a_8	1

Table 1.2: Coefficients of Denominator

The magnitude and the phase responses of this equivalent Butterworth LPF are as follows:

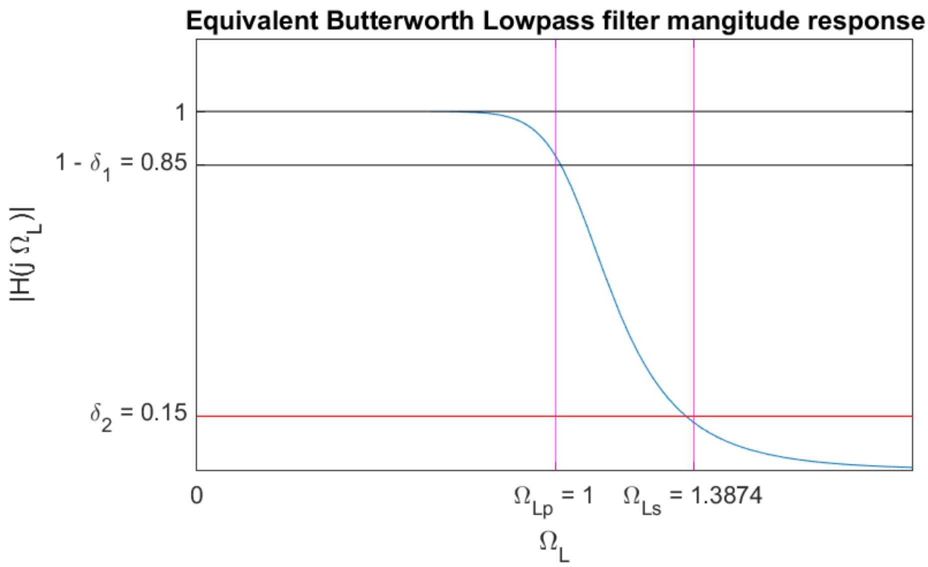

Figure 1.2: Magnitude Response

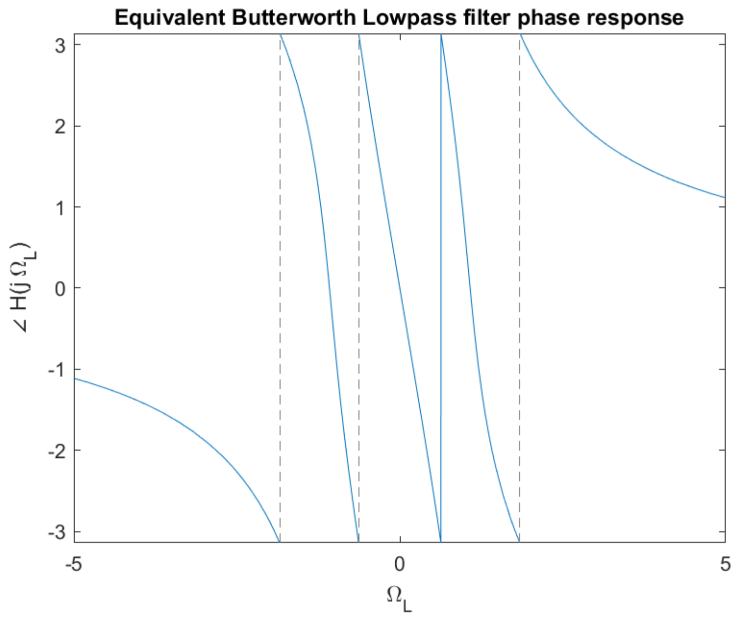

Figure 1.3: Phase Response

1.7 Analog Bandpass Transfer Function

We now use the inverse transformation to convert the equivalent Butterworth lowpass transfer function to get the Bandpass transfer function using the relation:

$$s_L \longleftarrow \frac{s^2 + \Omega_0^2}{B\,s}$$

where B and Ω_0 have the values as we found in section 1.4

$$H_{LPF}\left(\frac{s^2 + \Omega_0^2}{B\,s}\right) = H_{BPF}(s) = \frac{1.8345}{168879.9214 + \sum_{k=-8}^{8} a_k\, s^k}$$

The coefficients a_k are as follows:

Coefficient	Value
a_1	182483.5084
a_{-1}	94310.6177
a_2	252765.02515
a_{-2}	67513.5454
a_3	204166.44531
a_{-3}	28183.5223
a_4	220581.2811
a_{-4}	15736.8035
a_5	122303.3771
a_{-5}	4509.4469
a_6	101839.6751
a_{-6}	1940.6124
a_7	30259.6314
a_{-7}	298.0042
a_8	18714.6217
a_{-8}	95.2525

Table 1.3: Coefficients of Denominator in the Analog BP transfer function

The magnitude and frequency response are as follows:

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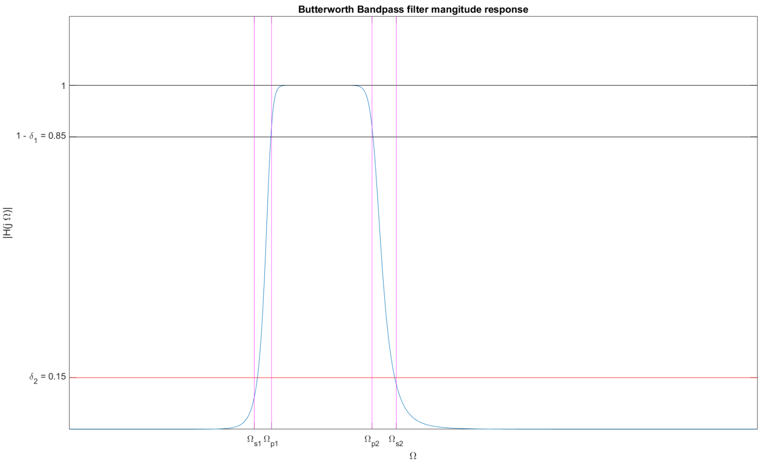

Figure 1.4: Magnitude response of the analog BPF

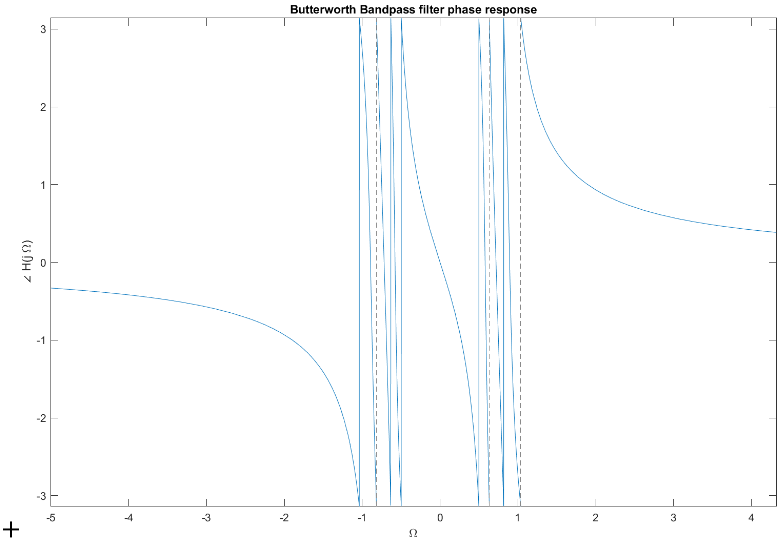

Figure 1.5: Phase response of the analog BPF

1.8 Discrete-Time Bandpass Transfer Function

We now make use of the Bilinear Transformation to convert the analog bandpass filter into a discrete bandpass filter in the normalized angular frequency domain. The transform is as follows:

$$s \longleftarrow \frac{1 - z^{-1}}{1 + z^{-1}}$$

The discrete time filter transfer function is

$$H_{BPF}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right) = H(z) = \frac{\sum_{k=0}^{16} b_k\, z^k}{\sum_{k=0}^{16} a_k\, z^k}$$

with coefficients

Coefficient	Value
a_{16}	0.0083
a_{15}	-0.0366
a_{14}	0.1197
a_{13}	-0.2688
a_{12}	0.5003
a_{11}	-0.7489
a_{10}	0.9662
a_9	-1.0495
a_8	0.9966
a_7	-0.8059
a_6	0.5697
a_5	-0.3389
a_4	0.1737
a_3	-0.0715
a_2	0.0244
a_1	-0.0057
a_0	0.0010

Table 1.4: Denominator $(\times 10^8)$

Coefficient	Value
b_{16}	1
b_{15}	0
b_{14}	-8
b_{13}	0
b_{12}	28
b_{11}	0
b_{10}	-56
b_9	0
b_8	70
b_7	0
b_6	-56
b_5	0
b_4	28
b_3	0
b_2	-8
b_1	0
b_0	1

Table 1.5: Numerator

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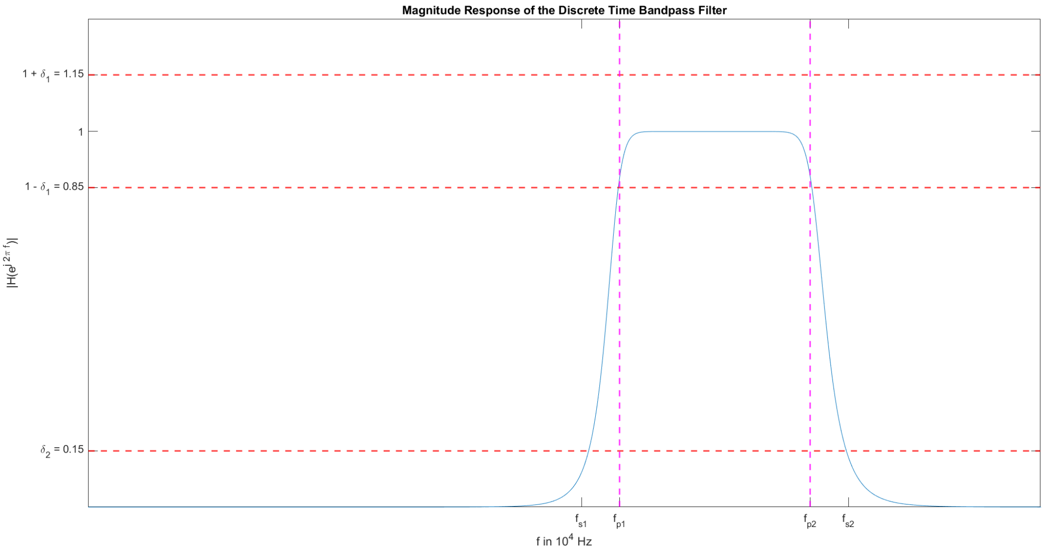

Figure 1.6: Magnitude Response of Discrete Time Bandpass Filter

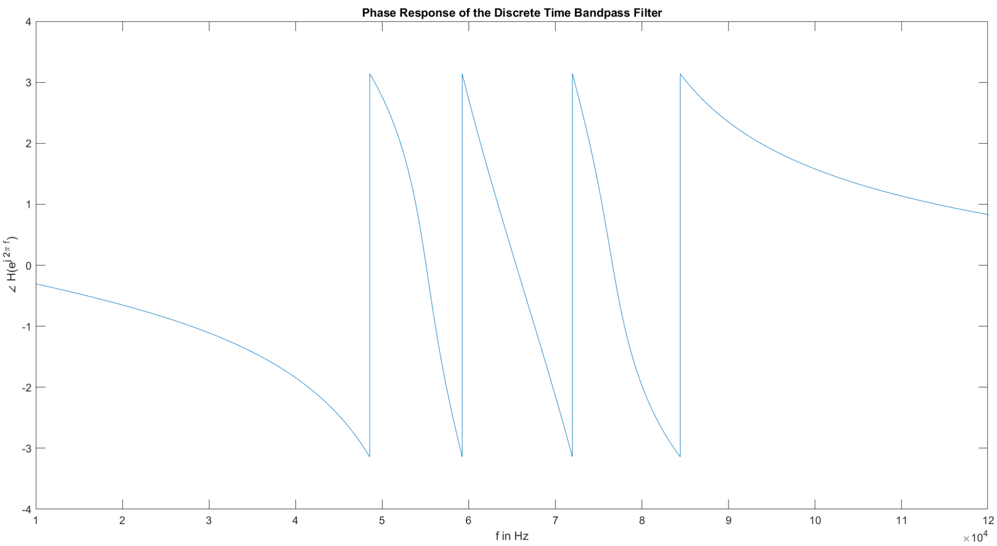

Figure 1.7: Phase Response of Discrete Time Bandpass Filter

2 Finite Impulse Response Bandpass Filter

In this part, we will use the windowing method to design a finite impulse response bandpass filter for the filter 44 using the same specifications as above. Recalling the specifications from section 1.2, we have the following normalized specifications:

- $\omega_{p1} = 2\pi\,\frac{55.8}{330} = 0.3382\,\pi$
- $\omega_{p2} = 2\pi\,\frac{75.8}{330} = 0.4594\,\pi$
- Transition bandwidth($\Delta\omega_T$) = $2\pi\,\frac{4}{330} = 0.0242\,\pi$
- $\omega_{s1} = \omega_{p1} - 0.0242\,\pi = 0.314\,\pi$
- $\omega_{s2} = \omega_{p2} + 0.0242\,\pi = 0.4836\,\pi$
- Passband and Stopband tolerance(δ) = 0.15

We will use a time-shifted **Kaiser Window** and multiply it with the impulse response of an ideal bandpass filter to get a realizable FIR filter.

2.1 Implementation Technique

While calculating the ideal bandpass filter response, the mean of the lower passband and stopband edges are used as one of the lower passband edge and the mean of the upper passband and stopband edges are used to find the upper passband edge. The same is mentioned in the following figure:

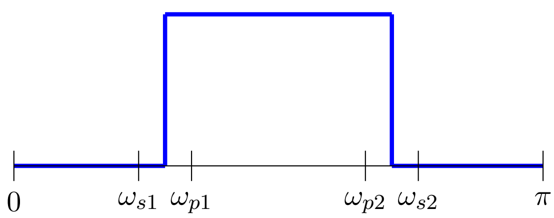

Figure 2.1: Ideal Bandpass Filter Magnitude Response

2.2 Kaiser Window Parameters

We have

$$A = -20\log_{10}(\delta) = 16.478$$

As $A < 21$, the shape parameter $\beta = 0$ and the Kaiser window in this case is essentially a rectangular window. Also, the window width M is

$$M \geq \left\lceil 1 + \frac{A-8}{2.285\ \Delta\omega_T} \right\rceil = 50$$

After tuning by hit and trail method so that the maximum error is within the tolerance, it has been found that a window width of $M = 66$ satisfies all the required specifications.

2.3 Discrete Time FIR Filter Transfer Function

The transfer function of the FIR filter obtained after adjusting for causality is as shown below. These are the coefficients of the Z transform of the impulse response of the FIR filter.

```
Columns 1 through 8

 -0.0177   -0.0031    0.0115    0.0070   -0.0020    0.0001    0.0012   -0.0095

Columns 9 through 16

 -0.0124    0.0084    0.0257    0.0067   -0.0264   -0.0241    0.0114    0.0274

Columns 17 through 24

  0.0056   -0.0134   -0.0058   -0.0010   -0.0144   -0.0073    0.0341    0.0436

Columns 25 through 32

 -0.0237   -0.0840   -0.0279    0.0904    0.0968   -0.0419   -0.1376   -0.0433

Columns 33 through 40

  0.1175    0.1175   -0.0433   -0.1376   -0.0419    0.0968    0.0904   -0.0279

Columns 41 through 48

 -0.0840   -0.0237    0.0436    0.0341   -0.0073   -0.0144   -0.0010   -0.0058

Columns 49 through 56

 -0.0134    0.0056    0.0274    0.0114   -0.0241   -0.0264    0.0067    0.0257

Columns 57 through 64

  0.0084   -0.0124   -0.0095    0.0012    0.0001   -0.0020    0.0070    0.0115

Columns 65 through 66

 -0.0031   -0.0177
```

Figure 2.2: Bandpass FIR Transfer Function

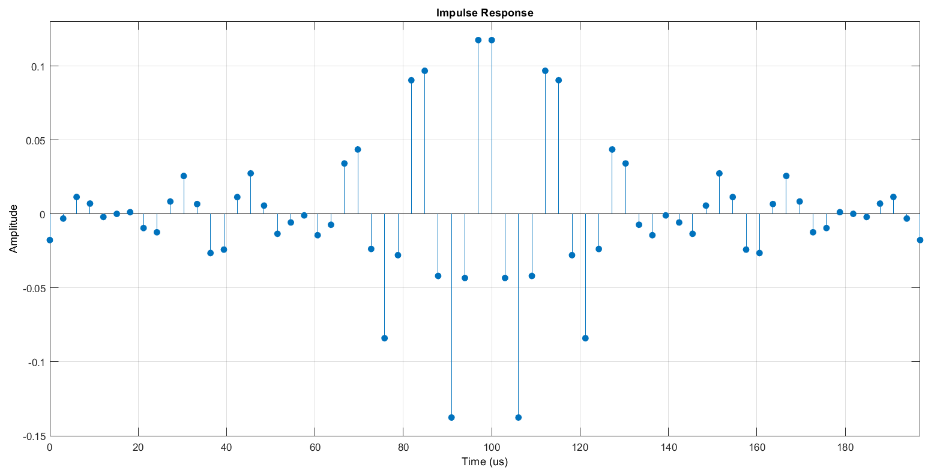

Figure 2.3: Bandpass FIR Impulse Response

2.4 Magnitude and Phase Response

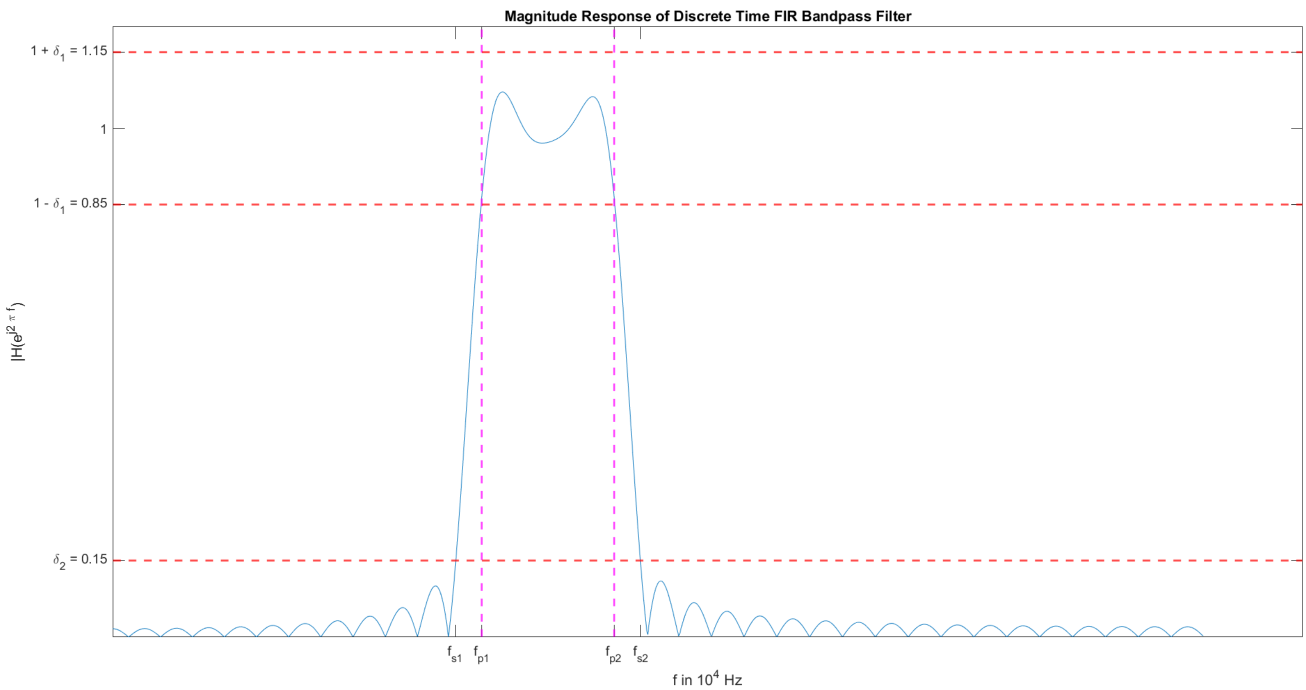

Figure 2.4: Magnitude Response of the FIR filter

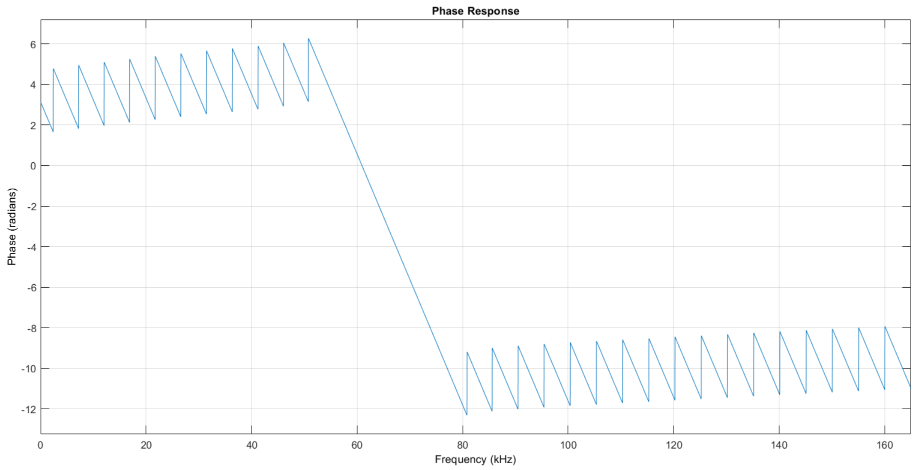

Figure 2.5: Phase Response of the FIR filter

2.5 Comparison with the IIR Filter

We can see that on comparing figure 1.7 and figure 2.5, we can see that the FIR filter gives a linear phase response for the frequencies in the passband range i.e from 55.8kHz to 75.8kHz.

Also, we can observe that the order of the IIR filter is 8 and the order of the FIR filter is 66 which is much higher. This once again proves the fact that *there is no free lunch*, i.e., we are getting a linear phase response at the expense of resources.

3 Infinite Impulse Response Bandstop Filter

The filter number $m = 44$. Hence,

$$\begin{aligned} q(m) &= \lfloor 0.1m \rfloor = 4 \\ r(m) &= m - 10\,q(m) = 44 - 40 = 4 \\ BL(m) &= 25 + 1.9q(m) + 4.1r(m) = 49\,\text{kHz} \\ BH(m) &= BL(m) + 20 = 69\,\text{kHz} \end{aligned}$$

3.1 Unnormalized Specifications

- Lower Passband Edge = 45 kHz
- Upper passband Edge = 73 kHz
- Transition band width = 4 kHz
- Lower Stopband Edge = 49 kHz
- Upper Stopband Edge = 69 kHz
- Passband tolerance = 0.15
- Stopband tolerance = 0.15
- Passband nature = equiripple
- Stopband nature = monotonic
- Sampling frequency = 260 kHz

3.2 Normalized Specifications

The sampling angular frequency $2\pi f_s$ should map to 2π after normalization. Hence the transformation used is

$$\omega = 2\pi \frac{f_s}{f}$$

The normalized specifications are as follows (following the notations used in the lecture)

- $\omega_{p1} = 2\pi \, \frac{45}{260} = 0.3461 \, \pi$
- $\omega_{p2} = 2\pi \, \frac{73}{260} = 0.5615 \, \pi$
- $\omega_{s1} = 2\pi \, \frac{49}{260} = 0.3769 \, \pi$
- $\omega_{s2} = 2\pi \, \frac{69}{260} = 0.5307 \, \pi$
- Transition bandwidth = $2\pi \, \frac{4}{260} = 0.0308 \, \pi$
- Passband and Stopband tolerance = 0.15
- Passband nature = equiripple
- Stopband nature = monotonic

3.3 Analog Filter Specifications using Bilinear Transform

We use the following transformation to move from the z domain to the s domain.

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

If we put $z = j\omega$ and $s = j\Omega$, we get

$$\Omega = \tan \frac{\omega}{2}$$

The specifications now are

- Passband nature = equiripple
- Stopband nature = monotonic
- $\Omega_{s1} = 0.6725$
- $\Omega_{s2} = 1.1014$
- $\Omega_{p1} = 0.6044$
- $\Omega_{p2} = 1.2146$
- $\delta_1 = \delta_2 = 0.15$

3.4 Analog Frequency Transformation

Here we use the **Bandstop to Lowpass** analog frequency transformation, which is as follows:

$$\Omega_L = \frac{B\,\Omega}{\Omega_0^2 - \Omega^2}$$

where

$$\Omega_0 = \sqrt{\Omega_{p1}\,\Omega_{p2}} = 0.8568$$

and

$$B = \Omega_{p2} - \Omega_{p1} = 0.6102$$

Using the transformation mentioned above, we now have

- Passband and Stopband nature = monotonic
- $\Omega_{Ls1} = 1.456$
- $\Omega_{Ls2} = -1.4031$
- $\Omega_{Lp1} = +1$
- $\Omega_{Lp2} = -1$
- $\delta_1 = \delta_2 = 0.15$

3.5 Equivalent Analog Lowpass Filter Specifications

Using the more stringent specifications from those obtained in section 3.4, we have

- Passband nature = equiripple
- Stopband nature = passband
- $\Omega_{Ls} = \min\{|\Omega_{Ls2}|, |\Omega_{Ls1}|\} = 1.4031$
- $\Omega_{Lp} = +1$
- $\delta_1 = \delta_2 = 0.15$
- $D_1 = \frac{1}{(1-\delta_1)^2} - 1 = 0.384$
- $D_2 = \frac{1}{\delta_2^2} - 1 = 43.44$

3.6 Analog Lowpass Magnitude Response

Since we have to make a Chebyshev filter, the general response is given as

$$H_{LPF}(s_L)\, H_{LPF}(-s_L) = \frac{1}{1 + \epsilon^2\, C_N^2(s_L / j\, \Omega_{Lp})}$$

Where the parameters ϵ and N are given by

$$\epsilon = \sqrt{D_1} = 0.6197$$

and

$$N = \left\lceil \frac{\cosh^{-1}(\sqrt{D_2/D_1})}{\cosh^{-1}(\Omega_{Ls}/\Omega_{Lp})} \right\rceil = 4$$

After using MATLAB to find and plot the poles, we have

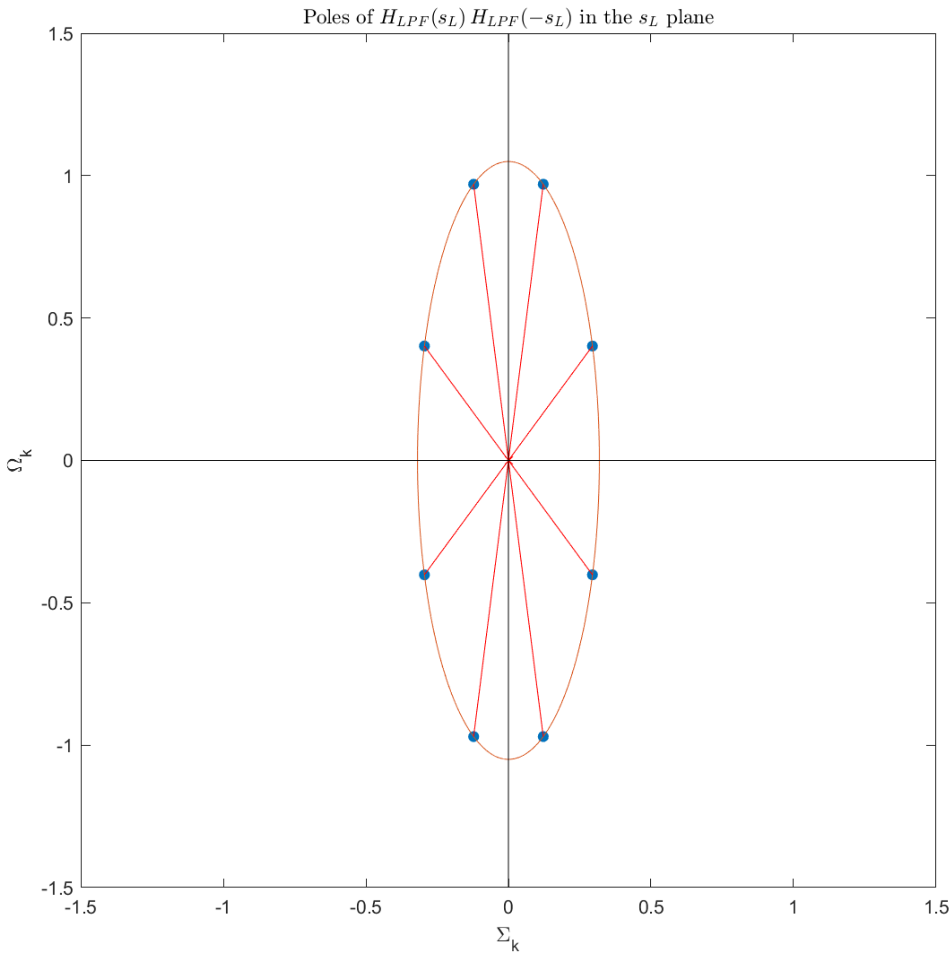

Figure 3.1: Poles of the equivalent lowpass filter

To have a realisable filter, we choose only those poles which lie on the left half of the complex s_L plane, which are as follows:

Pole	Value
s_1	$-0.1222 + j\,0.9698$
s_2	$-0.2949 + j\,0.4017$
s_3	$-0.2949 - j\,0.4017$
s_4	$-0.1222 - j\,0.9698$

Table 3.1: Poles on left half plane

Using the poles determined in table 3.1, the **equivalent analog lowpass transfer function** is given by

$$H_{LPF}(s_L) = \frac{A}{\prod_{k=1}^{N}(s_L - s_k)}$$

Substituting the constraint that the value of the magnitude response at the passband edge is $1 - \delta_1$, we have

$$H_{LPF}(s_L) = \frac{\prod_{i=1}^{4} s_i}{\sqrt{1+D_1}\left(\prod_{k=1}^{N}(s_L - s_k)\right)} = \frac{0.2373}{1.1764\, s_L^4 + 0.9814\, s_L^3 + 1.5858\, s_L^2 + 0.7345\, s_L + 0.2792}$$

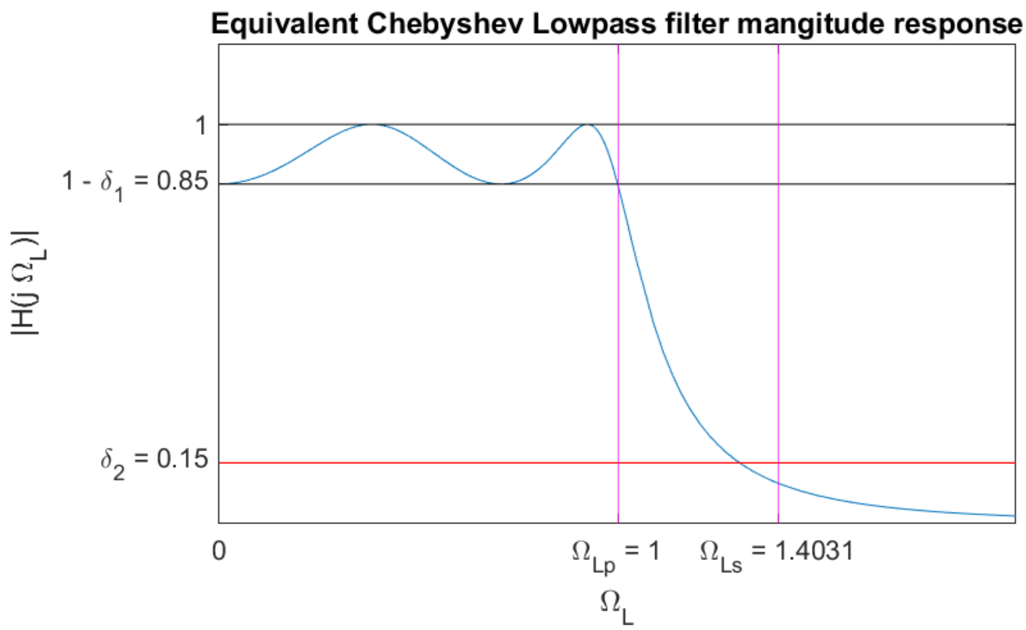

Figure 3.2: Magnitude Response

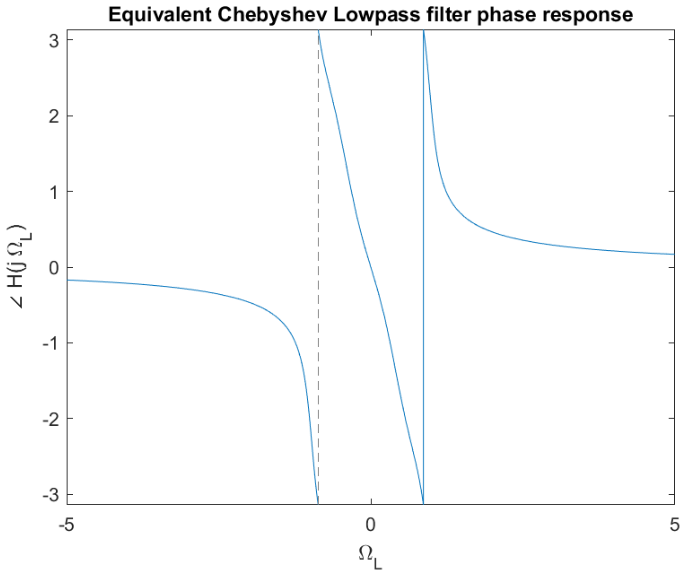

Figure 3.3: Phase Response

3.7 Analog Bandstop Transfer Function

We now use the inverse transformation to convert the equivalent Chebyshev lowpass transfer function to get the Bandstop transfer function using the relation:

$$s_L \longleftarrow \frac{B\,s}{s^2+\Omega_0^2}$$

where B and Ω_0 have the values as we found in section 3.4

$$H_{LPF}\left(\frac{B\,s}{s^2+\Omega_0^2}\right) = H_{BSF}(s) = \frac{1.3516\,s^8 + 3.9691\,s^6 + 4.3707\,s^4 + 2.1390\,s^2 + 3.9257}{4.6183 + \sum_{k=1}^{8} b_k\,s^k}$$

Coefficient	Value
b_8	1.5902
b_7	2.5527
b_6	8.0327
b_5	6.8921
b_4	1.1001
b_3	5.0595
b_2	4.3289
b_1	1.0099

Table 3.2: Denominator

The magnitude and frequency response are as follows:

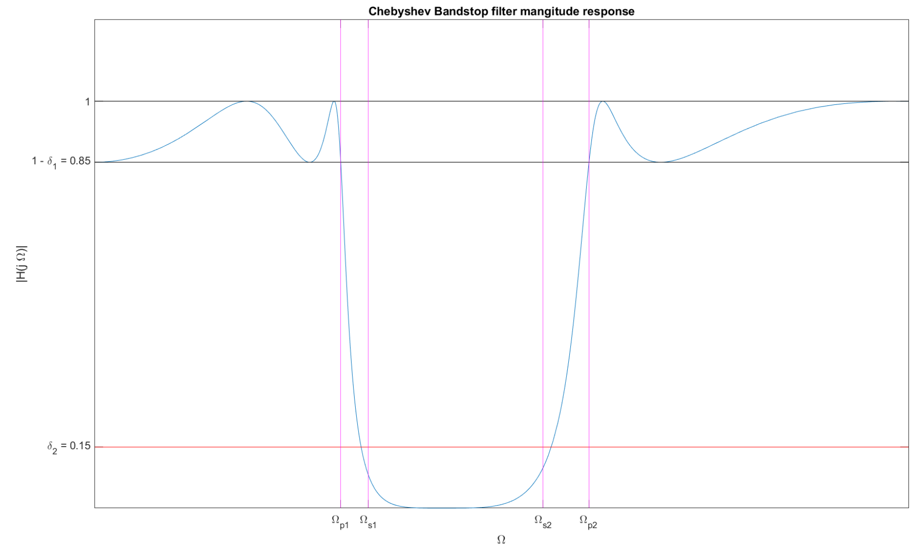

Figure 3.4: Magnitude Response of Chebyshev Bandstop filter

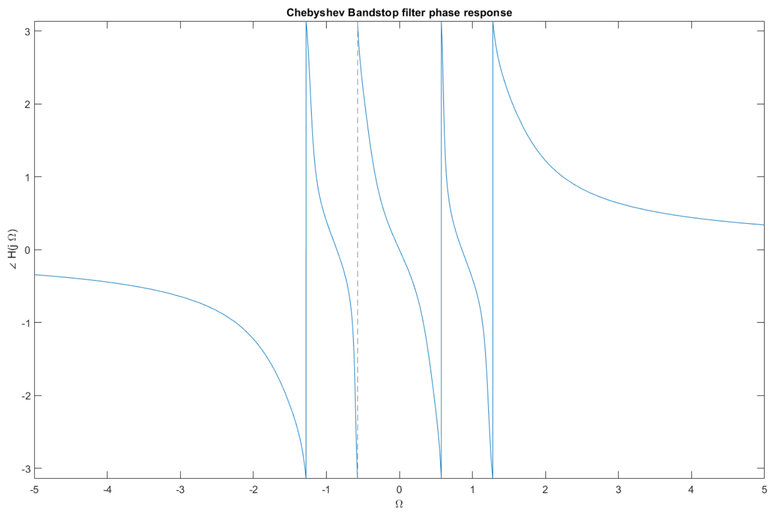

Figure 3.5: Phase Response of Chebyshev Bandstop filter

3.8 Discrete-Time Bandstop Transfer Function

We now make use of the Bilinear Transformation to convert the analog bandstop filter into a discrete bandstop filter in the normalized angular frequency domain. The transform is as follows:

$$s \longleftarrow \frac{1 - z^{-1}}{1 + z^{-1}}$$

The discrete time filter transfer function is

$$H_{BSF}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right) = H(z) = \frac{\sum_{k=0}^{8} b_k \, z^k}{\sum_{k=0}^{8} a_k \, z^k}$$

Coefficient	Value
a_8	4.0936
a_7	-3.6764
a_6	8.8842
a_5	-5.8977
a_4	8.6077
a_3	-3.7768
a_2	3.6893
a_1	-1.0919
a_0	9.9081

Table 3.3: Denominator

Coefficient	Value
b_8	1.2223
b_7	-1.4993
b_6	5.5789
b_5	-4.6390
b_4	8.7240
b_3	-4.6390
b_2	5.5789
b_1	-1.4993
b_0	1.2223

Table 3.4: Numerator

Please see the next page

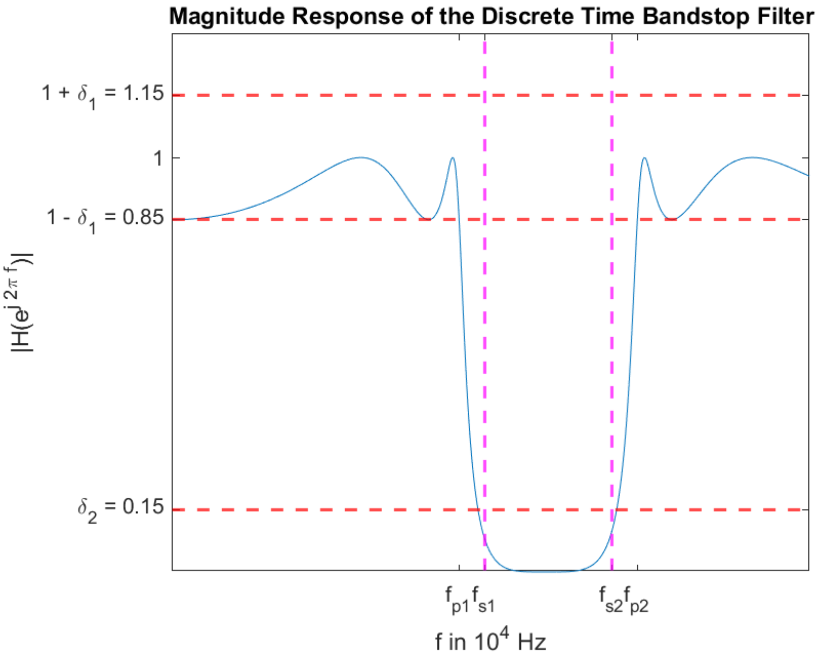

Figure 3.6: Magnitude Response of Discrete Time Bandstop Filter

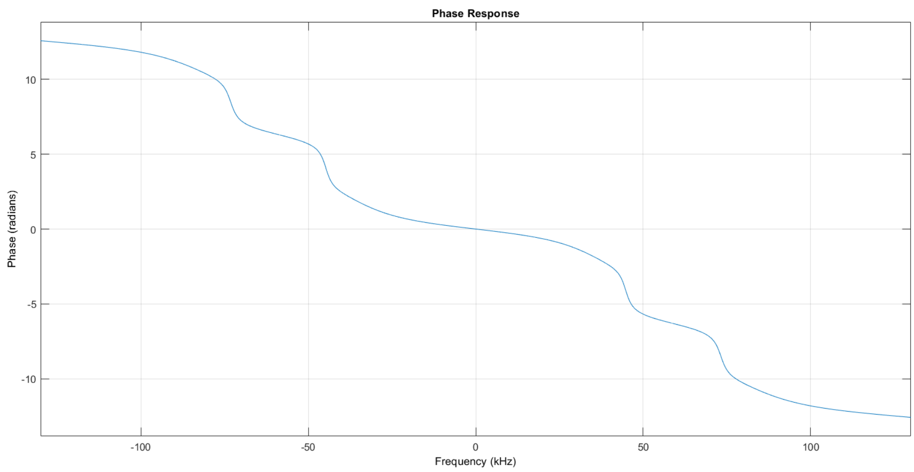

Figure 3.7: Phase Response of Discrete Time Bandstop Filter

4 Finite Impulse Response Bandstop Filter

In this part, we will use the windowing method to design a finite impulse response bandstop filter for the filter 44 using the same specifications as above. Recalling the specifications from section 3.2, we have the following normalized specifications:

- $\omega_{p1} = 2\pi \, \frac{45}{260} = 0.3461 \, \pi$
- $\omega_{p2} = 2\pi \, \frac{73}{260} = 0.5615 \, \pi$
- $\omega_{s1} = 2\pi \, \frac{49}{260} = 0.3769 \, \pi$
- $\omega_{s2} = 2\pi \, \frac{69}{260} = 0.5307 \, \pi$
- Transition bandwidth($\Delta\omega_T$) = $2\pi \, \frac{4}{260} = 0.0308 \, \pi$
- Passband and Stopband tolerance = 0.15

We will use a time-shifted **Kaiser Window** and multiply it with the impulse response of an ideal bandstop filter to get a realizable FIR filter.

4.1 Implementation Technique

While calculating the ideal bandstop filter response, the mean of the lower passband and stopband edges are used as one of the lower stopband edge and the mean of the upper passband and stopband edges are used to find the upper stopband edge. The same is mentioned in the following figure:

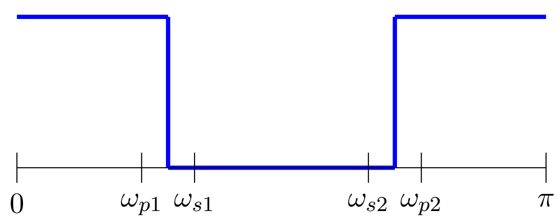

Figure 4.1: Ideal Bandstop Filter Magnitude Response

4.2 Kaiser Window Parameters

We have

$$A = -20\log_{10}(\delta) = 16.478$$

As $A < 21$, the shape parameter $\beta = 0$ and the Kaiser window in this case is essentially a rectangular window. Also, the window width M is

$$M \geq \left\lceil 1 + \frac{A-8}{2.285\ \Delta\omega_T} \right\rceil = 40$$

After tuning by hit and trail method so that the maximum error is within the tolerance, it has been found that a window width of $M = 53$ satisfies all the required specifications.

4.3 Discrete Time FIR Filter Transfer Function

The transfer function of the FIR filter obtained after adjusting for causality is as shown below. These are the coefficients of the Z transform of the impulse response of the FIR filter.

```
Columns 1 through 8

 -0.0188    0.0098    0.0156   -0.0020   -0.0028    0.0005   -0.0144   -0.0088

Columns 9 through 16

  0.0267    0.0228   -0.0271   -0.0326    0.0161    0.0273   -0.0030   -0.0028

Columns 17 through 24

  0.0018   -0.0346   -0.0232    0.0693    0.0674   -0.0839   -0.1220    0.0684

Columns 25 through 32

  0.1671   -0.0263    0.8154   -0.0263    0.1671    0.0684   -0.1220   -0.0839

Columns 33 through 40

  0.0674    0.0693   -0.0232   -0.0346    0.0018   -0.0028   -0.0030    0.0273

Columns 41 through 48

  0.0161   -0.0326   -0.0271    0.0228    0.0267   -0.0088   -0.0144    0.0005

Columns 49 through 53

 -0.0028   -0.0020    0.0156    0.0098   -0.0188
```

Figure 4.2: Bandstop FIR Transfer Function

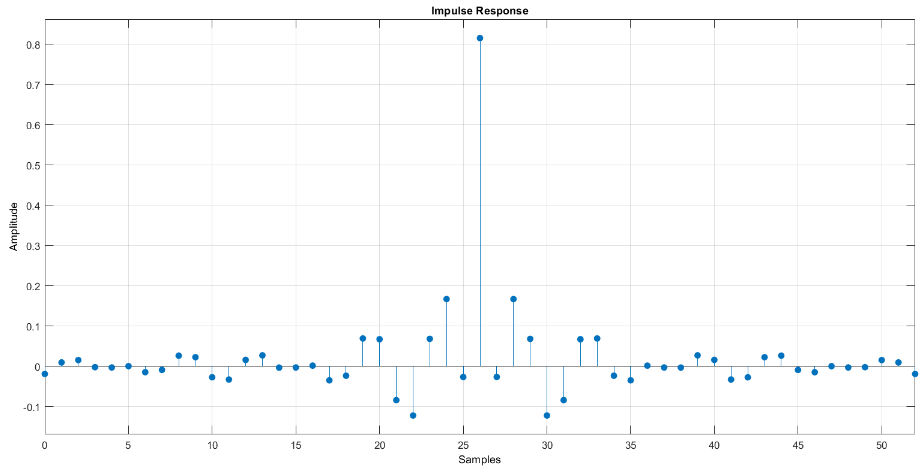

Figure 4.3: Bandstop FIR Impulse Response

4.4 Magnitude and Phase Response

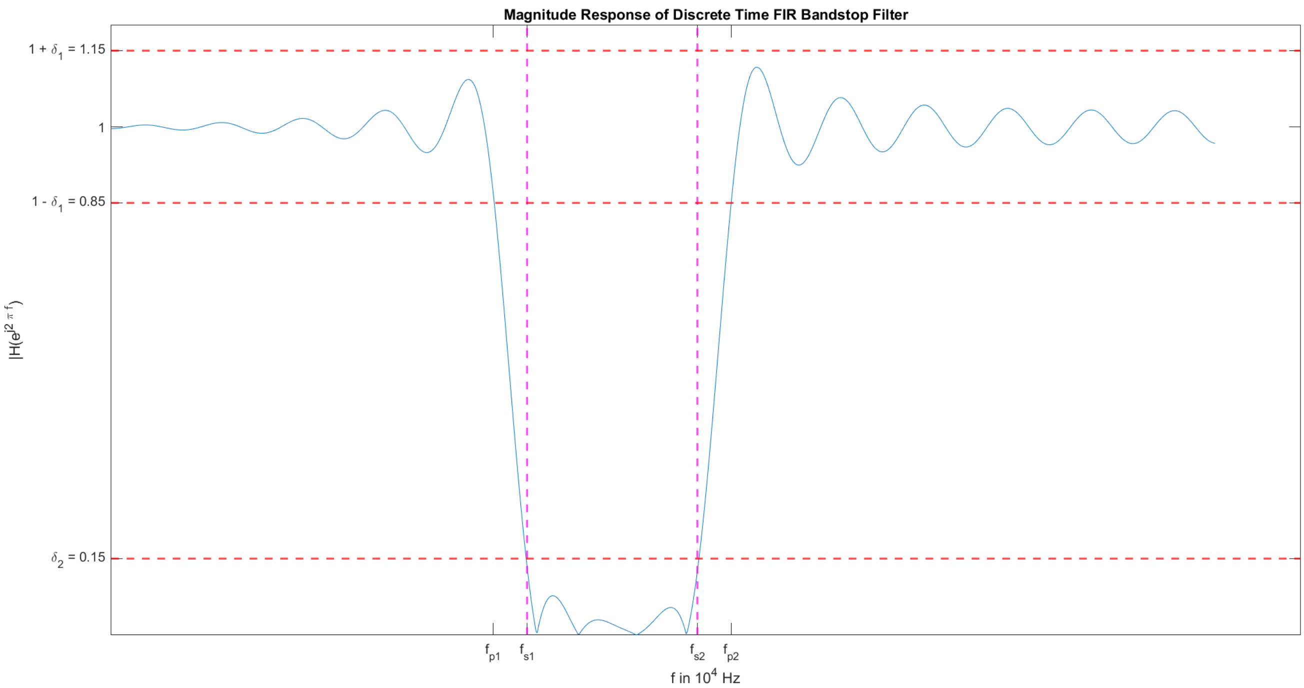

Figure 4.4: Magnitude Response of the FIR filter

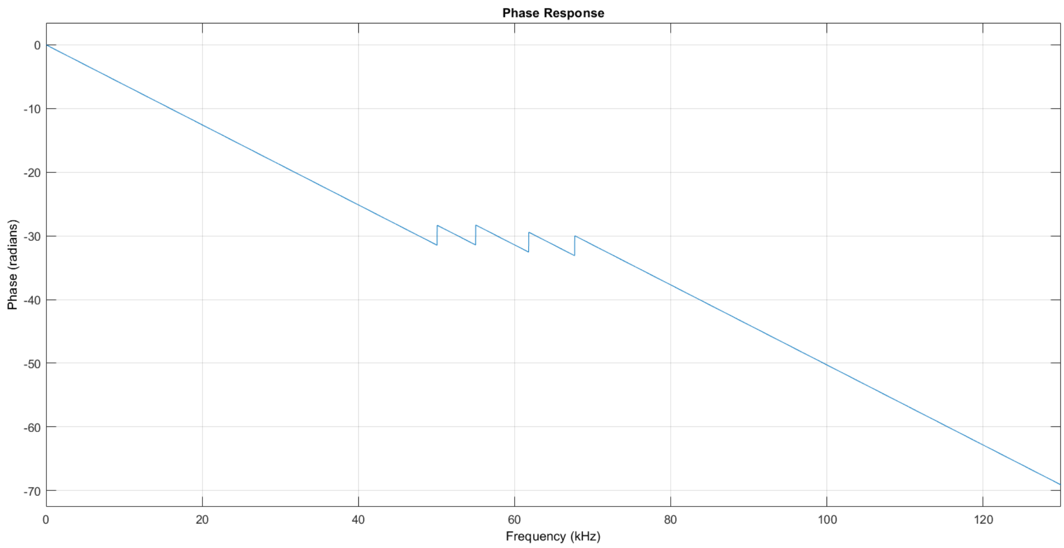

Figure 4.5: Phase Response of the FIR filter

4.5 Comparison with the IIR Filter

We can see that on comparing figure 3.7 and figure 4.5, we can see that the FIR filter gives a linear phase response for the frequencies in the passband range.

Also, we can observe that the order of the IIR filter is 4 and the order of the FIR filter is 53 which is much higher. This once again proves the fact that *there is no free lunch*, i.e., we are getting a linear phase response at the expense of resources.

5 Elliptical Bandpass filter design

The filter number $m = 44$. Hence,

$$\begin{aligned}
q(m) &= \lfloor 0.1m \rfloor = 4 \\
r(m) &= m - 10\,q(m) = 44 - 40 = 4 \\
BL(m) &= 25 + 1.7q(m) + 6.1r(m) = 55.8\,\text{kHz} \\
BH(m) &= BL(m) + 20 = 75.8\,\text{kHz}
\end{aligned}$$

5.1 Unnormalized Specifications

- Lower Passband Edge = 55.8 kHz
- Upper passband Edge = 75.8 kHz
- Transition band width = 4 kHz
- Passband tolerance = 0.15
- Stopband tolerance = 0.15
- Passband nature = monotonic
- Stopband nature = monotonic
- Sampling frequency = 330 kHz

5.2 Normalized Specifications

The sampling angular frequency $2\pi f_s$ should map to 2π after normalization. Hence the transformation used is

$$\omega = 2\pi \frac{f_s}{f}$$

The normalized specifications are as follows (following the notations used in the lecture)

- $\omega_{p1} = 2\pi \, \frac{55.8}{330} = 0.3382\,\pi$
- $\omega_{p2} = 2\pi \, \frac{75.8}{330} = 0.4594\,\pi$

- Transition bandwidth = $2\pi \frac{4}{330} = 0.0242\,\pi$
- $\omega_{s1} = \omega_{p1} - 0.0242\,\pi = 0.314\,\pi$
- $\omega_{s2} = \omega_{p2} + 0.0242\,\pi = 0.4836\,\pi$
- Passband and Stopband tolerance = 0.15
- Passband and Stopband nature = equiripple

5.3 Analog Filter Specifications using Bilinear Transform

We use the following transformation to move from the z domain to the s domain.

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

If we put $z = j\omega$ and $s = j\Omega$, we get

$$\Omega = \tan \frac{\omega}{2}$$

The specifications now are

- Passband and Stopband nature = monotonic
- $\Omega_{s1} = 0.5375$
- $\Omega_{s2} = 0.9498$
- $\Omega_{p1} = 0.5875$
- $\Omega_{p2} = 0.8799$
- $\delta_1 = \delta_2 = 0.15$

.

5.4 Analog Frequency Transformation

Here we use the **Bandpass to Lowpass** analog frequency transformation, which is as follows:

$$\Omega_L = \frac{\Omega^2 - \Omega_0^2}{B\Omega}$$

where

$$\Omega_0 = \sqrt{\Omega_{p1}\,\Omega_{p2}} = 0.7189$$

and

$$B = \Omega_{p2} - \Omega_{p1} = 0.2924$$

Using the transformation mentioned above, we now have

- Passband and Stopband nature = monotonic
- $\Omega_{Ls1} = -1.4501$
- $\Omega_{Ls2} = 1.3874$
- $\Omega_{Lp1} = -1$
- $\Omega_{Lp2} = +1$
- $\delta_1 = \delta_2 = 0.15$

5.5 Equivalent Analog Lowpass Filter Specifications

Using the more stringent specifications from those obtained in section 1.4, we have

- Passband and Stopband nature = monotonic
- $\Omega_{Ls} = \min\{\Omega_{Ls2}, |\Omega_{Ls1}|\} = 1.3874$
- $\Omega_{Lp} = +1$
- $\delta_1 = \delta_2 = 0.15$
- $D_1 = \frac{1}{(1-\delta_1)^2} - 1 = 0.384$
- $D_2 = \frac{1}{\delta_2^2} - 1 = 43.44$

5.6 Analog Lowpass Magnitude Response

The general transfer function for an **Elliptic** filter is given by

$$H_{LPF}(s_L)\,H_{LPF}(-s_L) = \frac{1}{1 + \epsilon^2\,F_N^2(\frac{s_L}{j\,\Omega_{Lp}})}$$

with

$$F_N(w) = \mathrm{cd}(NuK_1, k_1)$$

and

$$w = \mathrm{cd}(uK, k))$$

where $\mathrm{cd}(x, k)$ denotes the Jacobian elliptic function cd with modulus k and real quarter-period K.

5.6.1 Jacobian Elliptic Functions

The elliptic function $w = \mathrm{sn}(z, k)$ is defined as follows:

$$z = \int_0^{\phi} \frac{d\theta}{\sqrt{1 - k^2 \sin^2\theta}} = \int_0^{w} \frac{dt}{\sqrt{(1-t^2)(1-k^2 t^2)}} \quad , w = \sin\left(\phi(z,k)\right)$$

There are three more relevant elliptic functions, cn, dn, and cd defined as:

$$\begin{aligned} w &= \mathrm{cn}(z,k) = \cos\left(\phi(z,k)\right) \\ w &= \mathrm{dn}(z,k) = \sqrt{1 - k^2 \,\mathrm{sn}^2(z,k)} \\ w &= \mathrm{cd}(z,k) = \frac{\mathrm{cn}(z,k)}{\mathrm{dn}(z,k)} \end{aligned}$$

We define the *complete elliptic integral of first kind* $K(k)$ or simply K as the value of z when $\mathrm{sn}(z,k) = 1$ i.e. when $\phi = \pi/2$

$$K(k) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2 \sin^2\theta}}$$

Clearly $\mathrm{cd}(K, k) = 0$. Associated with an elliptic modulus k, there is a complementary modulus $k' = \sqrt{1-k^2}$ and its associated complete elliptic integral $K(k')$ denoted by $K'(k)$ or simply K'

$$K'(k) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k'^2 \sin^2\theta}} = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - (1-k^2)\sin^2\theta}}$$

The significance of K and K' is that sn and cd are *doubly-periodic functions* in the z plane with a real period $4K$ and a complex period $2jK'$. Some of the useful properties are as follows:

$$\mathrm{cd}(z + (2n-1)K, k) = (-1)^n \, \mathrm{sn}(z, k) \tag{1}$$

$$\mathrm{cd}(z + 2nK, k) = (-1)^n \, \mathrm{cd}(z, k) \tag{2}$$

$$\mathrm{cd}(z + j\,K', k) = \frac{1}{k\,\mathrm{cd}(z, k)} \tag{3}$$

$$\mathrm{cd}(j\,z, k) = \frac{1}{\mathrm{dn}(z, k')} \tag{4}$$

$$\mathrm{cd}(j\,K', k) = \frac{1}{k} \tag{5}$$

$\mathrm{cd}(z, k)$ has zeroes at $(2m+1)\,K + j\,2n\,K'$ and poles at $(2m+1)\,K + j\,(2n+1)\,K'$ for any integers m and n in the z plane.

5.6.2 Elliptic Filter Parameters

The order of the filter N is determining by putting a constraint that the magnitude response at the passband edge is not less than $1 - \delta_1$ and at the stopband edge not greater than δ_2.

$$N = \left\lceil \frac{K\,K_1'}{K'\,K_1} \right\rceil$$

where K, K', K_1, K_1' are defined for the complete elliptic integrals of modulus k and k_1 given by

$$k = \frac{\Omega_{Lp}}{\Omega_{Ls}} = 0.72077$$

$$k_1 = \sqrt{\frac{D_1}{D_2}} = 0.0940$$

We then have

Parameter	Value
K	1.8710
K'	1.8379
K_1	1.5743
K_1'	3.7566
N	3

Table 5.1: Elliptic Filter Parameters

We now have to find the poles of the the transfer function. The poles of the transfer function are given by the zeros of the denominator and the zeros of the transfer function are given by the poles of $F_N(\frac{s_L}{j\,\Omega_{Lp}})$. The zeros and poles of the transfer function are given by

$$\begin{aligned}
z_i &= j\,\Omega_{Lp}\,(k\zeta_i)^{-1}\ , \quad i = 1, 2, \ldots, \lfloor N/2 \rfloor \\
p_i &= j\,\Omega_{Lp}\,\mathrm{cd}\big((u_i - j\nu_0)\,K, k\big)\ , \quad i = 1, 2, \ldots, \lfloor N/2 \rfloor \\
p_0 &= j\,\Omega_{Lp}\,\mathrm{sn}(j\nu_0 K, k)
\end{aligned}$$

where

$$\begin{aligned}
\zeta_i &= \mathrm{cd}(u_i K, k) \\
u_i &= \frac{2i-1}{N}\ , \quad i = 1, 2, \ldots, \lfloor N/2 \rfloor \\
\nu_0 &= -\frac{j}{N\,K_1}\mathrm{sn}^{-1}\left(\frac{j}{\epsilon}, k_1\right)
\end{aligned}$$

As N is odd, the zeros and poles occur in conjugate pairs We then have

z_1	$j\,7.9193$
z_2	$-j\,7.9193$
p_0	-3.9154
p_1	$-0.7246 + j\,6.2431$
p_2	$-0.7246 - j\,6.2431$

Table 5.2: Poles and Zeros of $H_{LPF}(s_L)\,H_{LPF}(-s_L)$

$$H_{LPF}(s'_L) = \frac{1}{1 + 0.2554\,s'_L}\,\frac{1 + 0.0159\,s'^2_L}{1 + 0.0367\,s'_L + 0.0253\,s'^2_L}$$

$$H_{LPF}(s'_L) = \frac{1 + 0.0159\,s'^2_L}{1 + 0.2921\,s'_L + 0.0346\,s'^2_L + 0.00646\,s'^3_L}$$

Here, $s'_L = 2\pi\,s_L$. Hence

$$H_{LPF}(s_L) = \frac{1 + 0.6277\,s^2_L}{1 + 1.8353\,s_L + 1.3659\,s^2_L + 1.6024\,s^3_L}$$

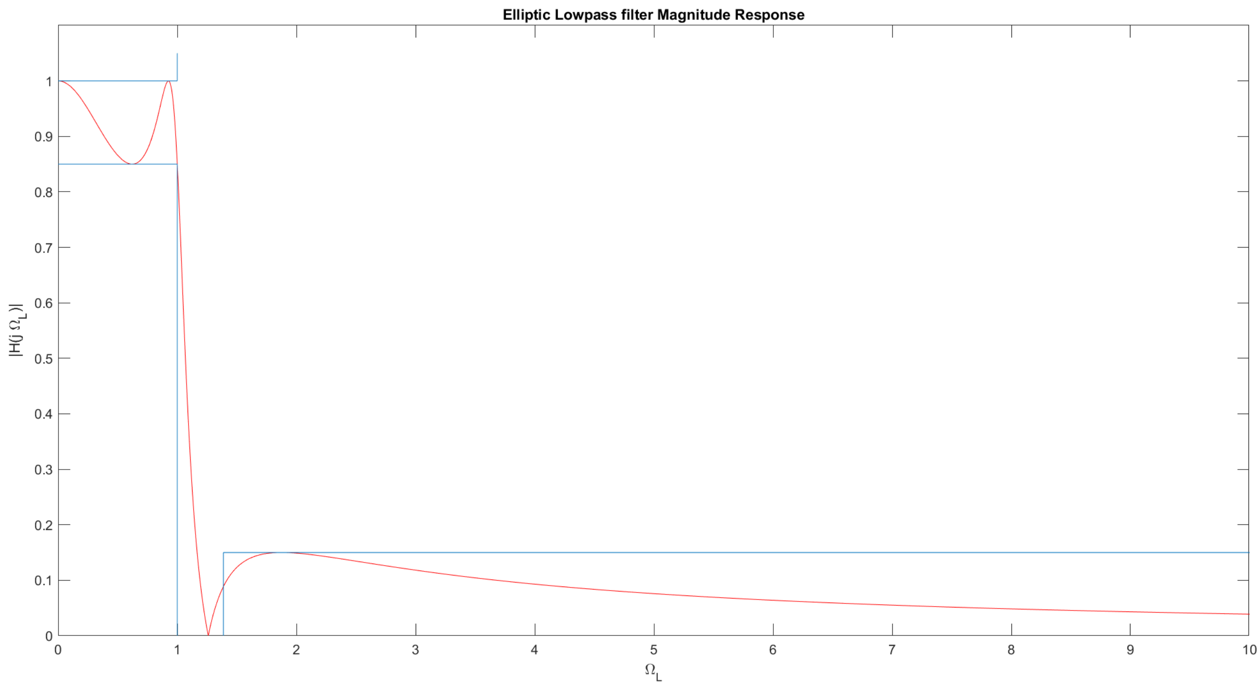

Figure 5.1: Magnitude Response of the equivalent Elliptic Lowpass Filter

5.7 Analog Bandpass Transfer Function

We now use the inverse transformation to convert the equivalent Elliptic lowpass transfer function to get the Bandpass transfer function using the relation:

$$s_L \longleftarrow \frac{s^2 + \Omega_0^2}{B\,\Omega}$$

where B and Ω_0 have the values found in section 5.4.

$$H_{LPF}\left(\frac{s^2 + \Omega_0^2}{B\,\Omega}\right) = H_{BPF}(s)$$

$$H_{BPF}(s) = \frac{0.2517\,s^5 + 0.2945\,s^3 + 0.0673\,s}{2.1973\,s^6 + 0.5476\,s^5 + 3.6232\,s^4 + 0.6005\,s^3 + 1.8732\,s^2 + 0.1464\,s + 0.3036}$$

The magnitude and frequency responses are as follows:

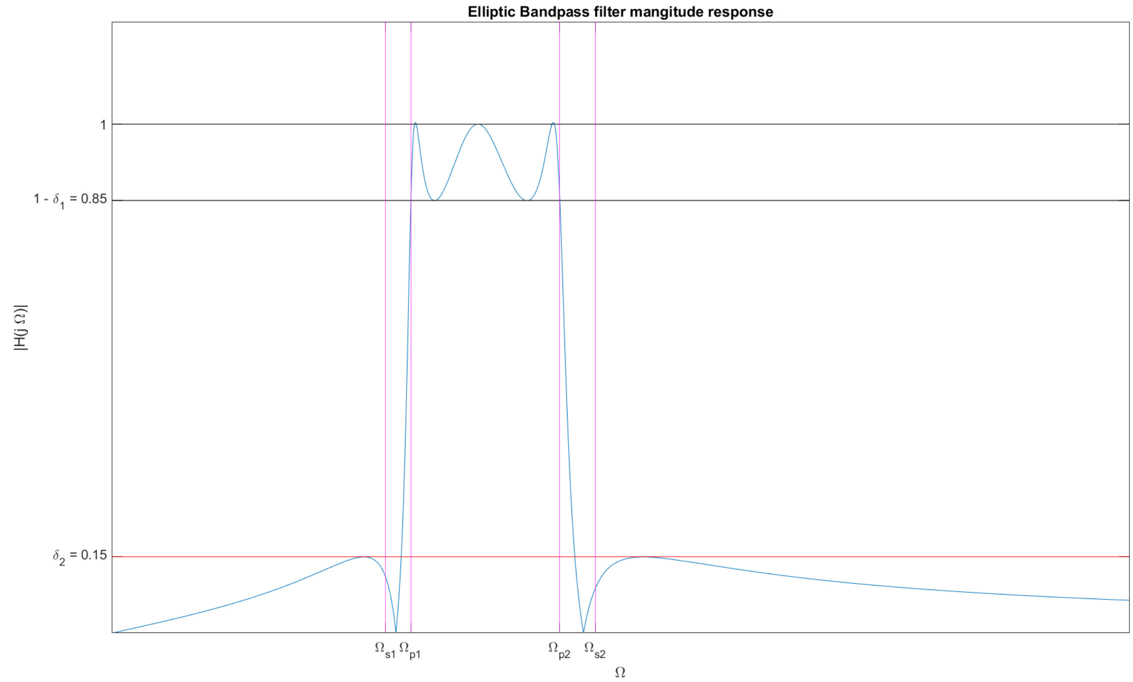

Figure 5.2: Magnitude Response of the analog BPF

5.8 Discrete-Time Bandpass Transfer Function

We now make use of the Bilinear Transformation to convert the analog bandpass filter into a discrete bandpass filter in the normalized angular frequency domain. The transform is as follows:

$$s \longleftarrow \frac{1 - z^{-1}}{1 + z^{-1}}$$

The discrete time filter transfer function is

$$H_{BPF}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right) = H(z)$$

$$H(z) = \frac{0.0613\, z^6 - 0.0738\, z^5 + 0.0711\, z^4 - 0.0711\, z^2 + 0.0738\, z - 0.0613}{0.9292\, z^6 - 1.6467\, z^5 + 3.3686\, z^4 - 3.0873\, z^3 + 3.0349\, z^2 - 1.3257\, z + 0.6703}$$

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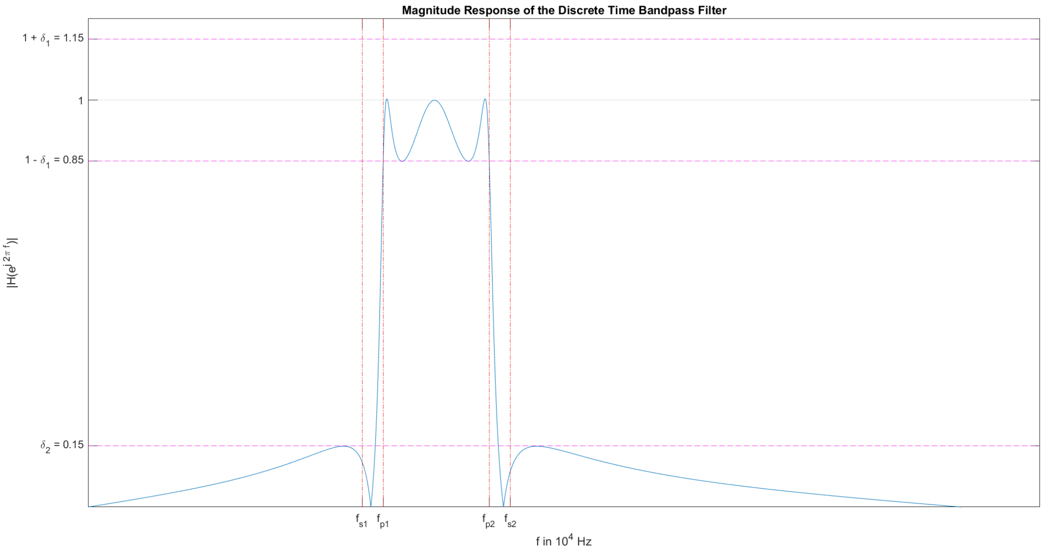

Figure 5.3: Magnitude Response of Discrete Time Bandpass Filter

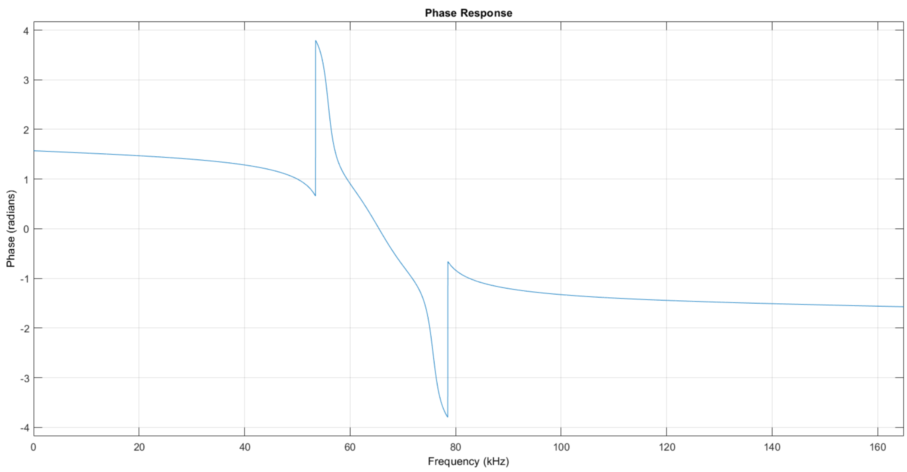

Figure 5.4: Phase Response of Discrete Time Bandpass Filter

6 Elliptic Bandstop filter design

The filter number $m = 44$. Hence,

$$\begin{aligned} q(m) &= \lfloor 0.1m \rfloor = 4 \\ r(m) &= m - 10\,q(m) = 44 - 40 = 4 \\ BL(m) &= 25 + 1.9q(m) + 4.1r(m) = 49\,\text{kHz} \\ BH(m) &= BL(m) + 20 = 69\,\text{kHz} \end{aligned}$$

6.1 Unnormalized Specifications

- Lower Passband Edge = 45 kHz
- Upper passband Edge = 73 kHz
- Transition band width = 4 kHz
- Lower Stopband Edge = 49 kHz
- Upper Stopband Edge = 69 kHz
- Passband tolerance = 0.15
- Stopband tolerance = 0.15
- Passband nature = equiripple
- Stopband nature = equiripple
- Sampling frequency = 260 kHz

6.2 Normalized Specifications

The sampling angular frequency $2\pi f_s$ should map to 2π after normalization. Hence the transformation used is

$$\omega = 2\pi \frac{f_s}{f}$$

The normalized specifications are as follows (following the notations used in the lecture)

- $\omega_{p1} = 2\pi\,\frac{45}{260} = 0.3461\,\pi$
- $\omega_{p2} = 2\pi\,\frac{73}{260} = 0.5615\,\pi$
- $\omega_{s1} = 2\pi\,\frac{49}{260} = 0.3769\,\pi$
- $\omega_{s2} = 2\pi\,\frac{69}{260} = 0.5307\,\pi$
- Transition bandwidth = $2\pi\,\frac{4}{260} = 0.0308\,\pi$
- Passband and Stopband tolerance = 0.15
- Passband nature = equiripple

- Stopband nature = equiripple

6.3 Analog Filter Specifications using Bilinear Transform

We use the following transformation to move from the z domain to the s domain.

$$s = \frac{1 - z^{-1}}{1 + z^{-1}}$$

If we put $z = j\omega$ and $s = j\Omega$, we get

$$\Omega = \tan \frac{\omega}{2}$$

The specifications now are

- Passband nature = equiripple
- Stopband nature = equiripple
- $\Omega_{s1} = 0.6725$
- $\Omega_{s2} = 1.1014$
- $\Omega_{p1} = 0.6044$
- $\Omega_{p2} = 1.2146$
- $\delta_1 = \delta_2 = 0.15$

6.4 Analog Frequency Transformation

Here we use the **Bandstop to Lowpass** analog frequency transformation, which is as follows:

$$\Omega_L = \frac{B\,\Omega}{\Omega_0^2 - \Omega^2}$$

where

$$\Omega_0 = \sqrt{\Omega_{p1}\,\Omega_{p2}} = 0.8568$$

and

$$B = \Omega_{p2} - \Omega_{p1} = 0.6102$$

Using the transformation mentioned above, we now have

- Passband and Stopband nature = monotonic
- $\Omega_{Ls1} = 1.456$
- $\Omega_{Ls2} = -1.4031$
- $\Omega_{Lp1} = +1$
- $\Omega_{Lp2} = -1$
- $\delta_1 = \delta_2 = 0.15$

6.5 Equivalent Analog Lowpass Filter Specifications

Using the more stringent specifications from those obtained in section 6.4, we have

- Passband nature = equiripple
- Stopband nature = equiripple
- $\Omega_{Ls} = \text{min}\{|\Omega_{Ls2}|, |\Omega_{Ls1}|\} = 1.4031$
- $\Omega_{Lp} = +1$
- $\delta_1 = \delta_2 = 0.15$
- $D_1 = \frac{1}{(1-\delta_1)^2} - 1 = 0.384$
- $D_2 = \frac{1}{\delta_2^2} - 1 = 43.44$

6.6 Analog Lowpass Magnitude Response

The general transfer function for an **Elliptic** filter is given by

$$H_{LPF}(s_L)\, H_{LPF}(-s_L) = \frac{1}{1 + \epsilon^2\, F_N^2(\frac{s_L}{j\,\Omega_{Lp}})}$$

with

$$F_N(w) = \mathrm{cd}(NuK_1, k_1)$$

and

$$w = \mathrm{cd}(uK, k))$$

where $\mathrm{cd}(x, k)$ denotes the Jacobian elliptic function cd with modulus k and real quarter-period K.

6.6.1 Jacobian Elliptic Functions

The elliptic function $w = \mathrm{sn}(z, k)$ is defined as follows:

$$z = \int_0^{\phi} \frac{d\theta}{\sqrt{1 - k^2\, \sin^2\theta}} = \int_0^{w} \frac{dt}{\sqrt{(1-t^2)\,(1-k^2\,t^2)}} \quad , w = \sin\big(\phi(z, k)\big)$$

There are three more relevant elliptic functions, cn, dn, and cd defined as:

$$\begin{aligned} w &= \mathrm{cn}(z, k) = \cos\big(\phi(z, k)\big) \\ w &= \mathrm{dn}(z, k) = \sqrt{1 - k^2\, \mathrm{sn}^2(z, k)} \\ w &= \mathrm{cd}(z, k) = \frac{\mathrm{cn}(z, k)}{\mathrm{dn}(z, k)} \end{aligned}$$

We define the *complete elliptic integral of first kind* $K(k)$ or simply K as the value of z when $\mathrm{sn}(z, k) = 1$ i.e. when $\phi = \pi/2$

$$K(k) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2\, \sin^2\theta}}$$

Clearly $\text{cd}(K,k) = 0$. Associated with an elliptic modulus k, there is a complementary modulus $k' = \sqrt{1-k^2}$ and its associated complete elliptic integral $K(k')$ denoted by $K'(k)$ or simply K'

$$K'(k) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1-k'^2\sin^2\theta}} = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1-(1-k^2)\sin^2\theta}}$$

The significance of K and K' is that sn and cd are *doubly-periodic functions* in the z plane with a real period $4K$ and a complex period $2jK'$. Some of the useful properties are as follows:

$$\text{cd}(z+(2n-1)K,k) = (-1)^n\,\text{sn}(z,k) \tag{6}$$

$$\text{cd}(z+2nK,k) = (-1)^n\,\text{cd}(z,k) \tag{7}$$

$$\text{cd}(z+j\,K',k) = \frac{1}{k\,\text{cd}(z,k)} \tag{8}$$

$$\text{cd}(j\,z,k) = \frac{1}{\text{dn}(z,k')} \tag{9}$$

$$\text{cd}(j\,K',k) = \frac{1}{k} \tag{10}$$

$\text{cd}(z,k)$ has zeroes at $(2m+1)\,K + j\,2n\,K'$ and poles at $(2m+1)\,K + j\,(2n+1)\,K'$ for any integers m and n in the z plane.

6.6.2 Elliptic Filter Parameters

The order of the filter N is determining by putting a constraint that the magnitude response at the passband edge is not less than $1-\delta_1$ and at the stopband edge not greater than δ_2.

$$N = \left\lceil \frac{K\,K_1'}{K'\,K_1} \right\rceil$$

where K, K', K_1, K_1' are defined for the complete elliptic integrals of modulus k and k_1 given by

$$k = \frac{\Omega_{Lp}}{\Omega_{Ls}} = 0.7127$$

$$k_1 = \sqrt{\frac{D_1}{D_2}} = 0.0940$$

We then have

Parameter	Value
K	1.8609
K'	1.8474
K_1	1.5743
K_1'	3.7566
N	3

Table 6.1: Elliptic Filter Parameters

We now have to find the poles of the the transfer function. The poles of the transfer function are given by the zeros of the denominator and the zeros of the transfer function are given by the poles of $F_N(\frac{s_L}{j\,\Omega_{Lp}})$. The zeros and poles of the transfer function are given by

$$\begin{aligned} z_i &= j\,\Omega_{Lp}\,(k\zeta_i)^{-1}\ , \quad i = 1, 2, \ldots, \lfloor N/2 \rfloor \\ p_i &= j\,\Omega_{Lp}\,\mathrm{cd}\big((u_i - j\nu_0)\,K, k\big)\ , \quad i = 1, 2, \ldots, \lfloor N/2 \rfloor \\ p_0 &= j\,\Omega_{Lp}\,\mathrm{sn}(j\nu_0 K, k) \end{aligned}$$

where

$$\begin{aligned} \zeta_i &= \mathrm{cd}(u_i K, k) \\ u_i &= \frac{2i-1}{N}\ , \quad i = 1, 2, \ldots, \lfloor N/2 \rfloor \\ \nu_0 &= -\frac{j}{N\,K_1}\mathrm{sn}^{-1}\left(\frac{j}{\epsilon}, k_1\right) \end{aligned}$$

As N is odd, the zeros and poles occur in conjugate pairs We then have

z_1	$j\,7.9193$
z_2	$-j\,7.9193$
p_0	-3.9154
p_1	$-0.7246 + j\,6.2431$
p_2	$-0.7246 - j\,6.2431$

Table 6.2: Poles and Zeros of $H_{LPF}(s_L)\,H_{LPF}(-s_L)$

$$H_{LPF}(s_L') = \frac{1}{1 + 0.2554\,s_L'}\,\frac{1 + 0.0159\,s_L'^2}{1 + 0.0367\,s_L' + 0.0253\,s_L'^2}$$

$$H_{LPF}(s_L') = \frac{1 + 0.0159\,s_L'^2}{1 + 0.2921\,s_L' + 0.0346\,s_L'^2 + 0.00646\,s_L'^3}$$

Here, $s'_L = 2\pi\, s_L$. Hence

$$H_{LPF}(s_L) = \frac{1 + 0.6277\, s_L^2}{1 + 1.8353\, s_L + 1.3659\, s_L^2 + 1.6024\, s_L^3}$$

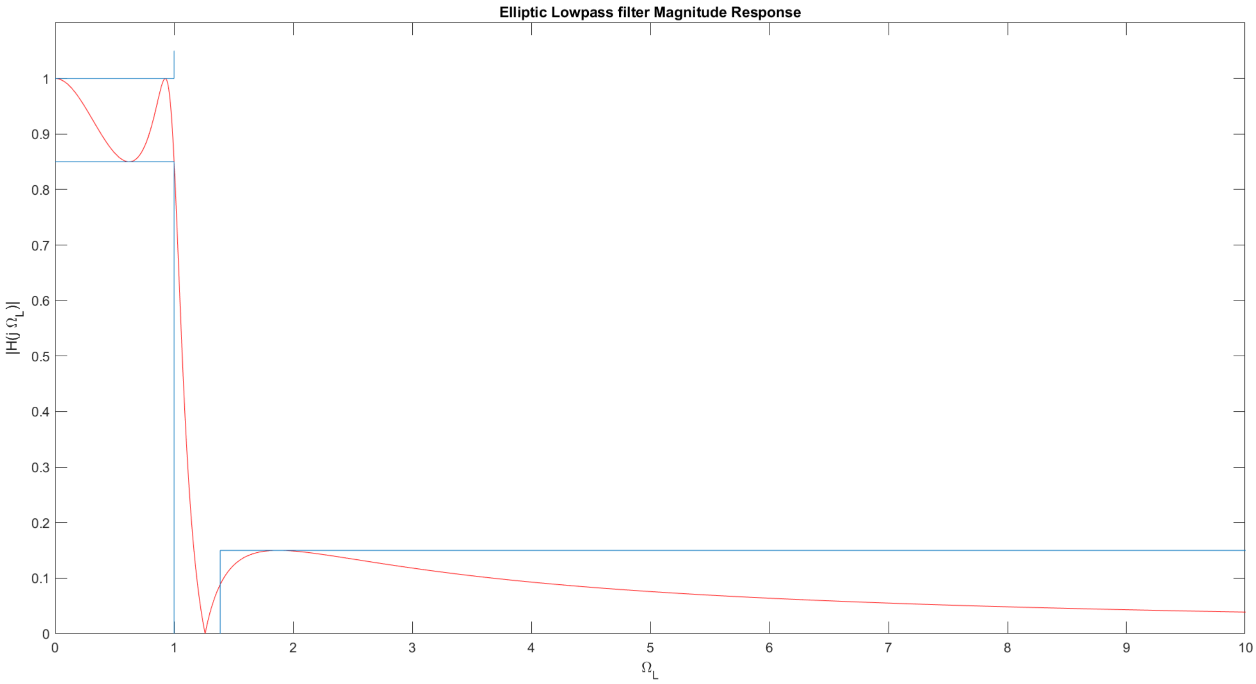

Figure 6.1: Magnitude Response of the equivalent Elliptic Lowpass Filter

6.7 Analog Bandstop Transfer Function

We now use the inverse transformation to convert the equivalent Elliptic lowpass transfer function to get the Bandstop transfer function using the relation:

$$s_L \longleftarrow \frac{B\, s}{s^2 + \Omega_0^2}$$

where B and Ω_0 have the values as we found in section 6.4

$$H_{LPF}\left(\frac{B\, s}{s^2 + \Omega_0^2}\right) = H_{BSF}(s)$$

$$H_{BSF}(s) = \frac{2.0569\, s^6 + 5.0122\, s^4 + 3.6807\, s^2 + 0.8145}{2.0569\, s^6 + 2.3037\, s^5 + 5.5777\, s^4 + 4.1324\, s^3 + 4.0959\, s^2 + 1.2423\, s + 0.8145}$$

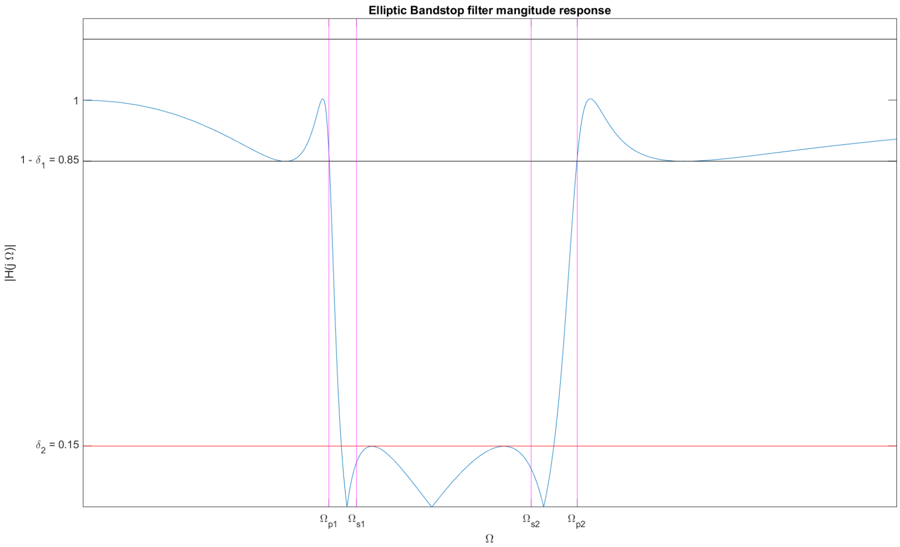

Figure 6.2: Magnitude Response of Elliptic Bandstop filter

6.8 Discrete-Time Bandstop Transfer Function

We now make use of the Bilinear Transformation to convert the analog bandstop filter into a discrete bandstop filter in the normalized angular frequency domain. The transform is as follows:

$$s \longleftarrow \frac{1 - z^{-1}}{1 + z^{-1}}$$

The discrete time filter transfer function is

$$H_{BSF}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right) = H(z)$$

$$H(z) = \frac{1.1564\, z^6 - 1.0117\, z^5 + 3.4378\, z^4 - 1.9521\, z^3 + 3.4378\, z^2 - 1.0117\, z + 1.1564}{2.0223\, z^6 - 1.4663\, z^5 + 3.8730\, z^4 - 1.8920\, z^3 + 2.8065\, z^2 - 0.6172\, z + 0.4867}$$

Please see the next page

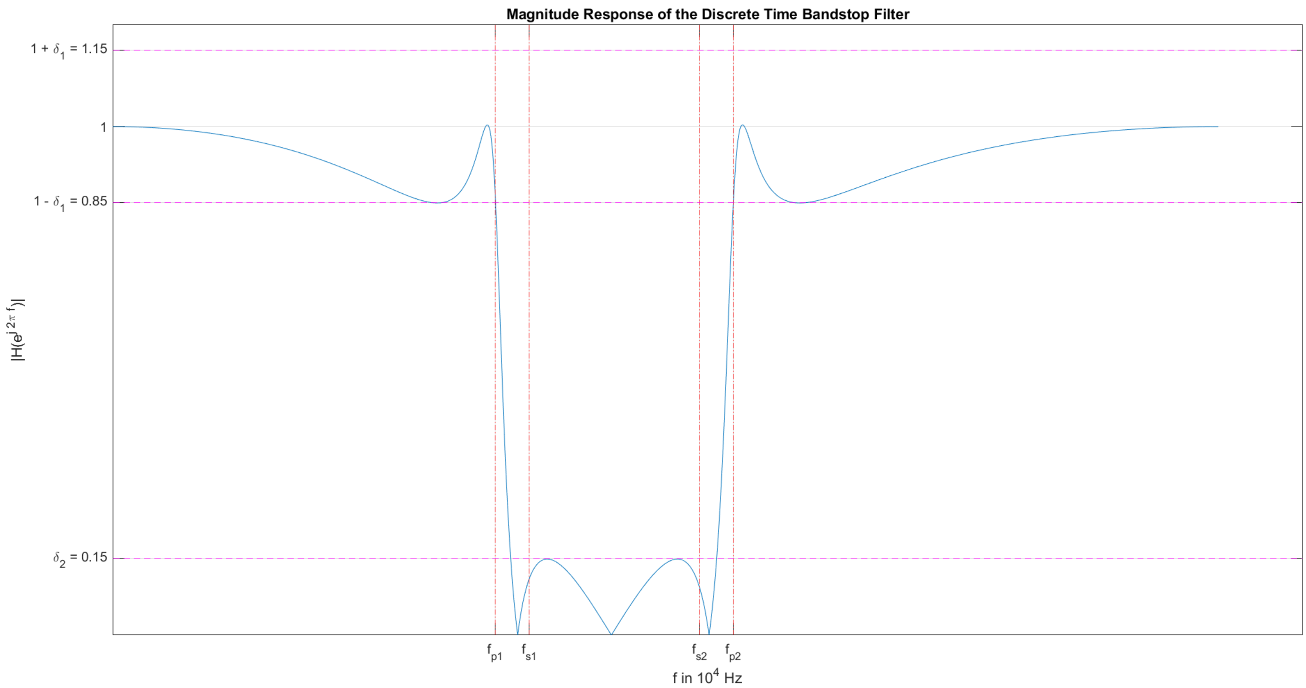

Figure 6.3: Magnitude Response of Discrete Time Bandstop Filter

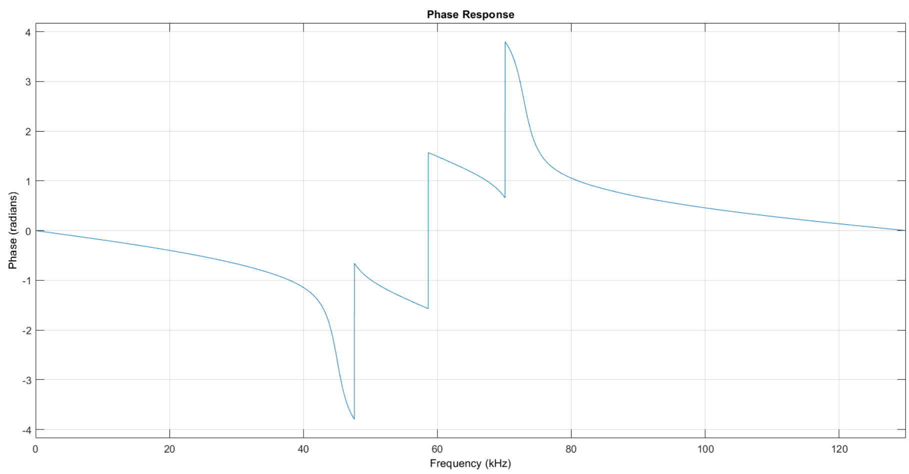

Figure 6.4: Phase Response of Discrete Time Bandstop Filter

7 Peer Review

7.1 Review Received

7.2 Review Given