EE 678 Activity 2 - RnD Project Report

Amol Girish Shah
Department of Electrical Engineering
Indian Institute of Technology Bombay
Mumbai, India
18D070006@iitb.ac.in

Mantri Krishna Sri Ipsit
Department of Electrical Engineering
Indian Institute of Technology Bombay
Mumbai, India
180070032@iitb.ac.in

I. Introduction

In the blooming era of innovations in the field of information technology, wireless communication systems are playing an important role. The way we communicate is changing very rapidly. Digital modulation techniques, in particular those involving multi-carrier modulation are seeming to be the most viable options for future mobile communication systems (5G). Waveforms with high spectral and temporal containment are becoming a requirement to allow communication with very low latency while making optimal use of resources available.

Future mobile systems will be characterized by a large range of possible use cases which will require high bandwidth rates, low latency, longer range etc. To efficiently support such diverse use cases, a flexible time-frequency resource allocation becomes necessary, [2] as illustrated below.

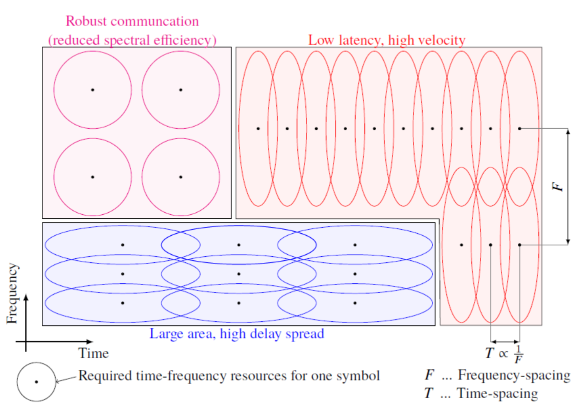

Fig. 1. Flexible assignment of the available time-frequency resources [3]

In such a multicarrier system, symbols are transmitted over a rectangular time-frequency grid. Note that the subcarrier spacing determines the shape in frequency and, correspondingly, in time. A high subcarrier spacing allows for low latency transmissions while a small subcarrier spacing increases the bandwidth efficiency. Furthermore, different subcarrier spacings allow to match the transmission system to specific channel conditions. A user at high velocities should employ a high subcarrier spacing. On the other hand, if multipath delay spread is the limiting factor, a small subcarrier spacing is the better choice. [3]

Two famous modulation schemes that have been proposed to solve this purpose are

- Orthogonal Frequency Division Multiplexing (OFDM) family
- Filter Bank Multicarrier (FBMC) family

In this paper we would like to present both these modulation schemes from a time-frequency perspective. The rest of the paper is organized as follows. In section II both the modulation schemes will be discussed. In section III, we introduce the Ambiguity function and Balian-Low theorem. We compare different modulation schemes from a time-frequency perspective in section IV and then we will provide our conclusion which is followed by acknowledgments and references.

II. Modulation Schemes for 5G

In multicarrier modulations, the message symbols are commonly transmitted over orthogonal pulses/subcarriers which overlap in time and frequency. These subcarriers usually occupy a very small bandwidth. This is an advantage when the communication is happening through a frequency-selective channel. The pulses are usually time and frequency translates of a prototype pulse which is also known as a prototype filter. Mathematically, the transmitted signal $s(t)$ in time domain can be expressed as

$$s(t) = \sum_{k=1}^{K} \sum_{l=1}^{L} g_{l,k}(t)\, x_{l,k} \tag{1}$$

where $x_{l,k}$ denotes the transmitted symbol at subcarrier position l and time position k and is often chosen from a QAM or PAM constellation. The transmitted basis pulse $g_{l,k}(t)$ is defined as

$$g_{l,k}(t) = p_{TX}(t - kT)\, e^{j2\pi\, lF\,(t-kT)}\, e^{j\theta_{l,k}} \tag{2}$$

which is a time and frequency translate of the transmitter prototype filter $p_{TX}(t)$ with T denoting the time spacing and F denoting the frequency spacing. The additional phase shift $\theta_{l,k}$ is used in the case of FBMC-OQAM. The received signal $r(t)$ is used to find the received symbols $y_{l,k}$

$$y_{l,k} = \langle r(t), q_{l,k}(t) \rangle = \int_{-\infty}^{\infty} r(t)\, q^*_{l,k}(t)\, dt \tag{3}$$

where $q_{l,k}(t)$ denotes the receive basis pulses making use of a different prototype filter $p_{RX}(t)$ given by

$$q_{l,k}(t) = p_{RX}(t - kT)\, e^{j2\pi\, lF\,(t-kT)}\, e^{j\theta_{l,k}} \tag{4}$$

A. Orthogonal Frequency Division Multiplexing (OFDM)

In OFDM, the transmit and receive prototype filters are rectangular pulses and the phase $\theta_{l,k} = 0$. The duration of $p_{TX}(t)$ chosen usually larger than that of $p_{RX}(t)$. This difference between the duration is known as Cyclic Prefic (CP). Mathematically,

$$p_{TX}(t) = \begin{cases} \frac{1}{\sqrt{T_0}} & \text{if } -\left(\frac{T_0}{2} + T_{CP}\right) \leq t \leq \frac{T_0}{2} \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

$$p_{RX}(t) = \begin{cases} \frac{1}{\sqrt{T_0}} & \text{if } -\frac{T_0}{2} \leq t \leq \frac{T_0}{2} \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

where T_0 denotes the pulse duration and T_{CP} denotes the duration of the cyclic prefix. The CP is needed for the receiver to correctly detect the message symbols even in the presence of a doubly dispersive channel. The use of rectangular transmit and receive pulses greatly reduce the computational complexity, but they are less bandwidth efficient because larger side lobes in its spectrum.

B. Filtered - OFDM

Traditional OFDM has some issues of inter-symbol interference (ISI) and inter-carrier interference (ICI) in multiple-input multiple-output (MIMO) transmissions. To overcome these, filtered OFDM (f-OFDM) was proposed. In this method, the multicarrier modulated signal is passed through a spectrum shaping filter with impulse response $f(t)$. This spectrum shaping filter is chosen so that its bandwidth is equal to the total frequency width of the assigned subcarriers and its time duration is a portion of the symbol duration. Mathematically, the transmitted signal $\tilde{s}(t)$ will now be

$$\tilde{s}(t) = s(t) * f(t) \tag{7}$$

At the receiver, a matched filter $f^*(-t)$ which is matched to $f(t)$ is used before passing it through a regular OFDM demodulator.

C. Filter Bank Multicarrier

Two key observations make Filter Bank Multicarrier Modulation - FBMC a viable choice for future wireless systems:

1) Flexible time-frequency allocation to efficiently support diverse user requirements and channel characteristics.
2) Low channel delay spread, especially in dense heterogeneous networks utilizing Multiple-Input and Multiple-Output (MIMO) beamforming and high carrier frequencies.

The first observation relates to Figure 1. In particular, low Out-Of-Band (OOB) emissions are required, so that the guard band between different use cases is relatively small. Conventional OFDM is not suited for that. Therefore windowing

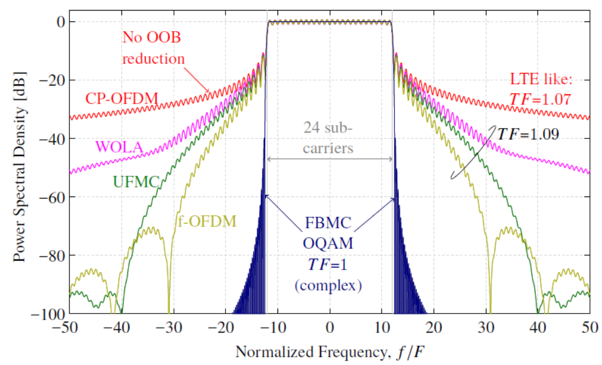

Fig. 2. FBMC has much better spectral properties

and filtering in OFDM is considered [4]. While windowing and filtering can indeed reduce the OOB emissions of pure OFDM, FBMC still performs much better, as shown in Figure 2 . Additionally, FBMC has a maximum symbol density of TF = 1 while in OFDM based schemes the symbol density is lower, as indicated by TF 1, additionally worsening the spectral efficiency. The second key observations relates to the finding that the measured delay spread is much smaller than "typically" assumed in simulations. Arguments for the lower delay spread are also provided, such as, decreasing cell sizes and "spatial filtering" of the environment through beamforming; these arguments will become even more significant in future mobile networks because of an increased network densification, application of massive two-dimensional antenna arrays and the push towards higher carrier frequencies, implying larger propagation path losses. The low delay spread guarantees that low-complexity one-tap equalizers are sufficient in FBMC to achieve a close to optimal performance.

1) FBMC-QAM: The prototype filter for FBMC-QAM is based on Hermite polynomials Hn(·) is: [5]

$$p(t) = \frac{1}{\sqrt{T_0}} \mathrm{e}^{-2\pi\left(\frac{t}{T_0}\right)^2} \sum_{\substack{i=\{0,4,8,\\12,16,20\}}} a_i H_i\left(2\sqrt{\pi}\frac{t}{T_0}\right)$$

Another prominent filter is the PHYDYAS prototype filter constructed by: [2]

$$p(t) = \begin{cases} \frac{1+2\sum_{i=1}^{O-1} b_i \cos\left(\frac{2\pi t}{OT_0}\right)}{O\sqrt{T_0}} & \text{if } -\frac{OT_0}{2} < t \leq \frac{OT_0}{2} \\ 0 & \text{otherwise} \end{cases}$$

The coefficients b_i depend on the overlapping factor O.

2) FBMC-OQAM: FBMC-OQAM is related to FBMC-QAM but has the same symbol density as OFDM without CP. To satisfy the Balian-Low theorem (See next section), the complex orthogonality condition $\langle g_{l_1,k_1}(t), g_{l_2,k_2}(t)\rangle = \delta_{(l_2-l_1),(k_2-k_1)}$ is replaced by the less strict real orthogonality condition $\Re\{\langle g_{l_1,k_1}(t), g_{l_2,k_2}(t)\rangle\} = \delta_{(l_2-l_1),(k_2-k_1)}$.
FBMC-OQAM works, in principle, as follows:

1) Design a prototype filter with $p(t) = p(-t)$ which is orthogonal for a time spacing of $T = T_0$ and a frequency spacing of $F = 2/T_0$, leading to $TF = 2$,

2) Reduce the (orthogonal) time-frequency spacing by a factor of two each, that is, $T = T_0/2$ and $F = 1/T_0$.
3) The induced interference, caused by the time-frequency squeezing, is shifted to the purely imaginary domain by the phase shift $\theta_{l,k} = \frac{\pi}{2}(l+k)$.

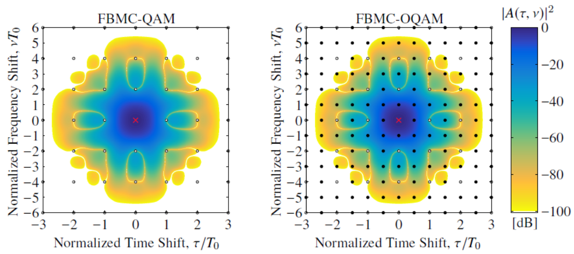

Fig. 3. Illustration of FBMC-OQAM: Starting from an FBMC-QAM system (left), the time spacing and the frequency spacing are both reduced by a factor of two, leading to a time spacing of $T = T_0/2$ and a frequency spacing of $F = 1/T_0$ (right). The so induced interference (black markers) is shifted to the purely imaginary domain by the phase shift $\theta_{l,k} = \frac{\pi}{2}(l+k)$ and can easily be canceled by taking the real part.

Starting from an FBMC-QAM system (left) the orthogonal time-frequency spacing of $T = T_0$ and $F = 2/T_0$ is reduced to $T = T_0/2$ and $F = 1/T_0$ (right). This causes interference, indicated by the black markers. To be specific, for $\theta_{l,k} = \frac{\pi}{2}(l+k)$ and $TF = 0.5$, the inner product can be expressed as

$$\langle g_{l+\Delta l,k+\Delta k}(t), q_{l,k}(t)\rangle = \underbrace{\mathrm{e}^{\mathrm{j}\frac{\pi}{2}(\Delta l+\Delta k)}\mathrm{e}^{-\mathrm{j}\frac{\pi}{2}\Delta k(2l+\Delta l}\Big)}_{\text{purely imaginary for odd }\Delta\mathrm{k},\Delta 1}$$

$$\underbrace{A(\Delta kT, \Delta lF)}_{0 \text{ if both } \Delta k\neq 0, \Delta l\neq 0 \text{ are even}} \quad .$$

The ambiguity function above guarantees orthogonality if both, Δk and Δl, are even. On the other hand, if either Δk or Δl is odd, $A(\cdot)$ no longer becomes zero. Then, the overall phase of above equation is purely determined by the exponential function since the ambiguity function is always real-valued because of $p(t) = p(-t)$. In particular, the exponential function becomes purely imaginary if either Δk or Δl is odd. Thus, the real orthogonality condition is satisfied.

III. Time - Frequency Analysis

As discussed in section II, the multicarrier modulation systems are mainly characterized by the transmit and receive prototype filters and the time and frequency spacing between them. For error less detection, we need the prototype filters to be orthogonal. This can be stated mathematically as follows:

$$\langle g_{l,k}, q_{n,m}\rangle = \delta_{ln}\delta_{km} \tag{8}$$

where δ_{ln} is the Kronecker delta function.

A. Ambiguity Function

The orthogonality condition mentioned in equation 8 above is captured in a more general sense using the ambiguity function defined as follows:

$$A(\tau,\nu) = \int_{-\infty}^{\infty} p_{TX}\left(t - \frac{\tau}{2}\right) p_{RX}^{*}\left(t + \frac{\tau}{2}\right) e^{j2\pi\nu t}\, dt \tag{9}$$

where t is a time delay and ν is a frequency shift. Using this function, we can reformulate equation 8 as follows:

$$\langle g_{l,k}, q_{n,m}\rangle \propto A((k-m)T, (l-n)F) \tag{10}$$

The surface of the ambiguity function can be plotted on a time - frequency grid. The zeroes of the ambiguity function correspond to the location of the transmit symbols on the time - frequency plane. This provides for a way to compare different modulation schemes using the symbol density which is a measure of efficiency.

B. Balian-Low Theorem

This theorem states the limitations which are fundamental in the case of multicarrier systems. The theorem states that mathematically it is impossible to satisfy the following four desired properties at the same time:

1) Maximum symbol density

$$TF = 1 \tag{11}$$

2) Time-localization

$$\sigma_t = \sqrt{\int_{-\infty}^{\infty} (t-\bar{t})^2 |p(t)|^2\, dt} < \infty \tag{12}$$

3) Frequency-localization

$$\sigma_f = \sqrt{\int_{-\infty}^{\infty} (f-\bar{f})^2 |P(f)|^2\, df} < \infty \tag{13}$$

4) (Bi)-orthogonality

$$\langle g_{l,k}, q_{n,m}\rangle = \delta_{ln}\delta_{km} \tag{14}$$

$$A((k-m)T, (l-n)F) = \delta_{ln}\delta_{km} \tag{15}$$

where $P(f)$ denotes the Fourier transform of the prototype filter $p(t)$, $\bar{t}$ denotes the time center and $\bar{f}$ denotes the frequency center of the prototype filter, and the prototype filter is normalized to have unit energy.

IV. Comparison between Modulation Schemes

The modulation schemes mentioned in section II will be compared from the perspective of Balian-Low theorem and their ambiguity surfaces. The comparisons made accurately justify the Balian-Low theorem. The calculations have been skipped for brevity.

A. OFDM

For traditional OFDM, we have

- $T = T_0 + T_{CP}$
- $F = 1/T_0$
- $TF = 1 + \frac{T_{CP}}{T_0} > 1$
- $\sigma_t = \frac{T_0 + T_{CP}}{2\sqrt{3}}$
- $\sigma_f = \infty$

Because of employing rectangular pulses, there is no frequency localization. This is expected because the Fourier transform of a rectangular pulse is a sinc pulse which has infinite support. The ambiguity surface plots re-inforce this fact.

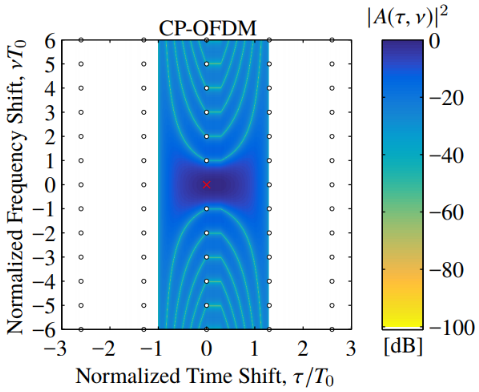

Fig. 4. The ambiguity function $20\log_{10}|A(\tau, v)|$ in case of CP-OFDM [2]

B. f-OFDM

Filtered OFDM has slightly higher symbol density than traditional OFDM. This can be expected because of the use of spectrum shaping filter. Although this leads to larger time spreading, on a whole it is more efficient.

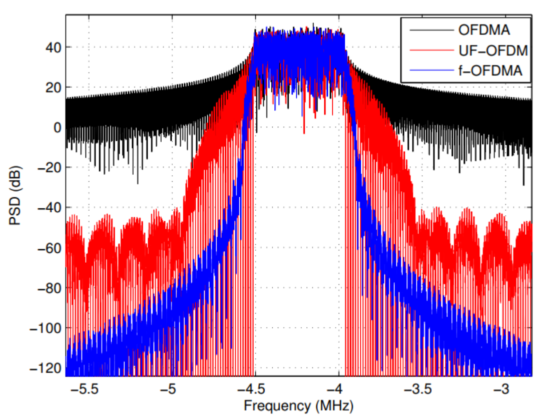

Fig. 5. Power Spectral Density of OFDM and f-OFDM [1]

C. FBMC QAM

From the prototype filter for FBMC QAM for which particular coefficents are selected from [6] gives following properties

$$T = T_0; \qquad F = 2/T_0 \quad TF = 2$$
$$\sigma_t = 0.2015T_0; \quad \sigma_f = 0.403/T_0$$

Orthogonality is observed for a time spacing of $T = T_0$ and a frequency spacing of $F = 2/T_0$. Compared to OFDM, the frequency localization is much better. Note that the Hermite pulse has the same shape in time and frequency, allowing to exploit symmetries. Furthermore, it is based on a Gaussian function and therefore has a good joint time-frequency localization of $\sigma_t\sigma_f = 1.02 \times 1/4\pi$, almost as good as the bound of $\sigma_t\sigma_f \geq 1/4\pi \approx 0.08$ (attained by the Gaussian pulse), making it relatively robust to doubly-selective channels.

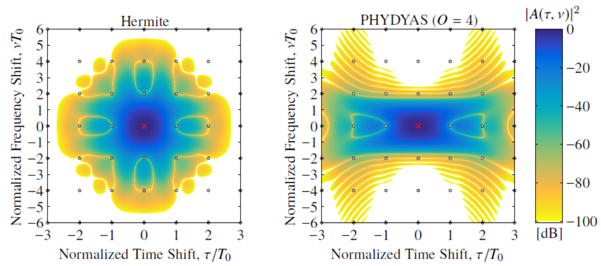

Fig. 6. The ambiguity function $20\log_{10}|A(\tau, v)|$ in case of FBMC-QAM [2]

D. FBMC OQAM

1) $T = T_0/2$
2) $F = 1/T_0$
3) $TF = 1$,

Although the time-frequency spacing of FBMC-OQAM is equal to $TF = 0.5$, only real-valued information symbols can be transmitted in such a system, leading to an equivalent time-frequency spacing of $TF = 1$ for complex-valued symbols. Very often, the real-part of a complex-valued symbol is mapped to the first time slot and the imaginary-part to the second time slot, thus the name offset-QAM. However, such self-limitation is not necessary. The main disadvantage of FBMC-OQAM is the loss of complex orthogonality. This implies particularities for some MIMO techniques, such as space-time block codes or maximum likelihood symbol detection , as well as for channel estimation [2]

E. Comparison table

	Maximum Symbol Density	Time-Localization	Frequency-Localization	(Bi)-Orthogonal	Independent Transmit Symbols
OFDM (without CP)	yes	yes	no	yes	yes
Windowed/Filtered OFDM	no	yes	yes	yes	yes
FBMC-QAM[1]	no	yes	yes	yes	yes
FBMC-OQAM	yes	yes	yes	real only	yes

Fig. 7. Comparison of different multicarrier schemes (for AWGN) [2]

V. Multiuser MIMO FBMC

- Consider $N_r \times N_t$ MIMO system
- The transmit signal at a single transmit antenna in vector form: $\boldsymbol{s} = \boldsymbol{G}\boldsymbol{x}$
- $\boldsymbol{G} \in \mathbb{C}^{LK \times LK}, \boldsymbol{s}, \boldsymbol{x} \in \mathbb{C}^{LK \times 1}$

- Received signal at a single receiver antenna: $\boldsymbol{r} = \tilde{\boldsymbol{h}}\boldsymbol{G}\boldsymbol{x} + \tilde{\boldsymbol{n}}$
- Received symbols at a single receiver antenna: $\boldsymbol{y} = \boldsymbol{G}^*\tilde{\boldsymbol{h}}\boldsymbol{G}\boldsymbol{x} + \boldsymbol{n} = \boldsymbol{h}\boldsymbol{x} + \boldsymbol{n}$
- As there are $N_r \times N_t$ possible channels,
- Considering a single active symbol, we then have

$$\overline{\mathrm{y}} = \boldsymbol{H}\overline{\boldsymbol{x}} + \boldsymbol{n}$$

- $\overline{\boldsymbol{y}} \in \mathbb{C}^{LN_r \times 1}$
- $\boldsymbol{H} \in \mathbb{C}^{LN_r \times LN_t}$
- $\overline{\boldsymbol{x}} \in \mathbb{C}^{LN_t \times 1}$
- Considering there are K users with N_r antennas each and a base station downlink with N_t antennas and repeating the above procedure for every user.

$$\overline{\boldsymbol{y}} = \boldsymbol{H}\overline{\boldsymbol{x}} + \boldsymbol{n}$$

- $\overline{\boldsymbol{y}} \in \mathbb{C}^{KLN_r \times 1}$
- $\boldsymbol{H} \in \mathbb{C}^{KLN_r \times LN_t}$
- $\overline{\boldsymbol{x}} \in \mathbb{C}^{LN_t \times 1}$
- Consider a precoding matrix $\boldsymbol{P}$ such that $\overline{\boldsymbol{x}} = \boldsymbol{P}\hat{\boldsymbol{x}}$
- $\hat{\boldsymbol{x}} = [\boldsymbol{x}_1, \ldots, \boldsymbol{x}_K]^\top, \boldsymbol{x}_i \in \mathbb{C}^{LK \times 1}$ is the message symbol vector corresponding to use $\bullet\overline{\boldsymbol{y}} = \boldsymbol{H}\boldsymbol{P}\hat{\boldsymbol{x}} + \boldsymbol{n}$
- If $\boldsymbol{H}\boldsymbol{P} = \boldsymbol{D}$ where $\boldsymbol{D}$ is a diagonal matrix, then there is no inter-user interference.
- In that case $\boldsymbol{P} = \boldsymbol{H}^\dagger\boldsymbol{D}$ which exists only when $\boldsymbol{H} \in \mathbb{C}^{KLN_r \times LN_t}$ has a full row rank- this is a suboptimal solution since each user is able to coordinate the processing of its own receiver outputs
- Consider a precoder $\boldsymbol{P}_i$ for each user i such that

$$\overline{\boldsymbol{x}} = \sum_{i=1}^{K} \boldsymbol{P}_i\hat{\boldsymbol{x}}_i$$

then for user j,

$$\begin{aligned}\overline{\boldsymbol{y}}_j &= \boldsymbol{H}_j\left(\sum_{i=1}^{K}\boldsymbol{P}_i\hat{\boldsymbol{X}}_i\right) + \boldsymbol{n}_j \\ &= \boldsymbol{H}_j\boldsymbol{P}_j\hat{\boldsymbol{x}}_j + \underbrace{\boldsymbol{H}_j\tilde{\boldsymbol{P}}_j\tilde{\boldsymbol{x}}_j}_{\text{IUI}} + \boldsymbol{n}_j\end{aligned}$$

where

$$\begin{aligned}\tilde{\boldsymbol{P}}_j &= [\boldsymbol{P}_1 \ldots \boldsymbol{P}_{j-1}\boldsymbol{P}_{j+1} \ldots \boldsymbol{P}_K] \\ \tilde{\boldsymbol{x}_j}^\top &= \left[\hat{\boldsymbol{x}}_1^\top \ldots \boldsymbol{x}_{j\hat{-}1}^\top\boldsymbol{x}_{j\hat{-}1}^\top \ldots \boldsymbol{x}_{\boldsymbol{K}}^\top\right]\end{aligned}$$

- **Block Diagonalization Algorithm** [7]
- Denote

$$\begin{aligned}\boldsymbol{H}_S &= \left[\boldsymbol{H}_1^\top\boldsymbol{H}_2^\top \ldots \boldsymbol{H}_K^\top\right]^\top \\ \boldsymbol{P}_S &= [\boldsymbol{P}_1\boldsymbol{P}_2 \ldots \boldsymbol{P}_K]\end{aligned}$$

- For zero IUI, we want $\boldsymbol{H}_S\boldsymbol{P}_S$ to be a block diagonal matrix
- This can be done either for Throughput Maximization or Power Control
- Null space, SVD, Shanon's channel capacity theorem and waterfilling algorithm are the concepts applied here
- This is a non-iterative algorithm so that the closed form solution can be directly implemented
- **Successive Optimization (SO) Algorithm** [7]
- Another non-iterative algorithm for Power Control to get closed-form solution
- Remove IUI for one user at a time, with the condition that the precoder of the current user j is optimized such that it does not interfere with the previous $j-1$ users' data
- Uses only the statistics of the interfering signals from previous steps and hence it is valid as long as this transmission procedure is a Stationary Random Process
- Here too the concepts of Null space, SVD, Shanon's channel capacity theorem and waterfilling algorithm are used

VI. Conclusion

In this report, we saw the time-frequency aspects of OFDM, f-OFDM and FBMC brought out. We also then covered the topic of MIMO FBMC which we studied along with related topics like Water-filling theorem. In this RnD project we learned a lot of new concepts - definitely many advanced concepts much more than we have learnt than any other course we have taken so far - starting with OFDM,Filtered OFDM, FBMC, types of FBMC, Alamouti coding, MIMO, Multiuser MIMO FBMC, Waterfilling Algorithm,etc. There was tremendous learning as all concepts required much pre-reading and background. We are very grateful to Mr. Murali Krishna Pavuluri who taught us various concepts, cleared our doubts and was a wonderful mentor. This RnD activity definitely gave us much exposure in the communications area. We are also thankful to Professor Vikram Gadre for proving this opportunity to us.

References

[1] J. Abdoli, M. Jia and J. Ma, "Filtered OFDM: A new waveform for future wireless systems," 2015 IEEE 16th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC), 2015, pp. 66-70, doi: 10.1109/SPAWC.2015.7227001.

[2] Ronald Nissel, "Filter Bank Multicarrier Modulation for Future Wireless Systems", Dissertation, TU Wien

[3] R. Nissel, S. Schwarz, and M. Rupp, "Filter bank multicarrier modulation schemes for future mobile communications," IEEE Journal on Selected Areas in Communications, vol. 35, no. 8, pp. 1768–1782, 2017.

[4] X. Zhang, M. Jia, L. Chen, J. Ma, and J. Qiu, "Filtered-OFDM-enabler for flexible waveform in the 5th generation cellular networks," in IEEE Global Communications Conference (GLOBECOM), pp. 1–6, 2015.

[5] R. Haas and J.-C. Belfiore, "A time-frequency well-localized pulse for multiple carrier transmission," Wireless Personal Communications, vol. 5, no. 1, pp. 1–18, 1997.

[6] R. Nissel and M. Rupp, "On pilot-symbol aided channel estimation in FBMCOQAM"in IEEE International Conference on Acoustics, Speech and Signal Processing(ICASSP), pp. 3681–3685, Shanghai, China, March 2016.

[7] Zero-Forcing Methods for Downlink Spatial Multiplexing in Multiuser MIMO Channels,Quentin H. Spencer, Student Member, IEEE, A. Lee Swindlehurst, Senior Member, IEEE, and Martin Haardt, Senior Member, IEEE; IEEE TRANSACTIONS ON SIGNAL PROCESSING, VOL. 52, NO. 2, FEBRUARY 2004