# Design notes

## Structure's geometry:

- Cylinder:
  - thickness $t_{cylinder} := 1.2\ m$
  - radius $R_{cylinder} := 22\ m$
  - height $H_{cylinder} := 41.6\ m$
- Dome:
  - thickness $t_{dome} := 1\ m$
  - radius $R_{dome} := 22\ m$

*Relative humidity:* $RH := 50\%$

## Materials properties:

*Concrete:*

- characteristic strength $f_{ck} := 37.5\ MPa$
- elastic modulus $E_c := 29000\ MPa$
- Poisson's ratio $\nu_c := 0.2$
- thermal expansion coefficient $\alpha_c := 10 \cdot 10^{-6}\ C^{-1}$
- density $\rho_c := 2500\ \frac{kg}{m^3}$

*Passive steel*

- characteristic yielding stress $f_{yk} := 390\ MPa$
- elastic modulus $E_s := 200\ GPa$
- ultimate strain $\varepsilon_{uk} := 5\%$
- the extrados reinforcement comprises 40cm²/m per direction.
- the intrados reinforcement comprises 25cm²/m per direction.
- stirrup section of 10cm²/m².
- covering is 50mm on the stirrup head.

*Metal liner* *P265GH black steel*

- minimal yielding stress $f_{l.min} := 255\ MPa$
- average yielding stress $f_{l.avg} := 320\ MPa$
- elastic modulus $E_l := 210\ GPa$
- Poisson's ratio $\nu_l := 0.28$
- thermal expansion coefficient $\alpha_l := 12 \cdot 10^{-6}\ C^{-1}$
- density $\rho_l := 7850\ \frac{kg}{m^3}$

$A_{passive.steel} := 65\ \frac{cm^2}{m}$

*thickness of the liner on the cylindrical part* $t_{liner.cylinder} := 7\ mm$

*thickness of the liner on the dome* $t_{liner.dome} := 9\ mm$

*Prestressed steel - cement grouted tendons*

T15 strands (15.7mm in diameter) class 1860 MPa VSR
Each tendon comprises 55 strands (55T15)

- nominal section of a trand $A_p := 150\ mm^2$ (55 strands - $A_{55T15} := 55 \cdot A_p = 8250\ mm^2$ )
- guaranteed ultimate strength $f_{pk} := 1860\ MPa$
- guaranteed strength per strand $A_p \cdot f_{pk} = 279\ kN$
- yield stress $f_{p.y} := 1653\ MPa$
- anchorage slip $8\ mm$
- elastic modulus $E_p := 190\ GPa$
- relaxation coefficient (20 $°C$ ) $\rho_{1000} := 2.5\%$

The tendons are tensioned from the 2 extremities at $\sigma_0 := 0.8\ f_{pk} = 1488\ MPa$

- ducts diameter $\phi_{duct} := 165\ mm$
- friction coefficient on curve
  - horizontal cables $\mu_h := 0.18 \cdot \dfrac{1}{rad}$
  - vertical cables and dome $\mu_v := 0.16 \cdot \dfrac{1}{rad}$
- loss coefficient due to local angular deviations in the duct
  - horizontal cables and dome $k_h := 0.009 \cdot \dfrac{1}{m}$
  - vertical cables $k_v := 0.005 \cdot \dfrac{1}{m}$

The internal containment is sized to withstand during 60 years:

LOCA accident: $P_d := 0.40\ MPa$ (relative)

Containment pressure test: $P_t := 1.15\ P_d = 0.46\ MPa$

*Average radius of tendons*

$R_{ext.tendon.v} := R_{dome} + t_{dome} - 1.5\ \phi_{duct} = 22.753\ m$

$R_{int.tendon.v} := R_{dome} + t_{dome} - 2.5\ \phi_{duct} = 22.588\ m$

$$R_{avg.v} := \frac{R_{ext.tendon.v} + R_{int.tendon.v}}{2} = 22670\ mm$$

$R_{ext.tendon.h} := R_{cylinder} + t_{cylinder} - 1.5\ \phi_{duct} = 22.953\ m$

$R_{int.tendon.h} := R_{cylinder} + t_{cylinder} - 3.5\ \phi_{duct} = 22.623\ m$

$$R_{avg.h} := \frac{R_{ext.tendon.h} + R_{int.tendon.h}}{2} = 22788\ mm$$

## Dome tendons

Necessary vertical prestress: $F_{V.min} := \dfrac{P_d \cdot R_{dome}}{2} + f_{l.avg} \cdot t_{liner.dome} + A_{passive.steel} \cdot f_{yk} = 9.815 \ \dfrac{MN}{m}$

1)Friction losses

$$\alpha := \frac{\pi}{2}$$

$$R_{avg.v} = 22670 \ mm$$

$$\sigma_{0.5\pi} := \sigma_0 \cdot e^{-\left(\mu_v \cdot \alpha + k_v \cdot \mu_v \cdot \left(\alpha \cdot R_{avg.v} + H_{cylinder}\right)\right)} = 1088 \ MPa$$

2)Elastic losses

The concrete stress due to prestress at the end of life is: $\sigma_f := \dfrac{F_{V.min}}{t_{dome}} = 9.815 \ MPa$

If we assume about 20 % of losses, the concrete stress due to prestress at the construction is about:

$$\sigma_i := 1.2 \ \sigma_f = 11.78 \ MPa$$

The elastic losses are: $L_e := \dfrac{1}{2} \cdot \dfrac{\sigma_i}{E_c} \cdot E_p = 39 \ MPa$

Stresses after construction: $\sigma_{0.5\pi.c} := \sigma_{0.5\pi} - L_e = 1049 \ MPa$ maximum tensile stress applied to the tendon minus the immediate losses occurred during the stressing process

3)Relaxation losses

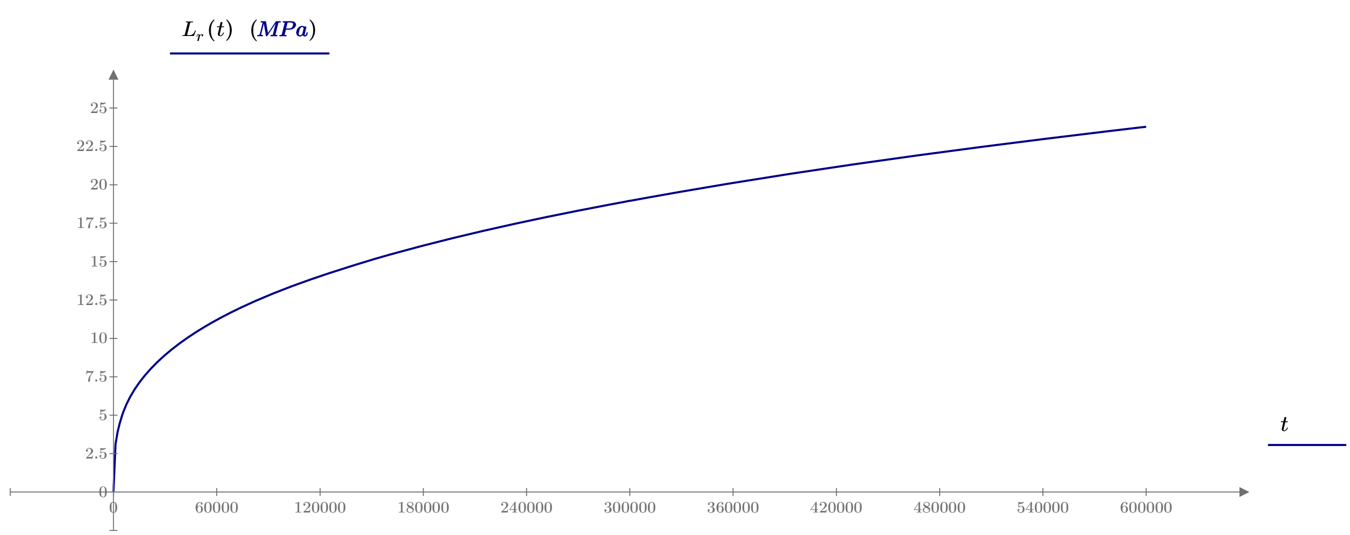
Class 2: wire or strand - low relaxation

The design calculations for the losses due to relaxation of the prestressing steel is based on the value of $\rho_{1000}$, the relaxation loss (in %) at 1000 hours after tensioning and at a mean temperature of 20 $°C$

$\mu := \dfrac{\sigma_{0.5\pi.c}}{f_{pk}} = 0.564$ $\quad t_e := 24 \cdot 365.5 \cdot 60 = 526320$ the number of hours in 30 years

$$L_r(t) := 0.66 \cdot \rho_{1000} \cdot 100 \cdot e^{9.1 \cdot \mu} \cdot \left(\frac{t}{1000}\right)^{0.75 \cdot (1-\mu)} \cdot 10^{-5} \cdot \sigma_{0.5\pi.c}$$

$$\Delta\sigma_\rho := L_r\left(t_e\right) = 22.8 \ MPa$$



4) Concete differed strain losses

a) Shrinkage

We consider desiccation shrinkage of concrete during 60 years and the autogenous shrinkage is neglected.

$K := 18$ $\quad (f_{ck} < 55\ MPa)$

$\beta_{cd} := 0.021$ $\quad$ (concrete without silica)

$h_0 := 2 \cdot t_{dome} = 2000\ mm$

$$\varepsilon_{cd}(t) := \frac{K \cdot \left(72 \cdot \exp\left(-0.046 \cdot \frac{f_{ck}}{MPa}\right) + 75 - RH \cdot 100\right) \cdot 365.25 \cdot t \cdot 10^{-6}}{365.25 \cdot t + \beta_{cd} \cdot \left(\frac{h_0}{mm}\right)^2}$$

$\varepsilon_{v.ds} := \varepsilon_{cd}(61) - \varepsilon_{cd}(1) = 139.8 \cdot 10^{-6}$

b) Creep

- Basic creep (anisotropic)

$\varphi_{b0} := 1.4$

$$\beta_{bc} := 0.4 \cdot \exp\left(3.1\ \frac{f_{ck}}{f_{ck}}\right) = 8.88$$

$$\varphi_b(t) := \varphi_{b0} \cdot \frac{\sqrt{t}}{\sqrt{t} + \beta_{bc}} \qquad \varphi_b(60) = 0.65$$

$$\varepsilon_{v.i} := \frac{\sigma_i}{0.8\ E_c}\ \left(1 - \nu_c\right) = 406.1 \cdot 10^{-6}$$

$\varepsilon_{v.bc} := \varphi_b(60) \cdot \varepsilon_{v.i} = 264.9 \cdot 10^{-6}$

- Desiccation creep (isotropic)

$\varphi_{d0} := 3200$

$\varphi_d\left(t_f, t_i\right) := \varphi_{d0} \cdot \left(\varepsilon_{cd}\left(t_f\right) - \varepsilon_{cd}\left(t_i\right)\right)$

$\varphi_d(60, 1) = 0.441$

$\sigma_m := 2 \cdot \sigma_i = 23.56\ MPa$

$$\varepsilon_{v.dc} := \frac{\varphi_d(60, 1) \cdot \sigma_m}{0.8\ E_c} = 448.2 \cdot 10^{-6}$$

## Recapitulation:

Shrinkage + basic creep + desiccation creep: $\varepsilon_{v.dl} := \varepsilon_{v.ds} + \varepsilon_{v.bc} + \varepsilon_{v.dc} = 852.897 \cdot 10^{-6}$

Concrete differed prestress losses: $\Delta\sigma_{v.dl} := \varepsilon_{v.dl} \cdot E_p = 162.05\ MPa$

Final tension: $\sigma_{v.f} := \sigma_{0.5\pi.c} - 0.8 \cdot \Delta\sigma_\rho - \Delta\sigma_{v.dl} = 869.1\ MPa$

Spacing $e_d := \dfrac{\sigma_{v.f} \cdot A_{55T15}}{F_{V.min}} = 731\ mm$

## Pressure test

Force in case of pressure test: $N_{pressure} := \dfrac{P_t \cdot R_{dome}}{2} = 5.06\ \dfrac{MN}{m}$

Effect of the liner and rebars thrust:

Shrinkage + basic creep + desiccation creep: $\varepsilon_{v.dl} = 852.897 \cdot 10^{-6}$

Liner thrust: $\sigma_{liner} := \varepsilon_{v.dl} \cdot E_l = 179\ MPa$

Rebar thrust: $\sigma_{rebar} := \varepsilon_{v.dl} \cdot E_s = 171\ MPa$

$A_{S.liner} := \dfrac{t_{liner.dome} \cdot 1\ m}{1\ m} = 90\ \dfrac{cm^2}{m}$ $\quad N_{liner} := A_{S.liner} \cdot \sigma_{liner} = 1.61\ \dfrac{MN}{m}$

$A_{S.rebar} := (40 + 25)\ \dfrac{cm^2}{m} = 65\ \dfrac{cm^2}{m}$ $\quad N_{rebar} := A_{S.rebar} \cdot \sigma_{rebar} = 1.11\ \dfrac{MN}{m}$

Total pressure: $N_{total.test} := N_{pressure} + N_{liner} + N_{rebar} = 7.78\ \dfrac{MN}{m}$

Instead of $F_{V.min} = 9.815\ \dfrac{MN}{m}$ in case of LOCA

Spacing

$e_d := \dfrac{\sigma_{v.f} \cdot A_{55T15}}{N_{total.test}} = 922\ mm$ larger than needed for LOCA case

Length of a chord of arc corresponding to $\dfrac{\pi}{2}$ is $D_c := 2 \cdot R_{dome} \cdot \sin\left(\dfrac{\pi}{4}\right) = 31.113\ m$

Thus the number of tendons needed is $\dfrac{D_c}{e_d} = 33.761$

Because of large tension stresses in concrete obtained with 34 cables on FEM model we will proceed the analysis by considering 57 tendons on each direction spaced at 540mm. Then it will result a covered length of $56 \cdot 540\ mm = 30.24\ m$

## Ultimate pressure assessment

Ultimate membrane strength of section (with liner). The ultimate force is obtained assuming the failure occurs when the most "brittle" element fails; i.e. tendons when they reach a 3% tensile strain.

$\sigma_{tendons} := 1888\ MPa$ in the same time with:

$\sigma_{liner} := 279\ MPa$

$\sigma_{rebar} := 419\ MPa \quad (f_{yk} = 390\ MPa)$

$e_d := 540\ mm$

The failure force is obtained by:

Tendons section $S_{tendons} := A_{55T15} \cdot \dfrac{1}{e_d} = 152.778\ \dfrac{cm^2}{m}$ $\quad n_{tendons} := S_{tendons} \cdot \sigma_{tendons} = 28.844\ \dfrac{MN}{m}$

Liner section $S_{liner} := t_{liner.dome} = 90\ \dfrac{cm^2}{m}$ $\quad n_{liner} := S_{liner} \cdot \sigma_{liner} = 2.511\ \dfrac{MN}{m}$

Rebar section intrados $25\ \dfrac{cm^2}{m}$ extrados $40\ \dfrac{cm^2}{m}$

$S_{rebar} := 25\ \dfrac{cm^2}{m} + 40\ \dfrac{cm^2}{m} = 65\ \dfrac{cm^2}{m}$ $\quad n_{rebar} := S_{rebar} \cdot \sigma_{rebar} = 2.724\ \dfrac{MN}{m}$

$$N_{rupture} := n_{tendons} + n_{liner} + n_{rebar} = 34.079\ \frac{MN}{m}$$

Hence we obtain the ultimate pressure $P_{ultimate.dome} := \dfrac{N_{rupture}}{R_{dome}} = 1.549\ MPa$

This value is considered as median value with 50% confidence. Thus is determined with the average values of the cross sections and of the mechanical characteristics.

## Cylinder tendons

Necessary horizontal prestress:

$$F_{H.min} := P_d \cdot R_{cylinder} + f_{l.avg} \cdot t_{liner.cylinder} + A_{passive.steel} \cdot f_{yk} = 13.575 \ \frac{MN}{m}$$

1)Friction losses

$$\alpha := \frac{\pi}{2}$$

$$R_{avg.h} = 22788 \ mm$$

$$\sigma_{0.5\pi} := \sigma_0 \cdot e^{-\left(\mu_h \cdot \alpha + k_h \cdot \mu_h \cdot \alpha \cdot R_{avg.h}\right)} = 1058 \ MPa$$

2)Elastic losses

The concrete stress due to prestress at the end of life is: $\sigma_f := \dfrac{F_{H.min}}{t_{cylinder}} = 11.313 \ MPa$

If we assume about 20 % of losses, the concrete stress due to prestress at the construction is about:

$$\sigma_i := 1.2 \ \sigma_f = 13.58 \ MPa$$

The elastic losses are: $L_e := \dfrac{1}{2} \cdot \dfrac{\sigma_i}{E_c} \cdot E_p = 44 \ MPa$

Stresses after construction: $\sigma_{0.5\pi.c} := \sigma_{0.5\pi} - L_e = 1014 \ MPa$

3)Relaxation losses

$$\mu := \frac{\sigma_{0.5\pi.c}}{f_{pk}} = 0.545 \qquad t_e := 24 \cdot 365.5 \cdot 60 = 526320$$

$$L_r(t) := 0.66 \cdot \rho_{1000} \cdot 100 \cdot e^{9.1 \cdot \mu} \cdot \left(\frac{t}{1000}\right)^{0.75 \cdot (1-\mu)} \cdot 10^{-5} \cdot \sigma_{0.5\pi.c}$$

$$\Delta\sigma_\rho := L_r\left(t_e\right) = 20.2 \ MPa$$

4) Concete differed strain losses

a) Shrinkage

We consider desiccation shrinkage of concrete during 60 years and the autogenous shrinkage is neglected.

$K := 18 \qquad (f_{ck} < 55 \ MPa)$

$\beta_{cd} := 0.021$ (concrete without silica)

$h_0 := 2 \cdot t_{cylinder} = 2400 \ mm$

$$\varepsilon_{cd}(t) := \frac{K \cdot \left(72 \cdot \exp\left(-0.046 \cdot \dfrac{f_{ck}}{MPa}\right) + 75 - RH \cdot 100\right) \cdot 365.25 \cdot t \cdot 10^{-6}}{365.25 \cdot t + \beta_{cd} \cdot \left(\dfrac{h_0}{mm}\right)^2}$$

$$\varepsilon_{h.ds} := \varepsilon_{cd}(61) - \varepsilon_{cd}(1) = 103.86 \cdot 10^{-6}$$

b) Creep

- Basic creep (anisotropic)

$\varphi_{b0} := 1.4$

$\beta_{bc} := 0.4 \cdot \exp\left(3.1 \frac{f_{ck}}{f_{ck}}\right) = 8.88$

$\varphi_b(t) := \varphi_{b0} \cdot \frac{\sqrt{t}}{\sqrt{t} + \beta_{bc}}$ $\qquad \varphi_b(60) = 0.65$

Vertical prestress and dead load:

- vertical prestress $\qquad F_{V.min} = 9.815 \frac{MN}{m}$
- dead load

volume of the cylinder $\qquad V_{cylinder} := \pi \cdot \left(\left(R_{cylinder} + t_{cylinder}\right)^2 - R_{cylinder}{}^2\right) \cdot H_{cylinder} = 7088.64\ m^3$

volume of the dome $\qquad V_{dome} := \frac{4}{6} \pi \cdot \left(\left(R_{dome} + t_{dome}\right)^3 - R_{dome}{}^3\right) = 3181.39\ m^3$

circular length of the cylinder $\qquad CL_{cylinder} := 2\ \pi \cdot \left(R_{cylinder} + \frac{t_{cylinder}}{2}\right) = 142\ m$

dead load $\qquad G_{DL} := \frac{\rho_c \cdot (1 + 20\%) \cdot \left(V_{cylinder} + V_{dome}\right)}{CL_{cylinder}} = 216972.39\ \frac{kg}{m}$

dead load force $\qquad F_{DL} := G_{DL} \cdot g = 2.128\ \frac{MN}{m}$ $\qquad$ where $g = 9.807\ \frac{m}{s^2}$

The force applied by the vertical prestress and dead load is $F_{VT} := \frac{F_{V.min}}{2} + F_{DL} = 7.035\ \frac{MN}{m}$ immediately after the prestessing. Thus the vertical stress $\sigma_v := \frac{F_{VT}}{t_{cylinder}} = 5.863\ MPa$

$\varepsilon_{h.i} := \frac{\sigma_i}{0.8\ E_c} - \nu_c \cdot \frac{\sigma_v}{0.8\ E_c} = 534.6 \cdot 10^{-6}$

$\varepsilon_{h.bc} := \varphi_b(60) \cdot \varepsilon_{h.i} = 348.7 \cdot 10^{-6}$

- Desiccation creep (isotropic)

$\varphi_{d0} := 3200$

$\varphi_d\left(t_f, t_i\right) := \varphi_{d0} \cdot \left(\varepsilon_{cd}\left(t_f\right) - \varepsilon_{cd}\left(t_i\right)\right)$

$\varphi_d(60, 1) = 0.328$

$\sigma_m := \sigma_i + \sigma_v = 19.44\ MPa$

$\varepsilon_{h.dc} := \frac{\varphi_d(60, 1) \cdot \sigma_m}{0.8\ E_c} = 274.5 \cdot 10^{-6}$

## Recapitulation:

Shrinkage + basic creep + desiccation creep: $\varepsilon_{h.dl} := \varepsilon_{h.ds} + \varepsilon_{h.bc} + \varepsilon_{h.dc} = 727.088 \cdot 10^{-6}$

Concrete differed prestress losses: $\Delta\sigma_{h.dl} := \varepsilon_{h.dl} \cdot E_p = 138.15\ MPa$

Final tension: $\sigma_{h.f} := \sigma_{0.5\pi.c} - 0.8 \cdot \Delta\sigma_{\rho} - \Delta\sigma_{h.dl} = 859.5\ MPa$

Spacing $e_d := \dfrac{\sigma_{h.f} \cdot A_{55T15}}{F_{V.min}} = 722\ mm$

## Pressure test

Force in case of pressure test: $N_{pressure} := P_t \cdot R_{cylinder} = 10.12\ \dfrac{MN}{m}$

Effect of the liner and rebars thrust:

Shrinkage + basic creep + desiccation creep: $\varepsilon_{h.dl} = 727.088 \cdot 10^{-6}$

Liner thrust: $\sigma_{liner} := \varepsilon_{h.dl} \cdot E_l = 153\ MPa$

Rebar thrust: $\sigma_{rebar} := \varepsilon_{h.dl} \cdot E_s = 145\ MPa$

$A_{S.liner} := \dfrac{t_{liner.cylinder} \cdot 1\ m}{1\ m} = 70\ \dfrac{cm^2}{m}$ $\qquad N_{liner} := A_{S.liner} \cdot \sigma_{liner} = 1.07\ \dfrac{MN}{m}$

$A_{S.rebar} := (40 + 25)\ \dfrac{cm^2}{m} = 65\ \dfrac{cm^2}{m}$ $\qquad N_{rebar} := A_{S.rebar} \cdot \sigma_{rebar} = 0.95\ \dfrac{MN}{m}$

Total pressure: $N_{total.test} := N_{pressure} + N_{liner} + N_{rebar} = 12.13\ \dfrac{MN}{m}$

Instead of $F_{H.min} = 13.575\ \dfrac{MN}{m}$ in case of LOCA

Spacing

$$e_d := \frac{\sigma_{h.f} \cdot A_{55T15}}{N_{total.test}} = 584\ mm$$

Thus the number of tendons needed is $\dfrac{H_{cylinder}}{e_d} = 71.184$ - thus 72 vertical tendons

We will consider the tendons spaced at 580 mm. Then it will result a covered length of $71 \cdot 580\ mm = 41.18\ m$

## Ultimate pressure assessment

Ultimate membrane strength of section (with liner). The ultimate force is obtained assuming the failure occurs when the most "brittle" element fails; i.e. tendons when they reach a 3% tensile strain.

$\sigma_{tendons} := 1888\ MPa$ in the same time with:

$\sigma_{liner} := 279\ MPa$

$\sigma_{rebar} := 419\ MPa \quad (f_{yk} = 390\ MPa)$

$e_d := 580\ mm$

The failure force is obtained by:

Tendons section $S_{tendons} := A_{55T15} \cdot \frac{1}{e_d} = 142.241\ \frac{cm^2}{m}$ $\qquad n_{tendons} := S_{tendons} \cdot \sigma_{tendons} = 26.855\ \frac{MN}{m}$

Liner section $S_{liner} := t_{liner.cylinder} = 70\ \frac{cm^2}{m}$ $\qquad n_{liner} := S_{liner} \cdot \sigma_{liner} = 1.953\ \frac{MN}{m}$

Rebar section intrados $25\ \frac{cm^2}{m}$ extrados $40\ \frac{cm^2}{m}$

$S_{rebar} := 25\ \frac{cm^2}{m} + 40\ \frac{cm^2}{m} = 65\ \frac{cm^2}{m}$ $\qquad n_{rebar} := S_{rebar} \cdot \sigma_{rebar} = 2.724\ \frac{MN}{m}$

$$N_{rupture} := n_{tendons} + n_{liner} + n_{rebar} = 31.532\ \frac{MN}{m}$$

Hence we obtain the ultimate pressure $P_{ultimate.cylinder} := \frac{N_{rupture}}{R_{cylinder}} = 1.433\ MPa$

This value is considered as median value with 50% confidence. Thus is determined with the average values of the cross sections and of the mechanical characteristics.

Recalling the ultimate pressure in for the dome section $P_{ultimate.dome} = 1.549\ MPa$ can be conclude that with the current configuration the cylinder will fail before the dome at the ultimate pressure of **14.43 bars** (50% confidence).