Constructing an optimal observing strategy for PTA

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Outline

- Gravitational wave detection sensitivity
- Basic framework for optimization
- Results and hints
- By-products
- Recent progress and development of web user interface

Based on MNRAS 2012, 423, 2642

The optimal schedule for pulsar timing array observations

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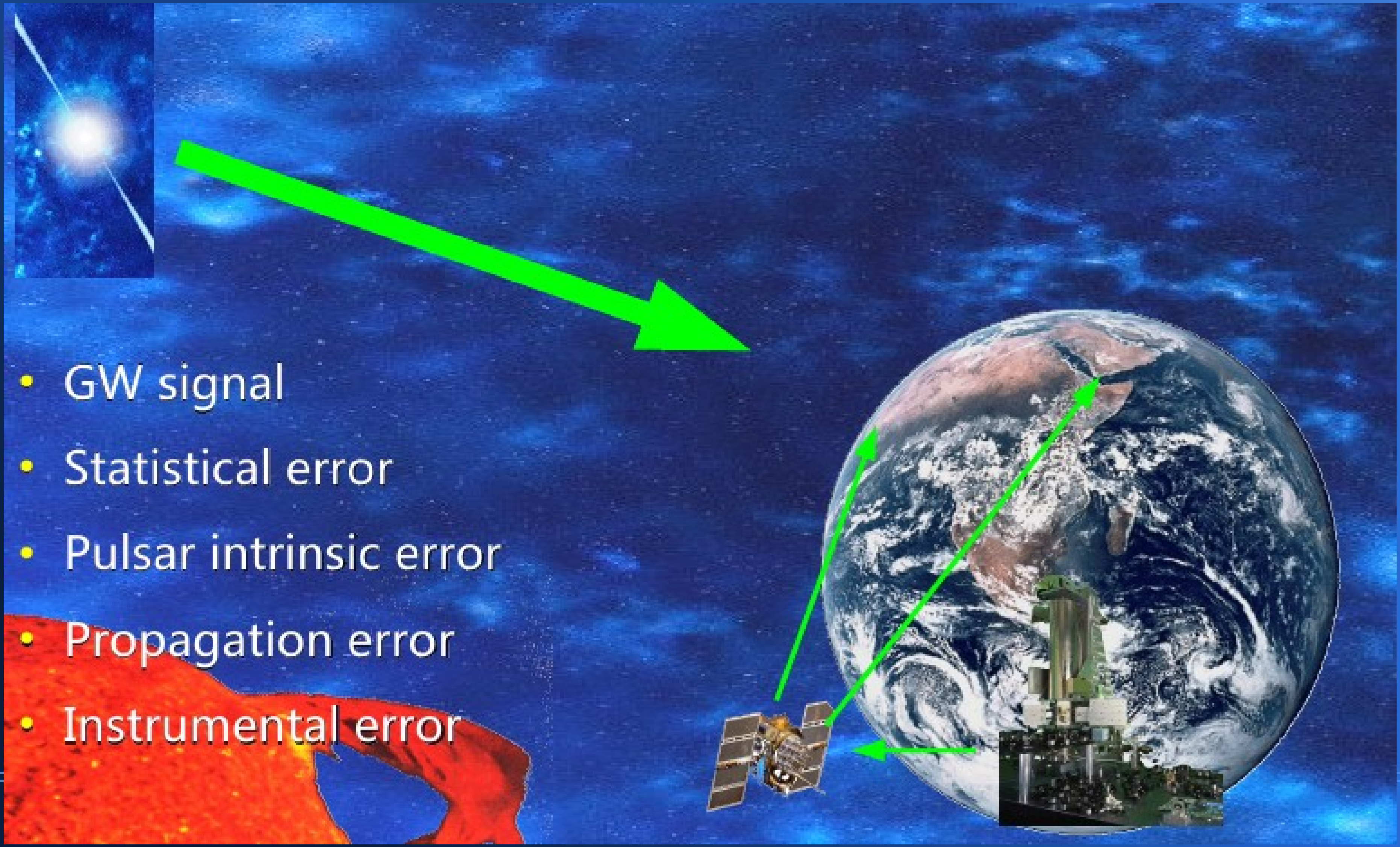
ABSTRACT
In order to maximize the sensitivity of pulsar timing arrays to a stochastic gravitational wave background, we present computational techniques to optimize observing schedules. The techniques are applicable to both single and multi-telescope experiments. The observing schedule is optimized for each telescope by adjusting the observing time allocated to each pulsar while keeping the total amount of observing time constant. The optimized schedule depends on the timing noise characteristics of each individual pulsar as well as the performance of instrumentation. Several examples are given to illustrate the effects of different types of noise. A method to select the most suitable pulsars to be included in a pulsar timing array project is also presented.

Key words: pulsar: general — gravitational wave

Great appreciation for the referee help for improving the results!

Timing signals



Phenomenological model of signal-noise

$$\boldsymbol{R} = \boldsymbol{s} + \boldsymbol{n}$$

$$\langle {}^{i}s_k \, {}^{j}s_{k'} \rangle = \sigma_{\mathrm{g}}^2 H({}^{ij}\theta)\gamma_{kk'}$$

$$\langle {}^{i}n_k \, {}^{j}n_{k'} \rangle = \left({}^{i}\sigma_{\mathrm{w}}^2 \delta_{kk'} + {}^{i}\sigma_{\mathrm{r}}^2 g_{kk'} \right) \delta_{ij}$$

PSR index

$${}^{i}s_k$$

Time index

Phenomenological model of signal-noise II

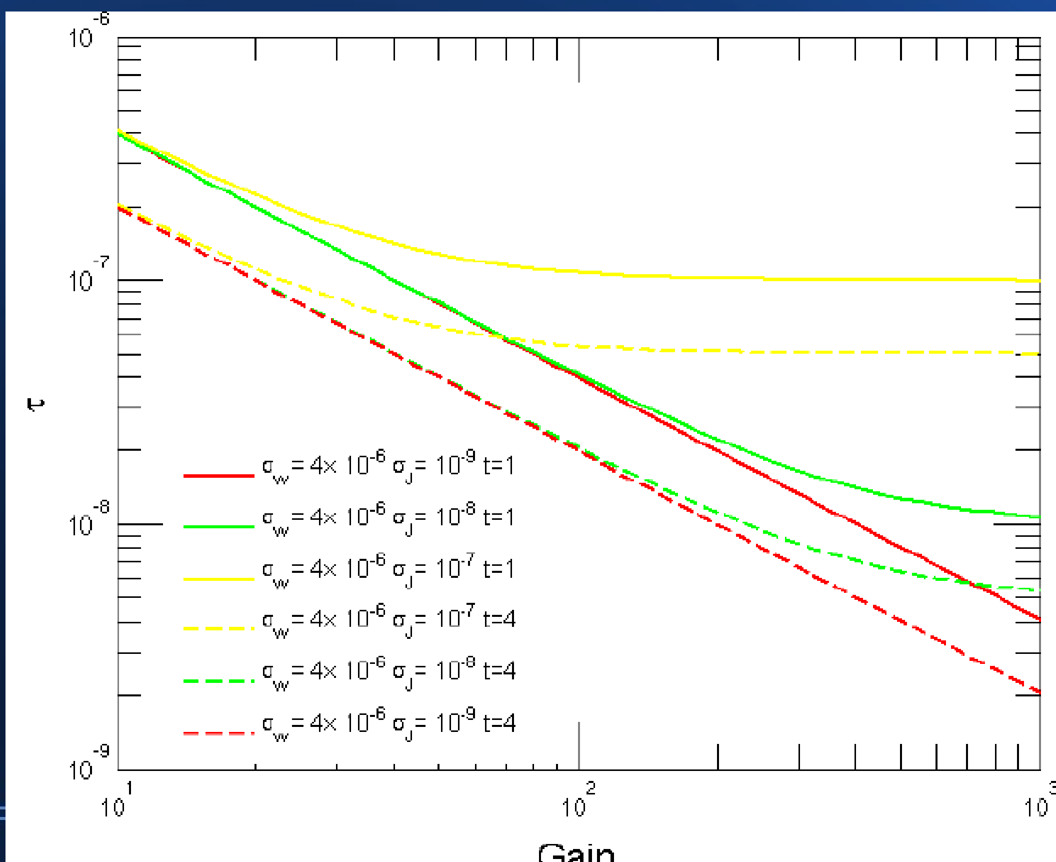
- we need the relation describing how signal or noise components respond to the schedule.
- For white noise components

$$ {}^{i}\sigma_{\mathrm{w}} = \left({}^{i}\sigma_0^2 \mathcal{G}^{-2} + {}^{i}\sigma_{\mathrm{J}}^2 \right)^{1/2} \left(\frac{{}^{i}\tau}{1\mathrm{hr}} \right)^{-1/2} $$

- Red noise components are assumed to be independent of schedule.

Gain dependents

- Describe how the noise respond to the system.



$$ {}^{i}\sigma_{\mathrm{w}} = \left({}^{i}\sigma_0^2 \mathcal{G}^{-2} + {}^{i}\sigma_{\mathrm{J}}^2 \right)^{1/2} \left(\frac{{}^{i}\mathcal{T}}{1\mathrm{hr}} \right)^{-1/2} $$

Small telescope can be equally useful compared to the large ones. They have different responsibility and contribute differently.

Other possibility of correcting jitter noise (Oslowski+2011)

GW detection sensitivity

- We need some GW detection sensitivity which is fast to compute.
- We use the Jenet's Statistics S

What is S?

$$ {}^{ij}c = \frac{1}{m}\sum_{k=1}^{m} {}^{i}R_k \, {}^{j}R_k $$

$$ S = \sqrt{M}\,\frac{\sum_{i-j\ \mathrm{pairs}}({}^{ij}c - \overline{c})(H({}^{ij}\theta) - \overline{H})}{\sqrt{\sum_{i-j\ \mathrm{pairs}}({}^{ij}c - \overline{c})^2 \sum_{i-j\ \mathrm{pairs}}(H({}^{ij}\theta) - \overline{H})^2}}, $$

Jenet+ 2003

$$ S = \sqrt{\frac{M(M-1)/2}{1 + [\chi(1+\overline{\zeta}^2) + 2(\sigma_n/\sigma_g)^2 + (\sigma_n/\sigma_g)^4]/N\sigma_\zeta^2}}. $$

Noise dependent version

$$S = \sqrt{\frac{M(M-1)/2}{1 + [\chi(1+\overline{\zeta^2}) + 2(\sigma_n/\sigma_g)^2 + (\sigma_n/\sigma_g)^4]/N\sigma_\zeta^2}}.$$

$$\langle S \rangle \simeq \sqrt{M}\left[1 + \frac{\sum_{i-j\ \mathrm{pairs}}\left({}^{ij}A + {}^{ij}B\right)}{M\Sigma_{\mathrm{H}}^2}\right]^{-\frac{1}{2}},$$

Note: the new M=M(M-1)/2

$$\begin{aligned}
{}^{ij}A &= \frac{1}{m^2}\sum_{kk'}\left[\left(1 + H({}^{ij}\theta)\right)\gamma_{kk'}^2\right. \\
&\quad \left. + \left({}^{i}\eta_{\mathrm{r},kk'} + {}^{j}\eta_{\mathrm{r},kk'}\right)\gamma_{kk'} + {}^{i}\eta_{\mathrm{r},kk'}\,{}^{j}\eta_{\mathrm{r},kk'}\right],
\end{aligned}$$

$$\boxed{{}^{ij}B = \frac{1}{m}\left({}^{i}\eta_{\mathrm{w}} + {}^{j}\eta_{\mathrm{w}} + {}^{i}\eta_{\mathrm{w}}\,{}^{j}\eta_{\mathrm{w}} + {}^{i}\eta_{\mathrm{r}}\,{}^{j}\eta_{\mathrm{w}} + {}^{j}\eta_{\mathrm{r}}\,{}^{i}\eta_{\mathrm{w}}\right)}$$

$$\begin{aligned}
{}^{i}\eta_{\mathrm{r},kk'} &= \frac{{}^{i}\sigma_{\mathrm{r}}^2\,{}^{i}g_{kk'}}{\sigma_{\mathrm{g}}^2} \\
{}^{i}\eta_{\mathrm{r}} &= \frac{{}^{i}\sigma_{\mathrm{r}}^2}{\sigma_{\mathrm{g}}^2} \\
{}^{i}\eta_{\mathrm{w}} &= \frac{{}^{i}\sigma_{\mathrm{w}}^2}{\sigma_{\mathrm{g}}^2}.
\end{aligned}$$

What we have now?

$$ {}^{i}\sigma_{\mathrm{w}} = \left({}^{i}\sigma_0^2 \mathcal{G}^{-2} + {}^{i}\sigma_{\mathrm{J}}^2 \right)^{1/2} \left(\frac{{}^{i}\tau}{1\mathrm{hr}} \right)^{-1/2} $$

Just minimize this one:

$$ \mathcal{L} = \sum_{i-j \text{ pairs}} {}^{ij}B \,. $$

$$
\begin{aligned}
{}^{i}\eta_{\mathrm{r},kk'} &= \frac{{}^{i}\sigma_{\mathrm{r}}^2 \, {}^{i}g_{kk'}}{\sigma_{\mathrm{g}}^2} \\
{}^{i}\eta_{\mathrm{r}} &= \frac{{}^{i}\sigma_{\mathrm{r}}^2}{\sigma_{\mathrm{g}}^2} \\
{}^{i}\eta_{\mathrm{w}} &= \frac{{}^{i}\sigma_{\mathrm{w}}^2}{\sigma_{\mathrm{g}}^2} \,.
\end{aligned}
$$

Constrains

- For single telescope case, it is easy

$$\boxed{\tau} = \sum_{i=1}^{N} {}^{i}\tau$$

Multiple telescopes

- Phased array addition (LEAP)
  - Just like the single telescope case
- Incoherent addition (IPTA)
  - We need extra information about the visibility of telescope to pulsars.

$$\boxed{{}^iO_\nu} = \text{whether one use} \begin{pmatrix} \text{Psr 1, Tel. 1} | & \text{Psr 1, Tel. 2} \ \ldots | & \text{Psr 1, Tel. } \mathrm{N_{tel}} \\ \hline \text{Psr 2, Tel. 1} | & \text{Psr 2, Tel. 2} \ \ldots | & \text{Psr 2, Tel. } \mathrm{N_{tel}} \\ \hline \ldots & \ldots & \ldots \\ \hline \text{Psr N, Tel. 1} | & \text{Psr N, Tel. 2} \ \ldots | & \text{Psr N, Tel. } \mathrm{N_{tel}} \end{pmatrix}$$

$${}^iO_\nu = \begin{pmatrix} 0 & 1 & \ldots & 1 \\ 1 & 0 & \ldots & 0 \\ \ldots & \ldots & \ldots & \\ 1 & 0 & \ldots & 0 \end{pmatrix}$$

Multi-telescope time

$$
{}^{i}P_{\nu} = \text{obs time of} \left(\begin{array}{c|c|c}
\text{Psr 1, Tel. 1} & \text{Psr 1, Tel. 2} \;\ldots & \text{Psr 1, Tel. N}_{\text{tel}} \\ \hline
\text{Psr 2, Tel. 1} & \text{Psr 2, Tel. 2} \;\ldots & \text{Psr 2, Tel. N}_{\text{tel}} \\ \hline
\ldots & \ldots & \ldots \\ \hline
\text{Psr N, Tel. 1} & \text{Psr N, Tel. 2} \;\ldots & \text{Psr N, Tel. N}_{\text{tel}} \\ \hline
\end{array} \right)
$$

$$
\boxed{\tau_{\nu}} = \sum_{i=1}^{N} {}^{i}P_{\nu}\, {}^{i}O_{\nu}
$$

Full problem

$$\text{Optimal schedule} = \operatorname{argmin} \mathcal{L}(\text{telescope time allocation})$$

Under constrains:

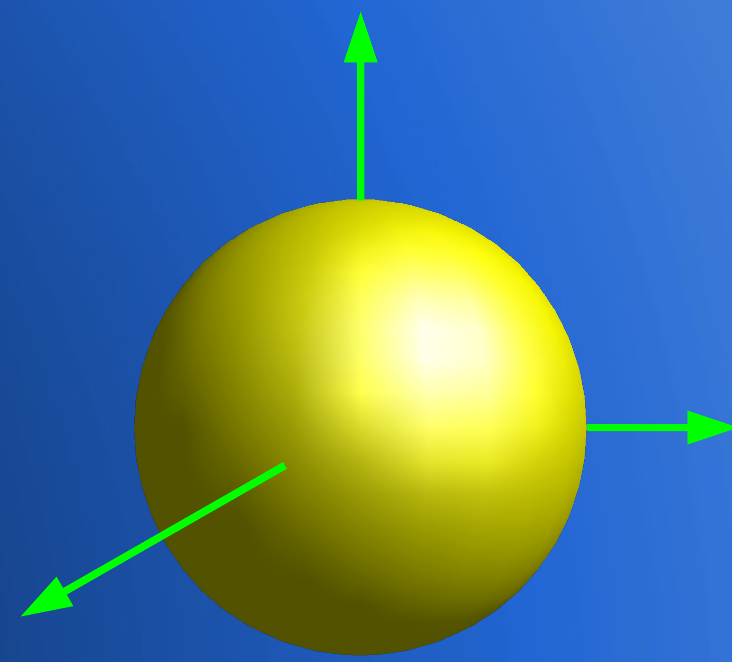
$$\boxed{\tau} = \sum_{i=1}^{N} {}^{i}\tau$$ τ>0

$$\boxed{\tau_\nu} = \sum_{i=1}^{N} {}^{i}P_\nu \, {}^{i}O_\nu$$ P>0

A trick

To be easy for numerical optimization:

$$R^2 = x^2 + y^2 + z^2$$

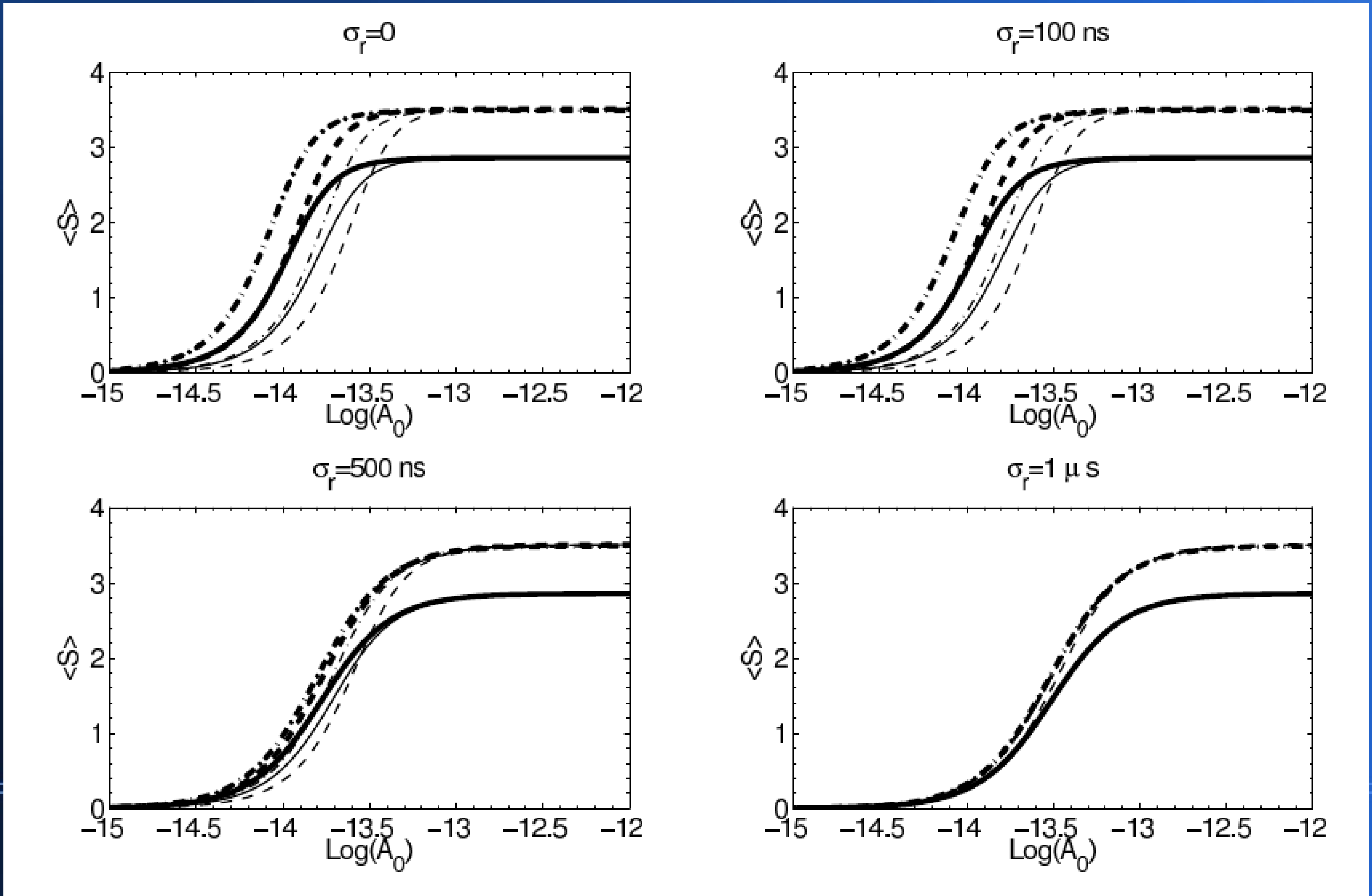


$$\tau = \sum_{i=1}^{N} {}^{i}\tau$$

$$\begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \cdots \\ \theta_{N-2} \\ \theta_{N-1} \end{pmatrix} = \begin{pmatrix} \arccos\left(\frac{\sqrt{{}^{1}\tau}}{\sqrt{\tau}}\right) \\ \arccos\left(\frac{\sqrt{{}^{2}\tau}}{\sqrt{\tau}\sin\theta_1}\right) \\ \arccos\left(\frac{\sqrt{{}^{3}\tau}}{\sqrt{\tau}\sin\theta_1\sin\theta_2}\right) \\ \cdots \\ \arccos\left(\frac{\sqrt{{}^{N-2}\tau}}{\sqrt{\tau}\prod_{i=1}^{N-3}\sin\theta_i}\right) \\ \arccos\left(\frac{\sqrt{{}^{N-1}\tau}}{\sqrt{\tau}\prod_{i=1}^{N-2}\sin\theta_i}\right) \end{pmatrix}$$

Results

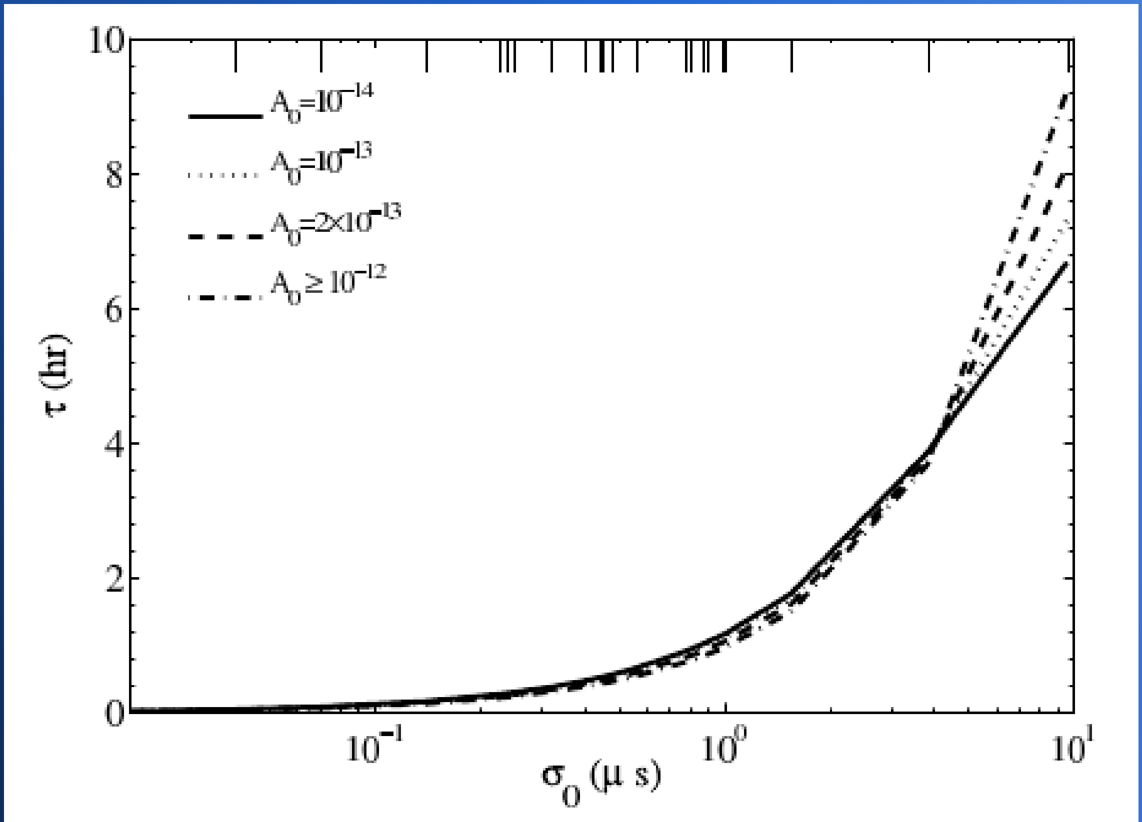
- How much improves?



How to distribute time (roughly)?

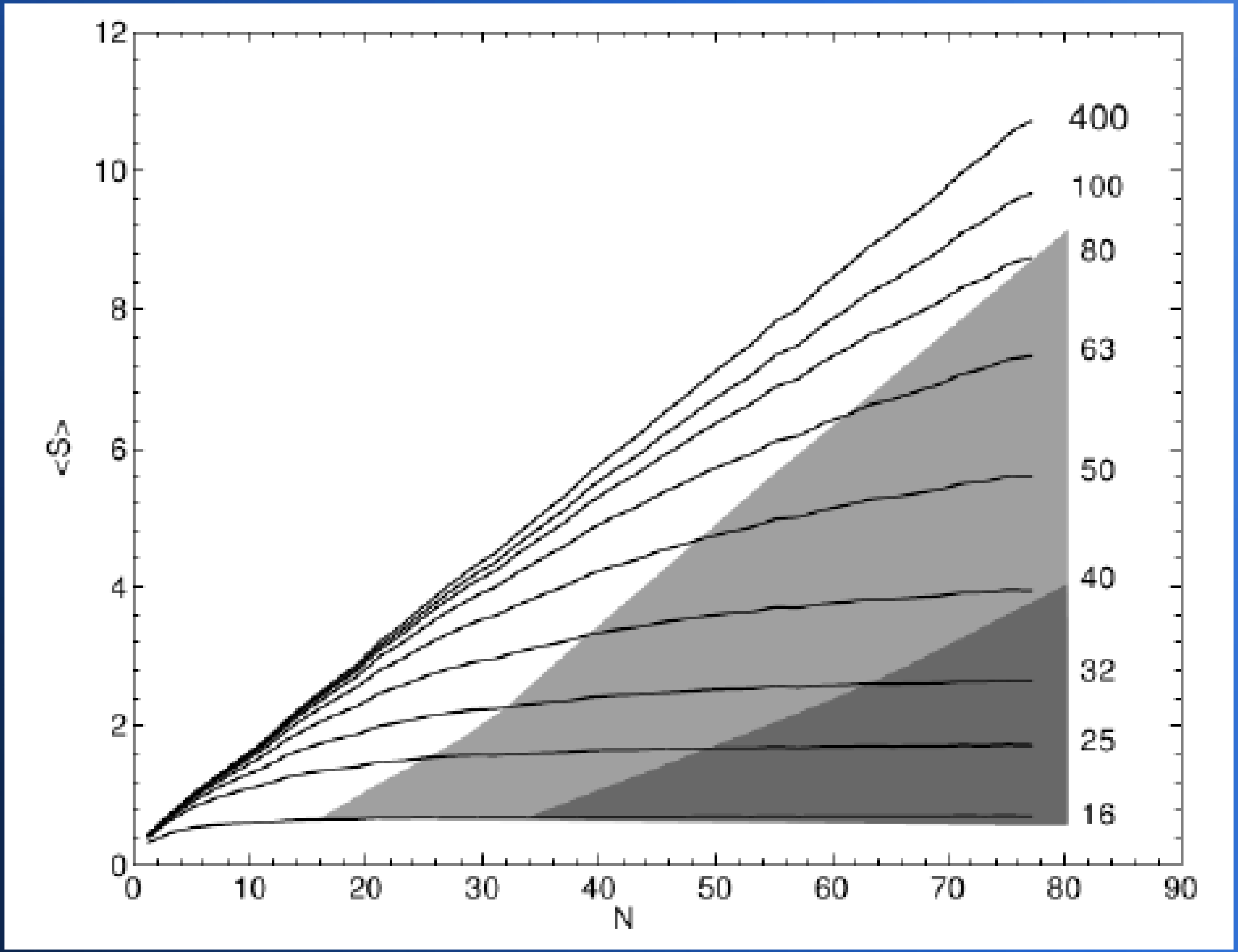
$$ {}^{i}\tau = \tau \left(\frac{\sqrt{{}^{i}Q}}{\sum_{j=1,\neq i}^{N} \sqrt{{}^{i}Q}} \right) $$

$$ {}^{i}Q = {}^{i}\sigma_0^2 \mathcal{G}^{-2} + \sigma_{\mathrm{J}}^2 $$



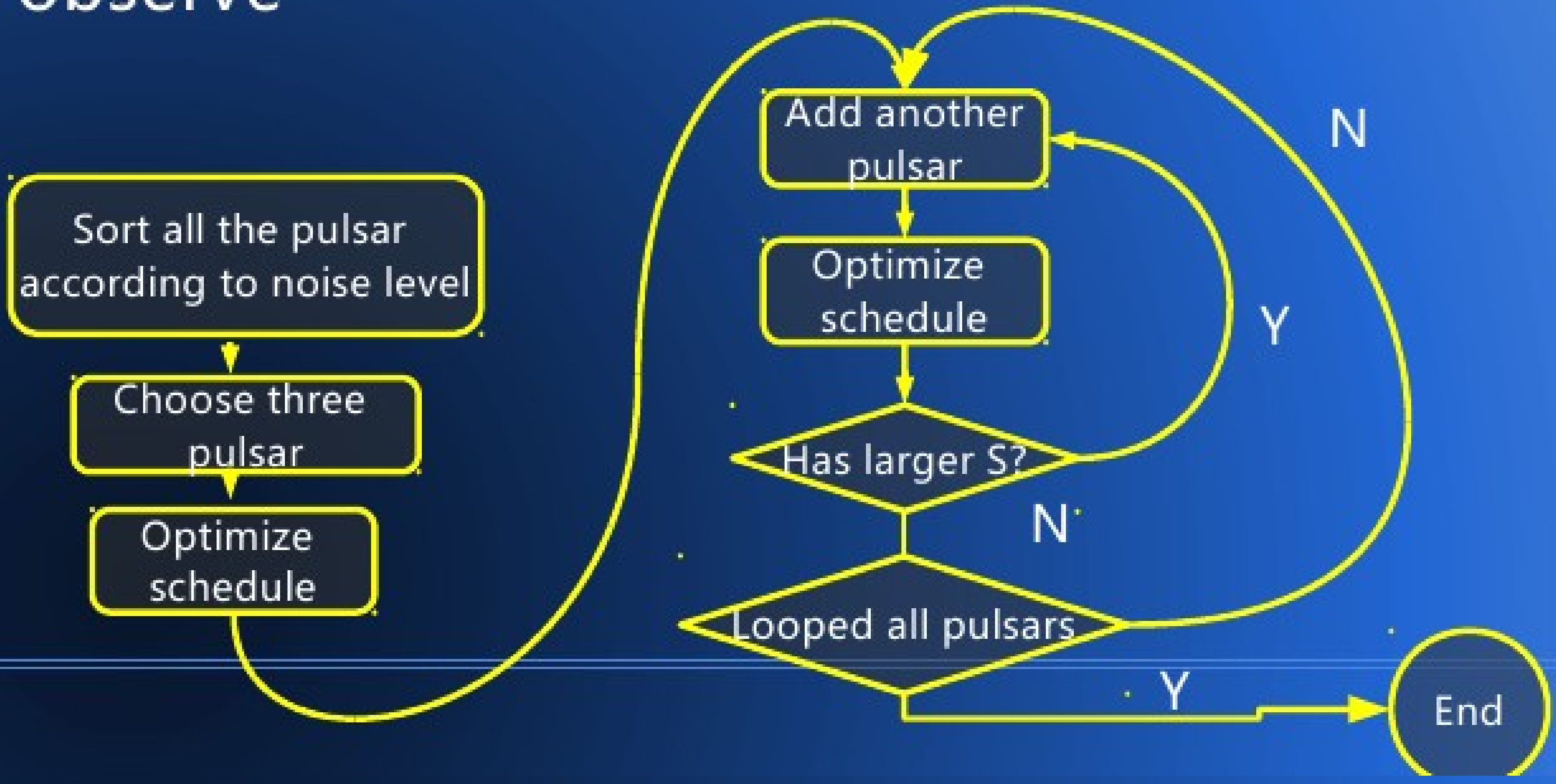
Should we be greedy?

- No!



To get best set of pulsars to obs

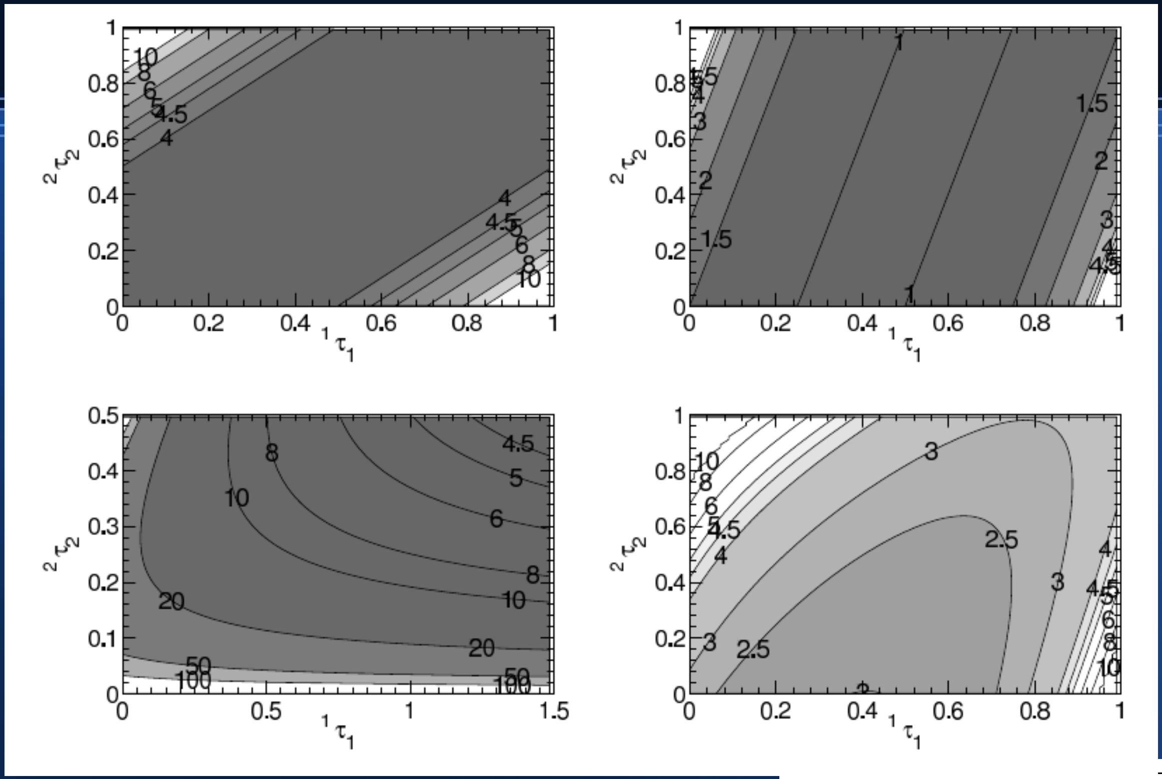
- Algorithm to find the pulsar samples to observe

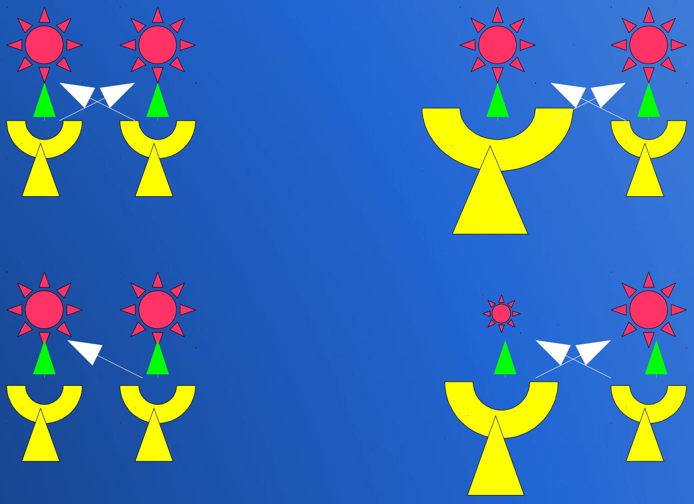


Freedom in optimization

- If two telescope can observe a same pulsar, one can exchange observing time on this object, as far as noise level is kept un-changed.
- Due to the limited telescope time or jitter noise, sometime, this is not possible.

Example of exchanging telescope time



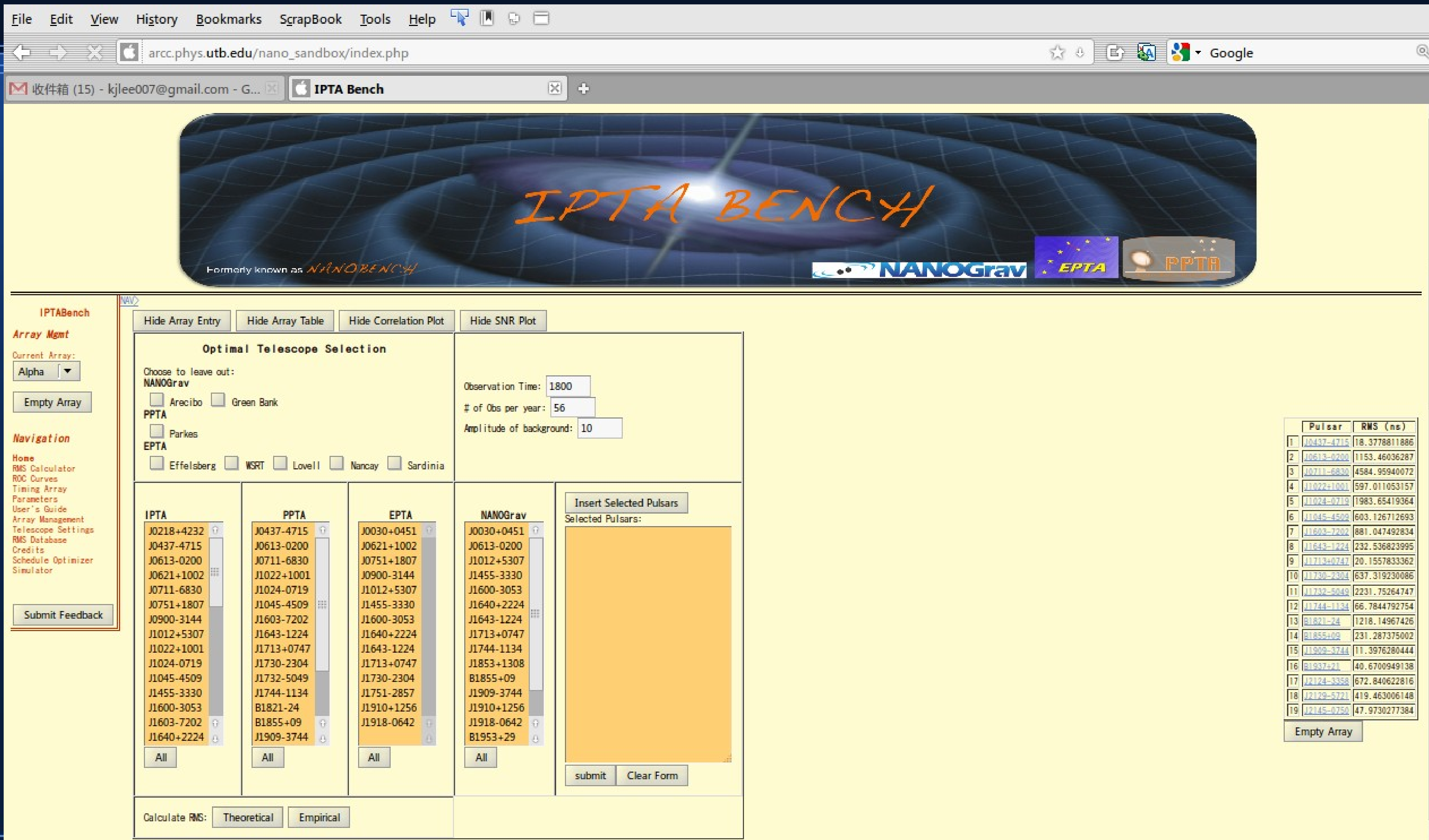


	case A:		case B:	
Radiometer noise	${}^1\sigma_0 = 10^{-7}$	${}^2\sigma_0 = 10^{-7}$	${}^1\sigma_0 = 10^{-7}$	${}^2\sigma_0 = 10^{-7}$
Jitter noise	${}^1\sigma_J = 0$	${}^2\sigma_J = 0$	${}^1\sigma_J = 0$	${}^2\sigma_J = 0$
Observation time	$\tau_1 = 1$	$\tau_2 = 1$	$\tau_1 = 1$	$\tau_2 = 1$
Telescope gain	$\mathcal{G}_1 = 1$	$\mathcal{G}_2 = 1$	$\mathcal{G}_1 = 2$	$\mathcal{G}_2 = 1$
Resource allocation matrix	${}^iO_\nu = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$		${}^iO_\nu = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$	
	case C:		case D:	
Radiometer noise	${}^1\sigma_0 = 10^{-7}$	${}^2\sigma_0 = 10^{-7}$	${}^1\sigma_0 = 10^{-7}$	${}^2\sigma_0 = 10^{-7}$
Jitter noise	${}^1\sigma_J = 0$	${}^2\sigma_J = 0$	${}^1\sigma_J = 10^{-7}$	${}^2\sigma_J = 0$
Observation time	$\tau_1 = 1.5$	$\tau_2 = 0.5$	$\tau_1 = 1$	$\tau_2 = 1$
Telescope gain	$\mathcal{G}_1 = 1$	$\mathcal{G}_2 = 1$	$\mathcal{G}_1 = 2$	$\mathcal{G}_2 = 1$
Resource allocation matrix	${}^iO_\nu = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$		${}^iO_\nu = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$	

Recent progress

- Jose Martinez (UTB) developed a web framework to the tools.

Homepage



Http://arcc.phys.utb.edu/nano_sandbox

Optimizer

Pulsar	RMS Receiver (s)
J0437-4715	1.29932154577E-8
J0613-0200	8.15518301099E-7
J0711-6830	3.24205532452E-6
J1022+1001	4.22150459676E-7
J1024-0719	1.40265529668E-6
J1045-4509	4.20195059321E-7
J1603-7202	6.22909451887E-7
J1643-1224	1.18078519387E-7
J1713+0747	1.4215821398E-8
J1730-2304	4.50606369605E-7
J1732-5049	1.57801577542E-6
J1744-1134	4.72237544937E-8
B1821-24	8.57120555888E-7
B1855+09	1.63508764021E-7
J1909-3744	8.03559198668E-9
B1937+21	5.88015198188E-9
J2124-3358	4.75770157565E-7
J2129-5721	2.96419827752E-7
J2145-0750	3.59195105536E-8

The calculations was done with the following parameters
Tsys=10K, BW=100MHz, T=1hr, G=1 K/Jy

Pulsars	Arecibo [-1,38]	GreenBank [-48,90]	Parkes [26,90]	Effelsberg [-30,90]	WSRT [-30,90]	Lovell [-35,90]	Nancay [-39,90]	Sardinia [-48,90]
J0437-4715			X					
J0613-0200		X	X	X	X	X	X	X
J0711-6830			X					
J1022+1001	X	X	X	X	X	X	X	X
J1024-0719		X	X	X	X	X	X	X
J1045-4509		X	X					X
J1603-7202			X					
J1643-1224		X	X	X	X	X	X	X
J1713+0747	X	X	X	X	X	X	X	X
J1730-2304		X	X	X	X	X	X	X
J1732-5049			X					
J1744-1134		X	X	X	X	X	X	X
B1821-24	X	X	X	X	X	X	X	X
B1855+09	X	X	X	X	X	X	X	X
J1909-3744		X	X				X	X
B1937+21	X	X	X	X	X	X	X	X
J2124-3358		X	X			X	X	X
J2129-5721			X					
J2145-0750		X	X	X	X	X	X	X

The total number of Pulsars is: 19

Observations Interval

Interval(days): 7
Duration (in weeks): 128
GW Characteristic Strain: -12

Telescope and Information

Telescope	Bandwidth [MHz]	System Temperature	Gain	Telescope Time [hrs]
Arecibo	300	30	13.2560625	24
GreenBank	8	20	1.99500	24
Parkes	100	25	0.700416	24
Effelsberg	112	24	1.53900	24
WSRT	160	29	1.4183424	24
Lovell	100	30	0.91015947	24
Nancay	128	35	1.2087648	24
Sardinia	100	25	0.700416	24

Results

Pulsars	Arecibo [-1,38]	GreenBank [-46,90]	Parkes [26,90]	Effelsberg [-30,90]	▮SRT [-30,90]	Lovell [-35,90]	Nancay [-39,90]	Sardinia [-46,90]
J0437-4715	0	0	0.06410933	0	0	0	0	0
J0613-0200	0	0.018024439	1.2726761e-05	16.426005	0.14982657	1.3043962	0.76609872	0.52646929
J0711-6830	0	0	12.432979	0	0	0	0	0
J1022+1001	4.7378995	0.0015524928	0.00046242621	1.5902349	0.014424352	2.0207003	2.4851966	0.0012532056
J1024-0719	0	0.04828203	6.691849e-06	4.678531	21.862225	6.2709268	3.7176933	1.0907023
J1045-4509	0	0.16901338	0.00041617707	0	0	0	0	22.347055
J1603-7202	0	0	2.9745556	0	0	0	0	0
J1643-1224	0	0.090604389	1.8793821e-06	0.4250262	0.14292907	6.0607324	0.36335848	0.0025097205
J1713+0747	10.591028	0.077536229	0.0002187739	0.073336972	0.013397967	2.9703166	0.035770846	0.0052676063
J1730-2304	0	0.061394913	1.3835251e-05	0.43226087	0.59579517	0.1333202	16.167427	0.007142793
J1732-5049	0	0	7.080247	0	0	0	0	0
J1744-1134	0	0.59852718	0.00085942536	0.042728901	0.46529539	0.46159167	0.32449194	0.0026027531
B1821-24	2.099218	0.00011005075	0.00027124723	0.14731965	0.16890144	1.4448698	0.1397908	0.0092290306
B1855+09	4.5820845	0.14778342	0.00053464003	0.0023261213	0.47855517	0.0086347036	0.00012167049	0.0039810775
J1909-3744	0	0.31997312	8.9891706e-05	0	0	0	3.3826808e-05	0.0002286825
B1937+21	1.9897699	0.14207766	3.9495733e-05	0.16162771	0.030899688	1.1400578	1.2794295e-05	0.0023424175
J2124-3358	0	22.311441	0.0012904238	0	0	0.15454382	3.7071969e-07	0.0011941498
J2129-5721	0	0	1.4428694	0	0	0	0	0
J2145-0750	0	0.013679377	0.0010216113	0.020603164	0.077750246	2.0299097	3.597583e-06	2.153196e-05

Simulator

Gravitational Wave

Observation Schedule

Ao: -14

alpha: 0.666667

$l-1=0$

Interval(days):

Duration (in weeks):

GW Characteristic Strain: -12

Pulsars Info

Pulsar	White Noise Level	Red Noise Characteristic Amplitude	Red Noise Power Spectrum Index
J0437-4715	18.3778811886		
J0613-0200	1153.46036287		
J0711-6830	4584.95940072		
J1022+1001	597.011053157		
J1024-0719	1983.65419364		
J1045-4509	603.126712693		
J1603-7202	881.047492834		
J1643-1224	232.536823995		
J1713+0747	20.1557833362		
J1730-2304	637.319230086		
J1732-5049	2231.75264747		
J1744-1134	66.7844792754		
B1821-24	1218.14967426		
B1855+09	231.287375002		
J1909-3744	11.3976280444		
B1937+21	40.6700949138		
J2124-3358	672.840622816		
J2129-5721	419.463006148		
J2145-0750	47.9730277384		

The calculations was done with the following parameters
Tsys=10K, BW=100MHz, T=1hr, G=1 K/Jy

The total number of Pulsars is: 19

J0030+0451.par

```
PSRJ J0030+0451
RAJ 00:30:27.4303000 1
DECJ +04:51:39.74000 1
UNITS TCB
F0 205.5306960881271075 1
F1 -4.29762032062e-16 1
PEPOCH 53000
POSEPOCH 53000
PMRA -5.2999999178224523112 1
PMDEC -1.9999999689896046457 1
PX 4.1666666666700000001 1
EPHVER 5
CLK UNCORR
EPHEM DE414
NITS 1
```

Submit

J0218+4232.par

```
PSRJ J0218+4232
RAJ 02:18:06.3510000 1
DECJ +42:32:17.43000 1
UNITS TCB
F0 430.46105967130733716 1
F1 -1.43399281654e-14 1
PEPOCH 53000
POSEPOCH 53000
PX 0.17094017094000000001 1
BINARY ELL1
PB 2.0288461154576600766 1
A1 1.9844400307691349556 1
TASC 49150.608918133652868 1
EPS1 -2.2e-05 1
EPS2 2.9999999999999999999e-06 1
EPHVER 5
CLK UNCORR
EPHEM DE414
```

Submit

Much more

- Noise calculator (together with Ryan Shannon, Delphine Perrodin etc.)
- GW detection sensitivity, ROC, etc..

Discussions

- Not all the noise parameters we need are measured, e.g. jitter noise, red noise parameters for each pulsar.
- We just tried to build a framework to do optimization, where we optimize the time respected to sensitivity.
- Many alternatives:
  - Single source detection (e.g. Burt+2011), hot-spot issue, alternative modes of gravity, graviton mass
  - Optimize the 'cost'
  - Other detection algorithms (too many to list here!)
  - Other aims, pulsar time scale, solar system ephemeris
- Session splitting, management, observation redundancy (freq, time, geographical issues)

Other goodies in that paper (in the appendix)

- We analytically investigated the effects of the non-stationary red noise due to the p and pdot fitting as also noticed by Coles and van Haasteren.

$$
\begin{aligned}
\sigma'^2 &= S_0 T^{\beta-1} \Bigg[\\
&- \sum_{k=n+1}^{\infty} \frac{(2\pi)^{2k}(-1)^{k+n} f_{0,\mathrm{L}}^{2k+1-\beta}(1+n)\Gamma(k)\sin(\pi\beta)}{(1+2k-\beta)\Gamma(2+2k)\Gamma(k-n)\Gamma(2+k+n)} \\
&+ 2^{\beta-2}(1+n)\pi^{\beta-1} \frac{\Gamma(\frac{3+2m-\beta}{2})\Gamma(\frac{\beta-1}{2})\Gamma(\frac{1+\beta}{2})}{\Gamma(\frac{3+2n+\beta}{2})\Gamma(1+\beta)} \Bigg] \quad \text{(A10)}
\end{aligned}
$$

- Clock noise issues
- Approximated solution to optimization

Thanks.

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The Max-Planck institute für Radioastronomie (MpiFr) seeking for PhD students. Various astronomical research topics are available. Please check

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Or just send emails to ask.

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- Galactic masers: nature and application to astrophysics
- Intra-day variability
- Infrared interferometry of disks and jets of young stars
- Very-Long-Baseline Interferometry development
- Interstellar matter in galaxies
- Infrared interferometry of AGN
- Strong and weak gravitational lensing
- Galactic and extragalactic magnetic fields
- High precision astrometry
- Envelopes of evolved stars
- Radiative transfer modeling
- Star formation in the Milky Way and other galaxies
- Astro-particle physics
- Super massive binary black holes in AGN
- Observational cosmology
- Stellar populations
- Star clusters
- Satellite galaxies
- Galactic dynamics
- Pulsars and Neutron Stars
- Experimental gravitational physics
- Transient radio sky
- Evolution of massive binaries and their manifestations