Image Credit: John Sarkissian

A new algorithm for constraining the gravitational wave background:

Applications to the IPTA data challenge and PPTA DR1e

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PPTA

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Outline

- Signal model for timing pulsars
- Algorithm for constraining strength of background
  - Key point 1: At 95% confidence simulated TOAs are statistically and visually different than observations
- Application to the IPTA data challenge
- Application to the PPTA extended data sets
- Implications of the limit
- **Bottom line:** Best to-date constraint on GWB from PPTA, because of high quality, high cadence data recorded with long observing span.

Credit: Mike Keith

A Signal Model for GW Detection

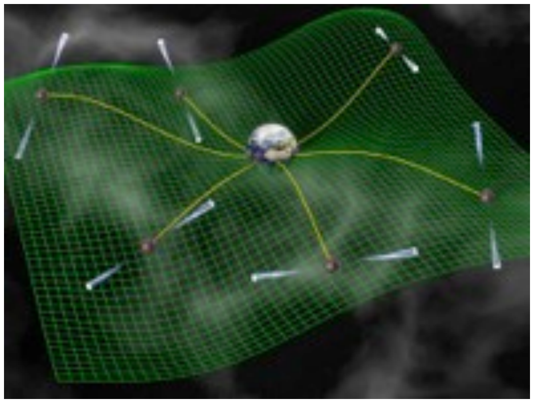
- **Partition uncorrected stochastic perturbations to TOAs by type of temporal variability.**
- **White noise:** uncorrelated between observations, e.g., radiometer noise, and pulse jitter (Oslowski et al. 2011, Shannon & Cordes 2012).
- **Red noise:** Correlated between observations, typically non-stationary.
  - Issue with detection: Red noise may have spectrally similar signature to GWB (Shannon & Cordes 2010)

Pulsar α

$$t_{\mathcal{M},\alpha}(t_i) = \underbrace{e_{\mathcal{M},\alpha}(t_i)}_{\text{Correlated GWs}} + \underbrace{p_{\mathcal{M},\alpha}(t_i)}_{\text{Uncorrelated GWs (Red Noise)}} + \underbrace{w_{\mathcal{M},\alpha}(t_i)}_{\text{White Noise}} + \underbrace{r_{\mathcal{M},\alpha}(t_i)}_{\text{Red Noise}}$$

After fit

Example: Detecting Gravitational Waves

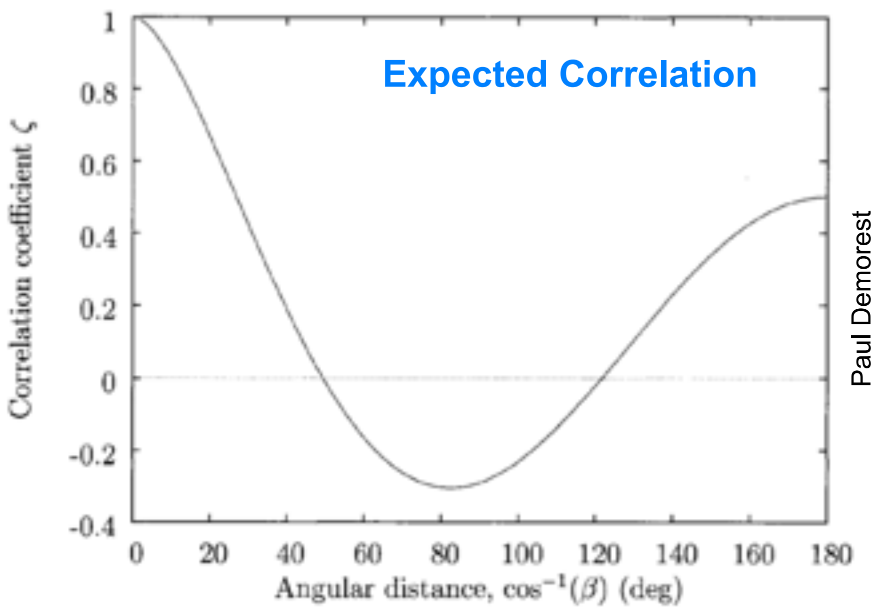


David Champion

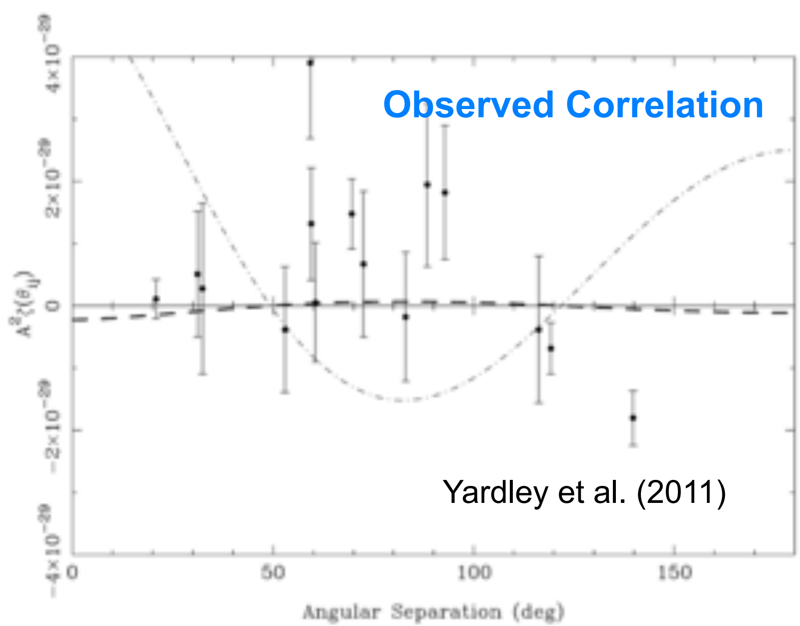
Need to identify signature of correlated GWs

Challenge: Other sources of noise can mimic gravitational wave signature (e.g., pulsar spin noise)

Solution: search for correlation in arrival times between multiple pulsars



Paul Demorest



Angle Between Pulsar Pairs

Differences Between Detecting and Constraining the GWB

Detecting GWB

- "Pulsar term" = nuisance
  - Angular signature is not manifested in pulsar end term
  - Consequence: S/N of correlation coefficient < 0.5
- Cross correlation
  - Sensitivity dominated by "baselines" between best pulsars

Constraining GWB

- "Pulsar term" = useable power
  - Pulsar term contains GWB that adds incoherently with Earth term
  - Power associated with GWB is large.
- Autocorrelation
  - Sensitivity dominated by best few pulsars

- **Key point: We have developed a technique to constrain the background using autocorrelation (power spectra)** that provides a superior bound to detection techniques that typically use cross correlations (cross spectra).

An Test Statistic for Constraining the GWB

Developed a **test statistic** (TS) based on power spectra analysis of residual time series

- Calculate power spectra density estimator for residual times of arrival (TOAs)
  - Use the spectral estimation methods developed by Coles, Hobbs, Manchester et al.
- Calculate a weighted sum of the spectral channels, weighting by the expected signal and a model for the noise.
- Combining pulsars: Weighted sum of each spectral bin in the PTA

Measured Power Spectrum

Inferred White Noise Level

TS for an individual pulsar:

$$S_p(f_i; A, \alpha) = \frac{\sum_i (P_{r,p}(f_i) - P_{w,p}) W_p(f_i; A, \alpha)}{\sum_i W_p(f_i; A, \alpha)}$$

Weighting: $$W_p(f_i; A, \alpha) = \frac{P_G(f_i; A, \alpha)}{[P_{m,p}(f_i)]^2}$$

Expected strength of GWB

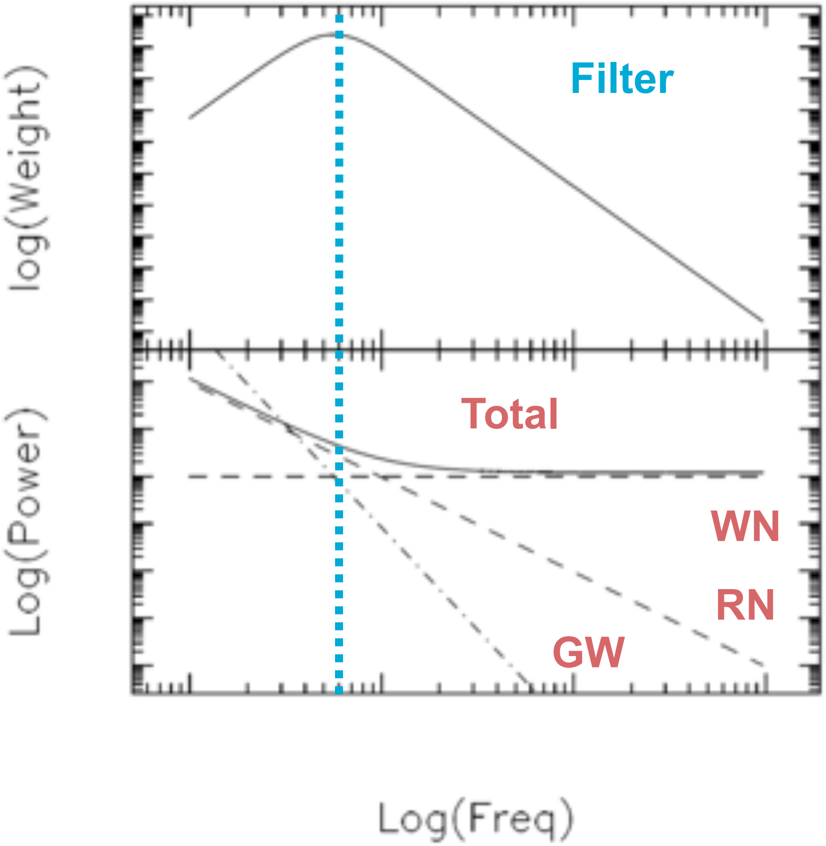
Model of observations

TS for a Pulsar Timing Array:

$$S_{\mathrm{PTA}}(A, \alpha) = \frac{\sum_p \sum_i (P_{r,p}(f_i) - P_{w,p}) W_p(f_i; A, \alpha)}{\sum_p \sum_i W_p(f_i; A, \alpha)}$$

Model for filter

- Filter shape depends on expected value of A.
- "Prewhitening" de-weights lowest frequency bins, because they have estimation error ~ SNR
- Peak of filter at S/N =1



$$W_p(f_i;\alpha) = \frac{P_G(f_i;A,\alpha)}{[P_{m,p}(f_i)]^2}$$

An Algorithm for Constraining the GWB

- In collaboration with Coles, Hobbs, Keith, Manchester and the PPTA collaboration.
  - `icLimit` plugin written within the tempo2 paradigm
- Calculate test statistic TS for real data (***observed TS***)
- Compare observed TS to ***TS derived from simulations (simulated TS)***
- Simulations:
  - Form ideal site TOAs
  - Add white noise consistent with observations
  - Add GW of strength (***A***) and spectral shape (***α***)~(e.g., ***α*** = -2/3)
  - On average, simulated TS increases as A increases
- Find value of **A** such that 95% of the time the simulated TS exceeds the observed TS: **This is the 95% confidence limit.**

Deriving a noise model

- Best limit will come with the best (signal + noise) model
- For PTAs, detection and bounding of the GWB occur is in the **strong signal** regime
  - Cannot assume that strength of GWB is much less than other noise contributions (e.g., red noise)
- When setting a limit with multiple pulsars, need to develop noise model that contains estimated strength of GWB
  - Jointly fit PSDE for all pulsars in order to estimate likely strength of GWB

Key Features of Algorithm

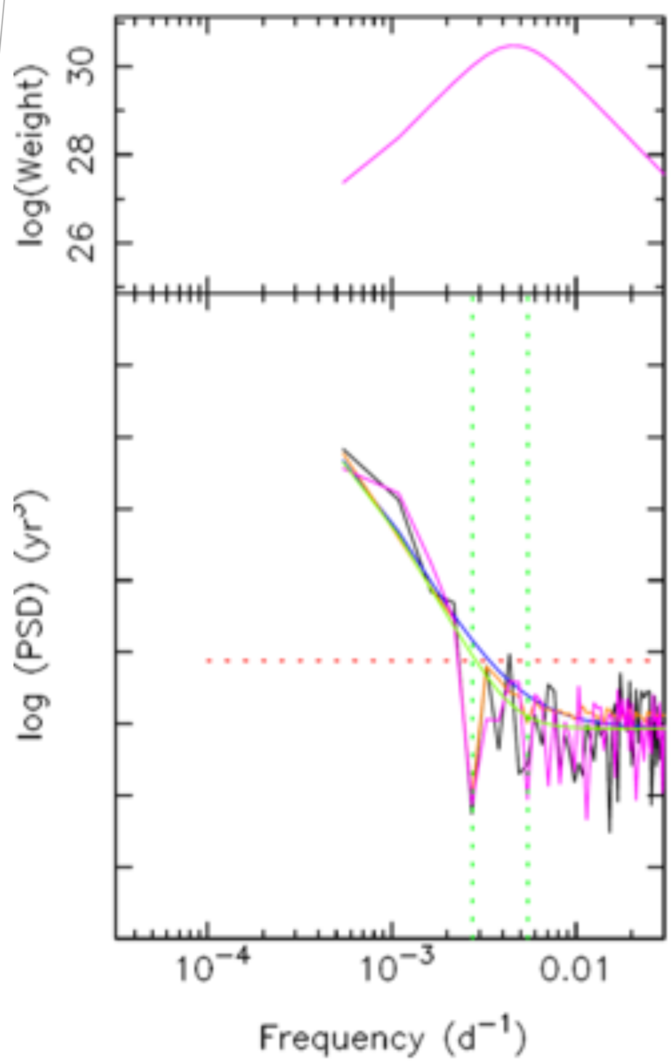
- Conservative:
  - Only add defensible noise to the *simulations*
  - Use filter that is more optimal for *observations*
  - Both of these will **deflate** simulated TS
    - Larger GW amplitude required to make simulated TS > observed TS

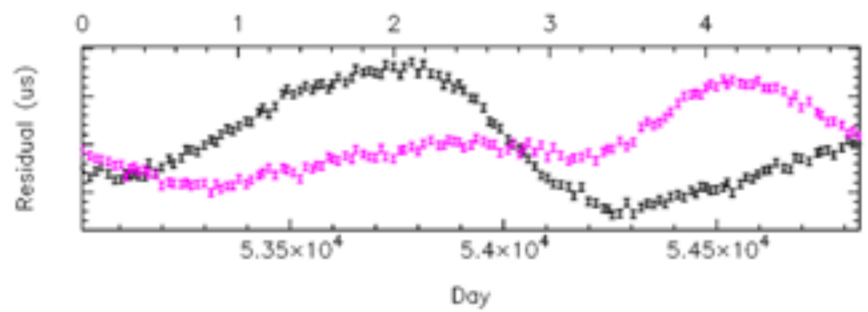
- Cholesky spectral modelling
  - Algorithm can identifying red noise in time series and appropriately model it in PSDE (mitigate spectral leakage)

- Automated fitting
  - Treat observations and simulations identically
  - Enables a comprehensive set of simulations that can be used for diagnostic purposes

Application: Data Challenge (single pulsar)

- Example: Data Challenge #3 (J1909-3744)

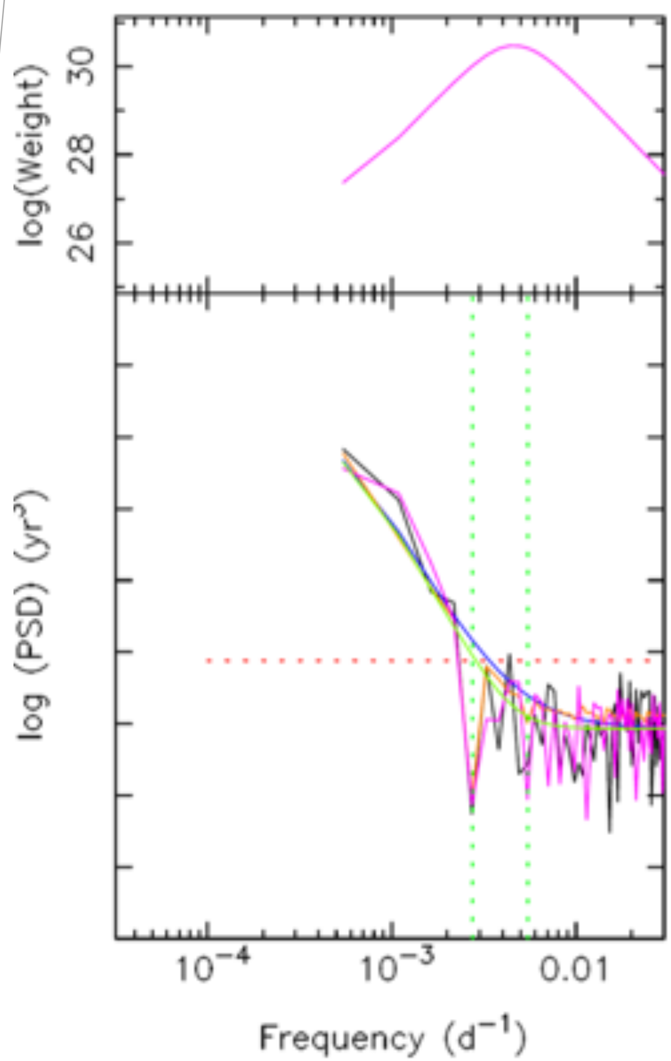


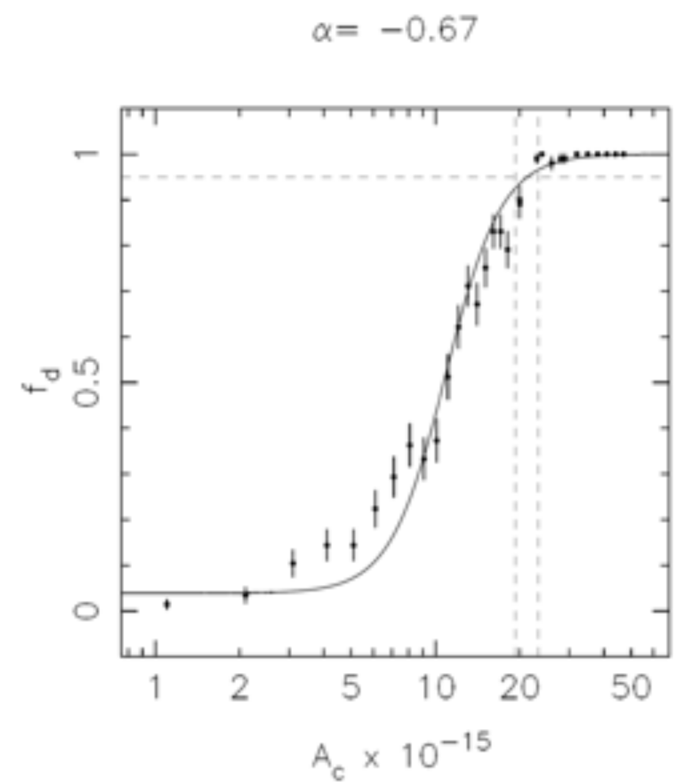


Black: real data
Purple: simulation ($A_{in} = 10^{-14}$)
Blue: expected model (GW +WN)
Orange: average
Green: model

Application: Data Challenge (single pulsar)

- Example: Data Challenge #3 (J1909-3744)

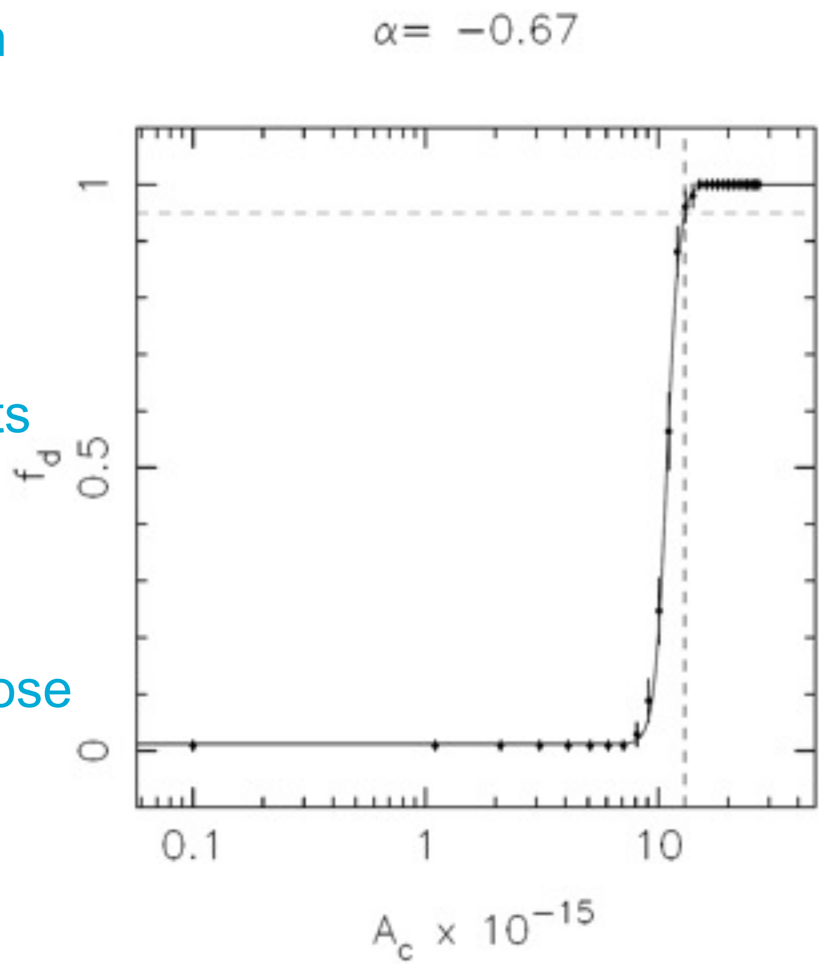




$A_{95} < 20 \times 10^{-15}$

Application: Data challenge (full simulated PTA)

- All of the pulsars from the open data challenge #3 (A_{in} 10^{-14})
- Limit: $A_{95} < 1.3 \times 10^{-14}$
- Limits from other open data sets
  - Dataset 1: 5.3×10^{-14}
  - Dataset 2: 6.0×10^{-14}
- Why is 95% confidence limit close to injected signal?



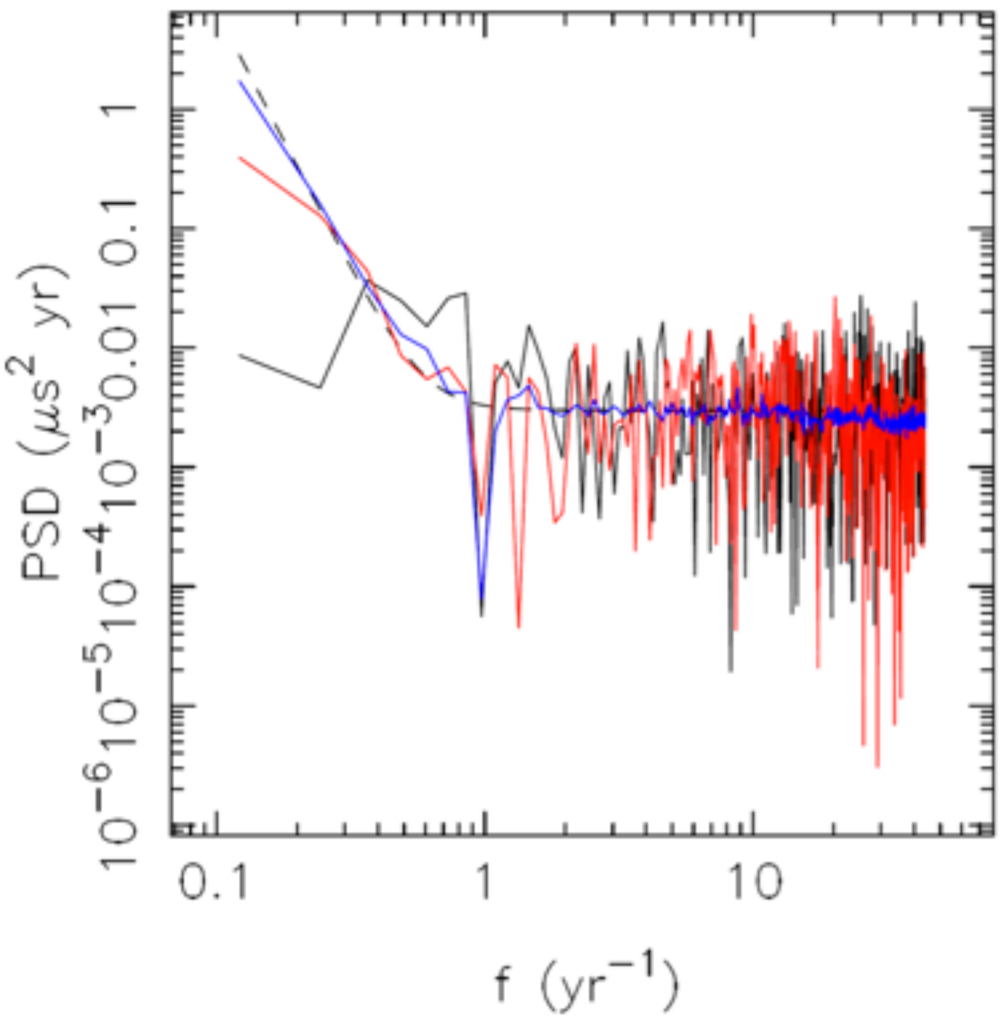
Example: J1909-3744 including a really strong GWB

- Compare PPTA observations of J1909-3744 to those with a really large GWB
  - (A ~ 6×10^{-15})

- Black: measured PSDE

Simulations: GW with 6×10^{-15}

- Blue: average of 100
- Red: 5th percentile



Constraining the GWB in the PPTA Data

- PPTA data release 1 “extended” data set
  - Increase timing baseline
- Include publicly available TOAs from Kaspi, Taylor, & Ryba (1994)
- Apply Dispersion Measure Correction if it improves timing (talk to Mike Keith)
  - Challenge: no DM correction in legacy data
- J0437-4715: use only “dr1”
  - Issue: “DM noise” leaks in from extended data sets and reduces sensitivity to GWs

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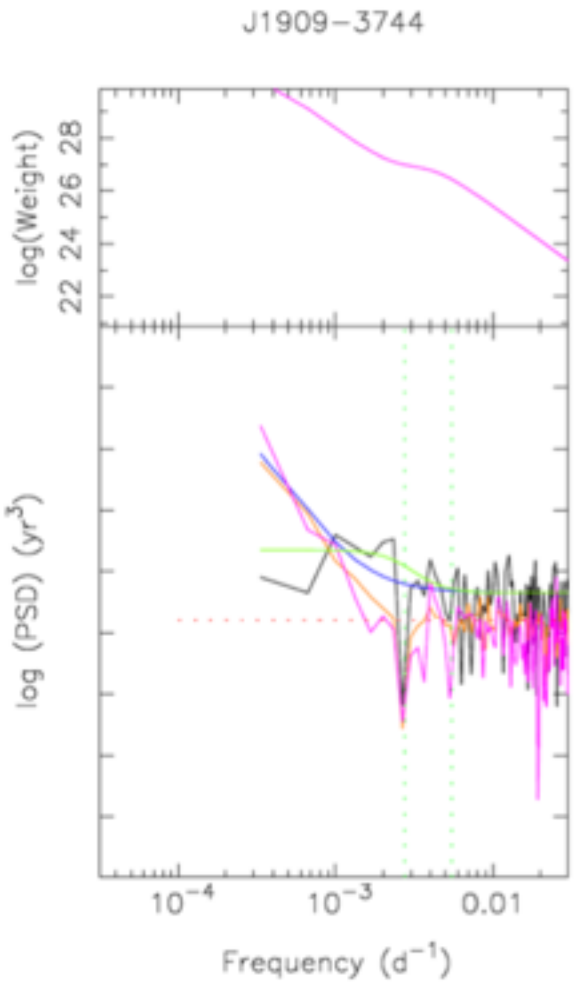
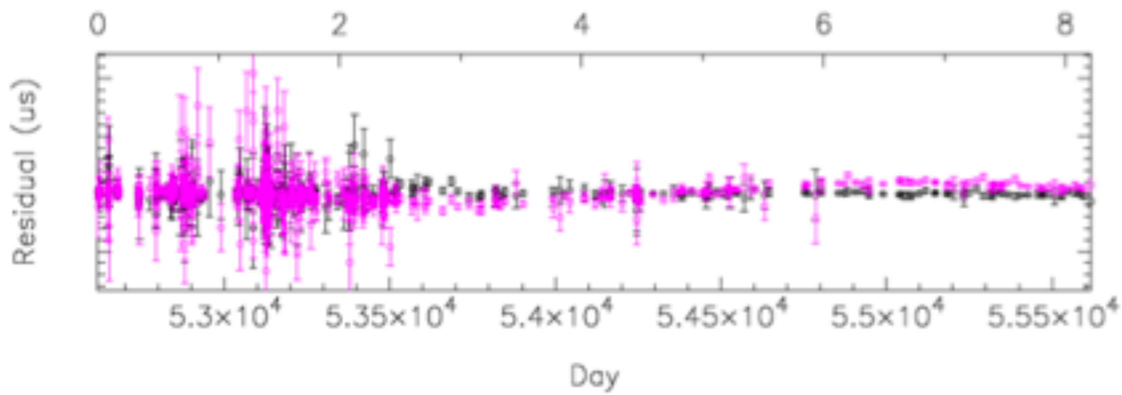
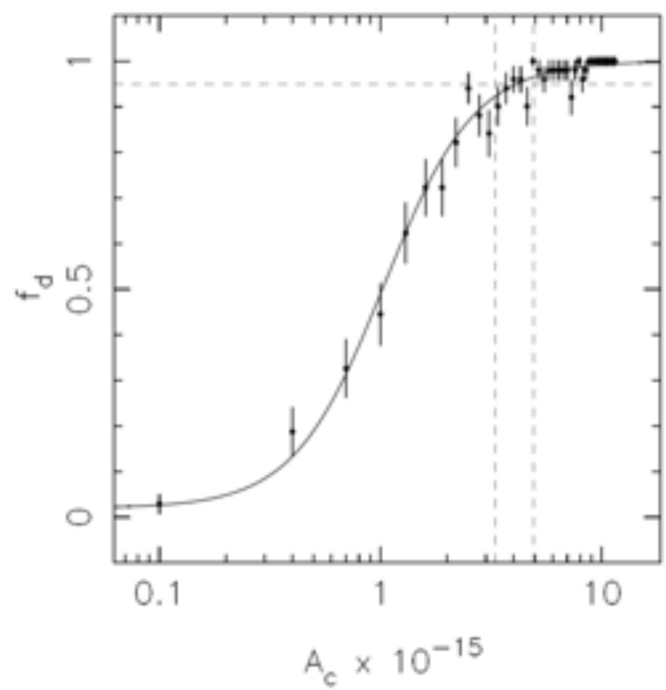
Example: Applying test statistic to J1909-3744





$A_{95} < 4.1 \times 10^{-15}$



Comparing Limits Across PPTA Pulsars

$$S_p(f_i; A, \alpha) = \frac{\sum_i (P_{r,p}(f_i) - P_{w,p}) W_p(f_i; A, \alpha)}{\sum_i W_p(f_i; A, \alpha)}$$

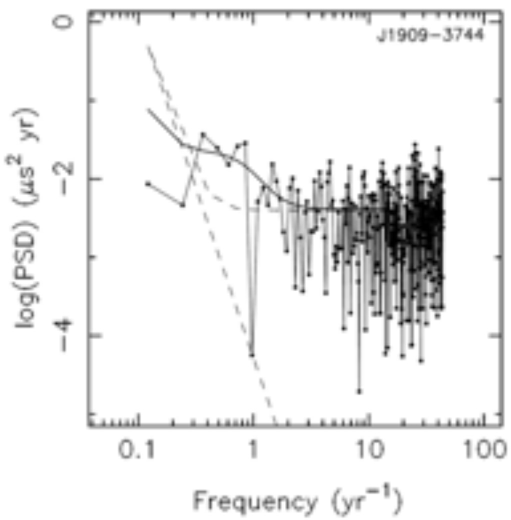
- Four pulsars have limits better best currently published limits
- The best four pulsars have comparable sensitivities
  - Sensitivity should be enhancec global analysis
- Limits from single pulsars vary markedly across PPTA pulsars

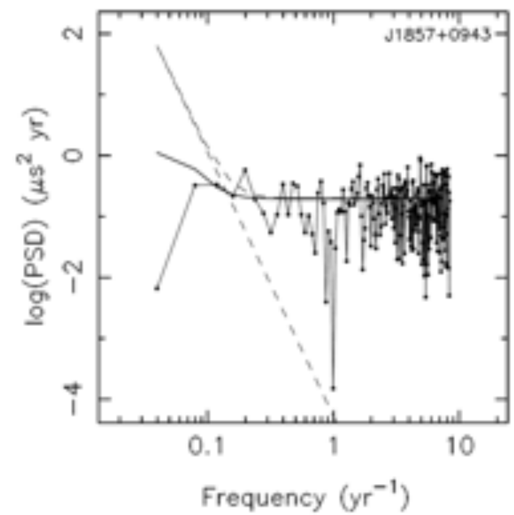
Preliminary Results

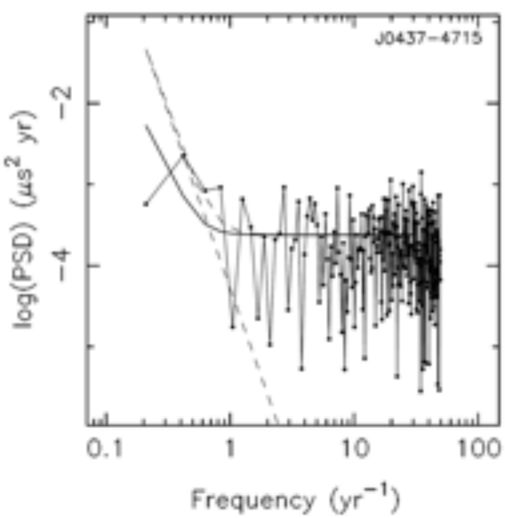
J1857+0943	3.3 ± 1.0
J1909-3744	4.1 ± 0.8
J0437-4715	4.2 ± 0.6
J1744-1134	5.9 ± 0.7
J1713+0747	7.3 ± 0.7
J0437-4715 (ext)	10 ± 2
J0613-0200	17 ± 2
J2124-3358	23 ± 3
J1022+1001	23 ± 6
J2145-0750	30 ± 2
J1730-2304	32 ± 3
J1600-3053	33 ± 3

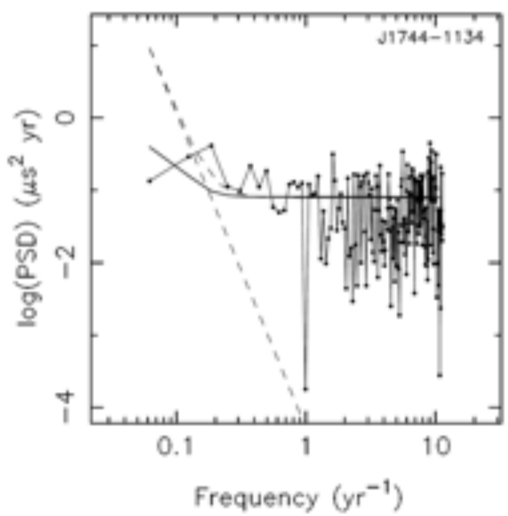
A limit from the PPTA DR1 data set

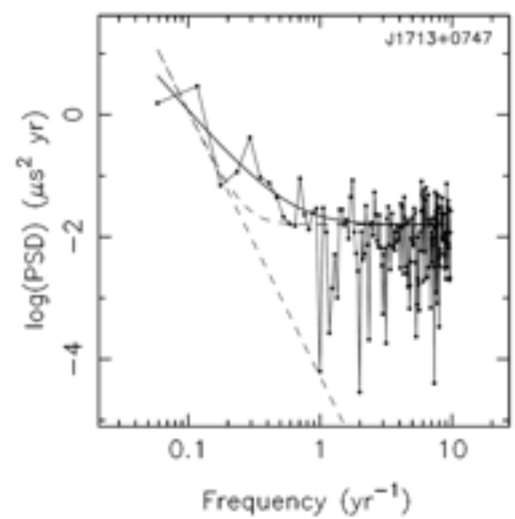
- Derive limit from five PPTA pulsars with consistent limits
- Augment PPTA data with publically available KTR observations





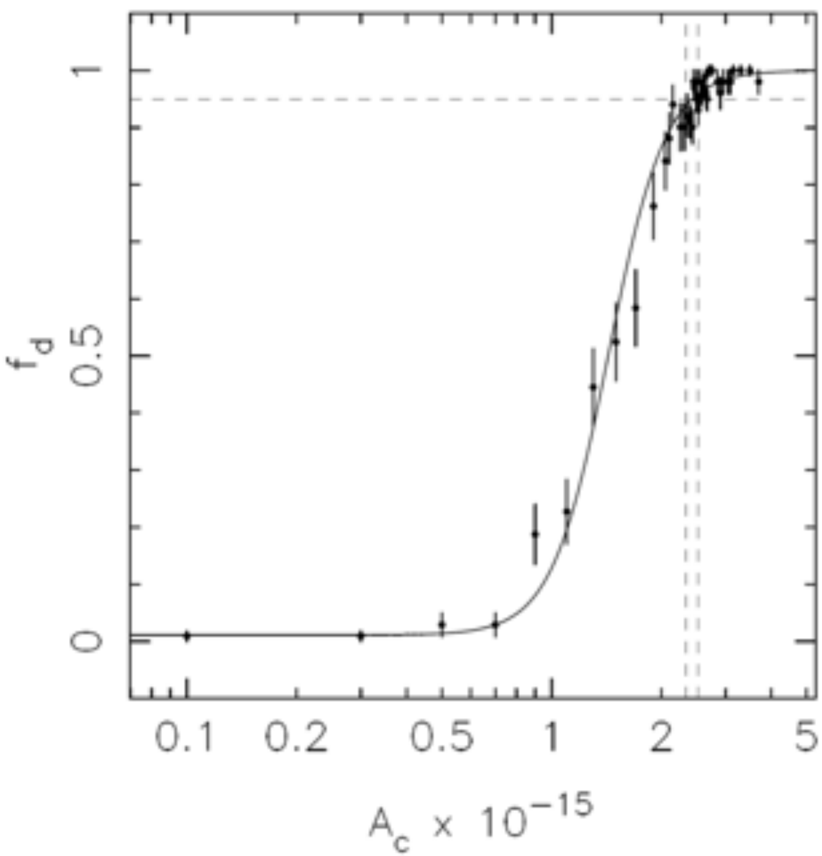






The Combined PPTA Limit

$$S_{\mathrm{PTA}}(A,\alpha) = \frac{\sum_p \sum_i (P_{r,p}(f_i) - P_{w,p}) W_p(f_i; A, \alpha)}{\sum_p \sum_i W_p(f_i; A, \alpha)}$$



Preliminary limit on the background:
With 95% confidence $A_{95} < (2.4 \pm 0.1) \times 10^{-15}$

Comparison to recent limits using other data sets

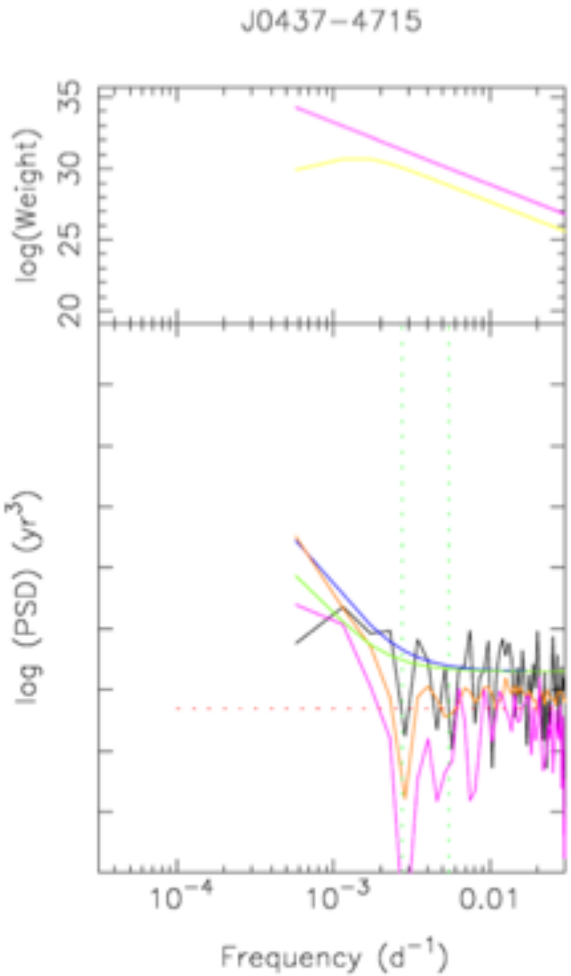
Why is PPTA limit better?

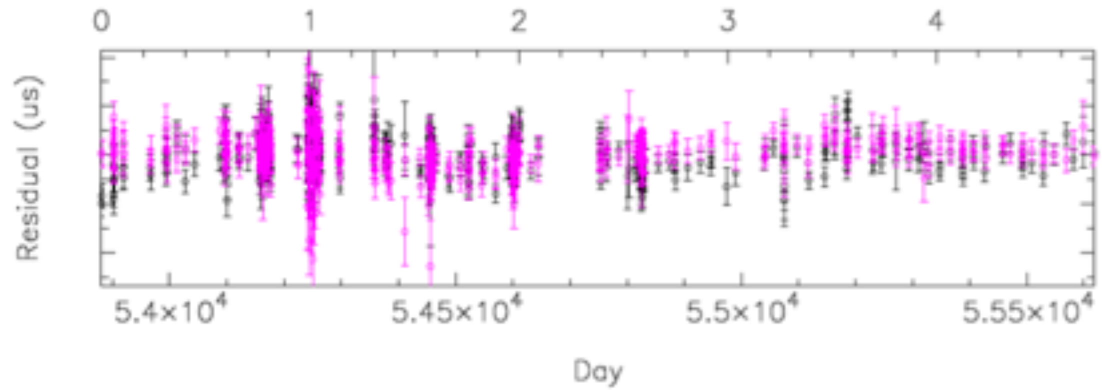
1. Higher quality data over longer observing span
2. Algorithm optimised for bounding.

($\alpha = -1$; $A_{95} < 10^{-15}$)

($\alpha = -7/6$; $A_{95} < 0.7 \times 10^{-15}$)

Reality Check: Pulsars at the limit

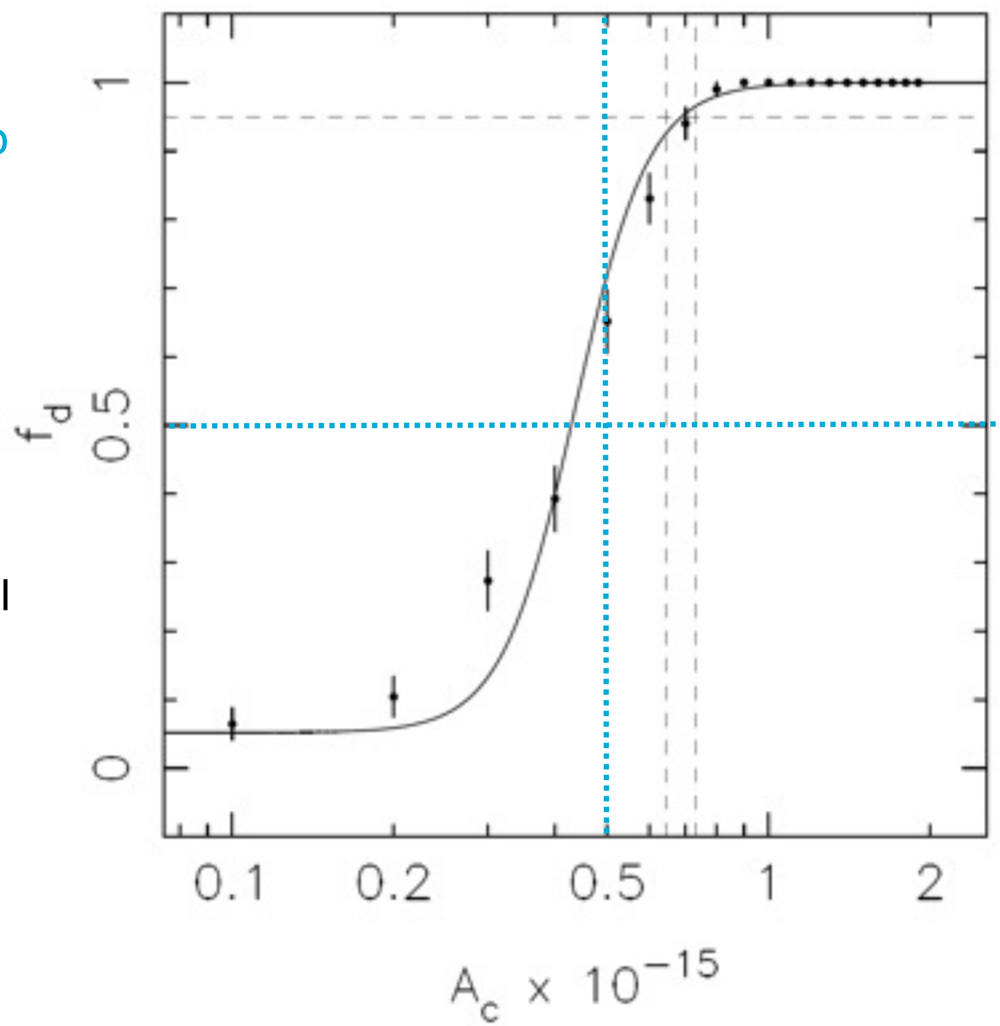




Black: real data
Purple: simulation
Blue: expected model (GW +WN)
Orange: average
Green: model

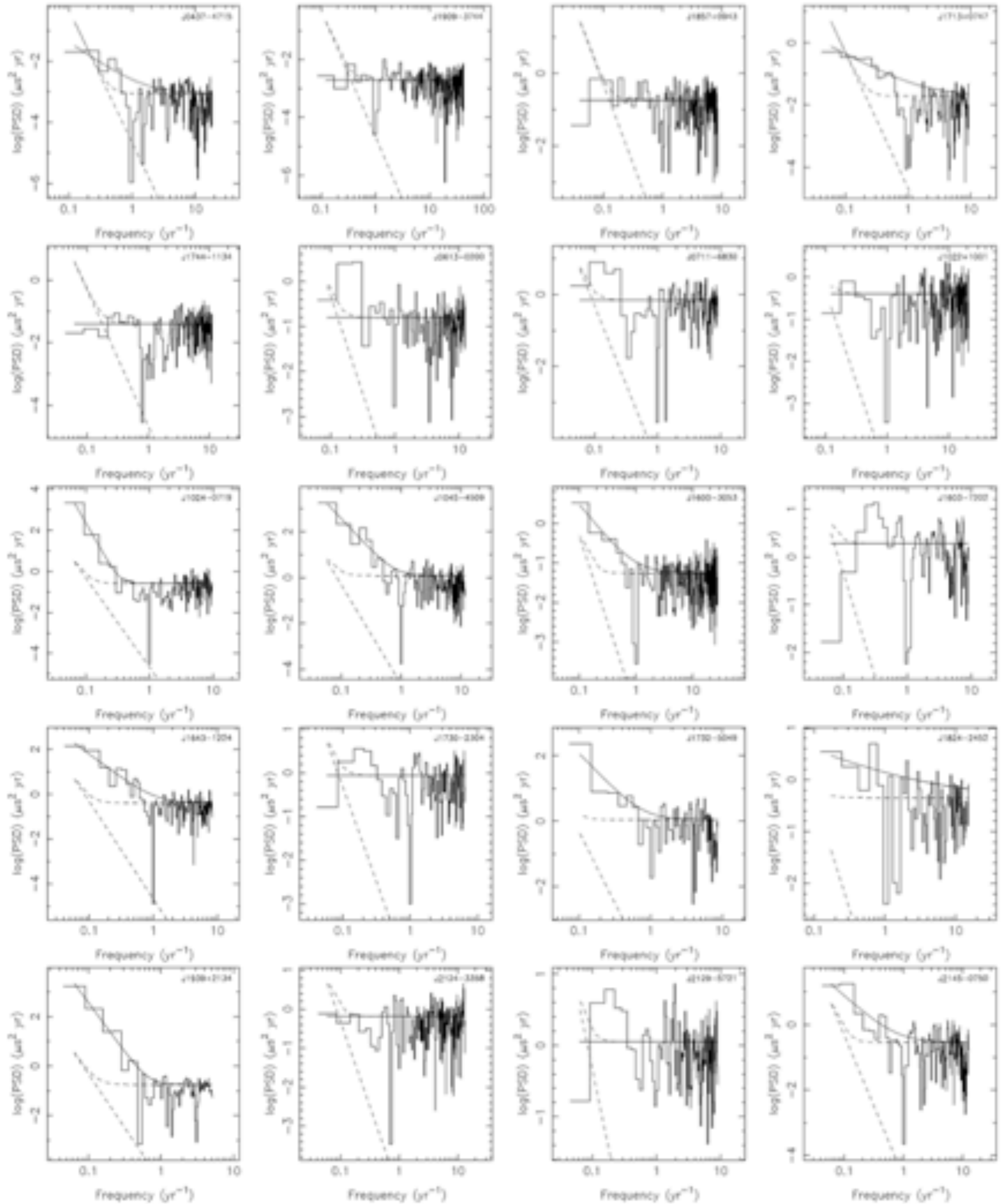
Reality Check: Application of Algorithm to Simulated Data

- Simulate PTA observations with white noise and GWB
- Similar number of pulsars to PPTA
- Ideal simulations
  - 95% of time $A_{95} > A_{in}$
- Realistic simulations
  - Red noise at modelled level + 3×10^{-15}
  - 98% of time $A_{95} > A_{in}$
- B1855+09 Kaspi, Taylor, & Ryba data set



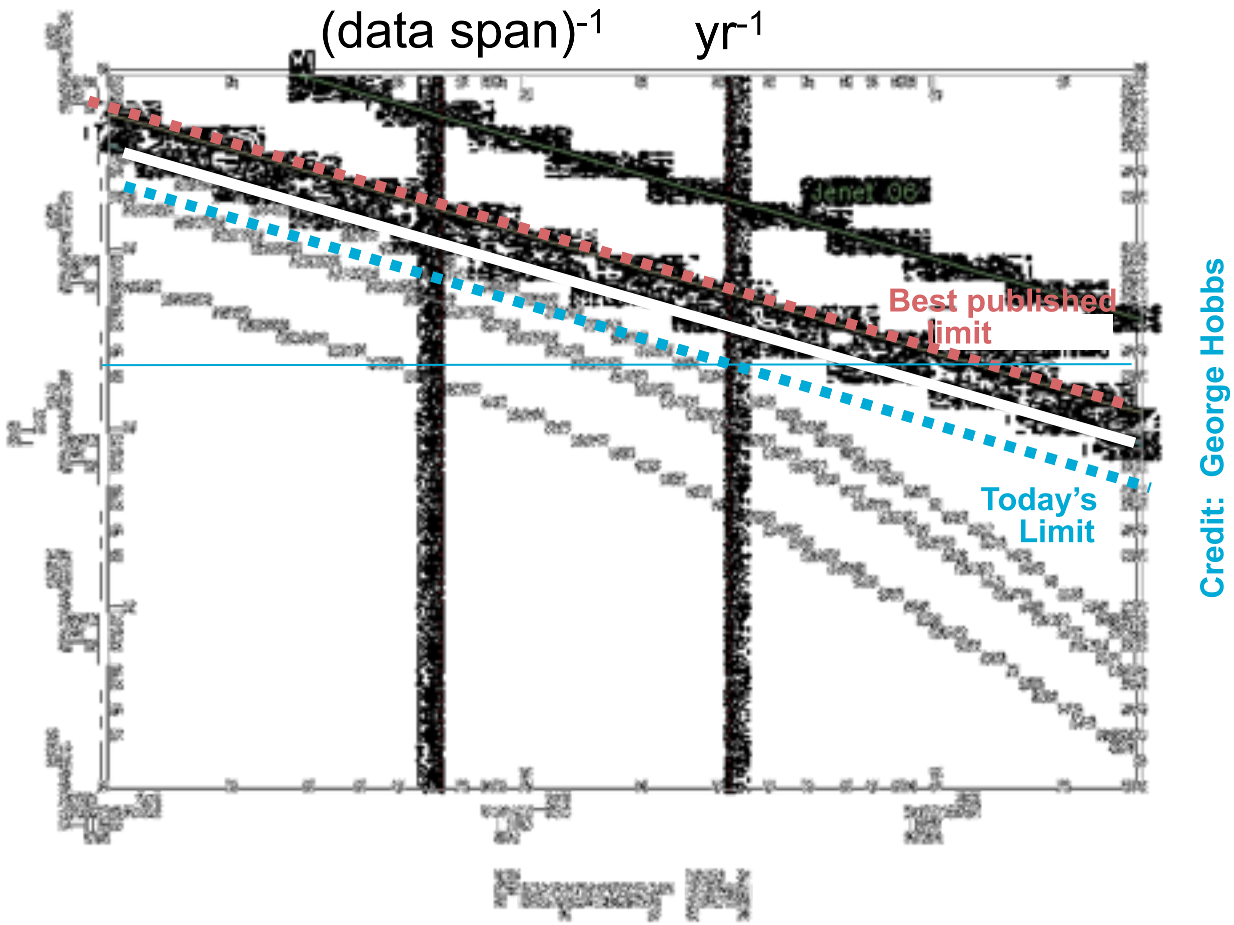
Reality Check: Power Spectra

- For the best pulsars, $A_c \sim 2.4\times10^{-15}$ will contribute significant power in lowest frequency bins
- With Cholesky spectral estimation, we are able to recover most of the power in the low frequency bins



Implications of Limit

Strain as function of frequency for MBH background predicted by Sesana et al.



Bottom line: We have entered an interesting portion of phase space!

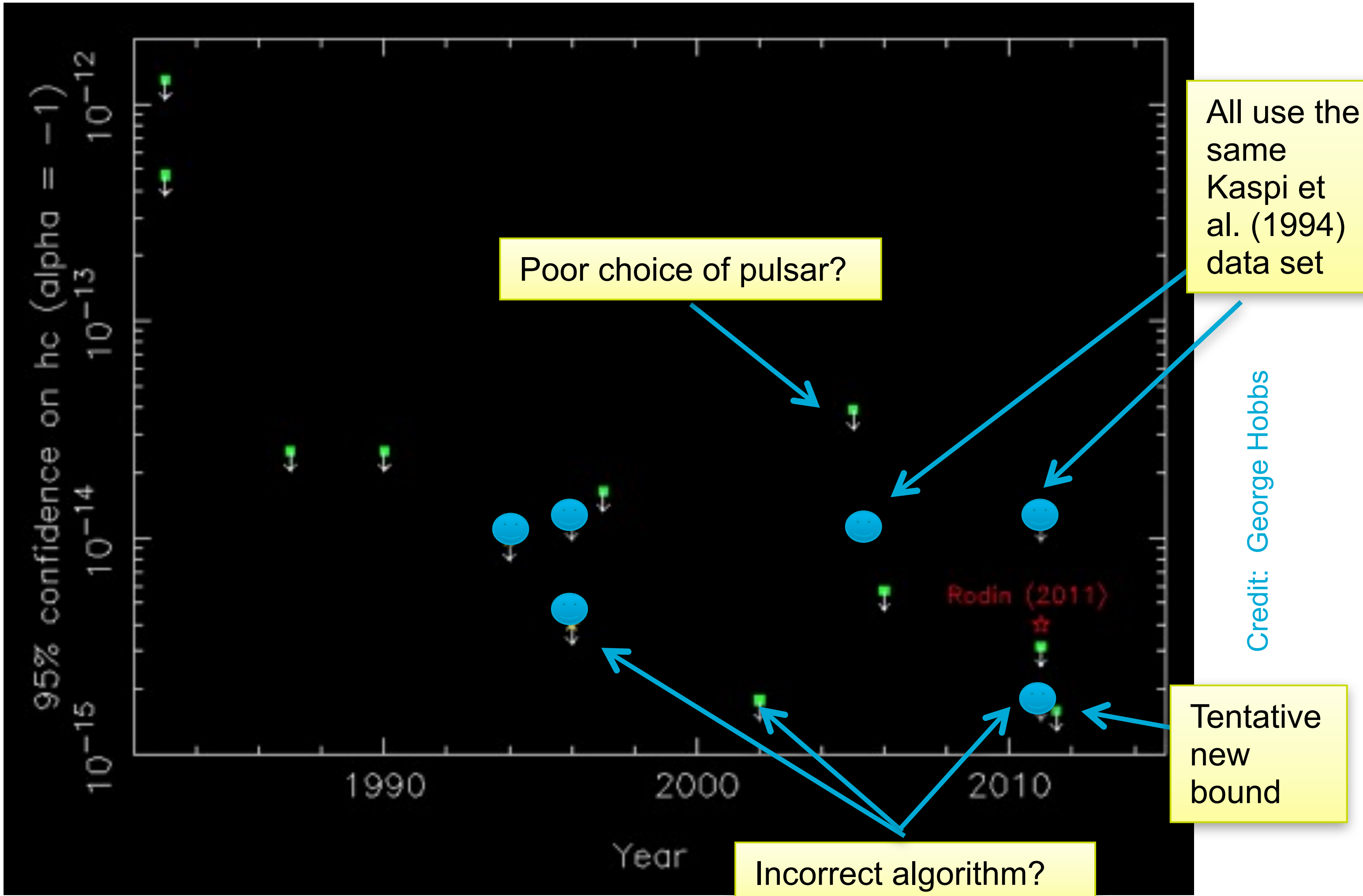
Recommendations for data challenge

- What is the purpose of the data challenge?
  - Engage scientists outside the PTA
  - Compare algorithms being implemented within the IPTA
- Need a data challenge to compare different algorithms
- Add more realistic noise, cadence
- Add more realisations

Towards a better constraint and detection

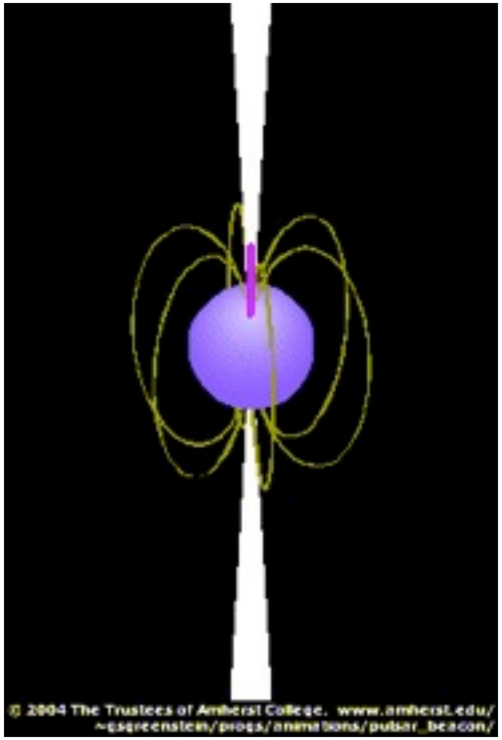
- Improve noise model
  - Incorporate effects of DM noise in model of legacy data
- International Pulsar Timing Array Data sets
  - Limit should be better?
  - Improvements for both J1857+0943 (no large gap) and J1713+0747 (more DM correction)
- Longer term: continue to improve timing precision
  - Wide bandwidth timing instrumentation (more sensitive, better DM correction, higher throughput)
  - SKA: Time (more/better) pulsars to a higher precision
  - Longer timing baselines

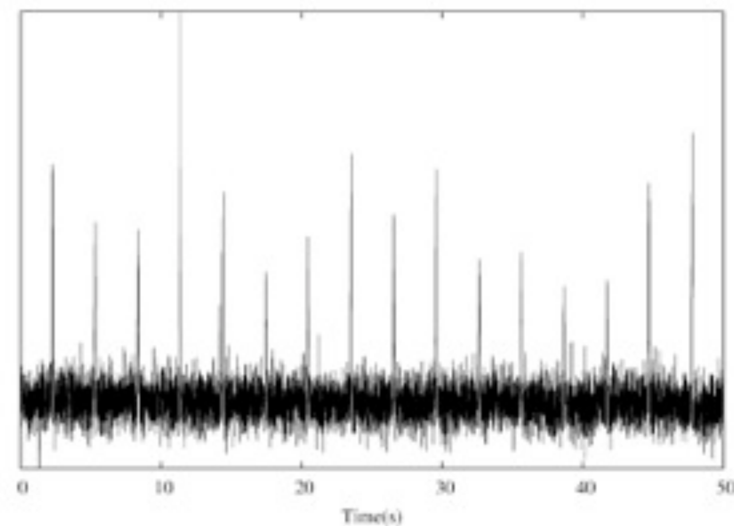
Comparing published limits of GWB strength



Use same data set

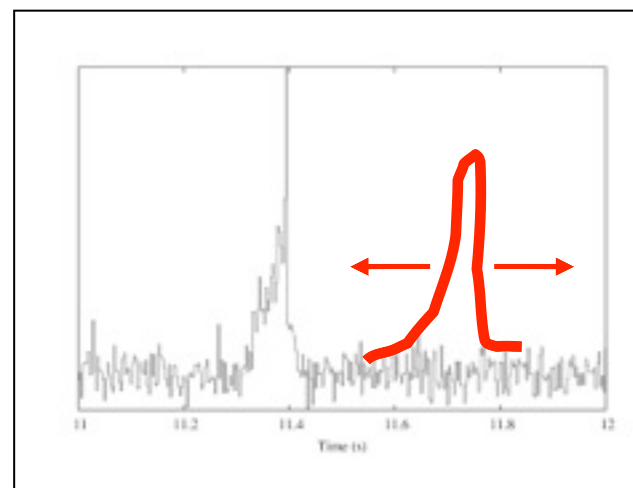
Pulsar Basics



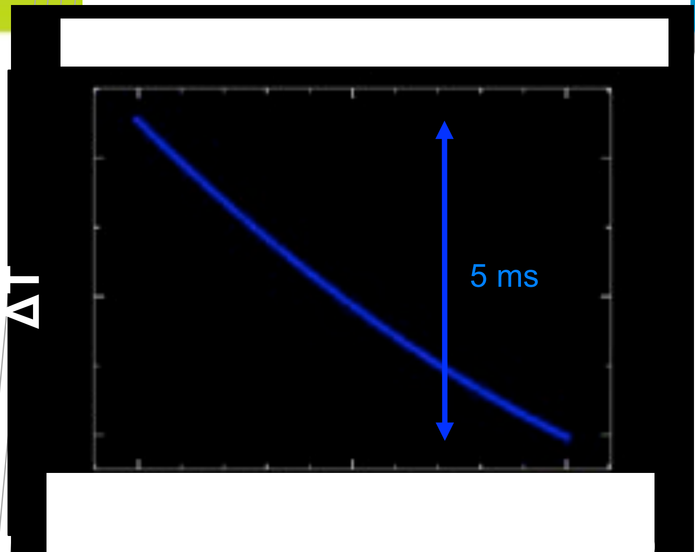


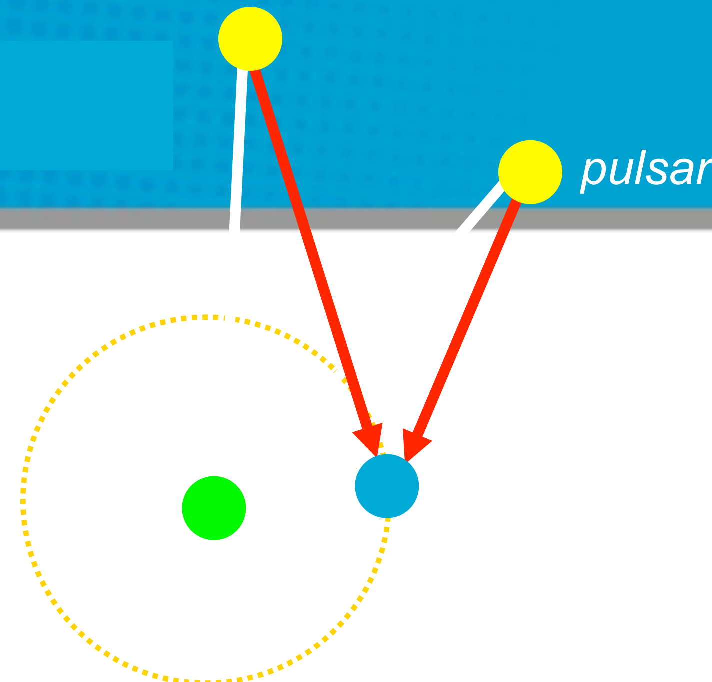
Beware: *Frequency* will be used in three contexts:
Radio frequency
Pulsar spin frequency
Fluctuation frequency

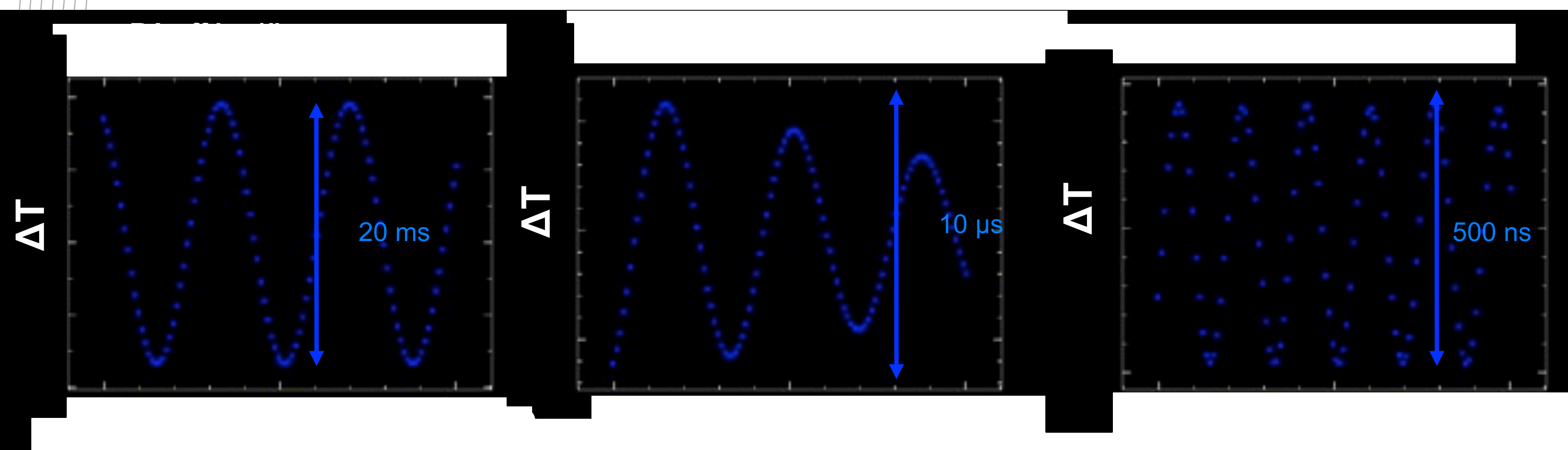
Convolve template with average pulse profile to determine accurate TOA



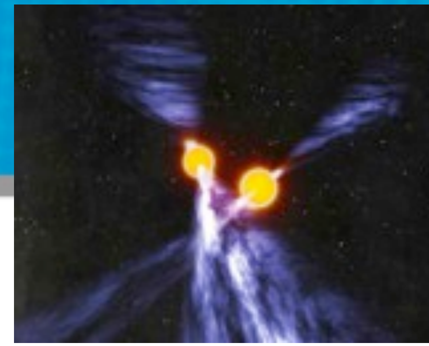
Amplitudes of Deterministic Contributions

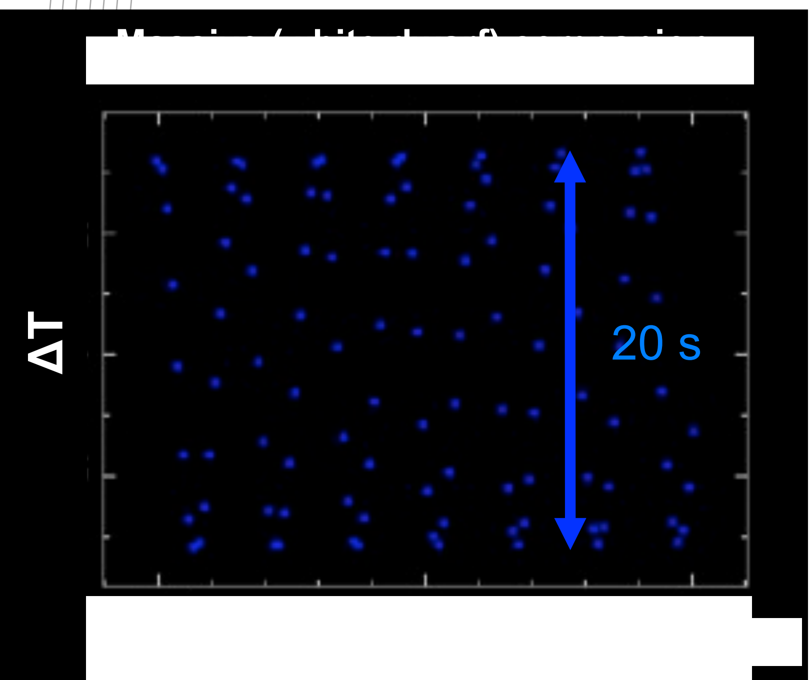


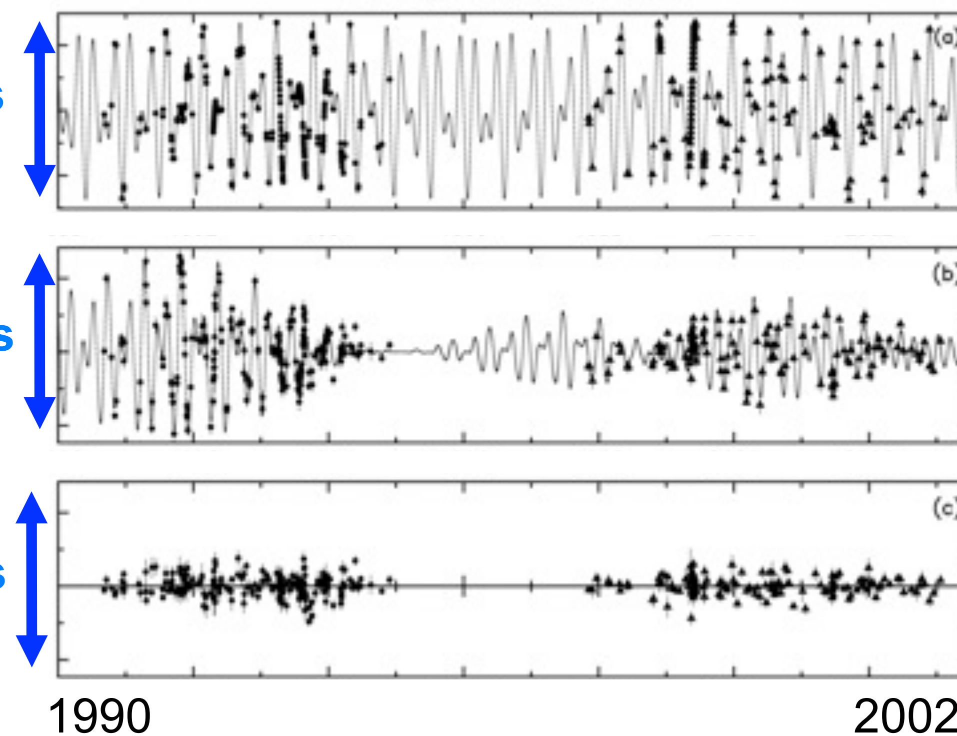




TOA Variations due to Pulsar Companions







Gravitational Waves and Pulsar Timing

Accelerating masses emit gravitational waves.

$$\partial_\mu \partial^\mu \left(h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h^\alpha{}_\alpha \right) = -16\cancel{\pi} T_{\mu\nu}$$

h: dimensionless strain, is related to the gravitational red/blueshift of signal, analogous to vector potential of E&M. Changes light travel time between pulsar and Earth

Propagating GW can be decomposed into two terms:
Earth end and pulsar end.

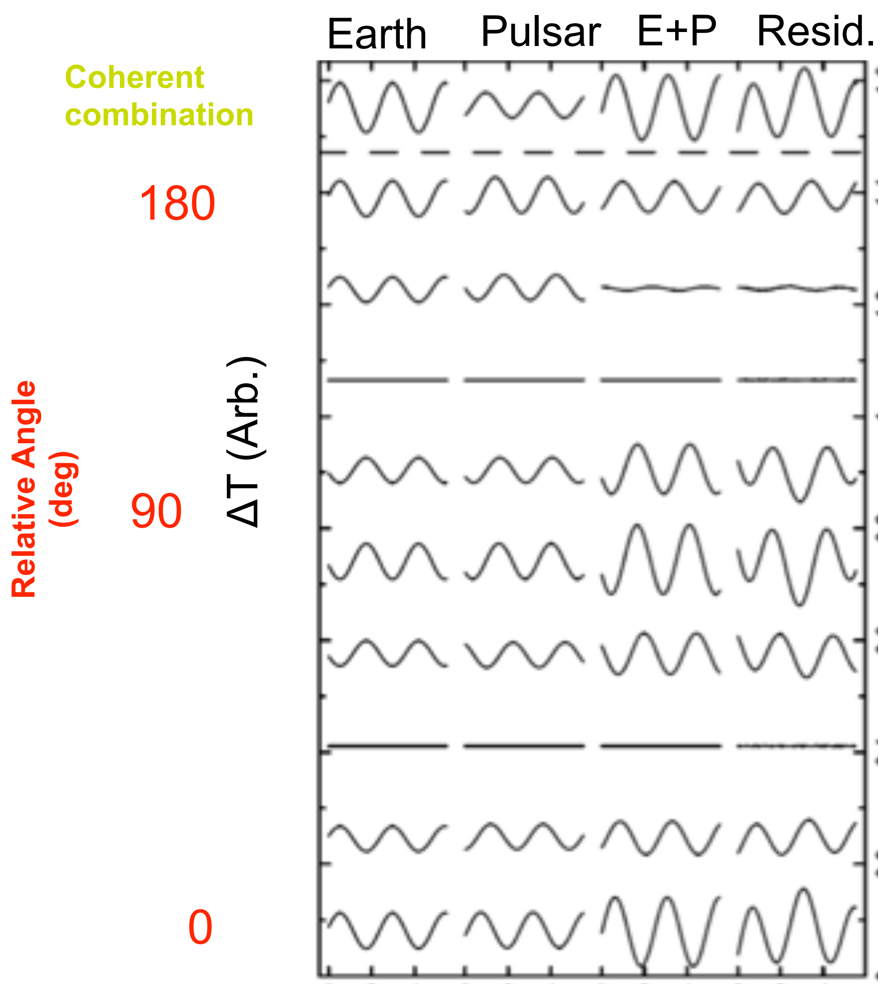
Angles between GW and Pulsar

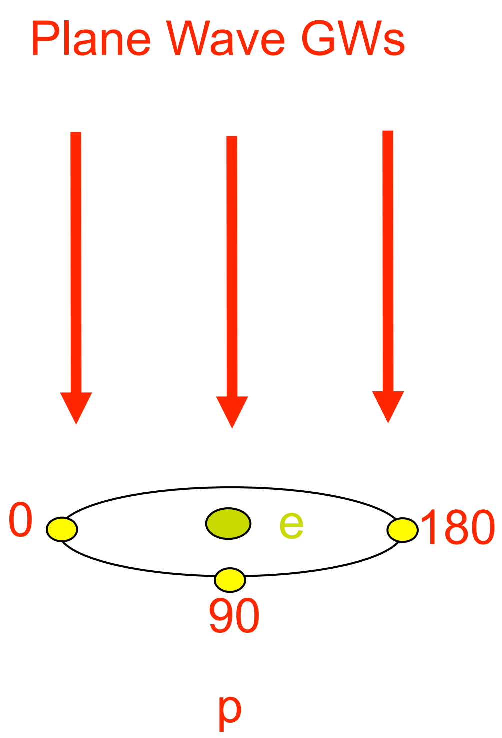
$$R_{\mathrm{GW}}(t) = \frac{1}{2\omega}(1-\cos\theta)(A_+\cos(2\phi) + A_\times \sin(2\phi))\left[\sin(\omega t + \Phi_\oplus) - \sin(\omega t + \Phi_{\mathrm{PSR}})\right]$$

$$A_{+,\times} \propto h(\omega)$$

$$\Phi_{\mathrm{PSR}} = \Phi_\oplus + \frac{\omega D}{c}$$

Example: Single GW Source

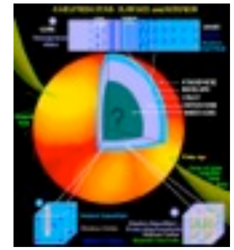


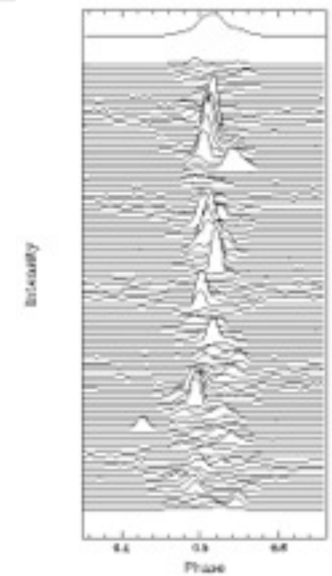


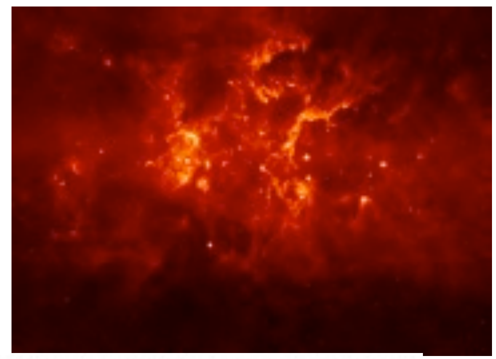
Contributions to Pulsar Arrival Times

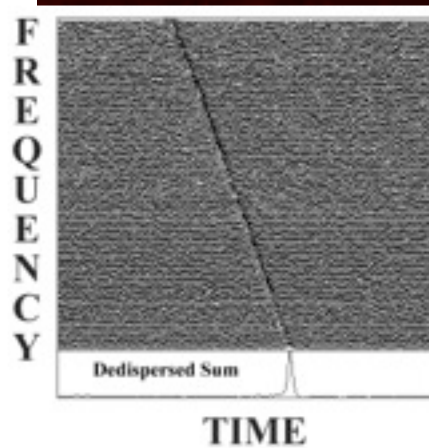
- Pulsar spin down
- *Stochastic spindown variations*
- Intrinsic variation in shape and/or phase of emitted pulse (jitter)
- *Reflex motion from companions.*
- Pulsar position, proper motion, distance
- *Gravitational Waves*
- *Warm electrons in the ISM*
- Location of Solar System Barycentre
- Irregularities in time standards

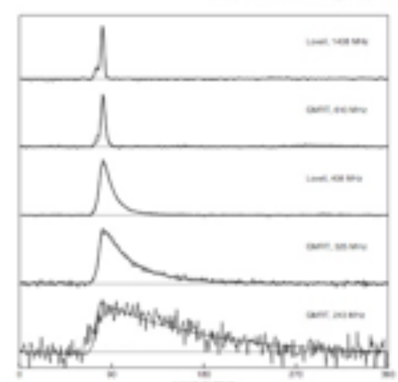
Pulsar











Earth

Sources of GWs in Pulsar Band

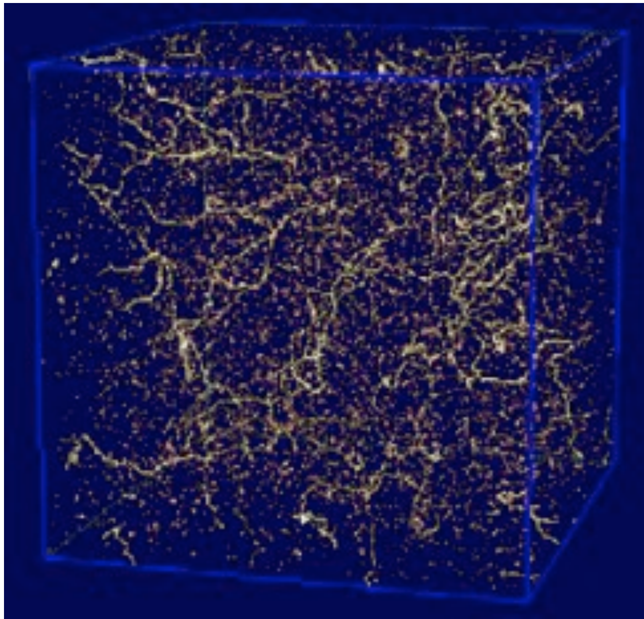
Pulsars are sensitive to GWs with frequencies between 3 and 300 nHz (get TOAs weekly for 10 years)

•**Massive Black Hole Binaries**
(Jaffe & Backer 2003, Sesana et al. 2009)

•**Cosmic Strings and Super Strings**
(Damour & Vilenkin 2005)

•**Cosmological QCD Phase Transition (with caveats)**
(Witten 2007, Caprini et al. 2010).

•**Primordial (Inflationary) GWs.**



The Stochastic Background

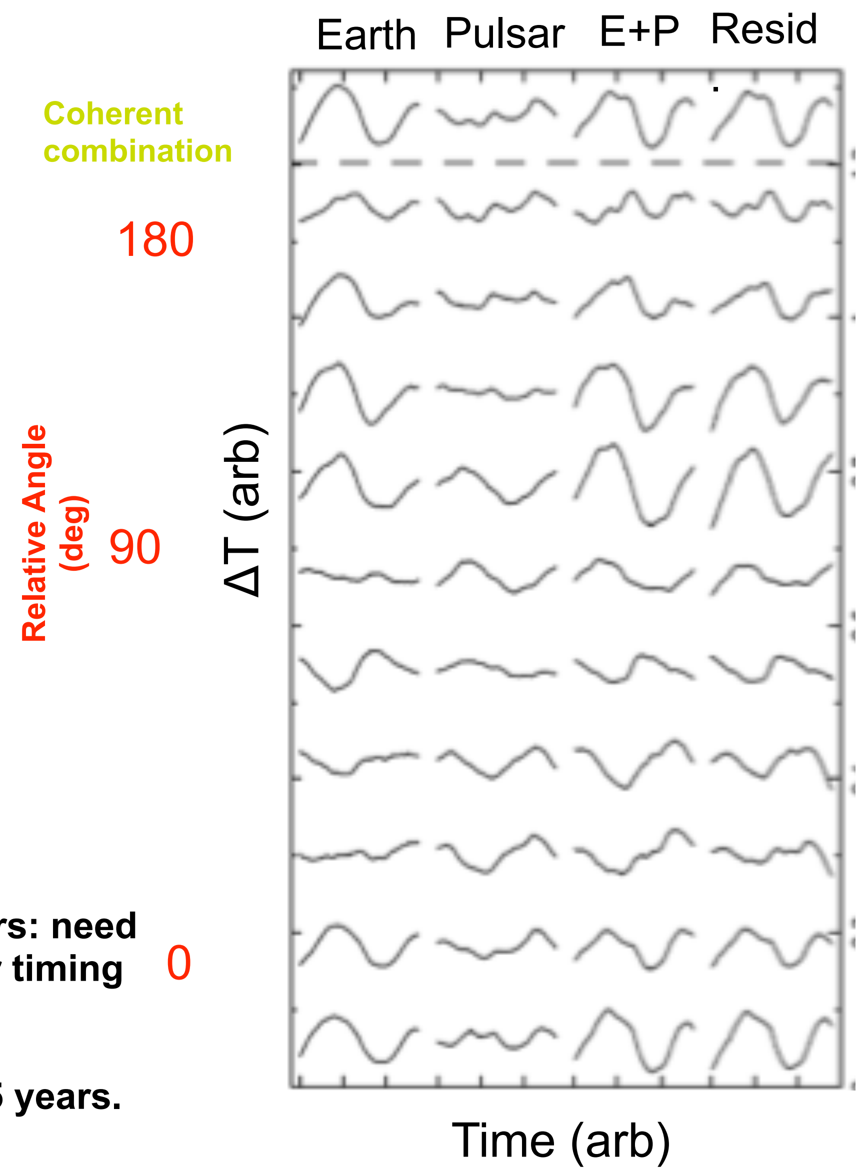
The stochastic background is the superposition of GWs from many sources

Background induces a **red** signal in residuals (more power at lowest frequency of GWs).

Characterized by a strain amplitude spectrum.

For MBH binary the spectrum is expected to be:

$$h_c(f) = 10^{-15}\left(\frac{f}{1\ \mathrm{yr}^{-1}}\right)^{-2/3}$$



RMS contributions to residuals ~20 ns over 5 years: need to combine signal from a group of pulsars (pulsar timing array, PTA).

RMS contribution is approximately 400 ns over 15 years.

The Parkes Pulsar Timing Array

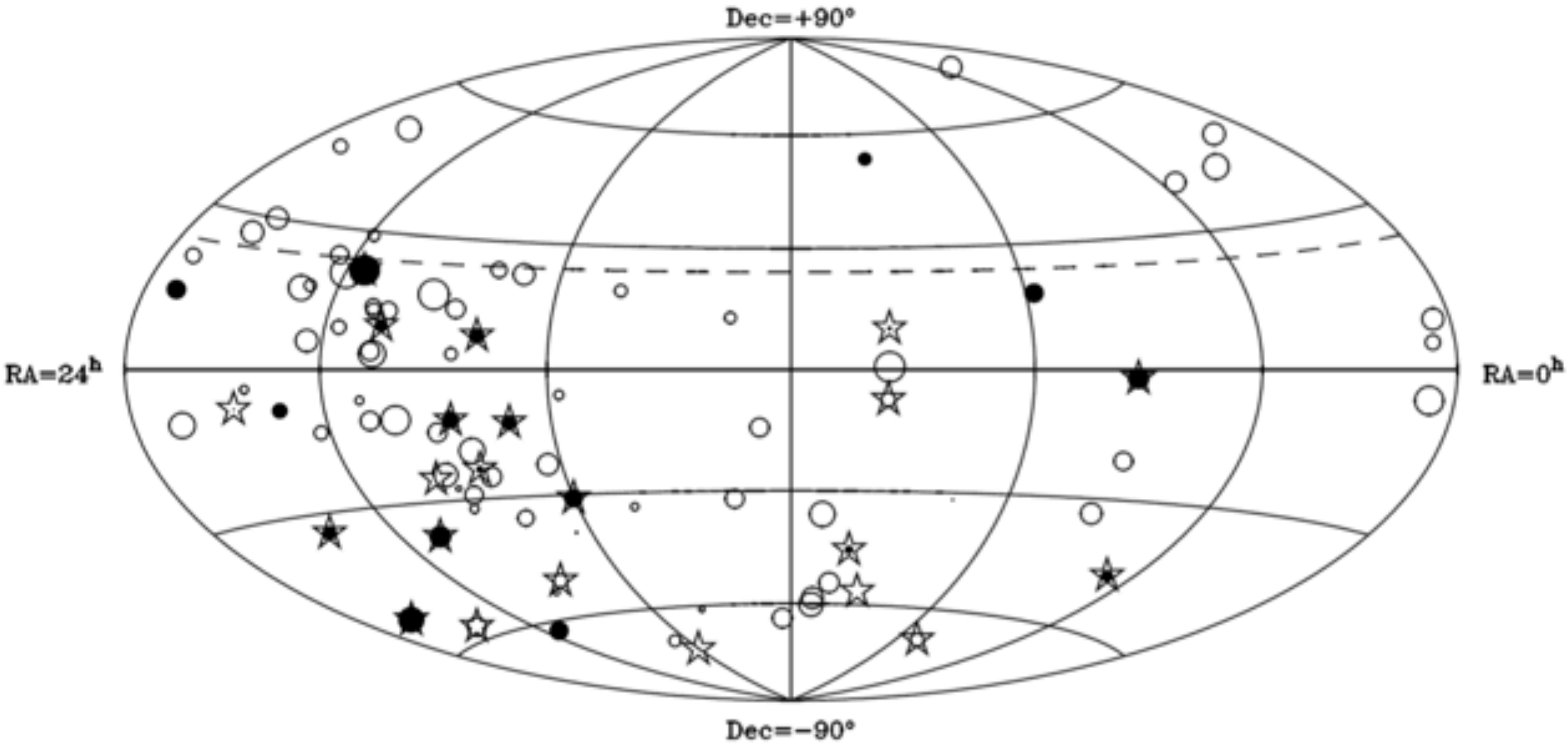
Collaboration primarily between CASS, Swinburne, American, Chinese-based Scientists

Observing strategy:

- Observing campaigns once every 2-3-4 weeks at Parkes Radio Telescope
- Observe 20 pulsars per session in three frequency bands
  - Correction for interstellar propagation effects
  - 20cm receiver and 10/50cm receiver
  - Data recorded with multiple backend signal processors that using different architectures

- Partial List of Goals:
  - **Detect gravitational waves**
  - Improve time standards/solar system ephemeris
  - Understand Interstellar Medium

MSPs and the PPTA Pulsars



Manchester et al. submitted

Features of the algorithm

- Treat simulated data and real data identically
- Developed to work with highly inhomogenous data
  - Combine data from pulsars with wildly different sensitivities to GWB
  - Data sets of varying length
  - Corrects for spectral leakage in the presence of red noise