# Designing Precoders to mitigate Inter-User Interference in MIMO FBMC Channels

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# Overview

# Orthogonal Frequency Division Multiplexing[1]

- Transmit Signal: $x(t) = \sum_{n} \sum_{k \in \mathcal{K}} s_k[n] p_T(t - nT) e^{j2\pi(t-nT)f_k} = \sum_{n} \sum_{k \in \mathcal{K}} s_k[n] p_{T,k}(t - nT)$
- $\mathcal{K}$ denotes the set of active symbol indices
- $s_k[n]$ denote the data symbols, $k$ is the subcarrier index
- $p_T(t)$ is the transmitting prototype filter, a rectangular pulse here
- $T$ is the symbol time
- $p_{T,k}(t) = p_T(t) e^{j2\pi t f_k}$
- The receiving prototype filter $p_R(t)$ is also a rectangular pulse with duration $\tau \leq T$ to maintain the orthogonality condition
- Bad spectral containment per subcarrier leading to inefficient bandwidth utilization

# Filter Bank Multi Carrier[2], [3]

- Transmit Signal: $x(t) = \sum_{k \in \mathcal{K}} \sum_{l=1}^{L} g_{l,k}(t) s_{l,k}$
- $s_{l.k}$ is the data symbol at the time frequency location $(l, k)$
- $g_{l,k}(t) = p(t - kT)e^{j2\pi lF(t-kT)}e^{j\theta_{l,k}}$ denotes the basis pulse which is a time and frequency shifted protptype filter $p(t)$
- $T$ is the symbol time and $F$ is the subcarrier spacing
- The receiving prototype filter is matched to the symmetric transmitting prototype filter, and hence both are the same and satisfy orthogonality in real domain

# MIMO FBMC[3]

- Consider $N_r \times N_t$ MIMO system
- The transmit signal at a single transmit antenna in vector form: $\boldsymbol{s} = \boldsymbol{G}\boldsymbol{x}$
- $\boldsymbol{G} \in \mathbb{C}^{LK \times LK}$, $\boldsymbol{s}, \boldsymbol{x} \in \mathbb{C}^{LK \times 1}$
- Received signal at a single receiver antenna: $\boldsymbol{r} = \tilde{\boldsymbol{h}}\boldsymbol{G}\boldsymbol{x} + \tilde{\boldsymbol{n}}$
- Received symbols at a single receiver antenna: $\boldsymbol{y} = \boldsymbol{G}^* \tilde{\boldsymbol{h}}\boldsymbol{G}\boldsymbol{x} + \boldsymbol{n} = \boldsymbol{h}\boldsymbol{x} + \boldsymbol{n}$
- As there are $N_r \times N_t$ possible channels,

# MIMO FBMC

- Considering a single active symbol, we then have

$$\bar{\boldsymbol{y}} = \boldsymbol{H}\bar{\boldsymbol{x}} + \boldsymbol{n}$$

- $\bar{\boldsymbol{y}} \in \mathbb{C}^{LN_r \times 1}$
- $\boldsymbol{H} \in \mathbb{C}^{LN_r \times LN_t}$
- $\bar{\boldsymbol{x}} \in \mathbb{C}^{LN_t \times 1}$

# Multiuser MIMO FBMC

- Considering there are $K$ users with $N_r$ antennas each and a base station downlink with $N_t$ antennas and repeating the above procedure for every user

$$\bar{\boldsymbol{y}} = \boldsymbol{H}\bar{\boldsymbol{x}} + \boldsymbol{n}$$

- $\bar{\boldsymbol{y}} \in \mathbb{C}^{KLN_r \times 1}$
- $\boldsymbol{H} \in \mathbb{C}^{KLN_r \times LN_t}$
- $\bar{\boldsymbol{x}} \in \mathbb{C}^{LN_t \times 1}$

# Zero Forcing (ZF) Precoder

- Consider a precoding matrix $\boldsymbol{P}$ such that $\bar{\boldsymbol{x}} = \boldsymbol{P}\hat{\boldsymbol{x}}$
- $\hat{\boldsymbol{x}} = [\boldsymbol{x}_1, \ldots, \boldsymbol{x}_K]^\intercal$, $\boldsymbol{x}_i \in \mathbb{C}^{LK \times 1}$ is the message symbol vector corresponding to user i
- $\bar{\boldsymbol{y}} = \boldsymbol{H}\boldsymbol{P}\hat{\boldsymbol{x}} + \boldsymbol{n}$
- If $\boldsymbol{H}\boldsymbol{P} = \boldsymbol{D}$ where $\boldsymbol{D}$ is a diagonal matrix, then there is no inter-user interference.
- In that case $\boldsymbol{P} = \boldsymbol{H}^\dagger\boldsymbol{D}$ which exists only when $\boldsymbol{H} \in \mathbb{C}^{KLN_r \times LN_t}$ has a full row rank - Suboptimal solution since each user is able to coordinate the processing of its own receiver outputs[4]

# Inter-User Interference[4]

Consider a precoder $\boldsymbol{P}_i$ for each user i such that

$$\bar{\boldsymbol{x}} = \sum_{i=1}^{K} \boldsymbol{P}_i \hat{\boldsymbol{x}}_i$$

then for user j,

$$\begin{aligned} \bar{\boldsymbol{y}}_j &= \boldsymbol{H}_j \left( \sum_{i=1}^{K} \boldsymbol{P}_i \hat{\boldsymbol{x}}_i \right) + \boldsymbol{n}_j \\ &= \boldsymbol{H}_j \boldsymbol{P}_j \hat{\boldsymbol{x}}_j + \underbrace{\boldsymbol{H}_j \tilde{\boldsymbol{P}}_j \tilde{\boldsymbol{x}}_j}_{\text{IUI}} + \boldsymbol{n}_j \end{aligned}$$

where

$$\tilde{\boldsymbol{P}}_j = [\boldsymbol{P}_1 \ldots \boldsymbol{P}_{j-1}\ \boldsymbol{P}_{j+1} \ldots \boldsymbol{P}_K]$$

$$\tilde{\boldsymbol{x}}_j{}^\intercal = [\hat{\boldsymbol{x}}_1{}^\intercal \ldots \boldsymbol{x}_{\widehat{j-1}}{}^\intercal\ \boldsymbol{x}_{\widehat{j-1}}{}^\intercal \ldots \hat{\boldsymbol{x}}_K{}^\intercal]$$

# Block Diagonalization (BD) Algorithm[4]

- Denote

$$\boldsymbol{H}_S = [\boldsymbol{H}_1^\intercal \ \boldsymbol{H}_2^\intercal \dots \boldsymbol{H}_K^\intercal]^\intercal$$

$$\boldsymbol{P}_S = [\boldsymbol{P}_1 \ \boldsymbol{P}_2 \dots \boldsymbol{P}_K]$$

- For zero IUI, we want $\boldsymbol{H}_S \boldsymbol{P}_S$ to be a block diagonal matrix
- This can be done either for **Throughput Maximization** or **Power Control**
- Null space, SVD, Shanon's channel capacity theorem and waterfilling algorithm
- This is a non-iterative algorithm so that the closed form solution can be directly implemented
- *Implementation and simulation results for MIMO FBMC will be provided in the Activity 2 report later, if interested, the general algorithm can be studied in [4]*

# Successive Optimization (SO) Algorithm[4]

- Another non-iterative algorithm for Power Control to get closed-form solution
- Remove IUI for one user at a time, with the condition that the precoder of the current user $j$ is optimized such that it does not interfere with the previous $j-1$ users' data
- Uses only the statistics of the interfering signals from previous steps and hence it is valid as long as this transmission procedure is a Stationary Random Process
- Here too the concepts of Null space, SVD, Shanon's channel capacity theorem and waterfilling algorithm are used
- *Implementation and simulation results for MIMO FBMC will be provided in the Activity 2 report later, if interested, the general algorithm can be studied in [4]*

# References


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