Lexical Analysis

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Plan

- What is lexical analysis?
- Formal languages
- Regular languages
- Deterministic automata
- Nondeterministic automata
- Lexer-generation: `flex`

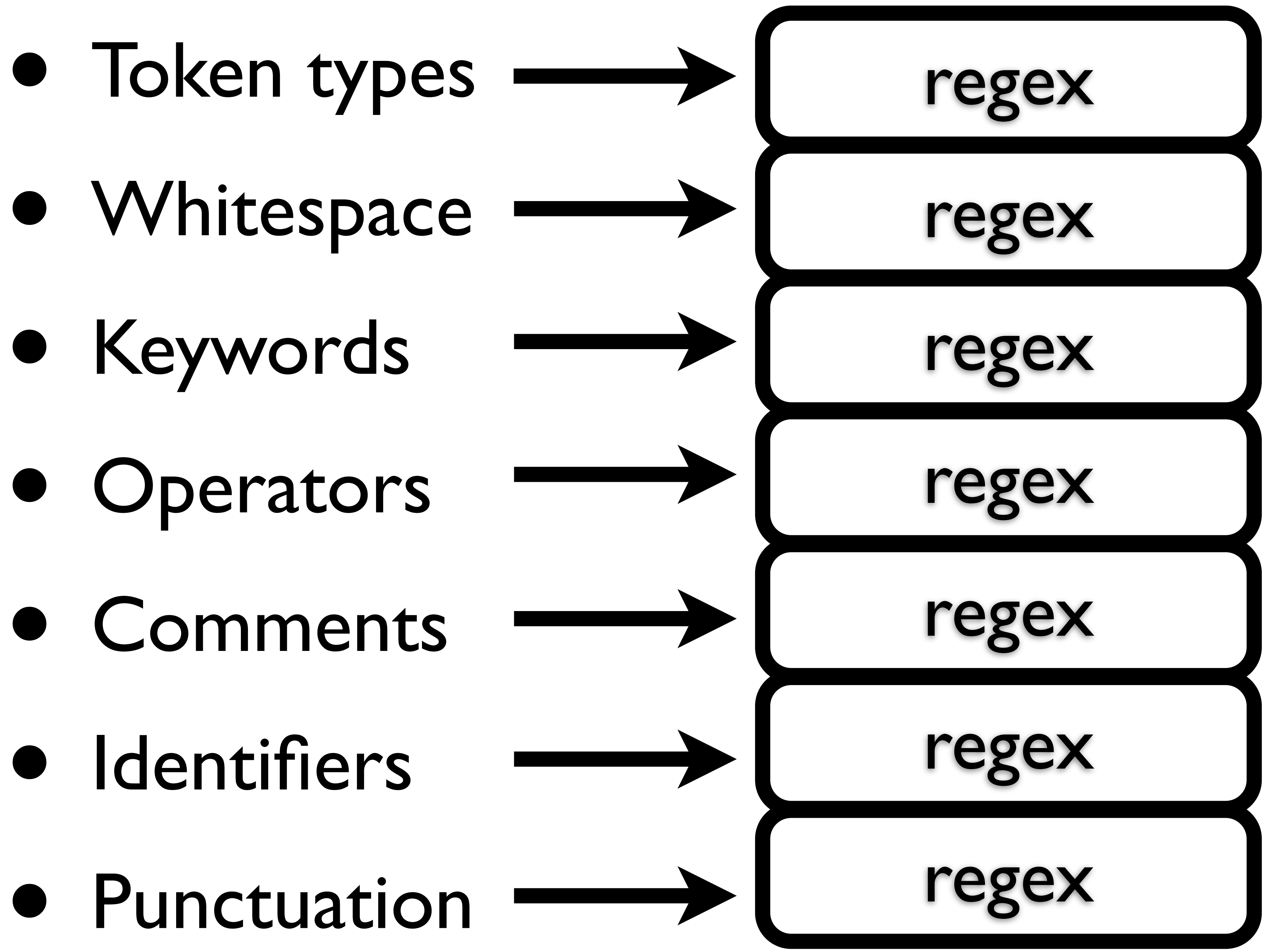
```
function id(x)
{
  return x ;  // comment
}
```

```
FUNCTION

IDENT(id)

LPAR

IDENT(x)

RPAR

LBRACE

RETURN

IDENT(x)

SEMI

RBRACE
```

Lexical specification

- Token types
- Whitespace
- Keywords
- Operators
- Comments
- Identifiers
- Punctuation

- Token types
- Whitespace
- Keywords
- Operators
- Comments
- Identifiers
- Punctuation


Token types
Whitespace
Keywords
Operators
Comments
Identifiers
Punctuation
regex
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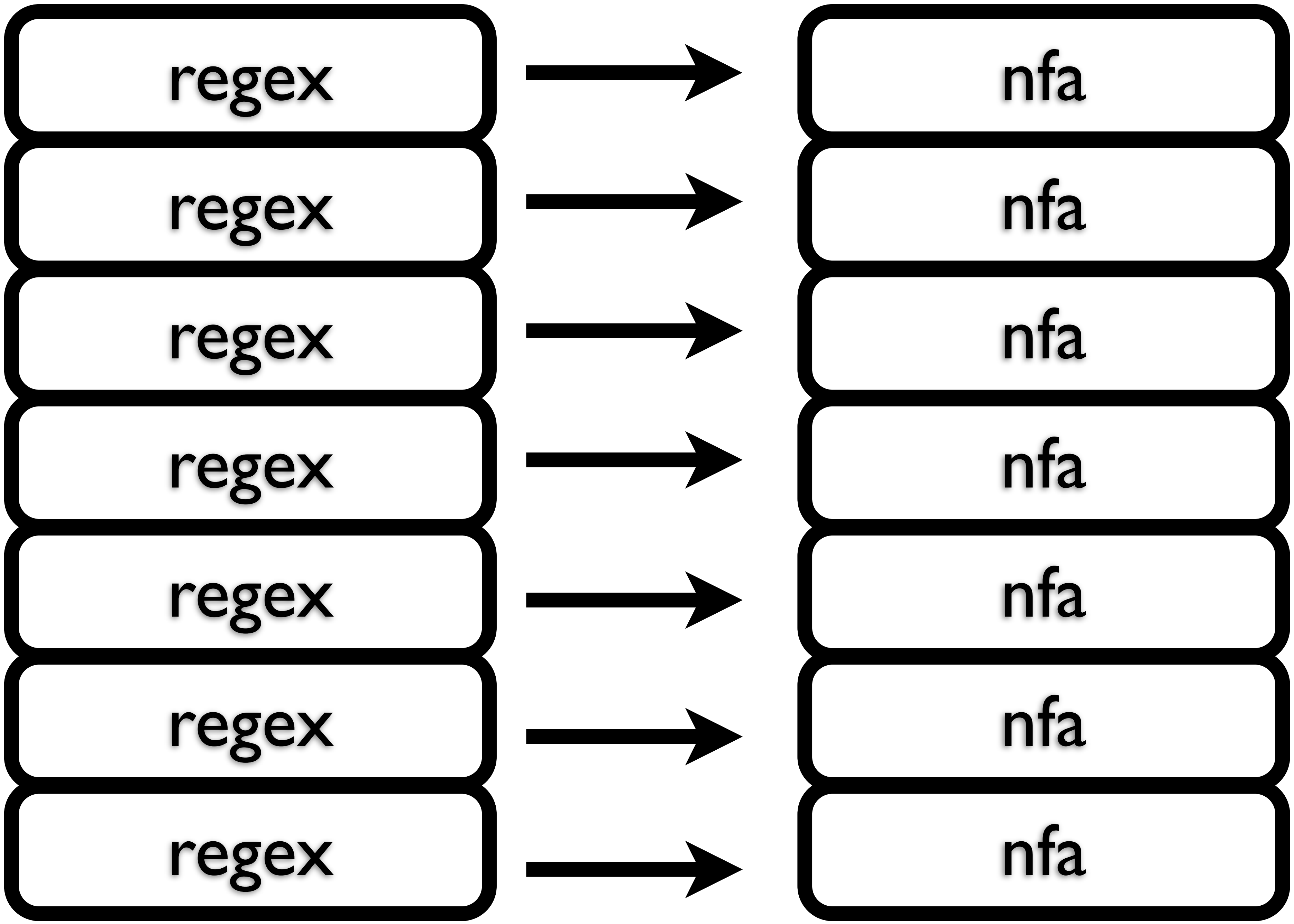

regex
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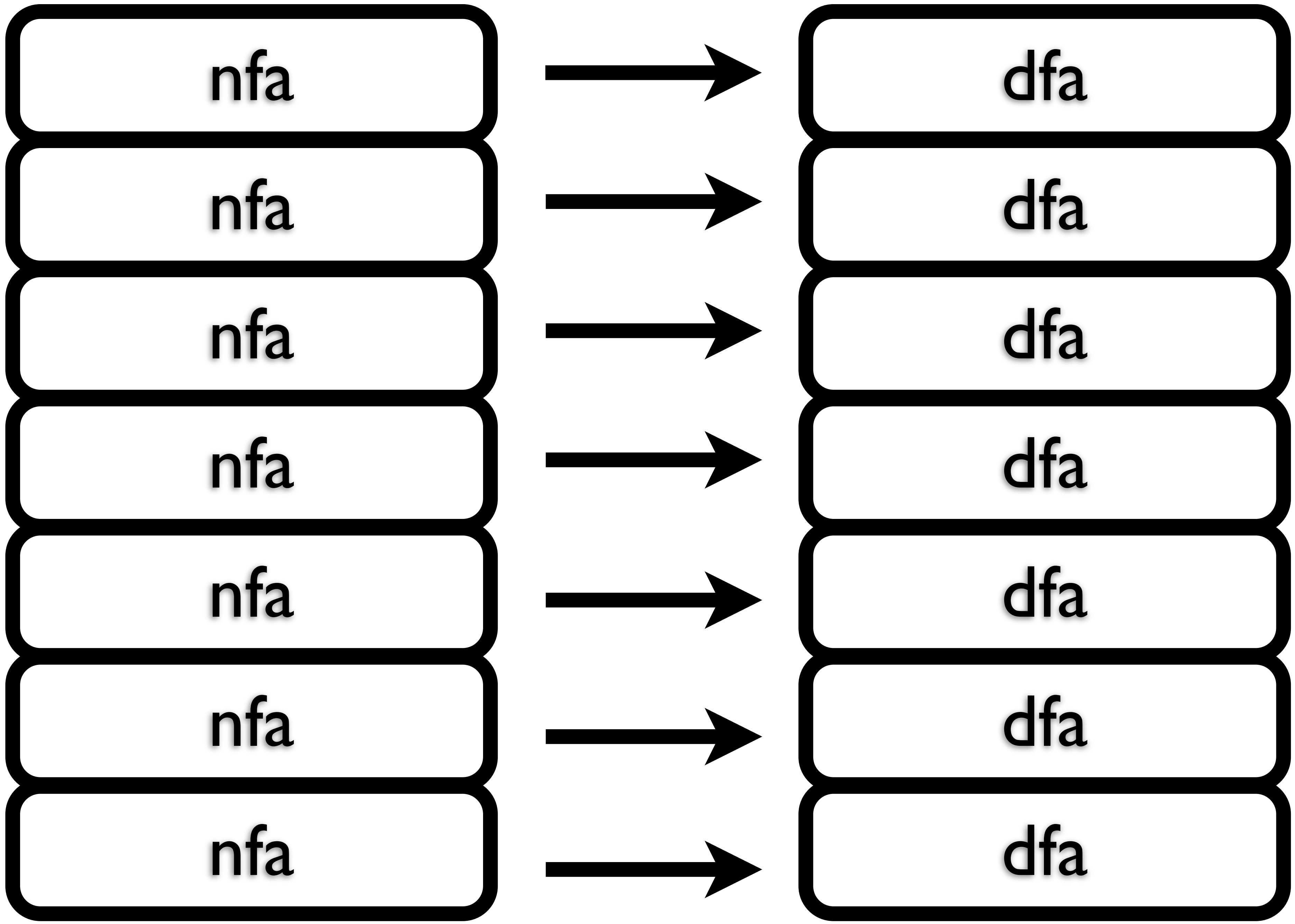

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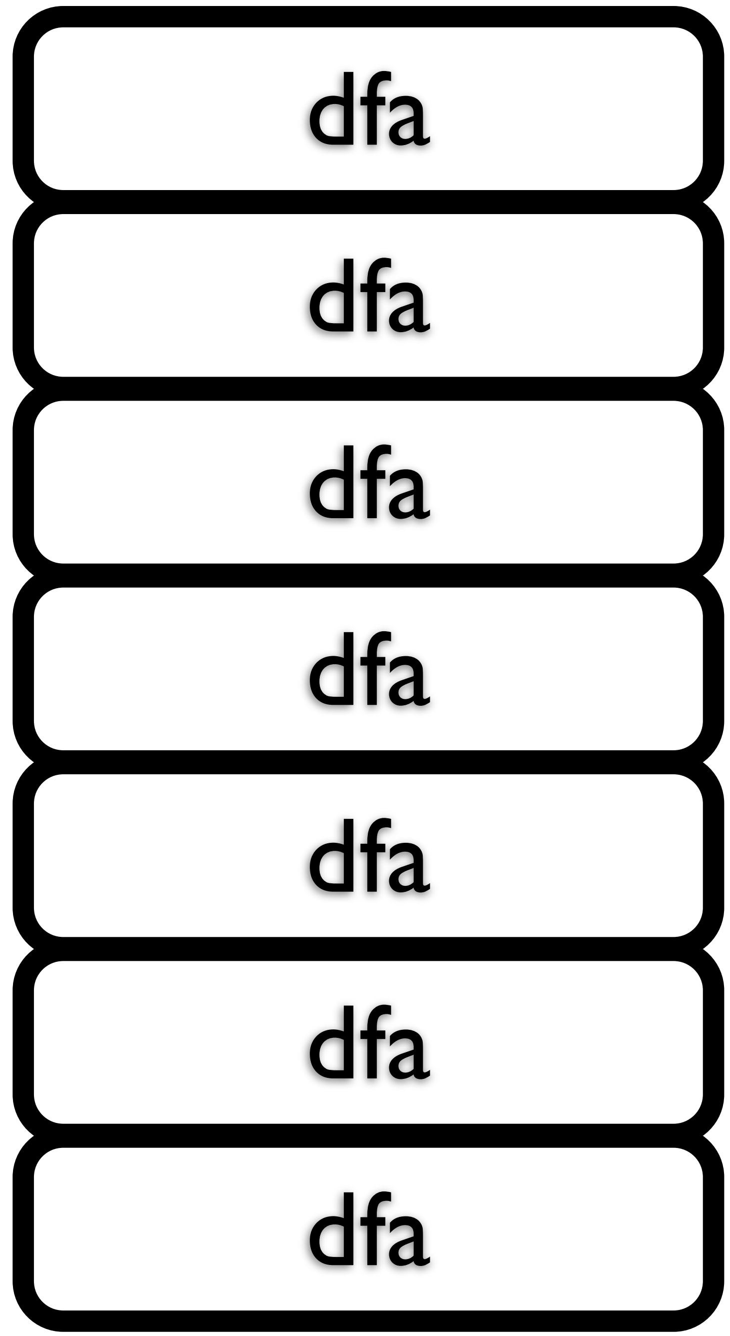
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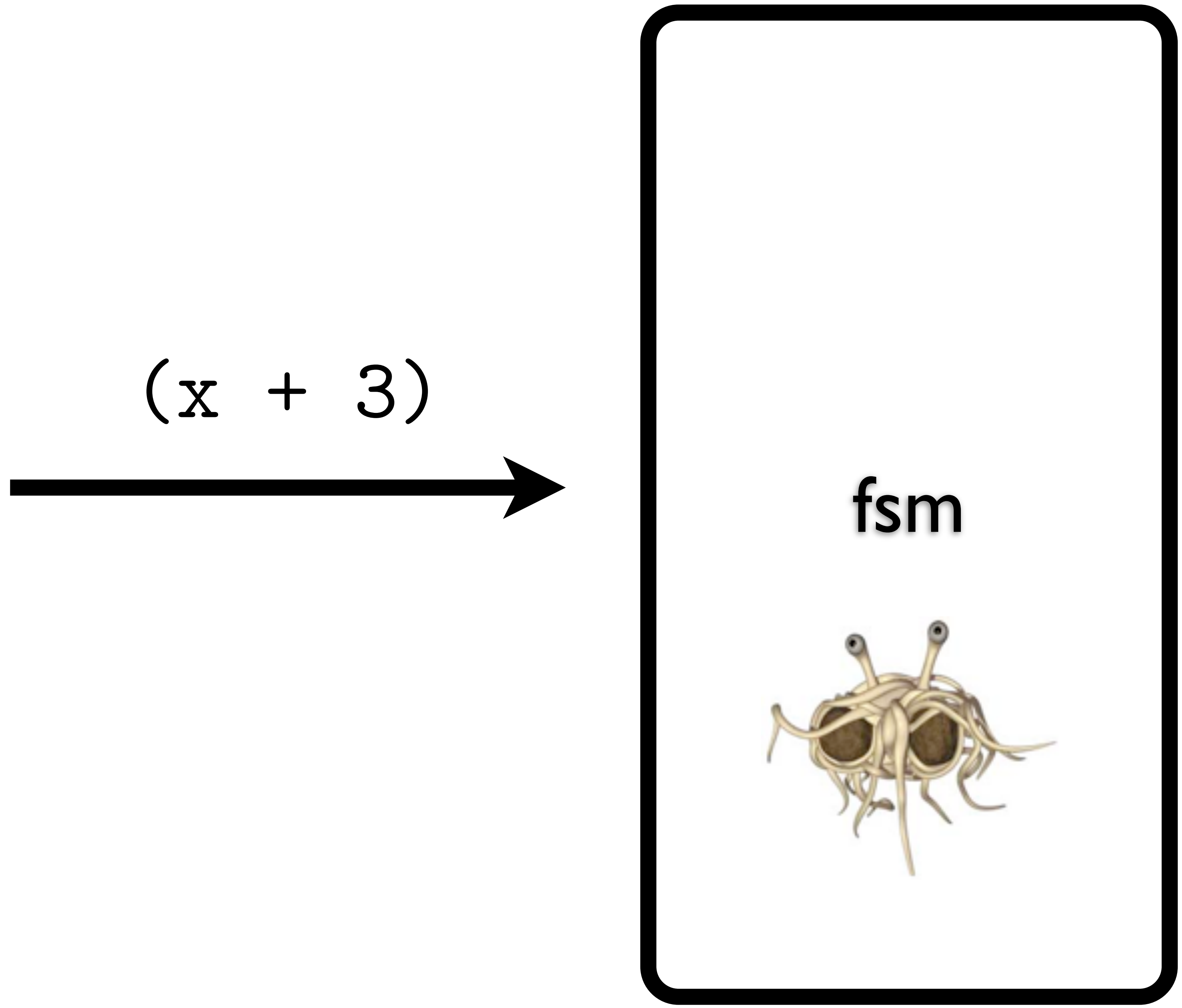
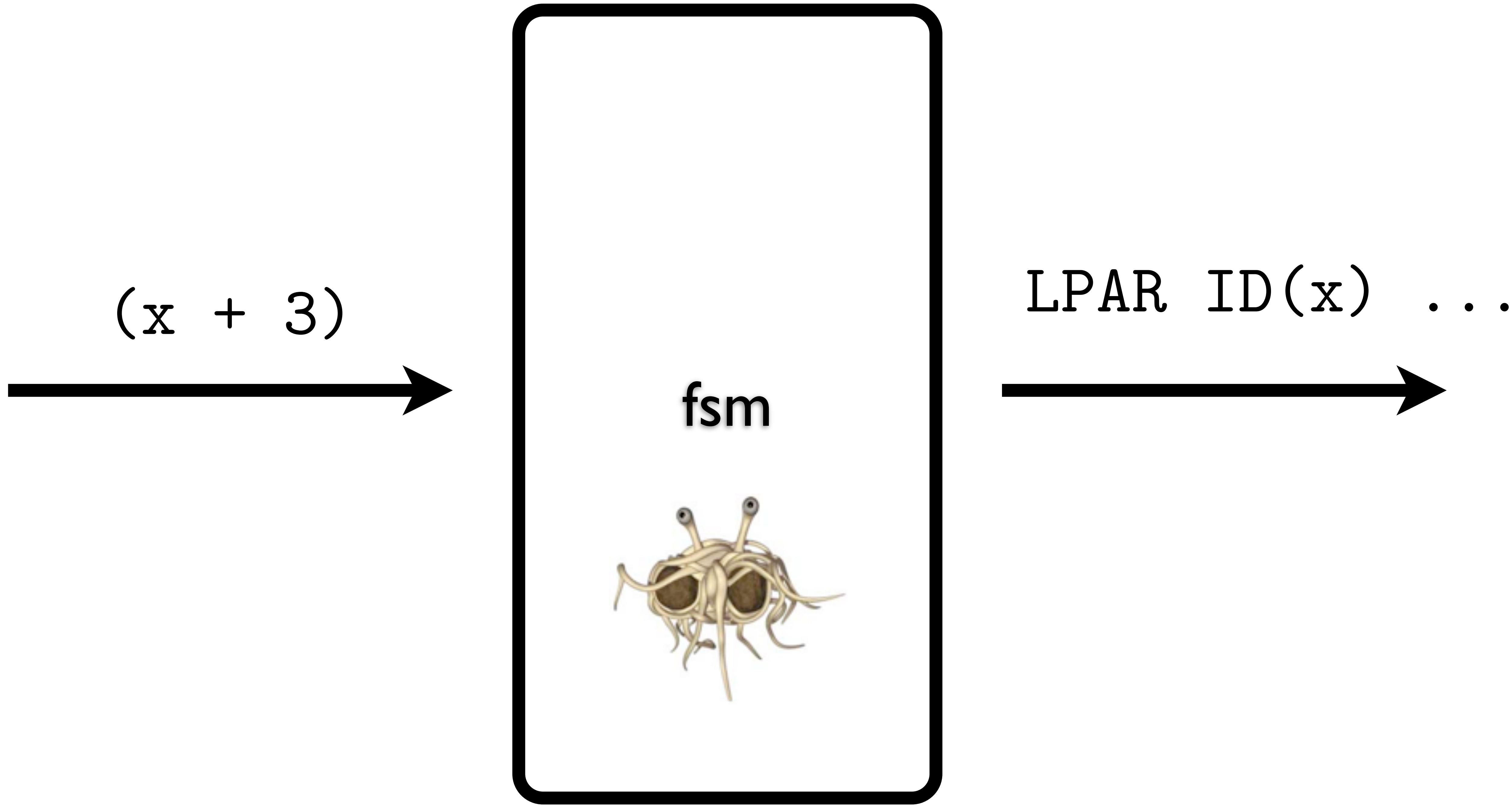
fsm


(x + 3)
fsm


(x + 3)
fsm
LPAR ID(x) ...

Formal languages

A **formal language** is a set of strings over an alphabet.

A **string** is a sequence
of characters from an alphabet.

Notation

A, Σ – Alphabets

L – Language

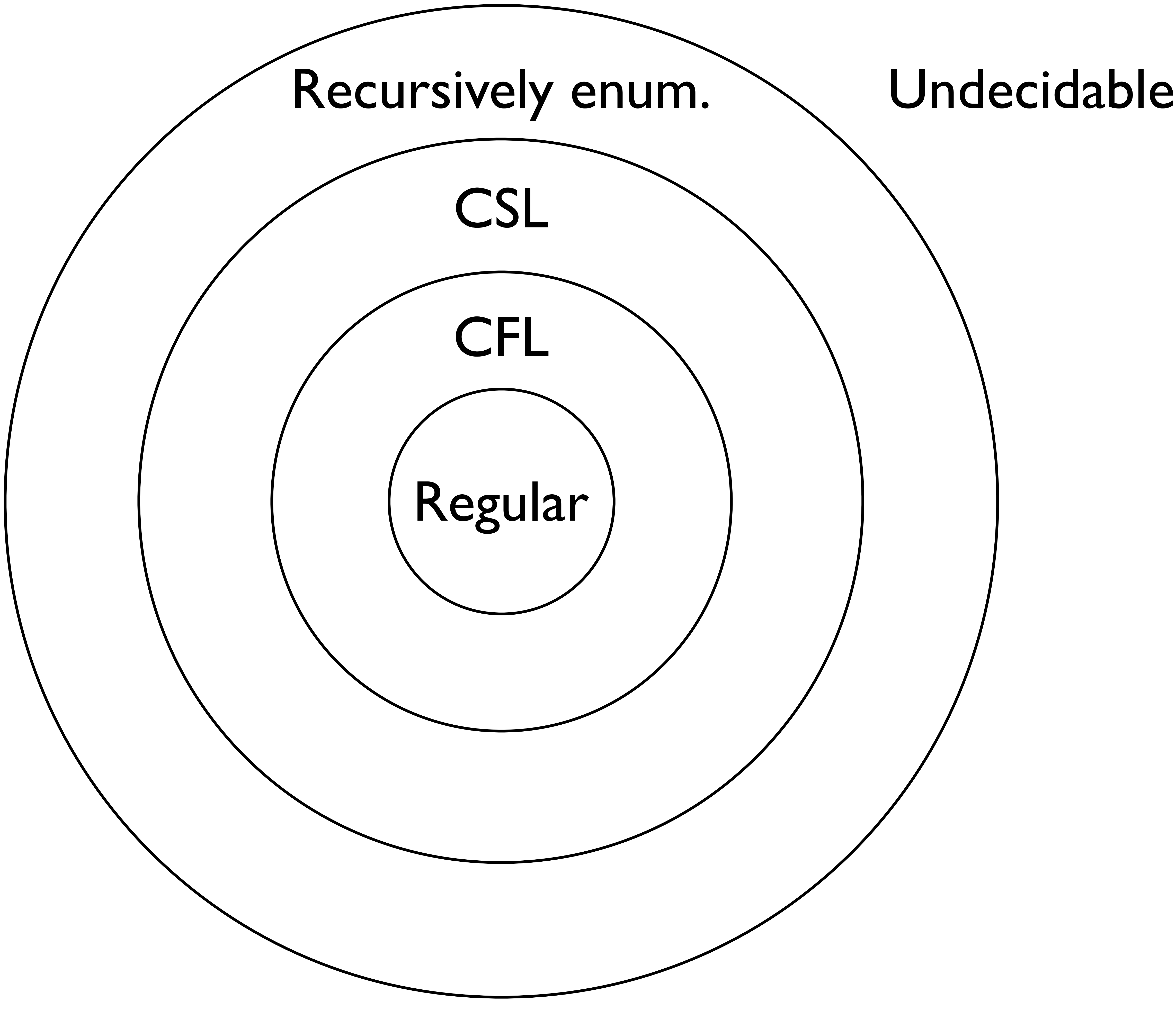
$L \subseteq A^*$ – Language over alphabet A

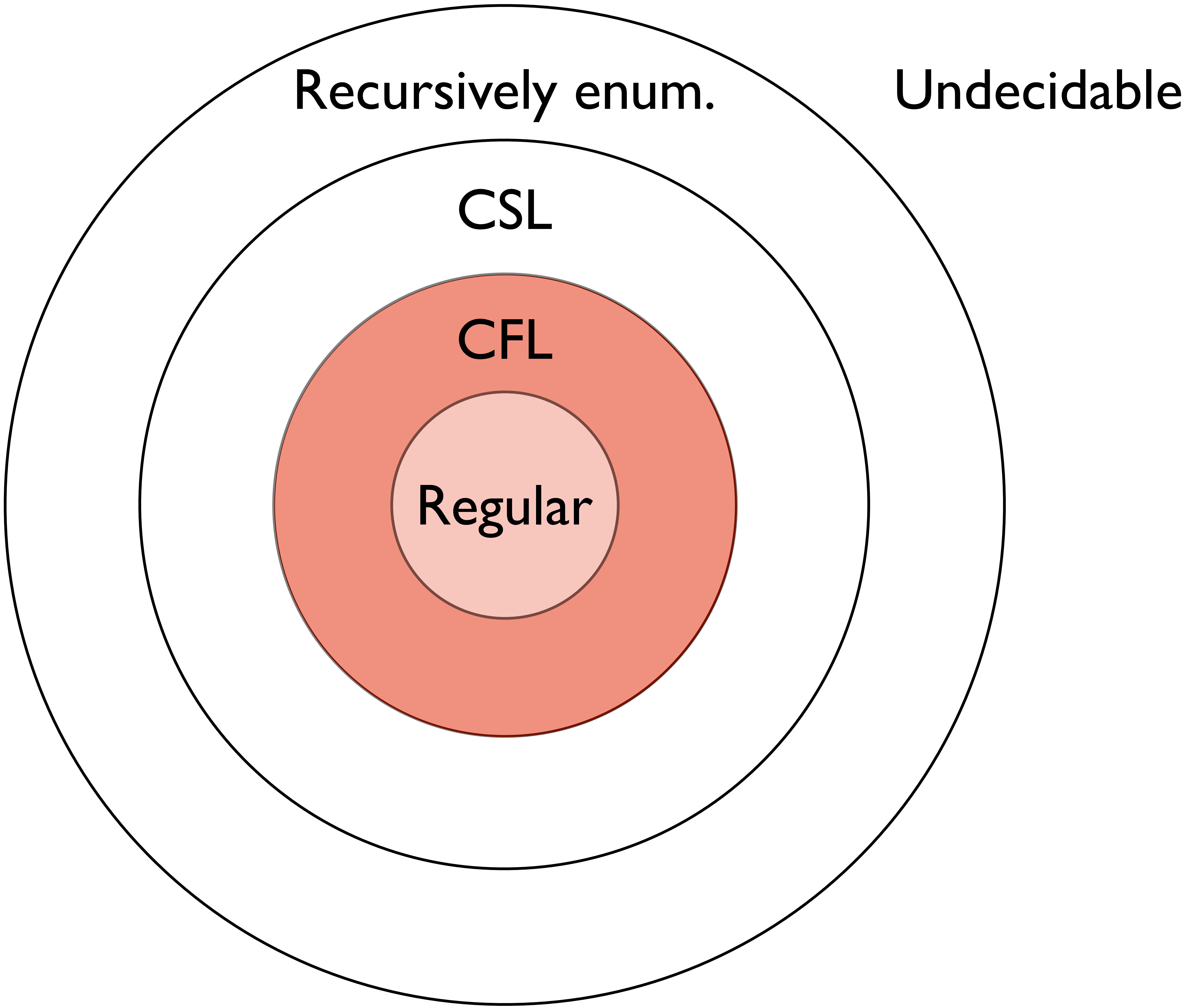
Examples

$$A = \{\texttt{a}, \texttt{b}\}$$

$$L_1 = \{\texttt{bab}, \texttt{abba}\}$$

$$L_2 = \{\texttt{a}, \texttt{aa}, \texttt{aaa}, \ldots\}$$


Recursively enum.
Undecidable
CSL
CFL
Regular


Recursively enum.
Undecidable
CSL
CFL
Regular

Atomic languages

$\emptyset$

ϵ

" "

$$\epsilon = \{\text{""}\}$$

$$\epsilon = ""$$

Primitive languages

c

$$c = \{\text{"}c\text{"}\}$$

Operations

Concatenation

$$L_1 \cdot L_2 = \{w_1 w_2 : w_1 \in L_1 \text{ and } w_2 \in L_2\}$$

Exercise

$$L_1 = \{\texttt{a}, \texttt{b}\}$$

$$L_2 = \{\texttt{c}, \texttt{d}\}$$

Union

$$L_1 \cup L_2 = \{w : w \in L_1 \text{ or } w \in L_2\}$$

Exercise

$$L_1 = \{\mathtt{a}, \mathtt{b}\}$$

$$L_2 = \{\mathtt{c}, \mathtt{d}\}$$

Exponentiation

$$L^n = \{w_1 \dots w_n : w_i \in L\}$$

$$L^0 = \epsilon$$

Exercise

$$\{\mathtt{a}, \mathtt{b}\}^3$$

Kleene star

$$L^\star = \bigcup_{n=0}^{\infty} L^n$$

Exercise

$$\{\mathtt{a}, \mathtt{b}\}^*$$

Concatenation,

union,

Kleene star.

Regular Languages

More regular operations!

Option

$$L^? \equiv L \cup \{\epsilon\}$$

Kleene plus

$$L^{+} = \bigcup_{n=1}^{\infty} L^{n}$$

Intersection

$$L_1 \cap L_2 = \{w : w \in L_1 \text{ and } w \in L_2\}$$

Difference

$$L_1 - L_2 = \{w : w \in L_1 \text{ and } w \notin L_2\}$$

Complement

$$\overline{L} = \{w : w \notin L\}$$

Reversal

$$L^R = \{\langle a_n, \ldots, a_1 \rangle : \langle a_1, \ldots, a_n \rangle \in L\}$$

Prefix

$$L^{\leq} = \{w_1 : \text{there exists } w_2 \text{ such that } w_1 w_2 \in L\}$$

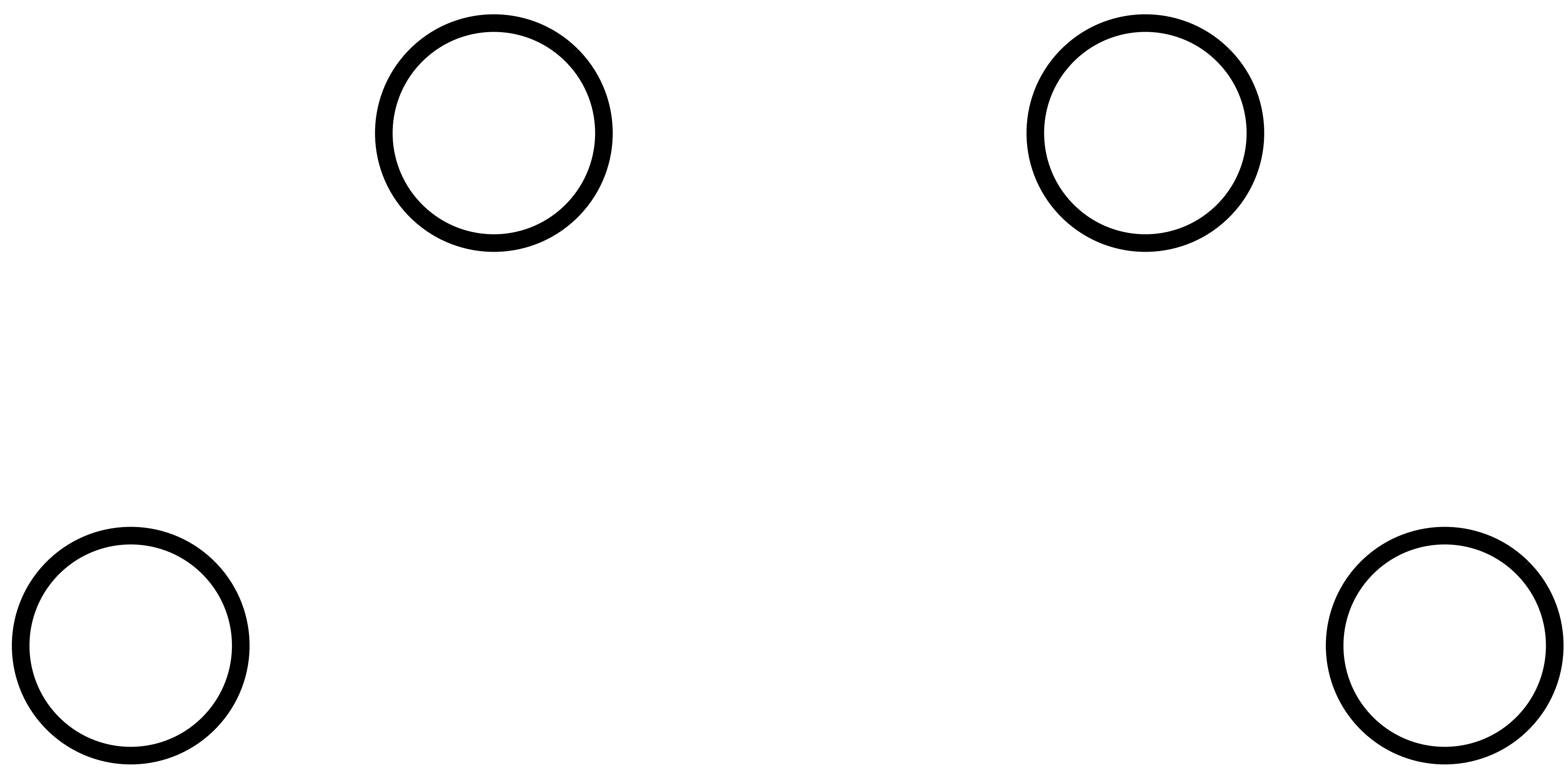
$$w \in L?$$

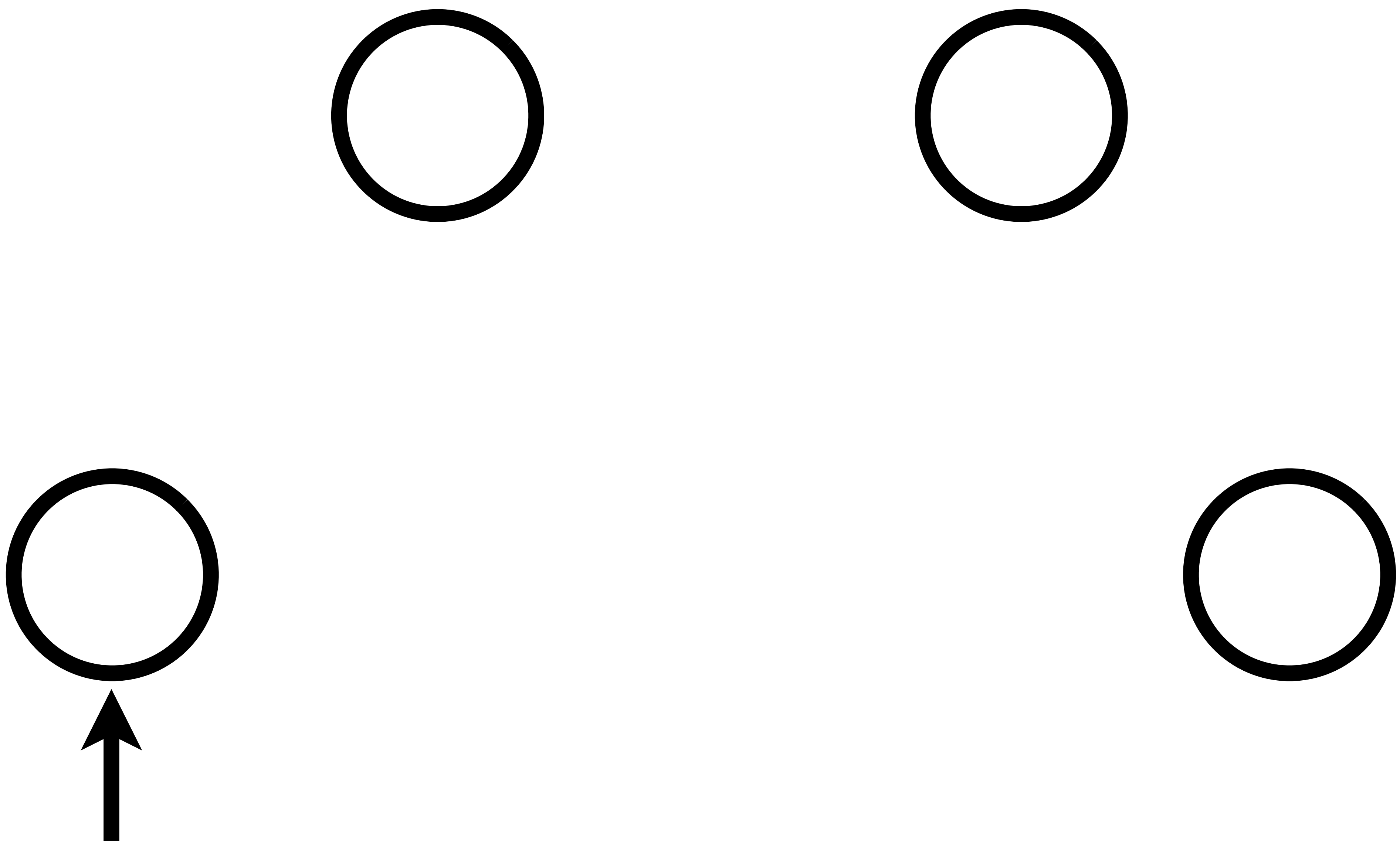
RegEx => NFA => DFA

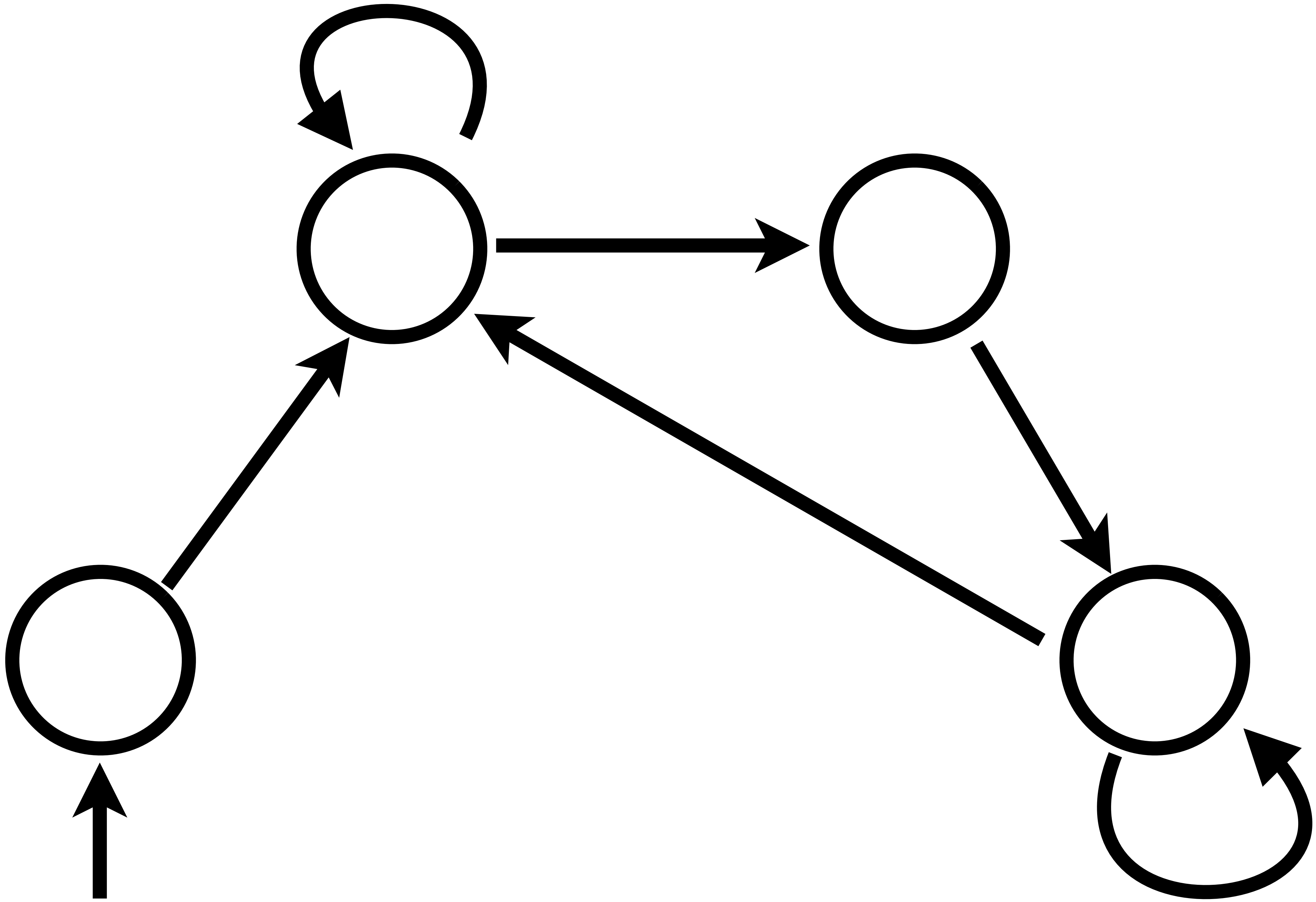
An **automaton** is a state machine that recognizes a formal language.

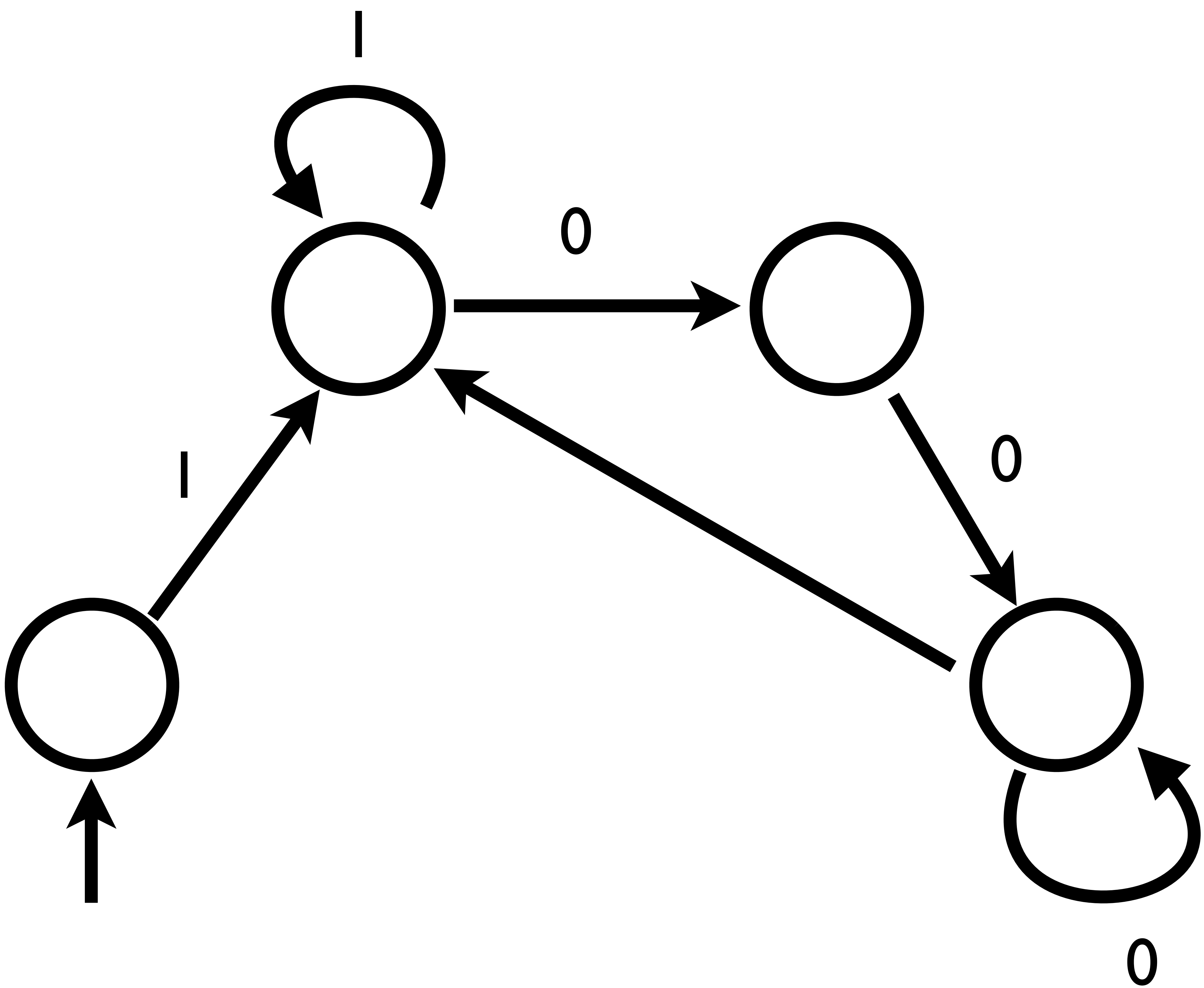
Formally, a **deterministic automaton** M is a 5-tuple (A, Q, q_0, δ, F) where:

- The set A is an alphabet.
- The set Q is a set of control states.
- The control state q_0 is the initial control state.
- The function $\delta : Q \times A \to Q$ determines the next state.
- The set F determines the final (accepting) states.

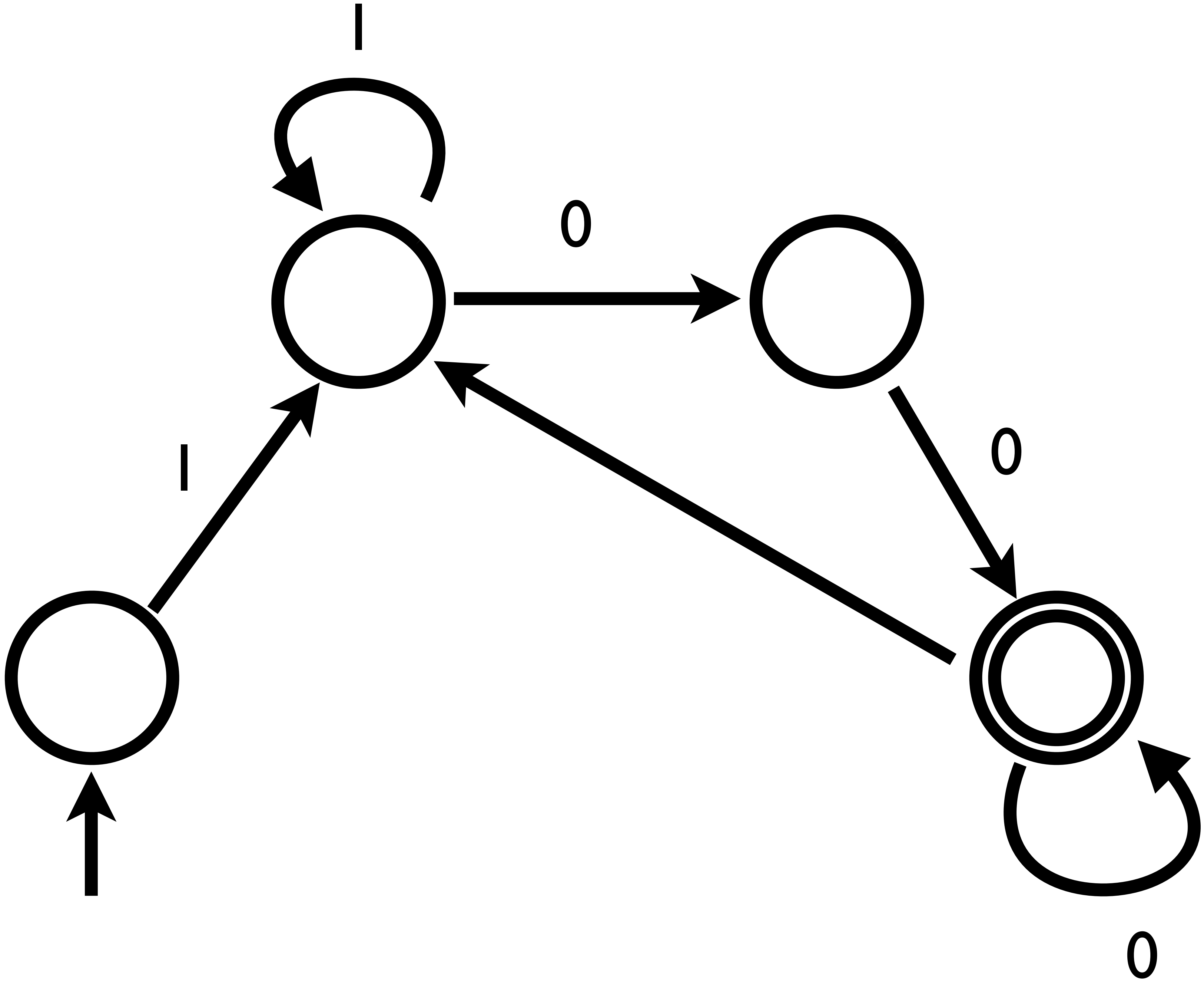








1
0
1
0
0


1
0
1
0
0

$$\mathcal{L}(A, Q, q_0, \delta, F) \subseteq A^*$$

If P_1 and $\ldots$ and P_n, then Q.

$$P_1 \quad \cdots \quad P_n$$

$$Q$$

$$\frac{P_1 \quad \cdots \quad P_n}{Q}$$

If P_1 and $\ldots$ and P_n, then Q.

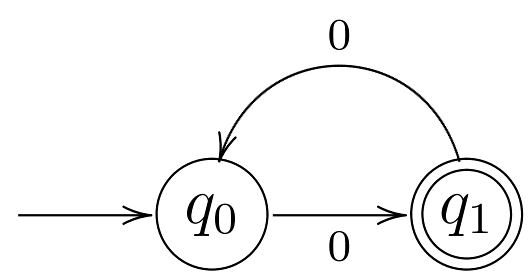
$$\frac{q_0 \in F}{\epsilon \in \mathcal{L}(A, Q, q_0, \delta, F)}$$

$$\frac{w \in \mathcal{L}(A, Q, \delta(q_0, c), \delta, F)}{cw \in \mathcal{L}(A, Q, q_0, \delta, F)}$$

Example 4.2. The automaton (A, Q, q_0, δ, F), where:

$$
\begin{aligned}
A &= \{0\} & \qquad \delta(q_0, 0) &= q_1 \\
Q &= \{q_0, q_1\} & \qquad \delta(q_1, 0) &= q_0 \\
F &= \{q_1\}\,, & &
\end{aligned}
$$

is depicted graphically as:



and it accepts exactly the set of strings with odd length—$\{0, 000, 00000, \ldots\}$.

Formally, a **nondeterministic automaton** M is also a 5-tuple (A, Q, q_0, δ, F) where:

- The set A is an alphabet.
- The set Q is a set of control states.
- The control state q_0 is the initial control state.
- The function $\delta : Q \times (A \cup \{\epsilon\}) \to \mathcal{P}(Q)$ determines the next state.
- The set F determines the final (accepting) states.

$$\delta : Q \times A \rightarrow Q$$

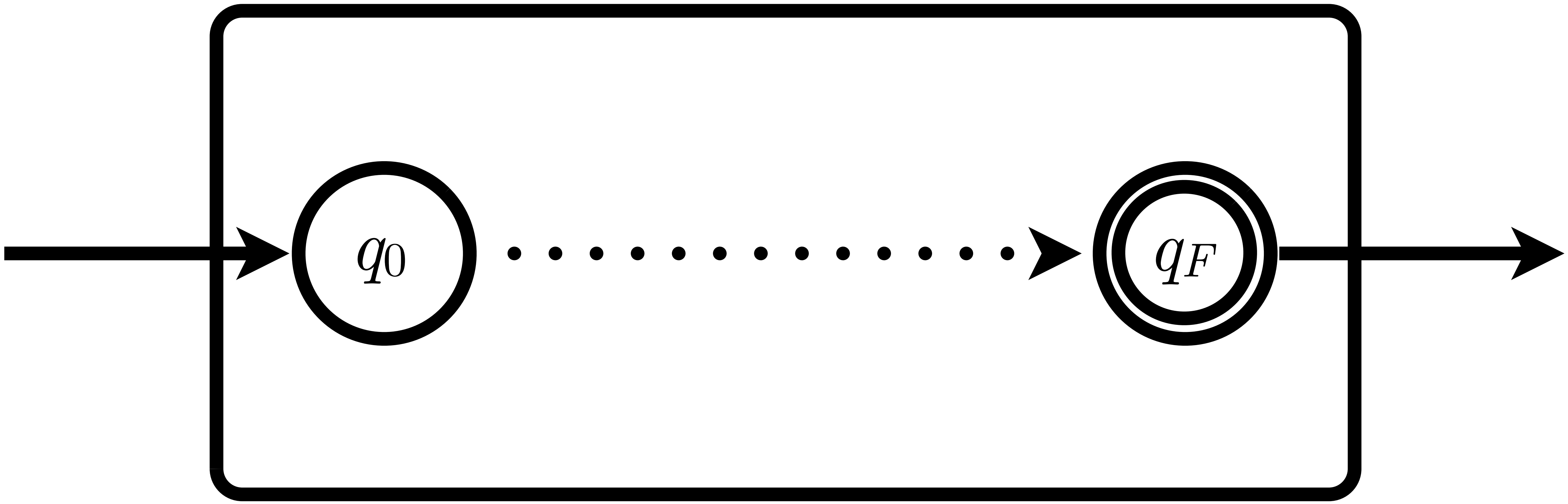
$$\delta : Q \times \left(A \cup \{\epsilon\}\right) \rightarrow \mathcal{P}(Q)$$

$$\frac{q_0 \in F}{\epsilon \in \boldsymbol{\mathcal{L}}(A, Q, q_0, \delta, F)}$$

$$\frac{w \in \mathcal{L}(A, Q, q', \delta, F) \qquad q' \in \delta(q_0, c)}{cw \in \mathcal{L}(A, Q, q_0, \delta, F)}$$

$$\frac{w \in \mathcal{L}(A, Q, q', \delta, F) \qquad q' \in \delta(q_0, \epsilon)}{w \in \mathcal{L}(A, Q, q_0, \delta, F)}$$

RegEx => NFA


q_0
q_F

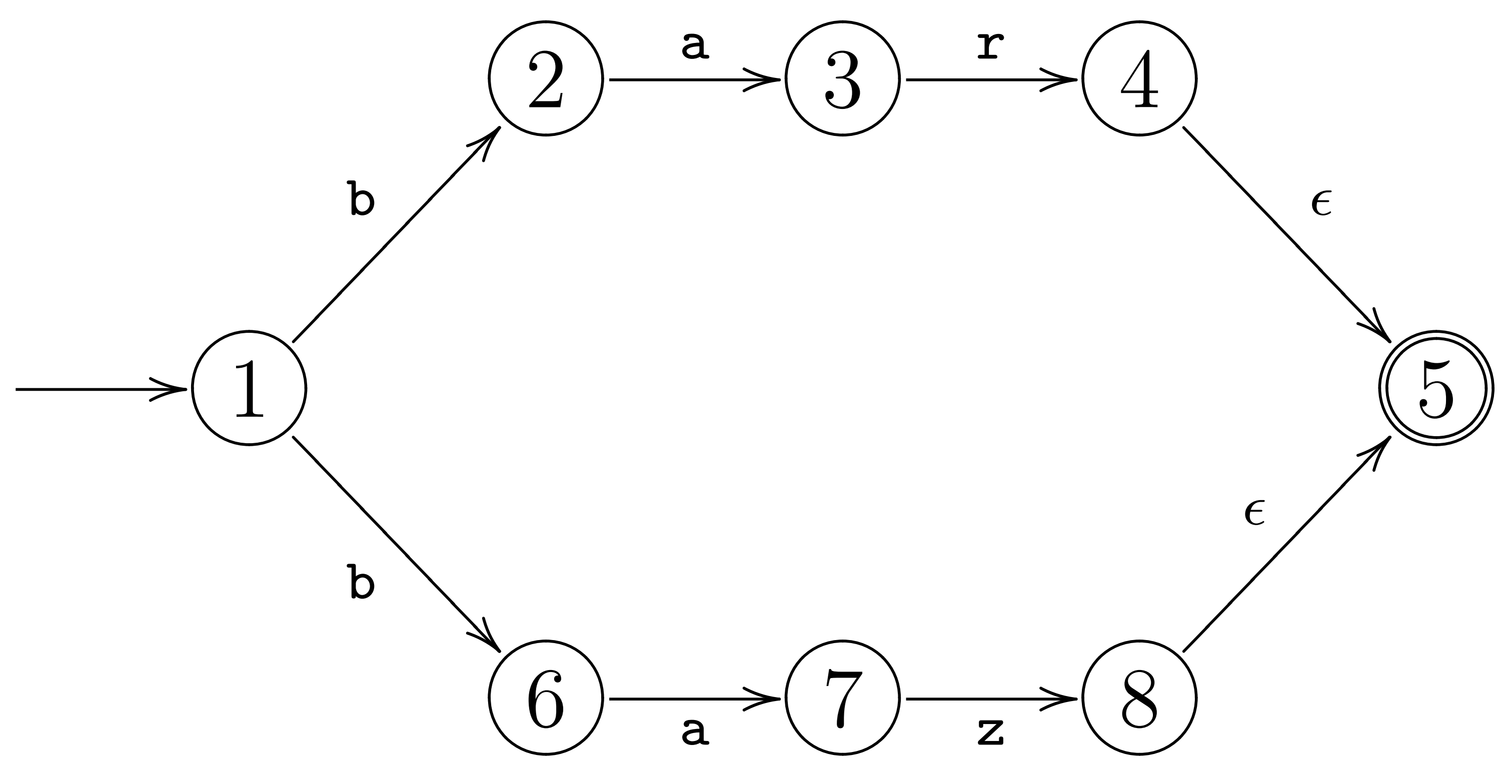
RegEx => NFA

- Empty language
- Empty-string singleton
- One-character singleton
- Concatenation
- Union
- Repetition

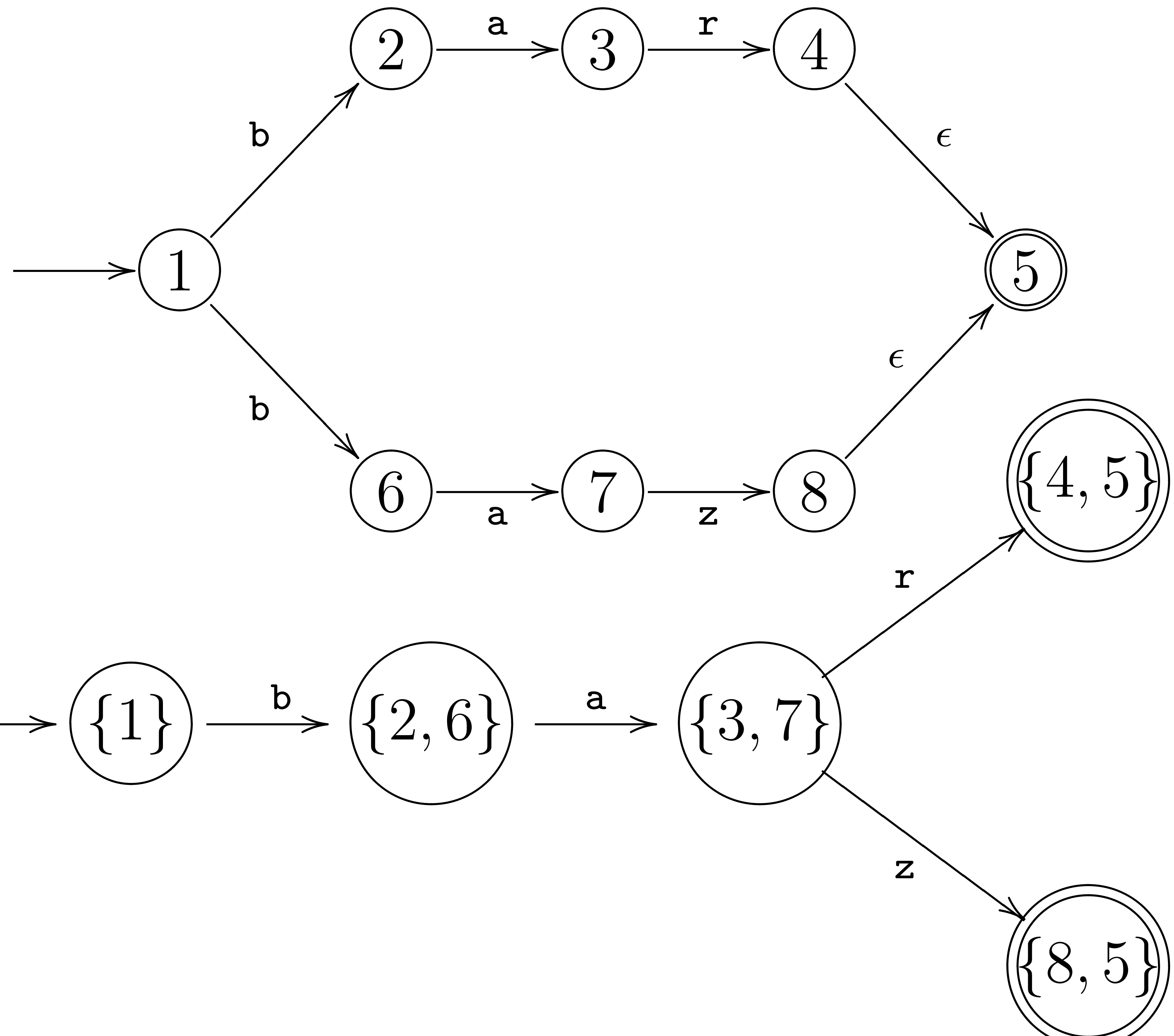
NFA => DFA

Thompson subset construction.

Example: NFA => DFA



Example: NFA => DFA



RegEx => NFA => DFA

lex

flex

lexical spec => lexer

- Identifiers match `[A-Za-z_][A-Za-z0-9_]*`
- Delimiters match `[();]`
- Operators match `[-*+/^=]`
- Integers match `-?([1-9][0-9]*|0)`
- Whitespace is ignored
- Comments match ^`#[^\n]*`

How lex works

foo.l

```
%%
[0-9]+   return 1;
[A-Z]+   return 2;
[ \t\n]  {}
%%
```

foo.l

```
%%
[0-9]+    return 1;
[A-Z]+    return 2;
[ \t\n]   {}
%%
```

lex.yy.c

```
int yylex() {
  ...
}
```

```
defs, opts, decs
%%
rules
%%
C code
```

Definitions

`name regex`

`{name}`

```
digit [0-9]
int   {digit}+

alpha  [A-Za-z_]
alnum  {alpha}|{digit}
id     {alpha}{alnum}*
```

Options

```
%option case-insensitive
```

```
%option yylineno
```

```
%option noyywrap
```

Declarations

```
%{
 C code
%}
```

`%s` $state_1$ $state_2$ ...

`%x` $state_1$ $state_2$ ...

Rules

pattern { *action* }

<state> pattern { action }

If in state $state$,
and $pattern$ matches the longest prefix,
then perform $action$.

```
%s state
%%
pat action
<state> pat action
```

```
%x state
%%
pat action
<state> pat action
```

Actions

{}

```
return token_type ;
```

BEGIN(*state*);

ECHO;

REJECT;

Example

- <MAIN> ‘‘(‘‘ { return(LPAR) ; }
- <MAIN> ‘‘)’’ { return(RPAR) ; }
- <MAIN> ‘‘/*’’ { BEGIN(COMMENT) ; }
- <COMMENT> ‘‘*/’’ { BEGIN(MAIN) ; }
- <COMMENT> . { }

Matching length

- Suppose the pattern: `f(oo)*`
- How many ways can match:

  `foooooooooooooooooooo`

Options

- Shortest match
- Longest match

Longest match

Given a string *w* and a set of regular expressions *R*, which regex can match the longest prefix of *w*?

Algorithm

```
NaïveRemoveLongestMatch(w ∈ A*,R ⊆ 2^A*)
  suffix ← w
  while (w ≠ ε)
   if ∃L ∈ R : ε ∈ L
      suffix ← w
   c:w ← w
   R ← D_c.R
  return suffix
```

Algorithm

```
RemoveLongestMatch(w ∈ A*,R ⊆ 2^A*)
  suffix ← w
  while (R ≠ ∅ or w ≠ ε)
   if ∃L ∈ R : ε ∈ L
      suffix ← w
   c:w ← w
   R ← D_c.R - {∅}
  return suffix
```

Example

- RegEx: fo*
- RegEx: foobar
- RegEx: foob
- Input string: foobarbaz

Examples

```
%{
 int num_lines = 0, num_chars = 0;
%}

%%
\n      ++num_lines; ++num_chars;
.       ++num_chars;
%%
int main(int argc, char* argv[]) {
   yylex();
   printf("# of lines = %d, # of chars = %d\n",
          num_lines, num_chars );
}
```

Questions?