Pulsar Intrinsic Noise in Precision Timing

Jim Cordes (Cornell)

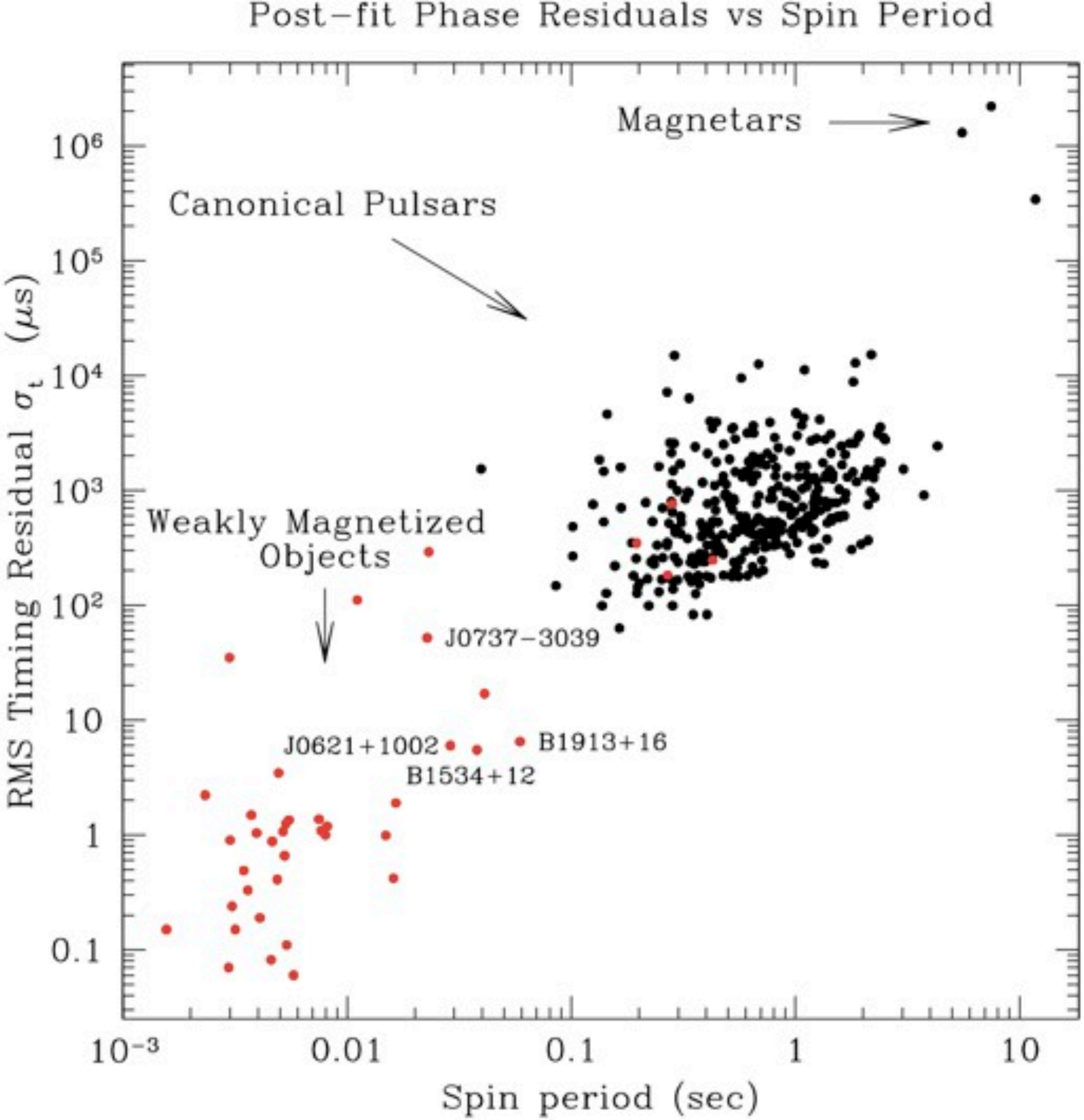
Red spin noise and pulse-shape variations

Diagnosis for PTA applications

Next steps

Fundamentals of Pulsar Timing

Clock Mechanism	Neutron Star Spin	• Differential rotation • Crust quakes • Torque variations in magnetosphere
Clock Ticks	Beamed radio emission at few x R_{NS}	• Variation of emission altitude with frequency • Temporal variations (phase jitter)
Modification by ISM	Cold plasma dispersion law + Faraday rotation (deterministic &	• Dispersive delays • Refraction + diffraction • Grav. Lensing, MW acceleration negligible
Telescope effects	Instrumental polarization, Radiometer noise	• TOA correction to SSBC • Time transfer



Post−fit Phase Residuals vs Spin Period
Magnetars
Canonical Pulsars
Weakly Magnetized Objects
J0737−3039
J0621+1002
B1913+16
B1534+12
RMS Timing Residual σ_t (μs)
Spin period (sec)
10^6
10^5
10^4
10^3
10^2
10
1
0.1
10^{-3}
0.01
0.1
1
10

Pulsar Intrinsic Causes of Timing Variations

Variations in Spin Rate	Clock Tick Variations
Glitches: • Discontinuities in $\nu, \dot{\nu}$ • Signature: (+, −) • NS crust + superfluid; crustquakes?	**Fast:** • Pulse to pulse amplitude/phase jitter • Drifting subpulses
Stochastic red noise: CPs: • Jumps in $\nu, \dot{\nu}$ • Signatures: (+,−), (−,+), … • Jump rates: small, large • NS+magnesphere •MSPs: ~ two objects + upper bounds	**Slow**: • Profile mode changes • Nulling **Slower:** • ~ month like intermittency • Pulse shape changes correlated $\Delta\dot{\nu}$

Also: pulse profile variations with frequency ("profile evolution")

Intrinsic Variations

- All of these variations have rms variations that are either achromatic or weakly chromatic
- Most are characterized by studies of canonical pulsars (P > 20 ms, B~10^{12} G)
- A few MSPs show red spin noise while others have upper limits (some not constraining)
- Pulse shape variations in MSPs not well studied
- “Timing noise” traditionally has meant “red spin noise” but recent usage has included any timing error
- → Deprecate “timing noise” and specify “red spin noise,” “white radiometer noise,” etc.

Spin Noise in Canonical Pulsars

- First recognized in the Crab pulsar
  - Boynton et al. 1972; Groth 1975; JMC 1980
  - Once mistaken for a planetary signature
  - Statistically similar to a random walk in spin frequency (T < 10 yr)
- Most CPs show red spin noise either as a smooth stochastic process or as a series of discrete steps in $\nu, \dot{\nu}$
  - Wide range of apparent rates (~1/week, 1/month)
  - Signatures are different from glitches
  - Power spectra $S(f) \sim f^{-\alpha}$, $\alpha \sim 4$ to 6
  - Some indication of quasiperiodicity
- "Intermittent pulsars:" smoking gun for correlation of discrete spin states with radio emission (Kramer + 2006)
- Discrete spin states (~ months) correlated with pulse shape states (Lyne et al. 2010)
  - Prospect of timing corrections based on pulse shapes

Characterizing Spin Noise in MSPs

- Red noise “obvious” in only two MSPs as of 2010
- Why is this? Might RN in MSPs be different than in CPs? Perhaps nonexistent in some MSPs?
- Global fit to CPs, MSPs, CPs+MSPs (including upper limits; Shannon & JMC 2010)

$$\hat{\sigma}_{\mathrm{RN},2} = C_2 \nu^{\alpha} |\dot{\nu}|^{\beta} T^{\gamma}$$

$$\hat{\sigma}_{\mathrm{RN},2} = 10^{0.7\pm0.17} \mu s \times \nu^{-1.4\pm0.1} |\dot{\nu}|^{1.1\pm0.1} T^{2\pm0.2}$$

There is significant spread (x5) about this best fit scaling law (→ additional parameters? NS mass? Temperature?)

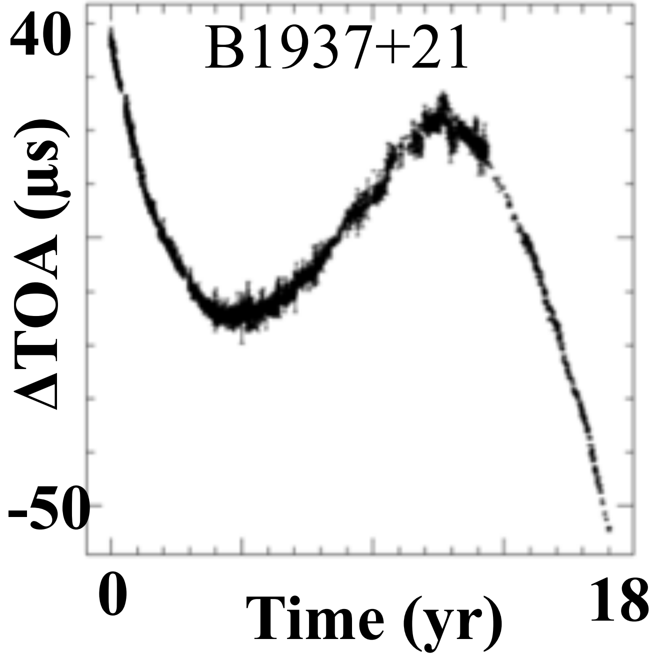
The scaling

$$\hat{\sigma}_{\mathrm{RN},2} \propto T^{2\pm0.2}$$

corresponds to a power spectrum

$$S_{RN} \propto f^{-5\pm0.4}$$

Compare with $f^{-4.3}$ spectrum for GW stochastic bg



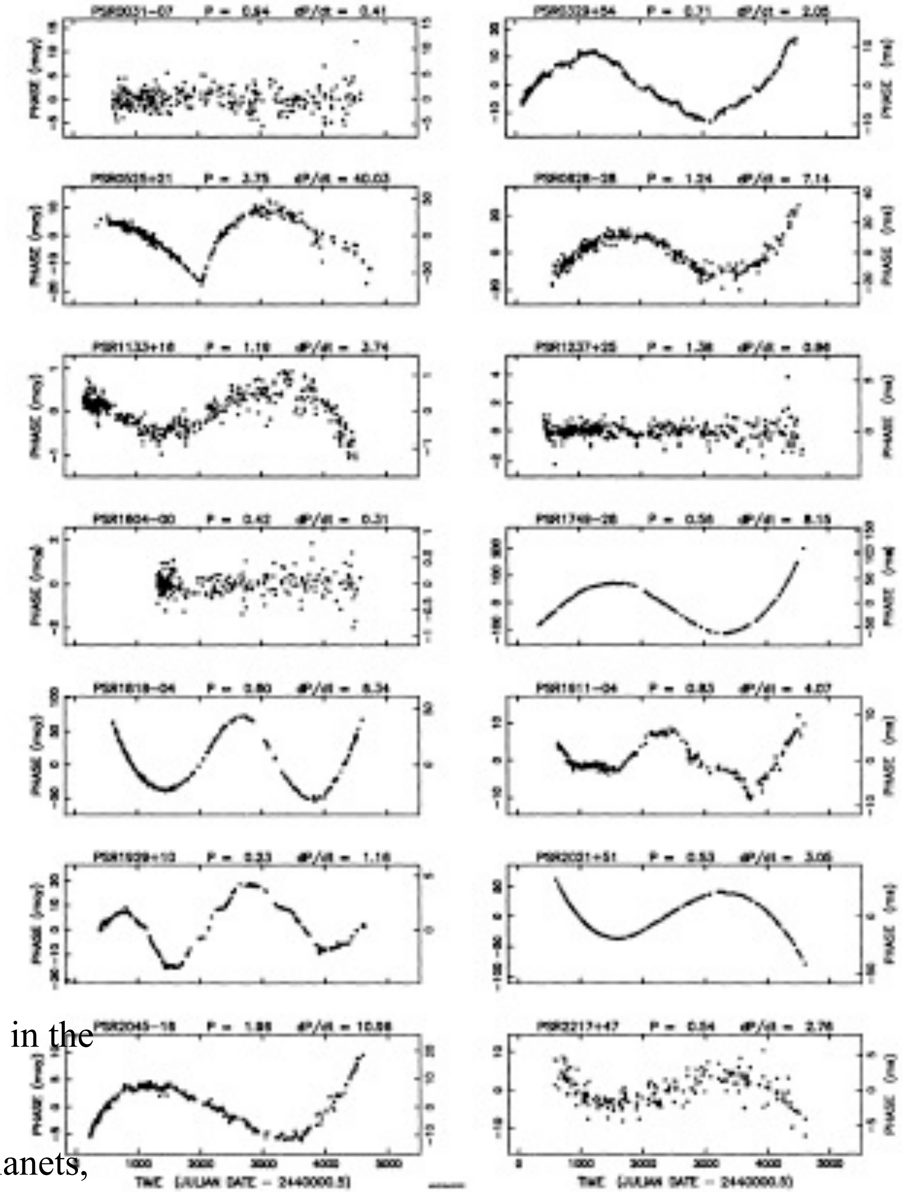


FIGURE I Phase residual curves $\mathcal{R}_2(t)$ for 14 pulsars from the JPL sample of Downs and Reichley (1983). Spin periods P (seconds) and derivatives $\dot{P}$ (in units of 10^{-15} s s^{-1}) are shown at the top of each panel.

SC10: scaling law for MSPs + CPs:

$$\boxed{\hat{\sigma}_{\mathrm{TN},2} = C_2 \nu^{\alpha} |\dot{\nu}|^{\beta} T^{\gamma}}$$

$$\boxed{\alpha = -1.4;\ \beta = 1.1;\ \gamma = 2.0}$$

For these pulsars, the residuals are mostly caused by spin noise in the pulsar:
torque fluctuations crust quakes superfluid-crust interactions
Other pulsars: excess residuals are caused by orbital motion (planets, WD, NS), ISM variations;
Potentially: BH companions, gwaves, etc.

Characterizing Spin Noise in MSPs

- Simplest interpretation (Occam):
  - Spin noise occurs in all objects according to their spin parameters (± spread)
- Red noise expected in all MSPs as better TOAs and longer spans T accrue
- Recent studies (since 2010) show the presence of "non-white" residuals in some PTA pulsars
  - PPTA objects (Hobbs et al)
  - NANOGrav objects (Demorest+ 2012; Perrodin+ 2012; Ellis+ 2012)
  - "non-white" ≠ red noise
  - Spectral characterization difficult at this stage

→ Suggests that "red noise" must be taken into account in statements of PTA sensitivity and forecasts of detection

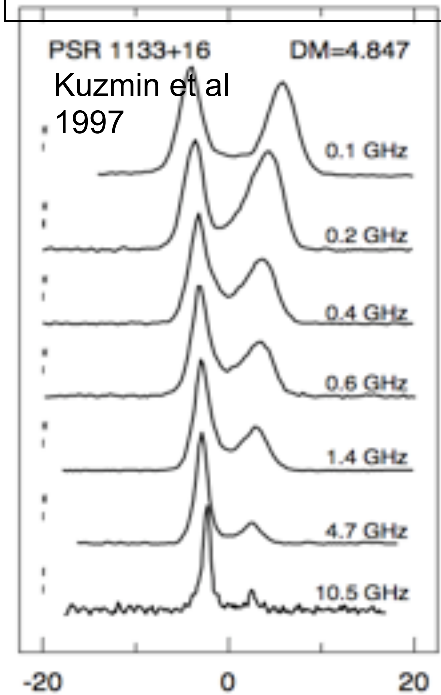
Open Questions about Spin Noise

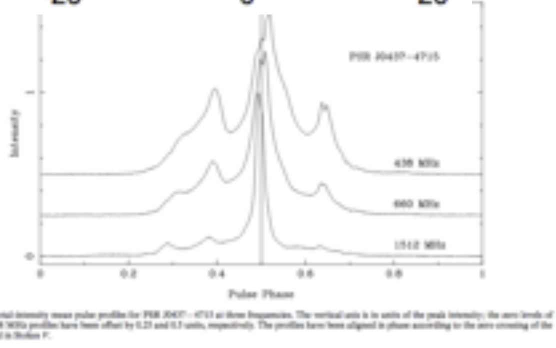
- How well can torque changes be modeled using pulse-shape changes in CPs?
- Does the pulse-shape/nu-dot correlation occur in MSPs? If so, can timing be corrected significantly?
- Is red noise in MSPs due to the same physical mechanisms as in CPs?
    - Pro: global fit unifies CPs & MSPs
    - Con: magnetospheres of MSPs and spin-down rates much smaller, so event triggers may differ
- Is the best mitigation method for red noise in MSPs simply <u>pulsar triage</u> after initial diagnosis?

Intrinsic Pulse-Shape Effects

Profile evolution vs ν

Kuzmin et al 1997





Polarization
(requires proper calibration)

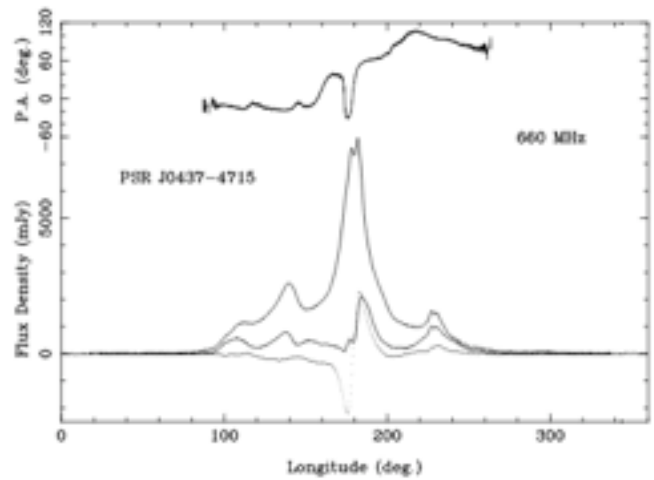
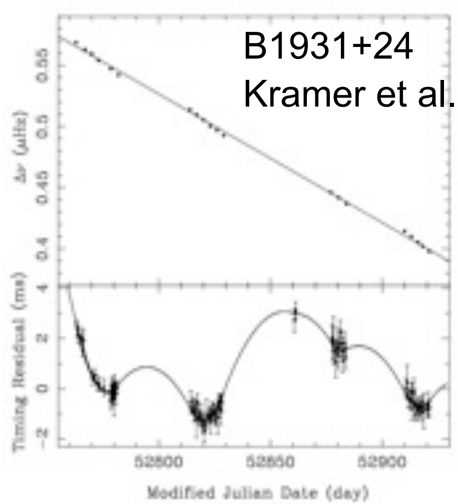


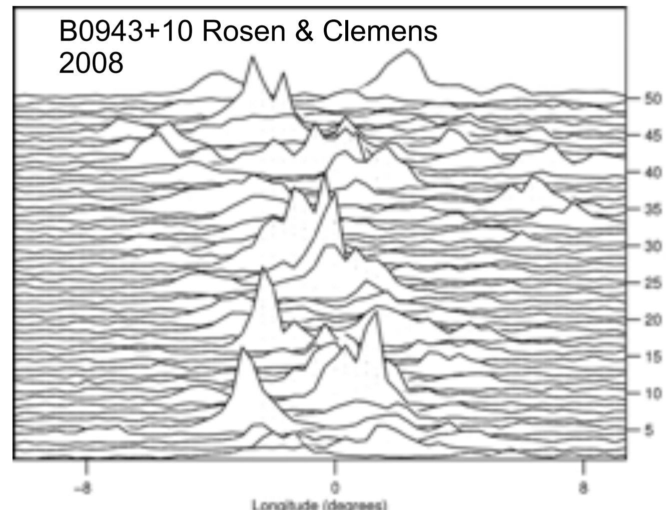
FIG. 2.—Mean pulse profile and polarization properties for PSR J0437−4715 at 660 MHz. See the caption to Fig. 1 for details.

B1931+24
Kramer et al. 2006



Phase jitter + amplitude modulations

B0943+10 Rosen & Clemens 2008



J1518+4904
Edwards & Stappers (2003)

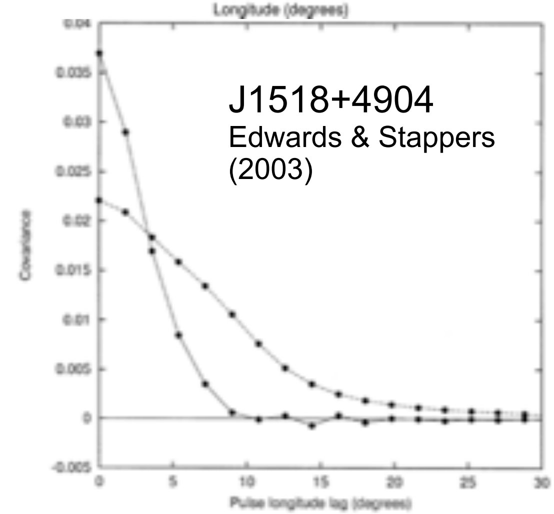


Fig. 9. Single-pulse ACF (solid line) and ACF of the average profile (dashed line) for PSR J1518+4904 at 1355 MHz.

Fast Pulse Shape Variations

- Well known since the discovery of pulsars
  - Mode changes (bi/tri-state integrated profiles)
  - Pulse to pulse phase/amplitude jitter
  - Drifting subpulses
  - Nulling (turn off of radio emission)
- TOA variations:
  - Mode changes: predictable offsets from fiducial phase
  - Jitter: broadband, stochastic variations
    - Single pulses long known to be broadband

“Mode” changes & Timing

B0329+54: well known three modes

- Discrete, fast changes between modes
- Timing offsets associated with each mode

Principal component analysis:
•Scatter plot of eigenvector dot products reveals the modes

Pulse-phase Jitter

- All well-studied canonical pulsars show ~100% variations in pulse phase and amplitude
    - intensity modulation index $m_I = \sigma_I/I \sim 1$
    - phase variations ~ widths of single pulses
- Crab giant pulses: jitter ~ 10x width at 1 GHz
- Millisecond pulsars:
    - J0437-4715: typical jitter (Jenet+ 1998; Oslowski+ 2011; Liu+2012)
    - J1713+0747: typical jitter Shannon et al., submitted
    - B1937+21 (aka J1939+2134) (Jenet+ 1998):
        - normal pulses show very little jitter
        - giant pulses show typical amounts of jitter
- J1518+4904 (41ms) (Edwards & Stappers 2003)
- B1534+12 (Stairs 2011, Snowshoe meeting)

δ[Pulse Shape] → δTOA

Measured profile

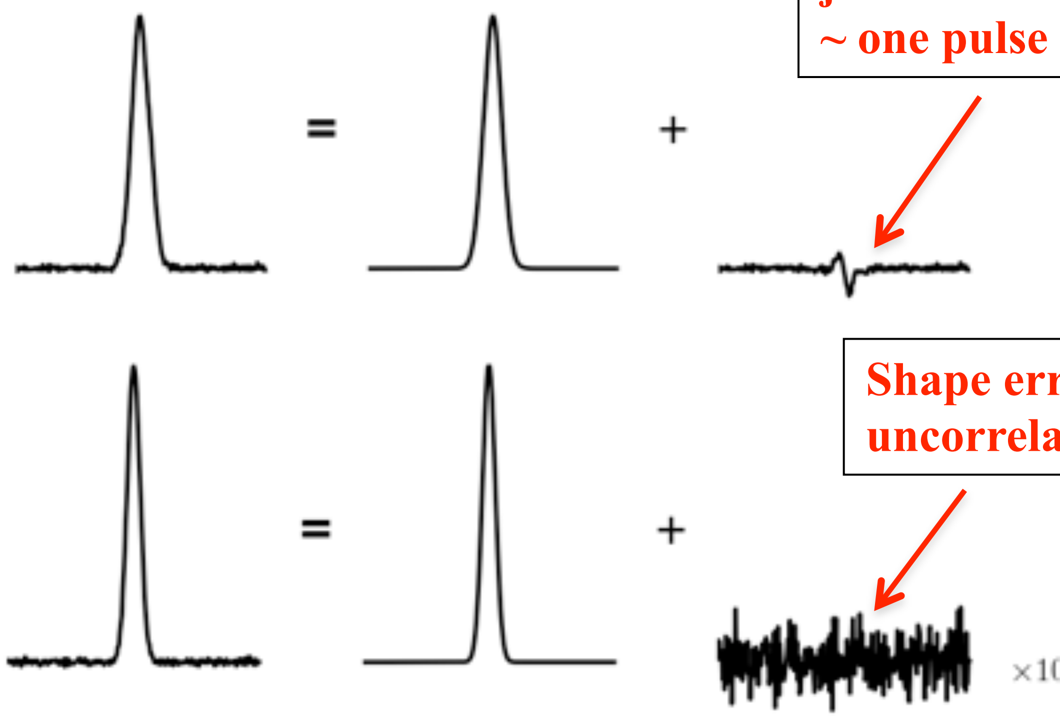
Template

Shape change from jitter correlated by ~ one pulse width

Jitter + additive noise

Additive noise only

Shape errors uncorrelated



The standard error σ_{fit} from template fitting is typically based on assuming white noise only (diagonal covariance matrix).
In practice, the TOA error σ_{TOA} is larger than σ_{fit}

CPs: rms white-noise > template fitting error,

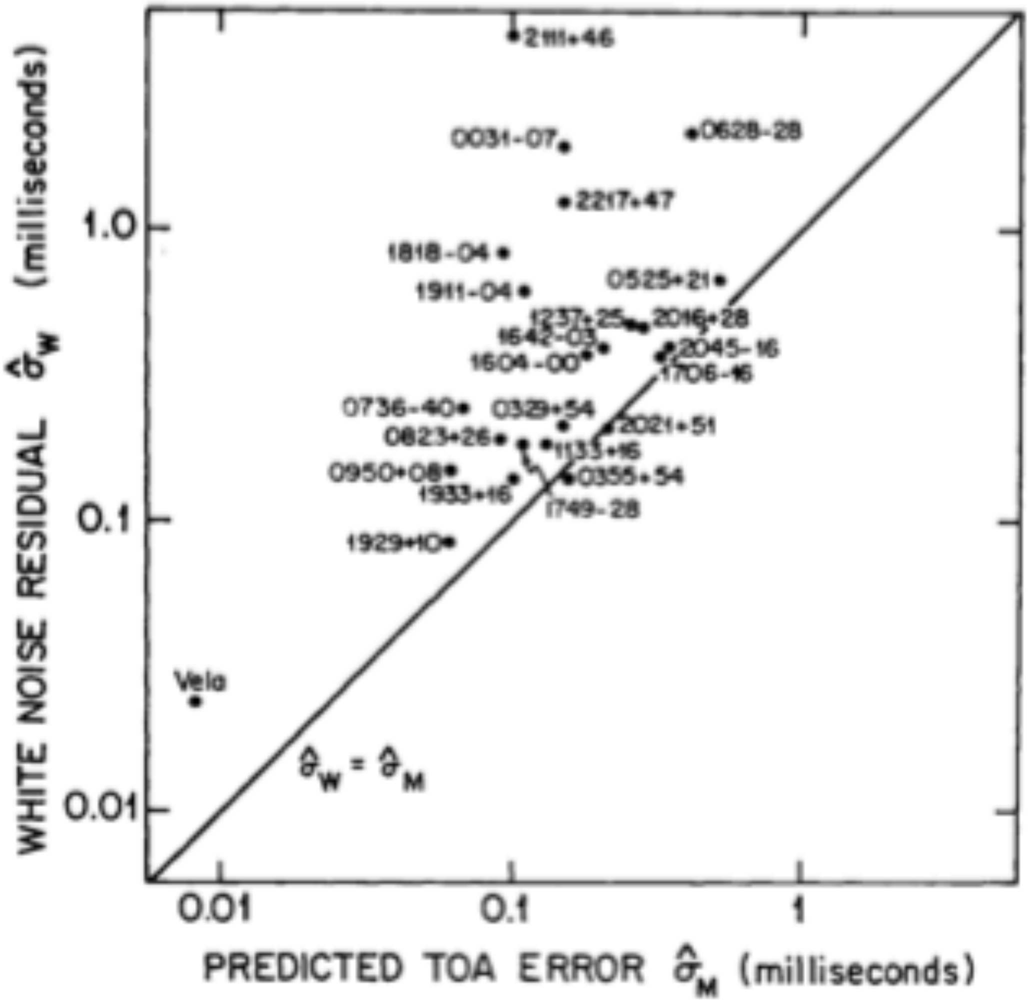


FIG. 3.—Root mean square white-noise residual from eq. (12) vs. rms TOA error due to additive noise from eq. (7)

Cordes & Downs 1985

Nearly all pulsars in the JPL sample show white-noise TOA errors > radiometer noise error

Interpreted as due to jitter.

Can TOA jitter be corrected in MSPs?

- Requires a significant correlation between a pulse-shape parameter and δTOA
  - For correlation coefficient ρ reduction in rms TOA is $\sigma_{TOA,\,c} = \sigma_{TOA}\,(1-\rho^2)^{1/2}$
  - e.g. need $\rho \geq 0.71$ to get reduction by factor ≤ ½
- J0437-4715: PCA analysis (Oslowski+2011) shows ~20% correctability
- A “large” correlation coefficient seems to require an asymmetric phase jitter distribution; but generally ρ may not be large enough

Timing Error from Pulse-Phase Jitter

$$U(\phi) \propto \int d\phi' \, f_\phi(\phi') a(\phi - \phi')$$

$$\begin{aligned}\Delta t_{\rm J} &= N_i^{-1/2}(1+m_I^2)^{1/2} P \langle \phi^2 \rangle^{1/2} \\ &= N_i^{-1/2}(1+m_I^2)^{1/2} P \left[\int d\phi \, \phi^2 f_\phi(\phi) \right]^{1/2}\end{aligned}$$

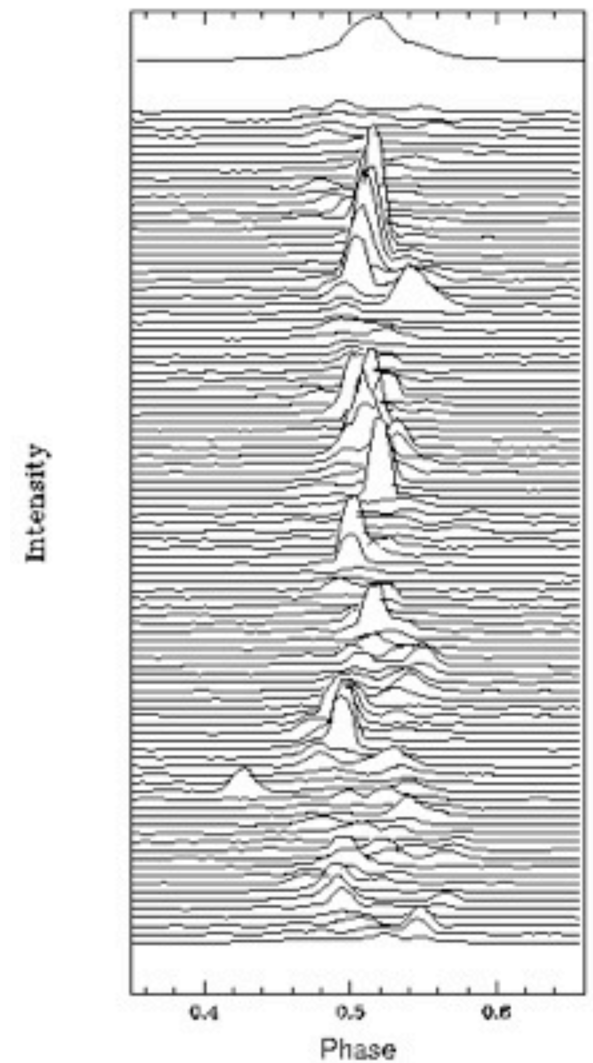
- f_ϕ = PDF of phase variation
- $a(\phi)$ = individual pulse shape
- N_i = number of independent pulses summed
- m_I = intensity modulation index ≈ 1
- f_J = fraction jitter parameter = ϕ_{rms} / W ≈ 1

Gaussian shaped pulse:

$$\Delta t_{\rm J} = \frac{f_J W_i (1+m_I^2)^{1/2}}{2(2N_i \ln 2)^{1/2}}$$

$N_6 = N_i / 10^6$

$$\boxed{\Delta t_{\rm J} = 0.28 \mu s \, W_{i,\rm ms} N_6^{-1/2} \left(\frac{f_J}{1/3}\right) \left(\frac{1+m_I^2}{2}\right)^{1/2}}$$



Jitter Analysis for J1713

Actual

Predicted
fitting error

Single pulses not needed to characterize jitter

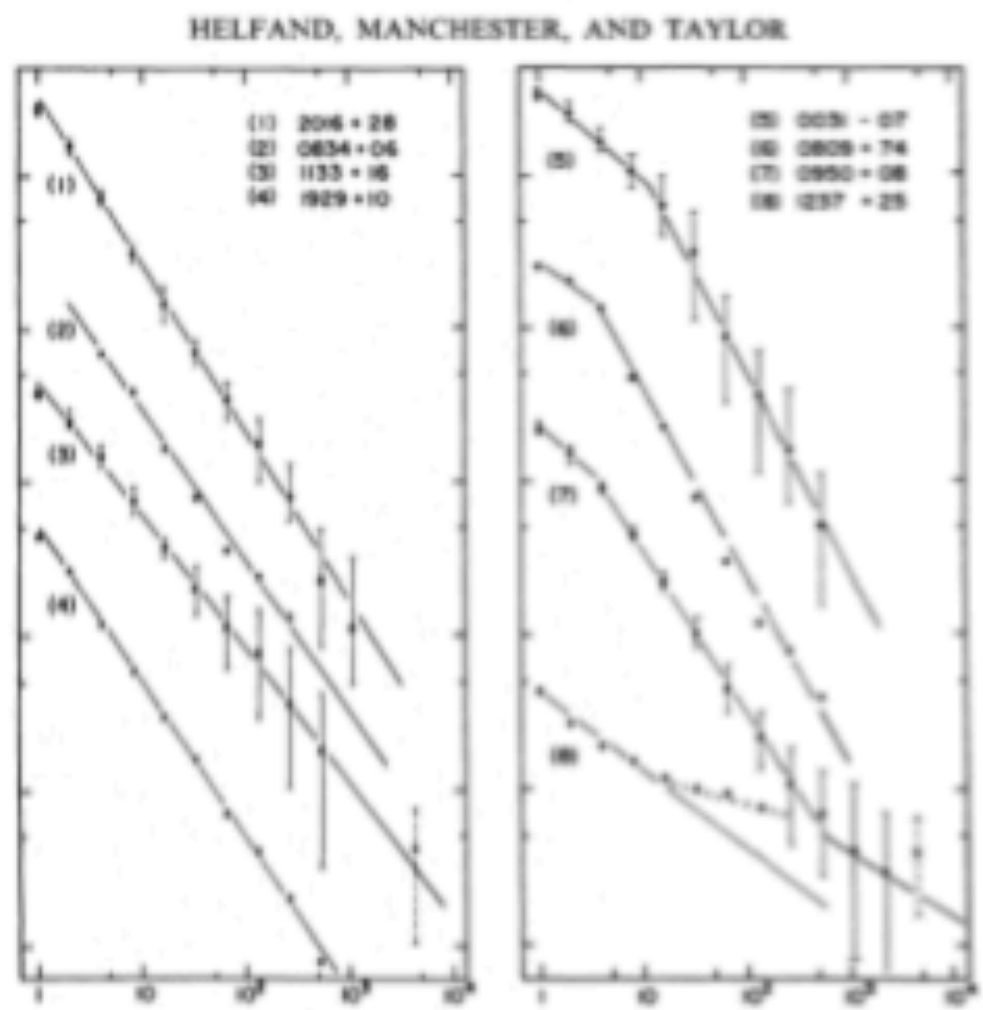
Shannon & JMC

Consistent analysis from ASP and WAPP data (Arecibo)

$f_J \sim 0.24$

Another approach: convergence to template

1-ρ



Number of pulses summed

ρ = correlation coefficient between template & pulse profile

1-ρ ~ N^{-1}
(if independent)

Relationship to jitter:

$$\rho(N=1) = 1 - f_J^2$$

Jitter vs Radiometer Noise Dominated TOAs

SEFD:

SKA

AO, LEAP, FAST

GBT, EVLA, Effelsberg

Parkes

JMC+RMS ‘10

Fraction of known MSPs that are jitter dominated calculated from ATNF catalog

Criterion: jitter dominated when SNR (single pulse) > 1

Implications

- Radiometer noise can be controlled:
  - Increase bandwidth, collecting area, integration time
  - Minimize pulse width (high frequencies)
- Jitter requires more integration time
- Red spin noise, if not correctable, can be mitigated by discarding noisy pulsars or summing over (many) more MSPs
- SKA: most <u>known</u> MSPs will be jitter (rather than noise) dominated
- Weak SKA-discovered MSPs may be noise dominated but will require ~SKA to time.
- Noise budget needed for every MSP.

Diagnosing Timing Residuals

- The BIG question is:

  "When will we detect GWs?"
- The answer requires knowing what is in the data that masks GWs.
- Another question is:

  "What new instruments will/do we need to detect GWs and begin GW astrophysics?"
- The answer to this question also requires knowing what contributes to the residuals.

→ Noise Budget Group in NANOGrav

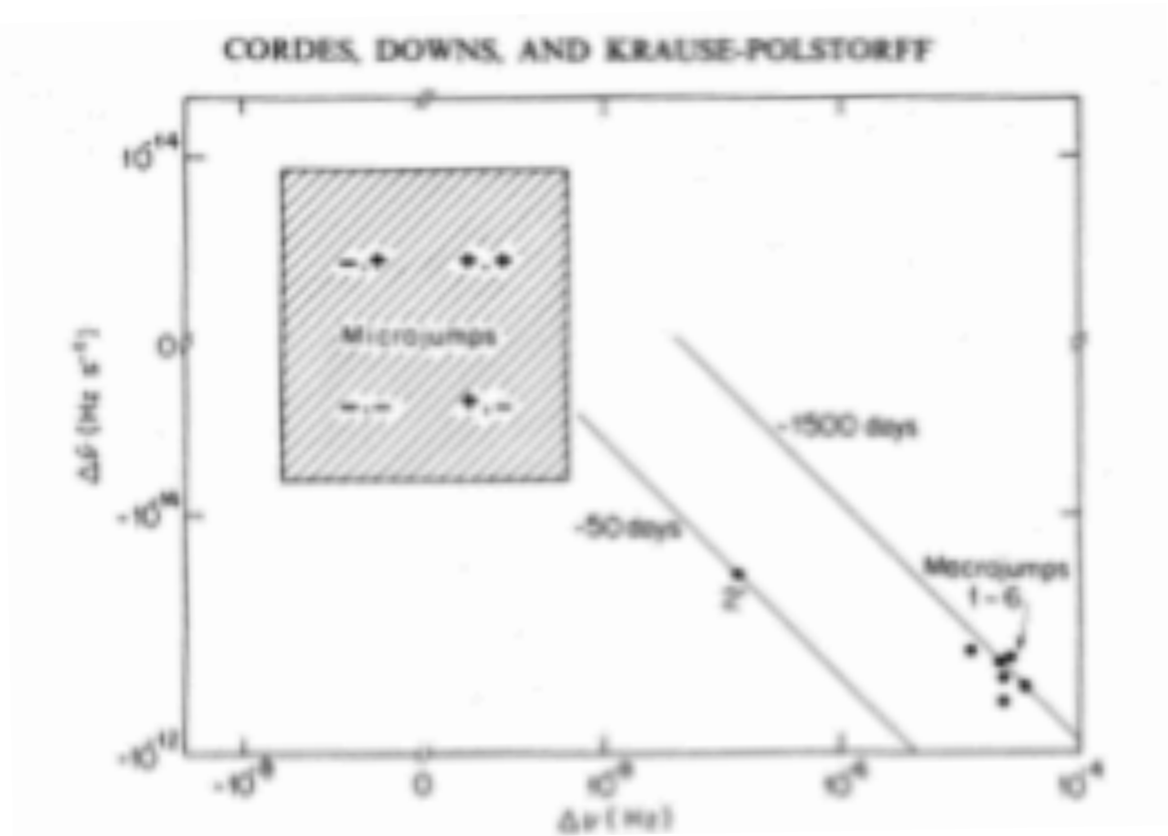
Timing Budget (short list)

Spin Noise	≤ 100 ns	Selected MSPs over 5 years	20 μs worst MSP Seconds worst CP
Pulse jitter	100 ns / $N_6^{-1/2}$	~ pulse width	~ λ independent, S/N independent
Interstellar Medium	δDM(t) Scattering broadening Refraction Scintillations	10 – 100 μs < 1 μs < 1 μs < 0.1 μs	Correctable ~ λ^2 Correctable ~λ^4 Correctable ~ λ^2,λ^4 Partly correctable, ~λ^4
Radiometer Noise	< 0.1 μs	$\propto \; W$	Integrate longer, narrower pulses, greater sensitivity
Template fitting errors	~ μs	λ-dependent pulse shapes	Correctable
Faraday rotation	< few ns	$\propto \lambda^2 \mathrm{RM}$	
Time transfer	< 10 ns		

Residuals Variance Budget

Category	Source	Correctability
White Noise	pulse jitter	Correctable?
	ISM/DISS (N_s)	Correctable?
	radiometer noise	**Not Correctable**
Red Noise	spin/torque noise	Correctable?
	asteroid belts	**Not Correctable**
	δDM(t)	**Correctable**
	ISM/DISS/RISS	**Partly Correctable**
Other	asteroid belts with $P_{orb} < T$ (bandpass noise)	**Not Correctable (low-mass asteroids)**
	etc.	?

On Deck


CORDES, DOWNS, AND KRAUSE-POLSTORFF
Micropjumps
-50 days
-1500 days
Macrojumps
1-6
Δν (Hz)

Gravitational Waves

- Stochastic background
  - Broadband in GW frequency
  - A red noise process
- Chirped/oscillatory signals from mergers
  - Narrowband
- Bursts (mergers, supernovae …)
  - Some better analyzed in the time domain

- How well can these be distinguished from other red and white noise processes?

TOA Signal Model

$$\Delta t_j(t) = e_j(t) + p_j(t) + r_j(t) + n_j(t)$$

correlated GW signal

uncorrelated GW signal

Red noise (spin, ISM)

White noise (ISM, radiometer)

All terms Gaussian random processes with zero mean

Simplify to:

$$\Delta t_j(t) = e_j(t) + m_j(t)$$

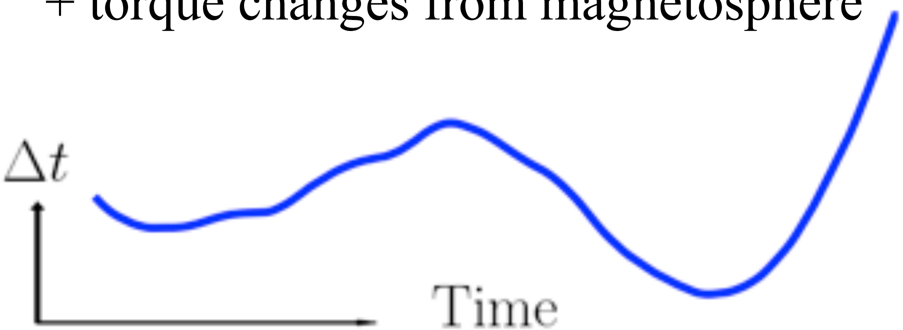
$e_j(t)$ correlated between LOS

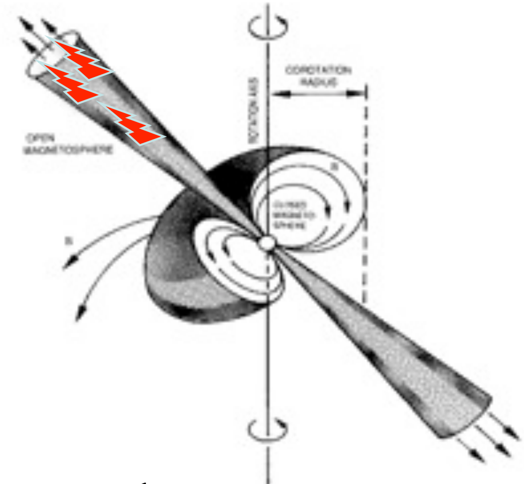
$m_j(t)$ uncorrelated between LOS

Pulsar Physics and Processes

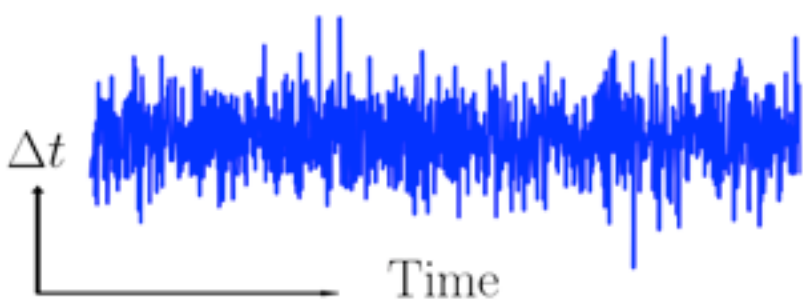
- Stochastic spin variations are slow and ~ scale-free; red power spectrum; random-walk like

A combination of neutron-star internal dynamics
+ torque changes from magnetosphere





- Stochastic emission
  - Fast changes from pulse to pulse (one pulsar ‘day’)
  - Uncorrelated → white noise in TOAs



<u>Key words</u>:
- Red and white noise
- Gaussian statistics
- Nonstationary process

<u>Key methods</u>:
- Autocorrelation function
- Power spectrum
- Structure function

2011 Snowshoe Talk

Papers Referred To

- "Assessing the Role of Spin Noise in the Precision Timing of MSPs" (SC10, ApJ)
- "A Measurement Model for Precision Pulsar Timing" (CS10, arXiv:1010.3785)
- "Minimum Requirements for Detecting a Stochastic Gravitational Background Using Pulsars" (CS11, submitted)
- "Asteroids Around Neutron Stars: Evidence from a MSP and Implications …" (Shannon et al. 2011, almost submitted)
- "Correcting for Interstellar Propagation in Precision Pulsar Timing" (SC, to be submitted)
- "Pulse Intensity Modulation and the Timing Stability of Millisecond Pulsars" (SC+, in preparation)
- "Coherent Sum Approach to GW Detection" (CS, in preparation)

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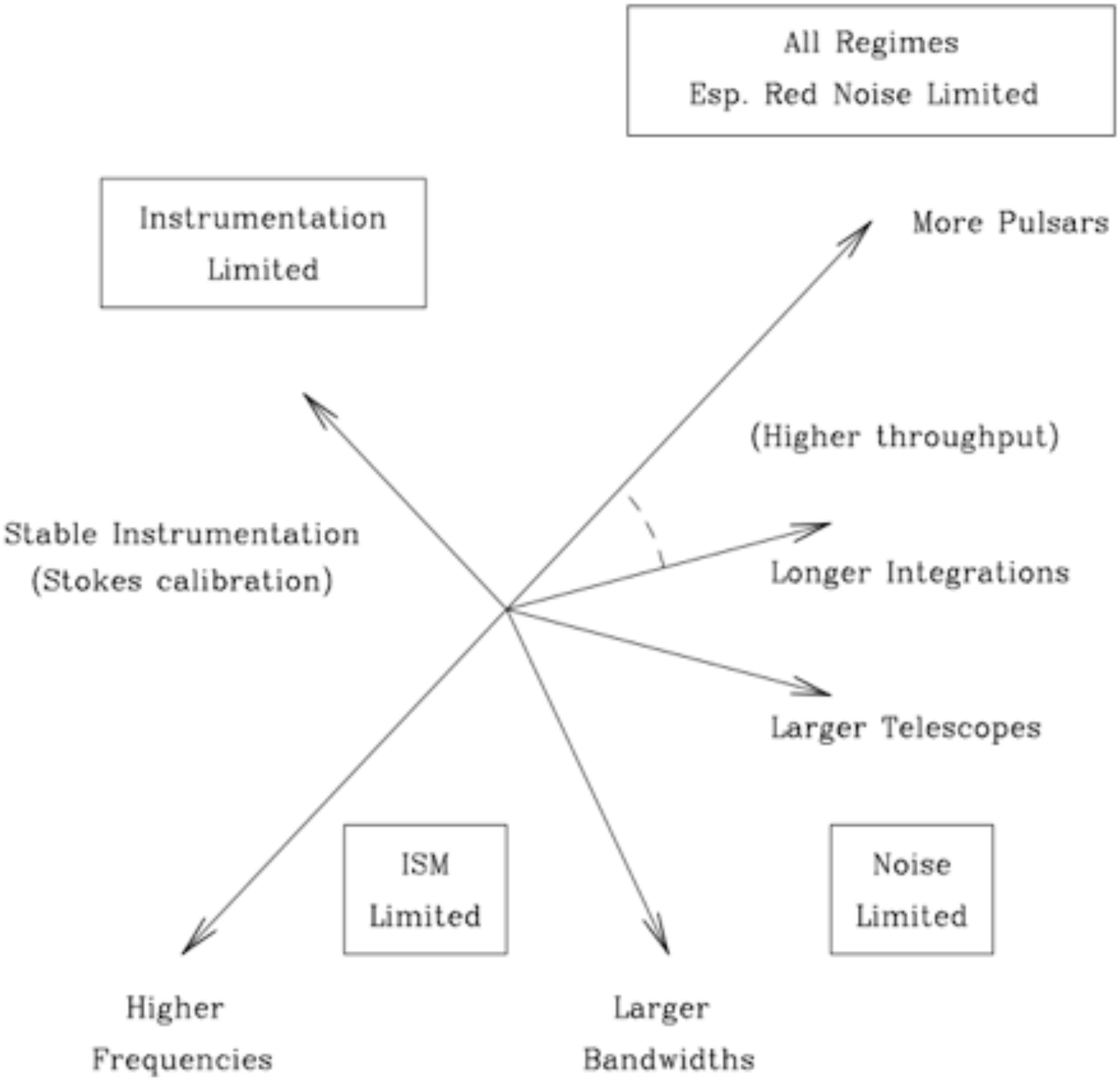
Improving Timing Precision

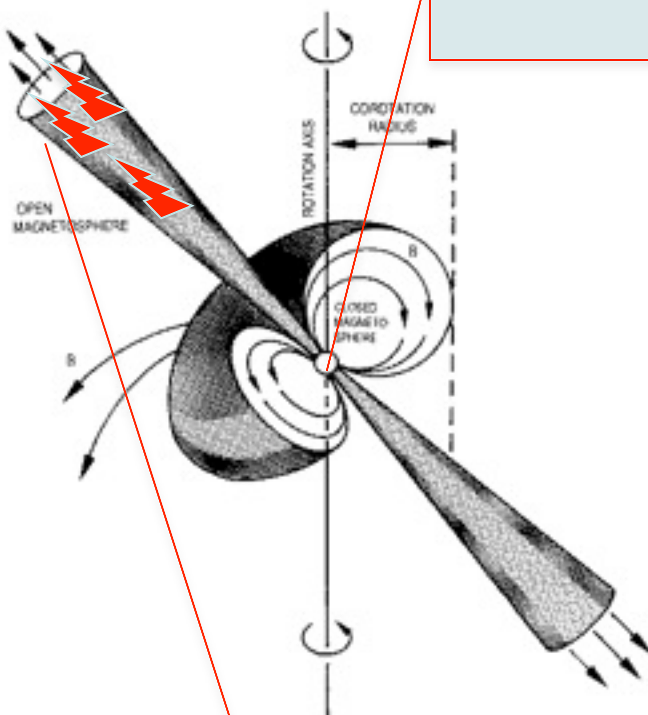


Depending on noise regime, better sensitivity comes from different improvements.

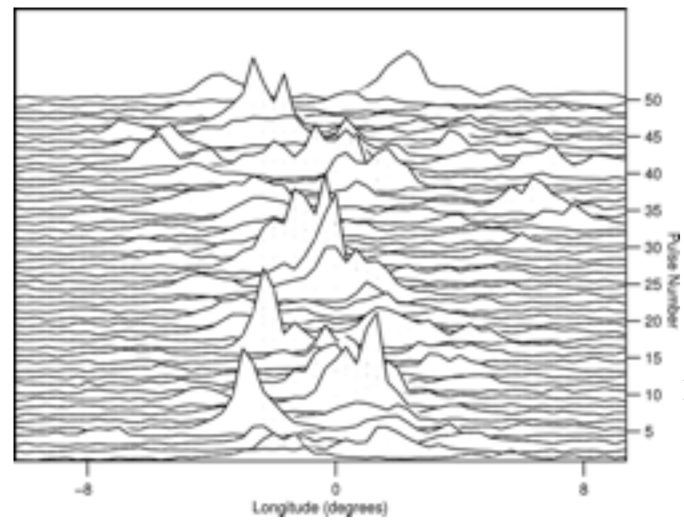
Increasing the number of *quality* MSPs is guaranteed to increase sensitivity to GWs

The clock is not perfect

The spinning NS = the clock
~ 10 km radius

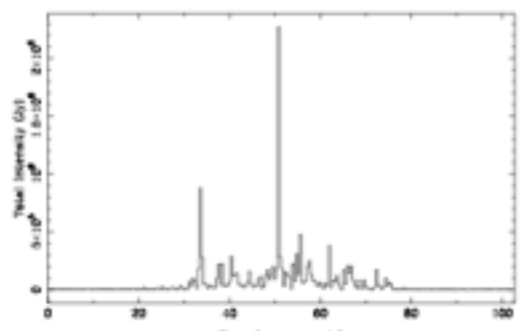


Relativistic emission regions
magnetosphere ~ 100 – 10^4 km



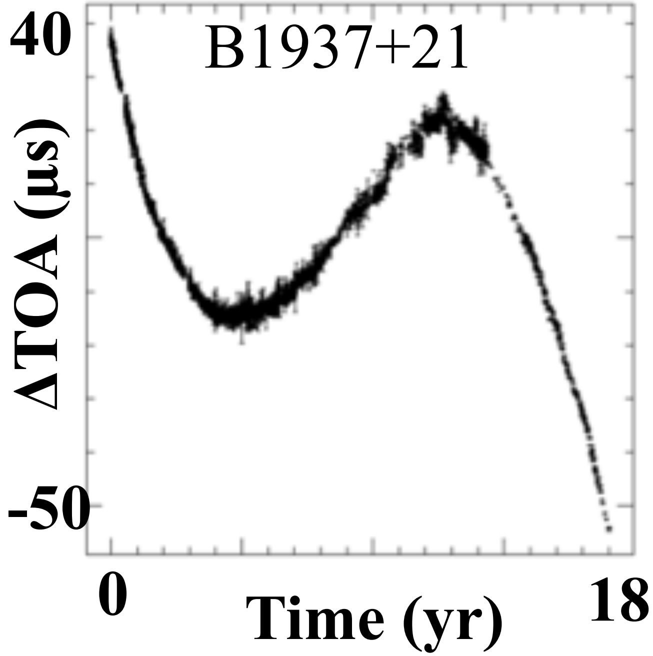
Single pulses: phase jitter + amplitude modulations

B0943+10 Rosen & Clemens 2



Crab pulsar shot pulses (ns)

Hankins & Eilek 2007



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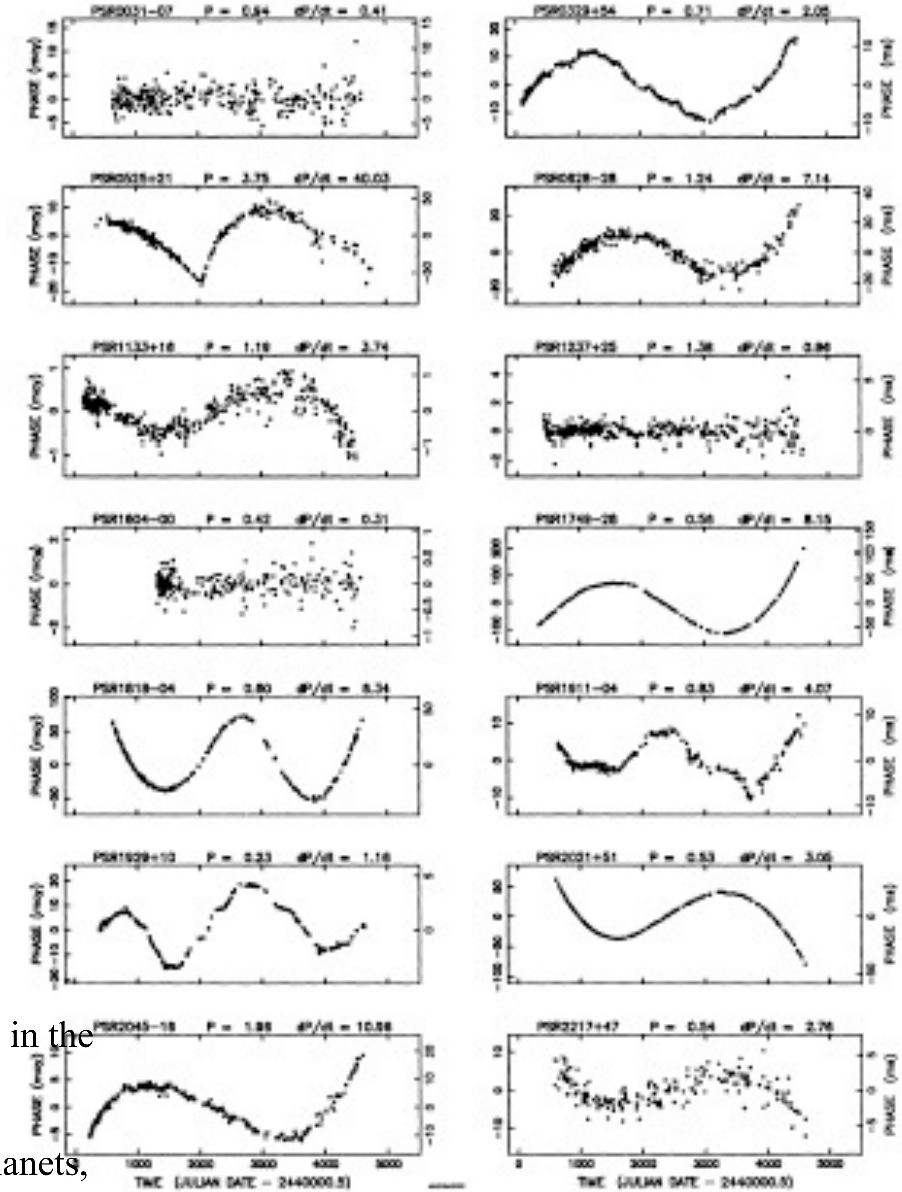

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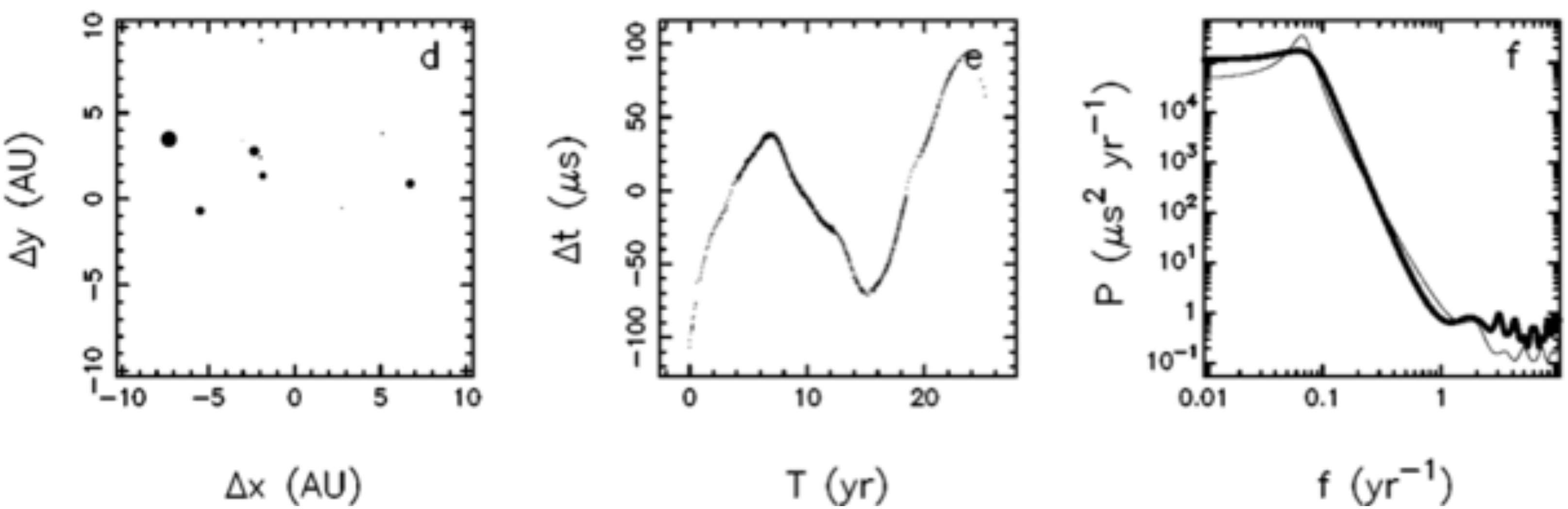
<u>Asteroid belt Interpretation to Timing Noise in B1937+21</u> (Shannon et al, almost submitted)

- Low mass circumpulsar system (total mass ~ 0.05 Earth masses)
- 10 -200 objects: Can't resolve periodicities of individual components.

$$\delta \mathrm{TOA}_{\mathrm{rms}} = 1.5\,\mathrm{ms} \left(\frac{a_{\mathrm{rms}} \sin i}{\sqrt{N_a}}\right)\left(\frac{M_{\mathrm{belt}}}{M_{\oplus}}\right)$$

Perturbations will look like a random walk for $T < P_{\mathrm{orb,max}}$

Data tools used



Spin noise vs asteroid noise in MSPs

- Asteroid noise likely relevant to isolated MSPs that formed disks from evaporated companions

  → Good news for precision timing:
  MSP +WD systems are probably dynamically clean

- If B1937+21 dominated by spin noise, then <u>all</u> MSPs will show red noise but with longer P, smaller Pdot favored

  → implications on choosing PTA pulsars

Timing Error from Radiometer Noise

rms TOA error from template fitting with additive noise:

$$\Delta t_{S/N} = \frac{\left[\int\int dt\,dt'\,\rho(t-t')U'(t)U'(t')\right]^{1/2}}{\mathrm{SNR}\int \mathrm{dt}\,[\mathrm{U'(t)}]^2} = \frac{W_{\mathrm{eff}}}{\mathrm{SNR}}\left(\frac{\Delta}{W_{\mathrm{eff}}}\right)^{1/2}$$

Low-DM pulsars: DISS (and RISS) will modulate SNR

Gaussian shaped pulse:

$$\Delta t_{S/N} = \frac{W}{(2\pi \ln 2)^{1/4}\mathrm{SNR}_1\sqrt{\mathrm{N}}}\left(\frac{\Delta}{W}\right)^{1/2}$$

$$\boxed{\Delta t_{S/N} = 0.69\mu s W_{\mathrm{ms}} N_6^{-1/2}\mathrm{SNR}_1^{-1}(\Delta/\mathrm{W})^{1/2}}$$

$N_6 = N / 10^6$

Interstellar pulse broadening, when large, increases $\Delta t_{S/N}$ in two ways:

- SNR decreases by a factor $W / [W^2+\tau_d^2]^{1/2}$
- W increases to $[W^2+\tau_d^2]^{1/2}$

→ Large errors for high DM pulsars and low-frequency

Pulse-phase Jitter

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  - intensity modulation index = $\sigma_I/I \sim 1$
  - phase variations ~ widths of single pulses
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- Millisecond pulsars:
  - J0437-4715: typical jitter (Jenet et al. 1998)
  - J1713+0747: typical jitter Shannon, JMC+ in preparation)
  - B1937+21 (aka J1939+2134):
    - normal pulses show very little jitter
    - giant pulses show typical amounts of jitter

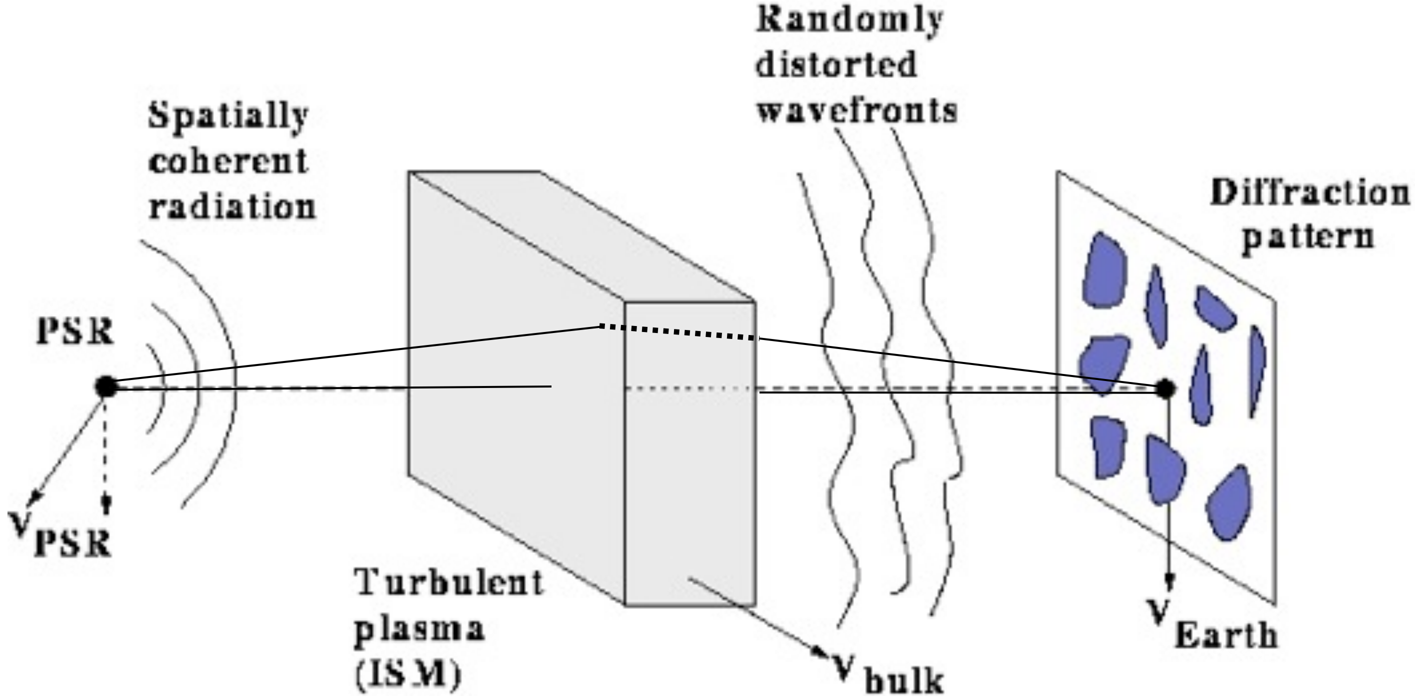
TOA Error Correction I

- Pulse shapes depart from templates
    - even with perfect Stokes calibration,etc.
    - yields extra δTOA
- If δTOA is correlated with shape parameter(s), TOAs can be corrected
    - TOA shift of unchanging pulse shape not useful
    - For correlation coefficient ρ reduction in rms TOA is $\sigma_{TOA,\ c} = \sigma_{TOA}\ (1\text{-}\rho^2)^{1/2}$
    - e.g. need $\rho \geq 0.71$ to get reduction by factor ≤ ½
- Principal component analysis (PCA) to characterize pulse shapes + correlate with δTOA.

PCA and TOA Correction

- Calculate pulse shape U(t) at each epoch
- Difference from average $\delta U(t) = U(t) - \langle U(t) \rangle$
- Covariance matrix of $\delta U(t)$
  - $C = \langle \delta U \, \delta U^t \rangle$ with δU = vector, t = transpose, j=epoch
- Diagonalize C to get eigenvalues, vectors
- Identify significant eigenvectors (typically only a few), e_k, k =1, … a few
- Dot products of e_k with δU_j for each epoch j characterize shape changes ($d_{jk} = e_k \cdot \delta U_j$)
- Can calculate correlation coefficient of δTOA_j with d_{jk} to assess correctability
- Simulation results (unpublished): no advantage to correction unless phase jitter distribution is asymmetric

TOA Variations from electron density variations

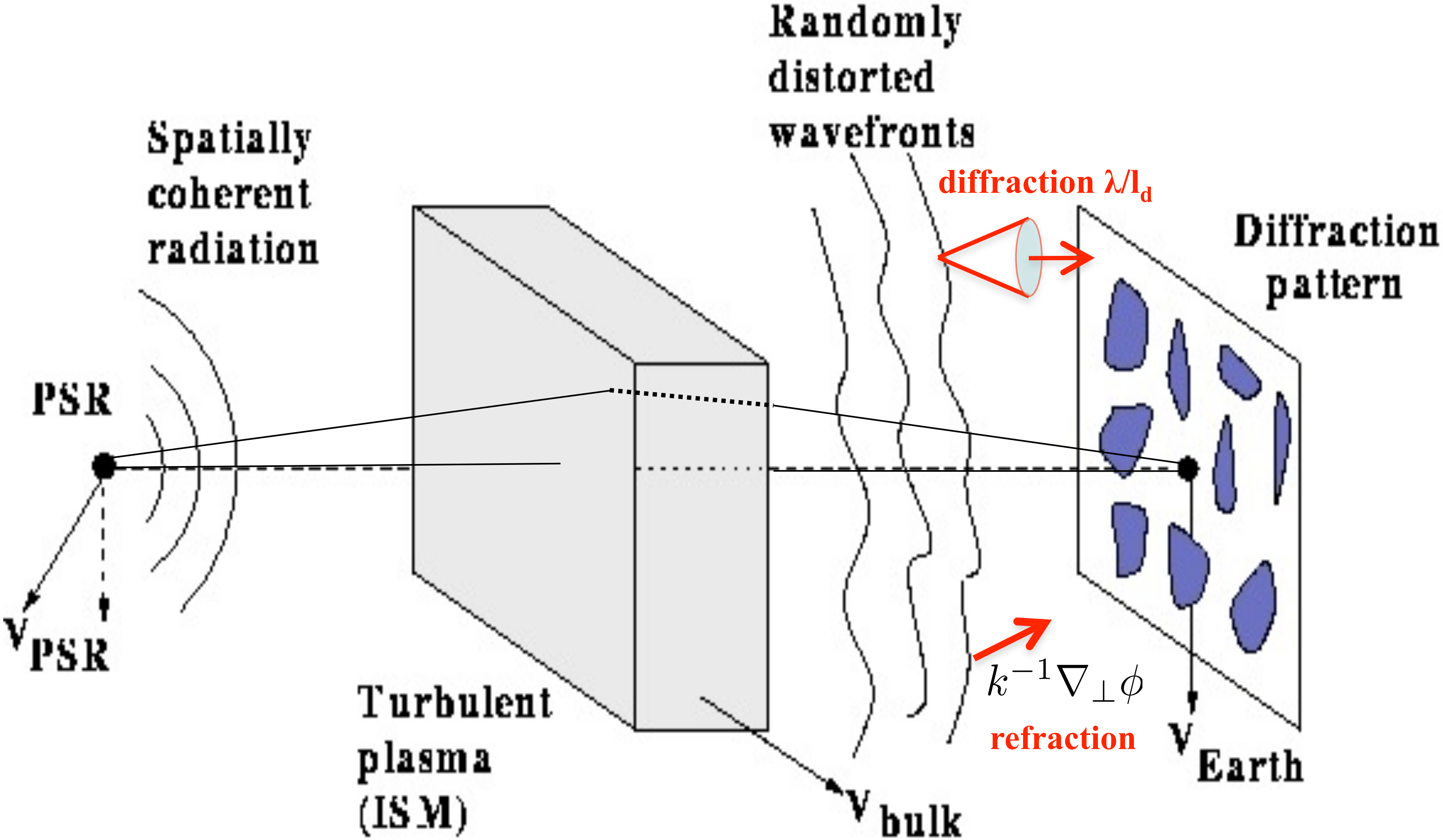


Electron density irregularities from ~100s km to Galactic scales

$$
\begin{aligned}
\phi_d &= -\lambda r_e \int_{\mathrm{LOS}} ds\, n_e(s) \\
n_e &= \overline{n}_e + \delta n_e \\
\Delta t_{\mathrm{DM}} &= \frac{\phi_d}{2\pi\nu} \propto \frac{\mathrm{DM}}{\nu^2}
\end{aligned}
$$

Trivial to correct if DM from mean electron density were the only effect!

TOA Variations from electron density variations



Dealing with the ISM [other than DM(t)]

Precision timing requires addressing interstellar propagation effects (one way or another).

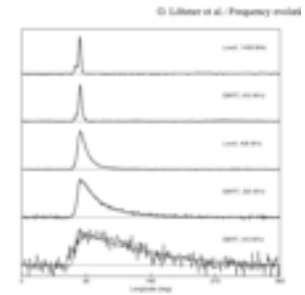
Two options:

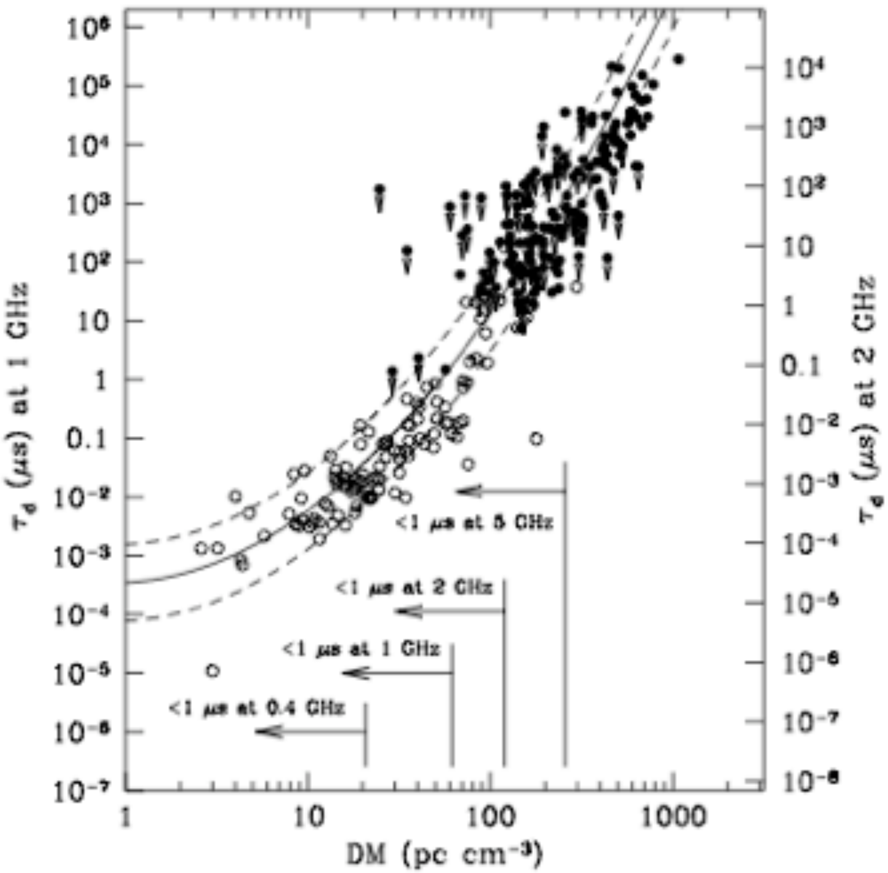
1. Restrict PTA observations to low DM pulsars and high frequencies and correct only for DM(t)
   - ✧ problem: too few high-quality, nearby MSPs (timing noise)

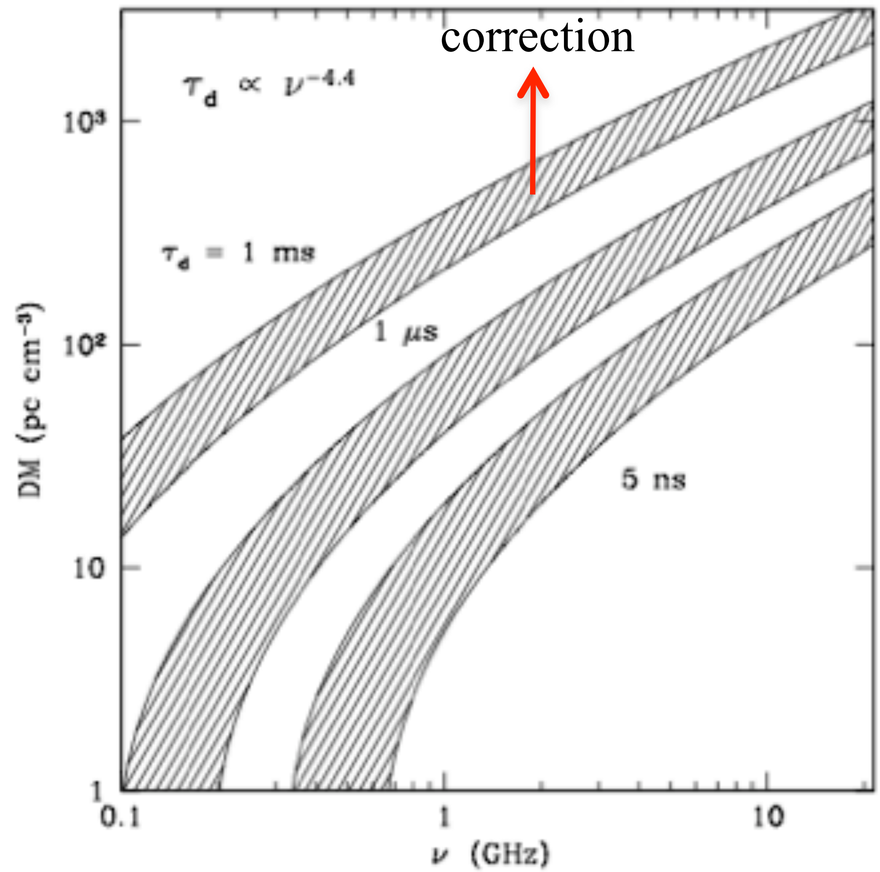
2. Aggressively correct ISM perturbations
   - ✧ DM(t), scattering, and refraction perturbations
   - ✧ can extend DM range of sample → more MSPs
   - ✧ Requires multi-frequency observations (≥ 4) or continuous over >2:1 range
   - ✧ TOA corrections = largely unexplored territory

ISM Perturbations

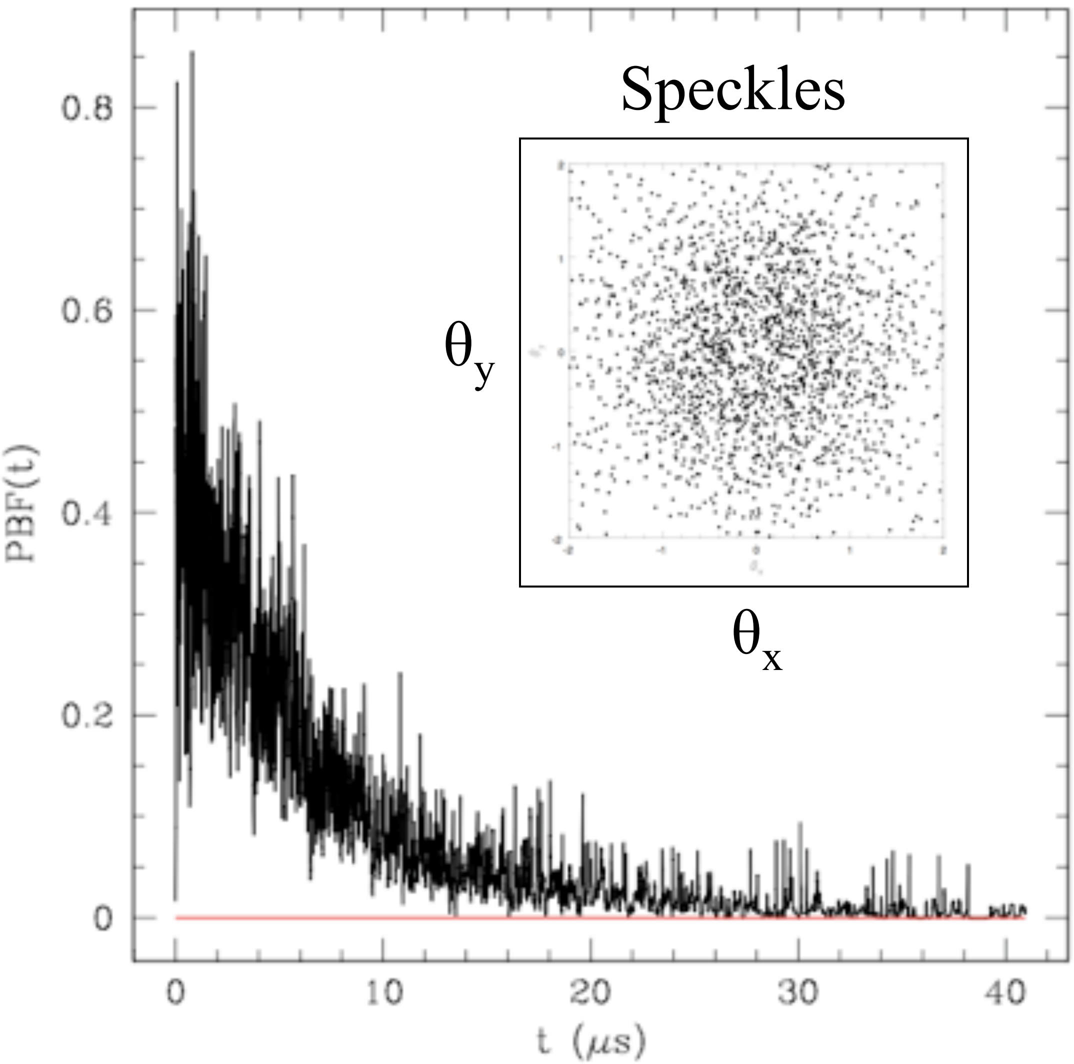
- DM variations: simple to deal with
- Multipath and refraction: many effects, with different frequency dependences
- Dominated by time-variable pulse broadening.





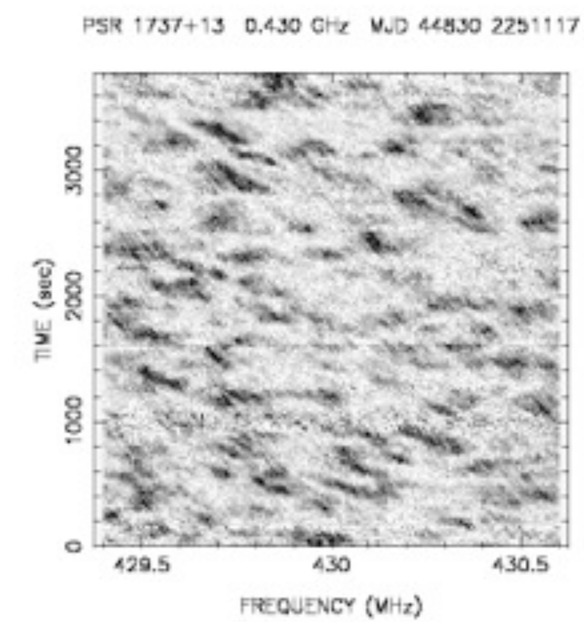


Stochasticity of the PBF

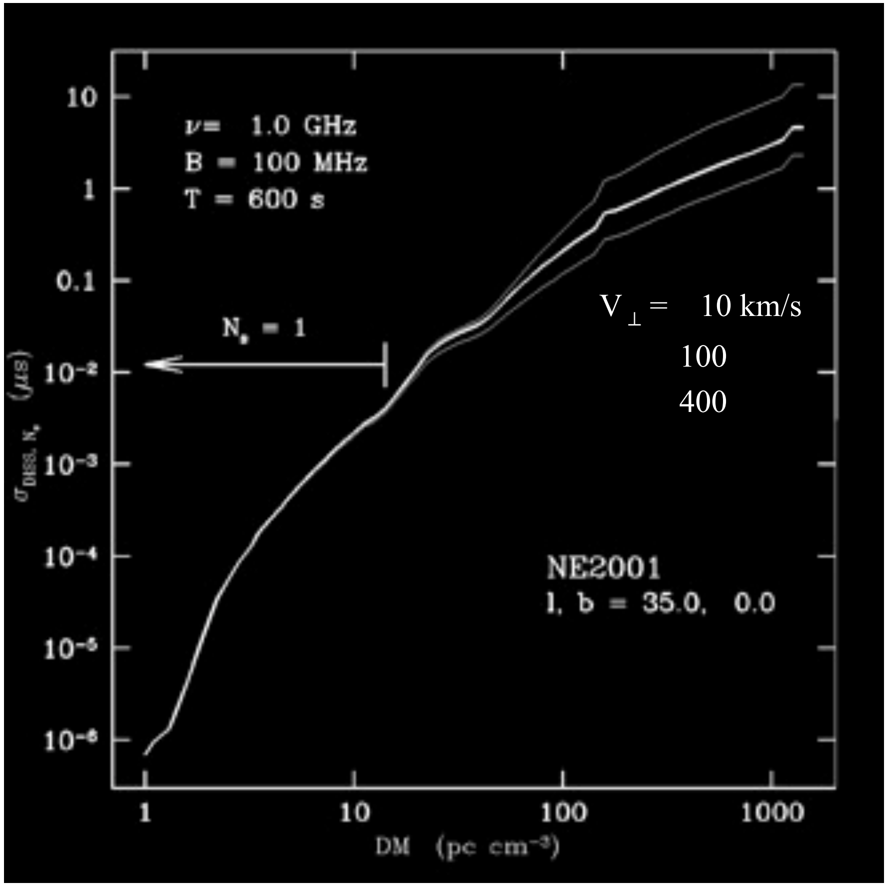


$N_{speckles} = N_{scintles}$

$N_{scintles}$ = number of bright patches in the time-bandwdith plane



Timing Error from Interstellar Scintillations:
white noise contribution



Calculations using the NE2001 electron-density model

N_s = number of scintles in the T-B plane used to construct an arrival time

$$\Delta t \propto \frac{\tau_d}{\sqrt{N_s}}$$

$$\tau_d \approx \frac{D\theta_d^2}{2c} \propto \lambda^{4.4}$$

$$N_s \approx \left(1 + \zeta\frac{B}{\Delta\nu_d}\right)\left(1 + \zeta\frac{T}{\Delta t_d}\right)$$
$$\zeta \approx 0.2$$

Refraction in the ISM

- Phase gradient in screen:
    - refraction of incident radiation
    - yields change in angle of arrival (AOA)
- Two timing perturbations:
    - extra delay

$$t_{\rm AOA} = \frac{1}{2c} D_{\rm eff}\theta_r^2 \approx 1.21\ \mu s\ D_{\rm eff}({\rm kpc})\theta_{\rm r}^2({\rm mas})$$

$$D_{\rm eff} = (D - D_s)\left(\frac{D}{D_s}\right)$$

- error in correction to SSBC

$$\Delta t_{\rm AOA,Bary} = c^{-1}\hat{n}\cdot\mathbf{r}_\oplus \approx c^{-1} r_\oplus \theta_r(t)\cos\Phi(t) \approx 2.4\ \mu s\,\theta_r({\rm mas})\cos\Phi({\rm t})$$

- Phase curvature in screen:
    - refractive intensity variation (RISS)
    - change in shape of ray bundle

Fitting multifrequency TOAs for:

(1) t_∞ only

(2) t_∞ and DM

(3) t_∞, DM, and pulse broadening delay

Use of wide bandwidths (e.g. 1-2 GHz) requires more terms than just DM fitting

100-m class

Arecibo-class

total error

random error

Notable TOA errors

White noise:

Radiometer noise: $\Delta t_{\mathrm{S/N}} = 0.69 \mu s W_{\mathrm{ms}} N_6^{-1/2} \mathrm{SNR}_1^{-1} (\Delta/\mathrm{W})^{1/2}$

Intrinsic jitter: $\Delta t_{\mathrm{J}} = 0.28 \mu s\, W_{i,\mathrm{ms}} N_6^{-1/2} \left(\frac{f_J}{1/3}\right) \left(\frac{1+m_I^2}{2}\right)^{1/2}$

PBF stochasticity: $\Delta t_{\delta PBF} \sim \tau_d / N_s^{-1/2}$ (ns - 100μs)

Slow ISM:

AOA (refraction): $t_{\mathrm{AOA}} = \frac{1}{2c} D_{\mathrm{eff}} \theta_r^2 \approx 1.21\ \mu s\ D_{\mathrm{eff}}(\mathrm{kpc}) \theta_{\mathrm{r}}^2 (\mathrm{mas})$

$$\Delta t_{\mathrm{AOA,Bary}} = c^{-1} \hat{n} \cdot \mathbf{r}_{\oplus} \approx c^{-1} r_{\oplus} \theta_r(t) \cos\Phi(t) \approx 2.4\ \mu s\, \theta_r(\mathrm{mas}) \cos\Phi(\mathrm{t})$$

Modulations of PBF: for τ_d from sub-μs to seconds: $\boldsymbol{\delta\tau_d \approx 0.2\tau_d}$:

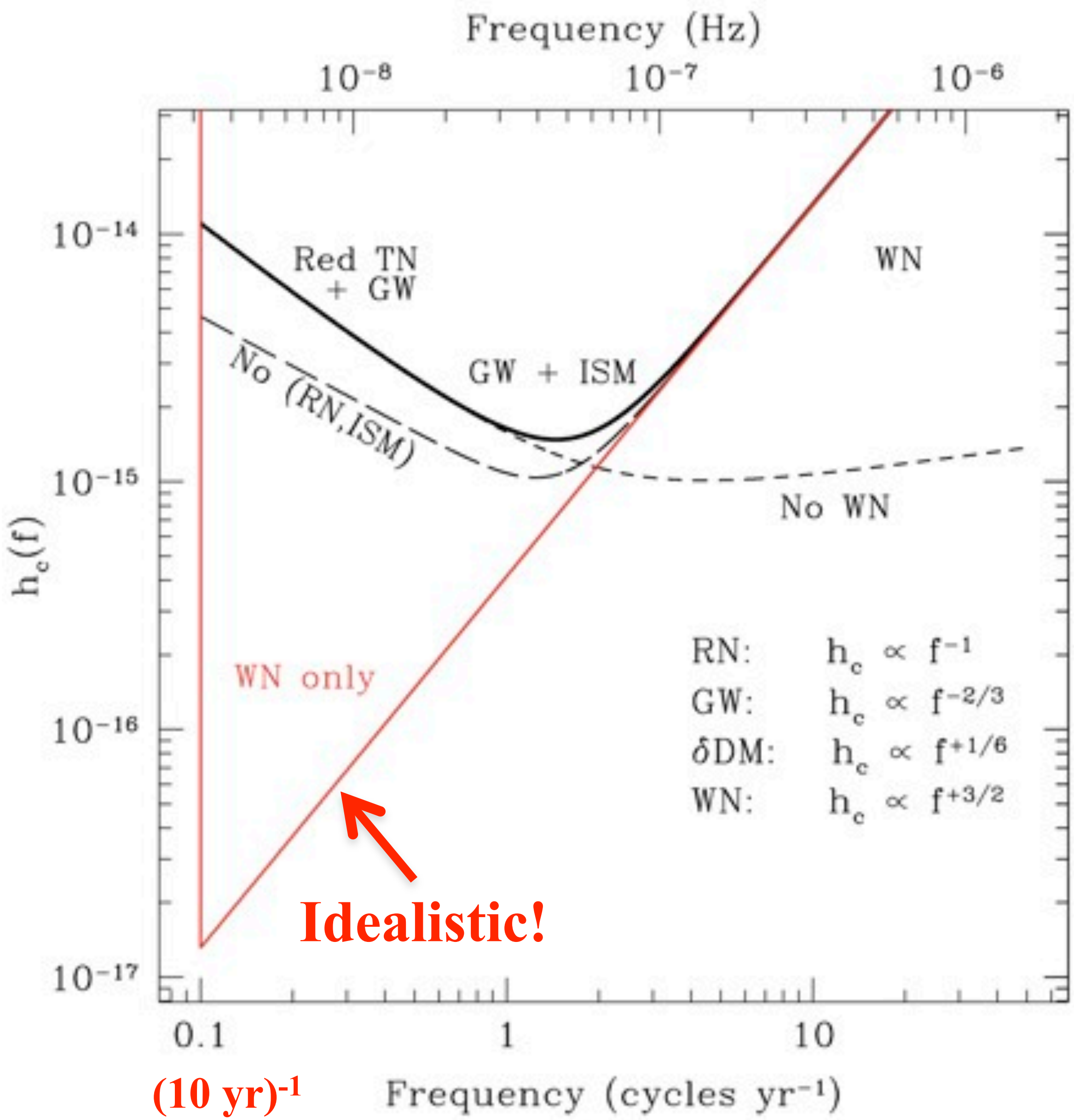
DM changes δDM(t): δDM(t) → tens of μs for nearby MSPs

Timing noise:

$$\sigma_{\mathrm{TN},2} \approx 10^{2.4}\ \mu s\ \nu^{-1.4\pm0.1}\ \dot{\nu}_{-15}^{1.0\pm0.1}\ T_{\mathrm{yr}}^{1.7\pm0.4}$$

RS+JMC '10

Noise Budget for Single Pulsar



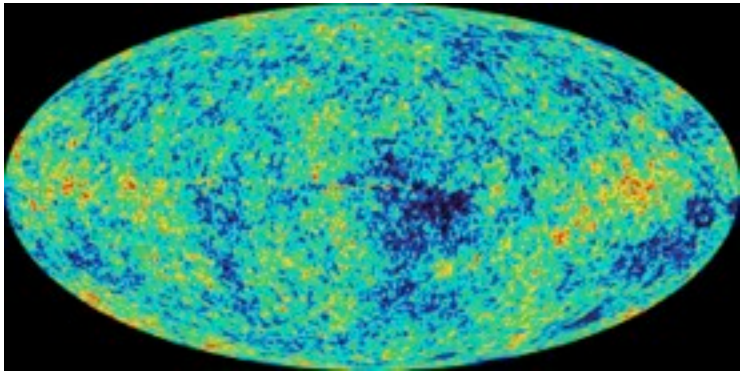
Schematic spectrum for $h_c(f)$
(strain per unit log f)

Detection is limited by
- spin noise in the pulsar
- pulse phase jitter
- ISM plasma effects
- radiometer noise (white)
- self noise in the GW bg (the "pulsar" term)

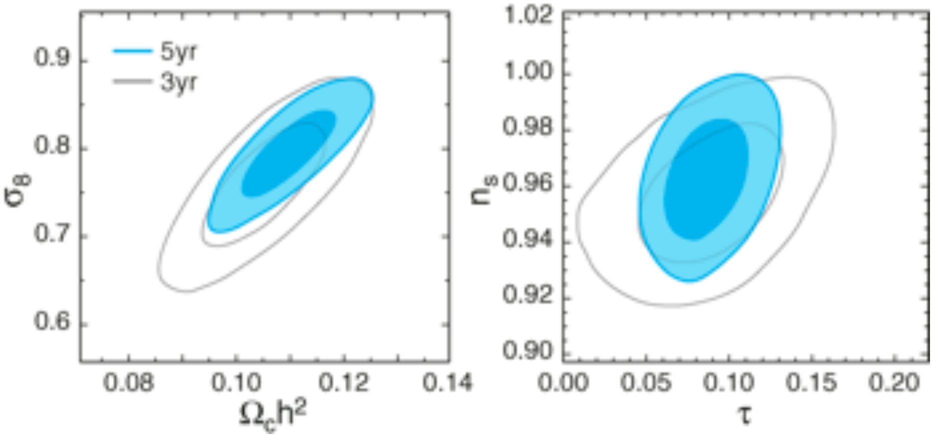
Recourse: use correlations between N_p pulsars

Detection: Analogy with the CMB

Data



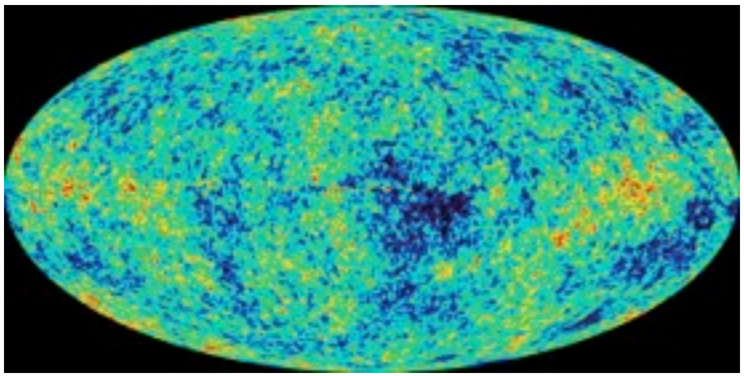
Inference



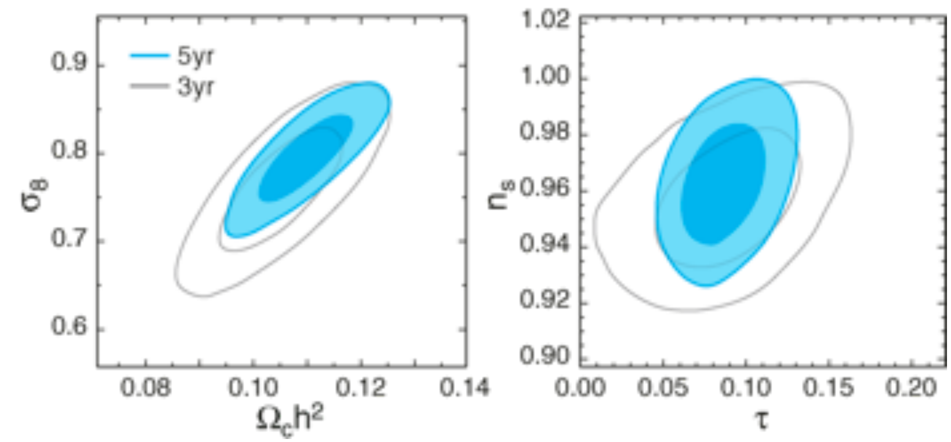
J. Dunkley, et al., 2009, ApJS, 180, 306-329

Detection: Analogy with the CMB

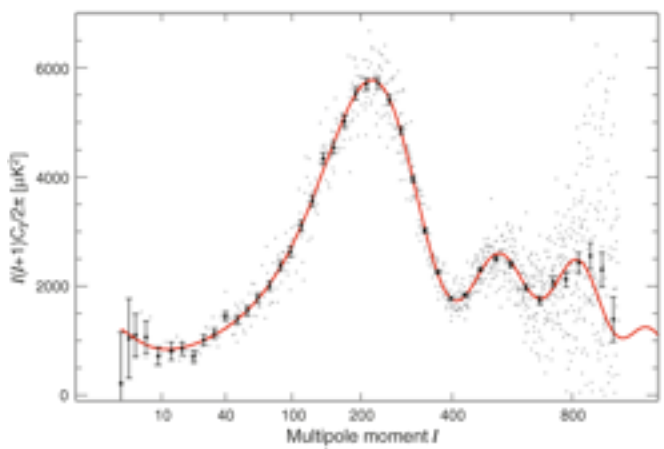
Data



Inference



Evidence!



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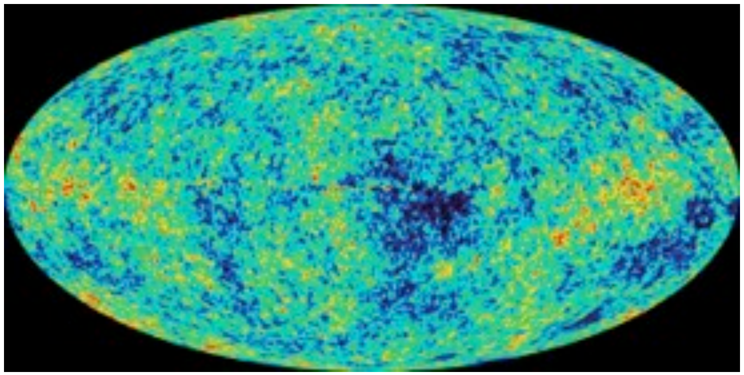
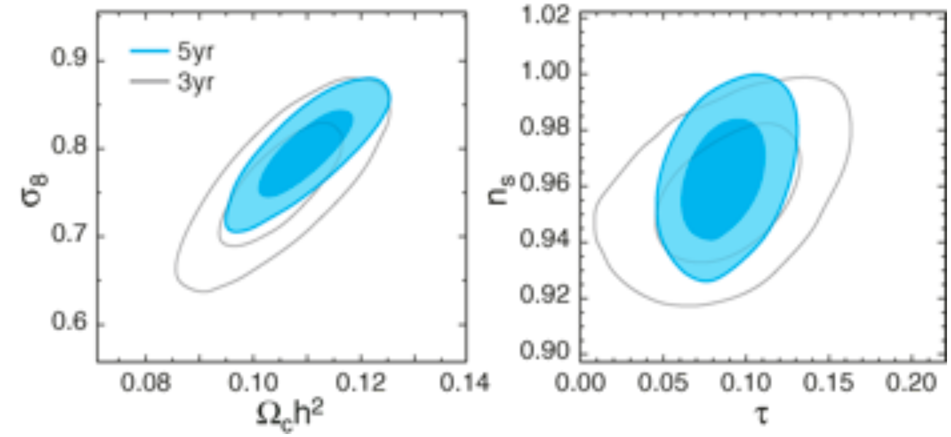
Detection: Analogy with the CMB

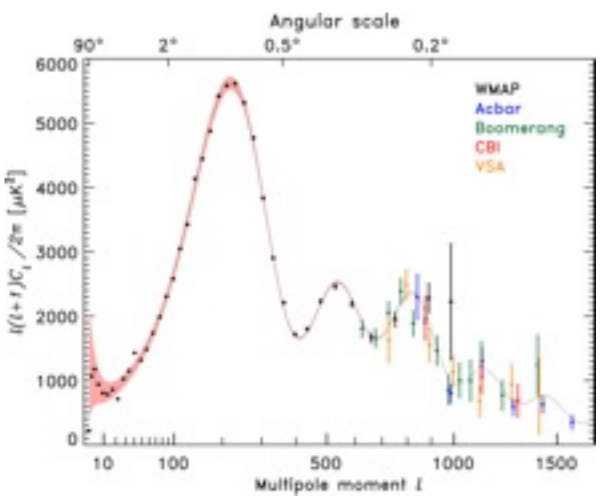
Data



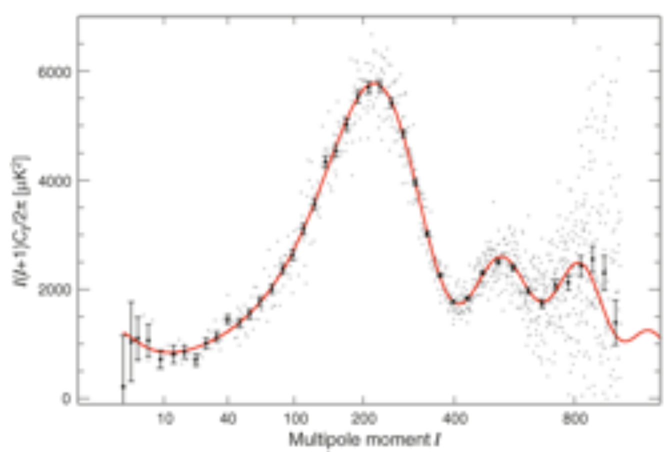
Inference



Confirmation



Evidence!



J. Dunkley, et al., 2009, ApJS, 180, 306-329

TOA Signal Model

$$\Delta t_j(t) = e_j(t) + p_j(t) + r_j(t) + n_j(t)$$

correlated GW signal

uncorrelated GW signal

Red noise (spin, ISM)

White noise (jitter, ISM, radiometer)

All terms Gaussian random processes with zero mean

Simplify to:

$$\Delta t_j = e_j(t) + u_j(t)$$

$e_j(t)$ correlated between LOS

$u_j(t)$ uncorrelated between LOS

Red Noise Processes

- For a residual spectrum $S_{\mathcal{R}}(f) = K f^{-\alpha_r}$
  - $\alpha_r=0$ white noise
  - $\alpha_r=1$ 1/f noise
  - $\alpha_r \approx 5$ global fit to MSPs + CPs (RMS +JMC'10)
  - $\alpha_r=13/3$ GW background from SMBHs ($h_c \sim f^{-2/3}$)

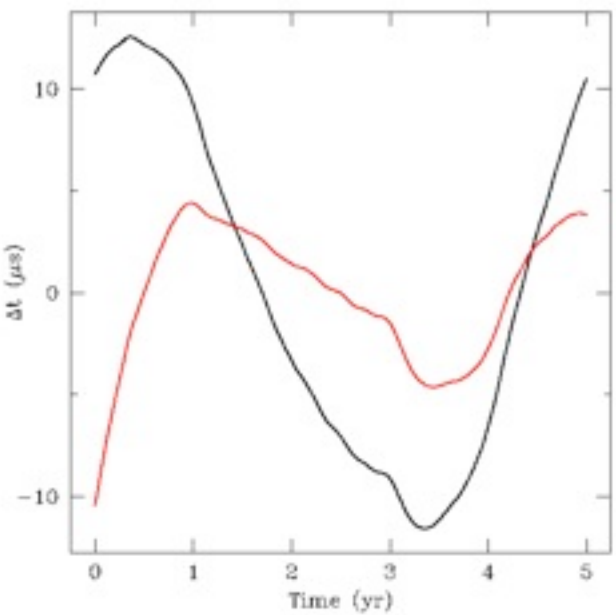
$$\sigma_{\mathcal{R}}(T) = C_{\mathrm{fit},\alpha_r} T^{(\alpha_r - 1)/2}$$

- RMS residual: scaling with data interval [0,T]
  - $T^{-1/2}$ white noise
  - T^{0} 1/f noise
  - T^{2} red timing noise

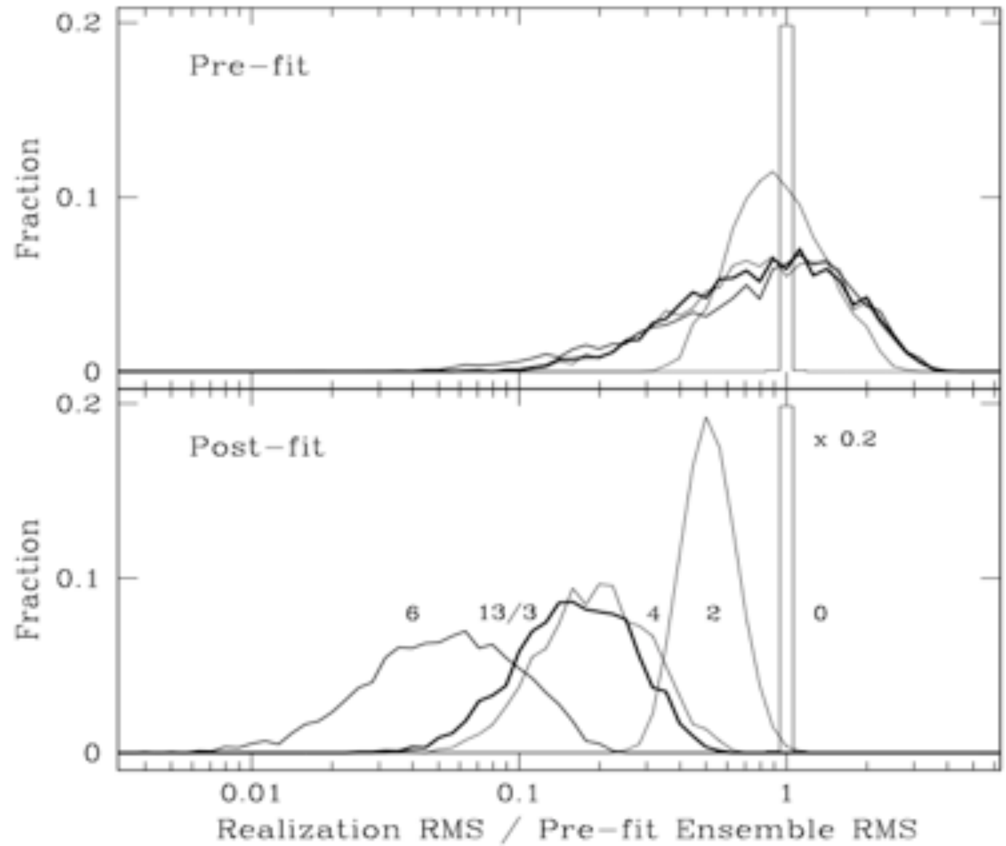
Similar scaling laws given in JMC'80 and Blandford et al. '84

Volatility of Red Noise

Simulated time series
(pre/post-fit = black/red)



Histograms of RMS:
Redder → greater variation



→ x10 variation in RMS
→ x100 in variance !

Pathologies of Red Noise
“The red menace”

- Red noise is <u>highly</u> nonstationary
  - moments depend on time span T of data
  - temporal correlation function <u>not</u> just a function of time lag
- Realization-to-realization variations of the RMS are huge (redder → larger range)
- Shape of <u>spectrum</u> will also vary significantly
- → Beware methods/limits that do not include the volatility

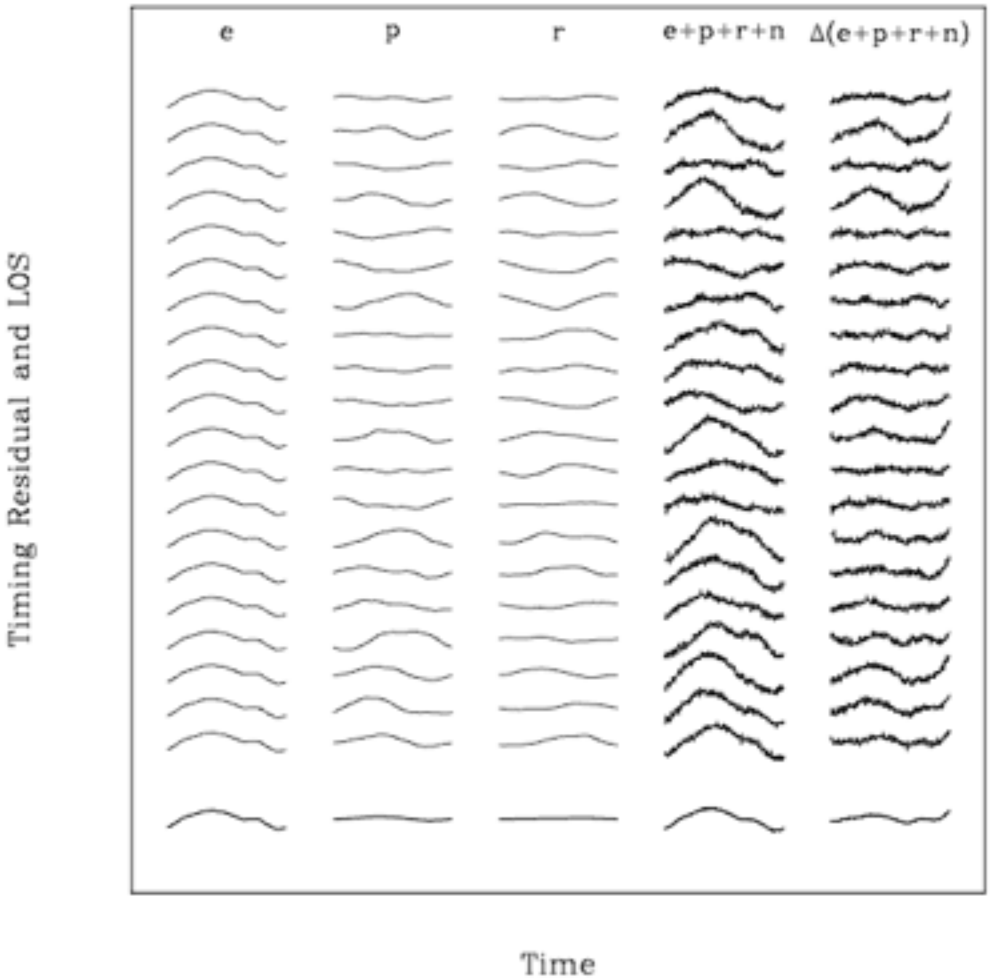
Minimum Requirements for Detection

- Assume the best possible PTA configuration:
  - All MSPs in the same direction → HD curve at maximum ($\theta=0$)
- Consider mixtures of GWs, RN, WN
- Results:
  - Red noise drastically increases the false-alarm fraction for a given detection fraction (ROC curves)
  - Detection of $A=10^{-15}\ yr^{-2/3}$ for SMBH bg requires:
    - 20 super-stable MSPs with rms red noise < 20 ns
    - 50-100 MSPs with larger red noise to detect and confirm detection with an independent sample

Simulated Timing Residuals

$$x(t) = e(t) + p(t) + r(t) + n(t)$$

GW terms: e(t) + p(t)
Red noise from pulsar spin + ISM: r(t)
White noise from pulsar + ISM + radiometer



Simulated 20 pulsar PTA

Extreme case of all pulsars in <u>same</u> direction at different unknown distances

Each time series = 5yr

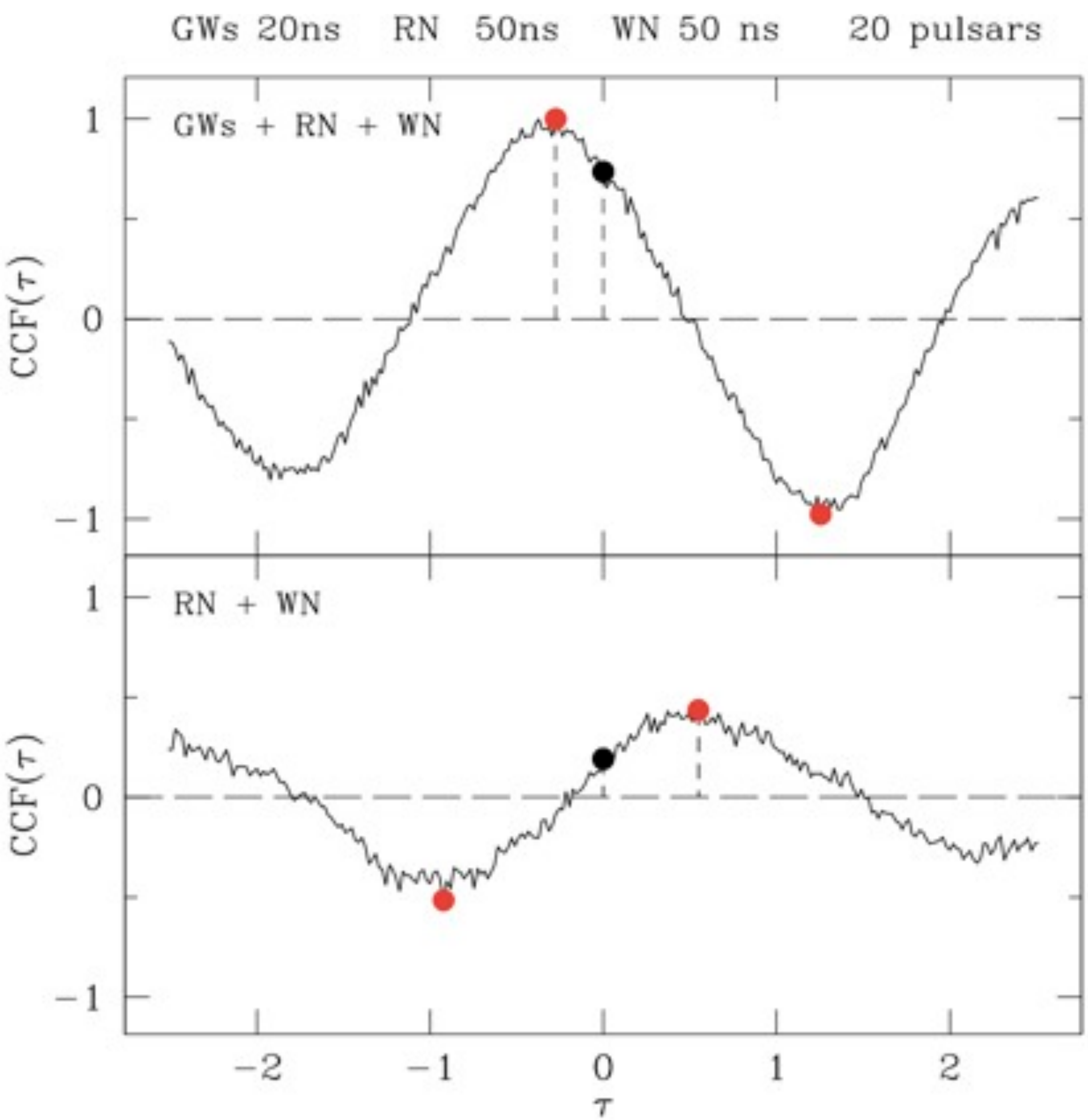
Temporal cross correlations = important diagnostic

GWs: 20ns RN: 20ns WN: 20ns (post fit)

These are <u>temporal</u> CCFs for the case where all pulsars are in the same direction (best case!)

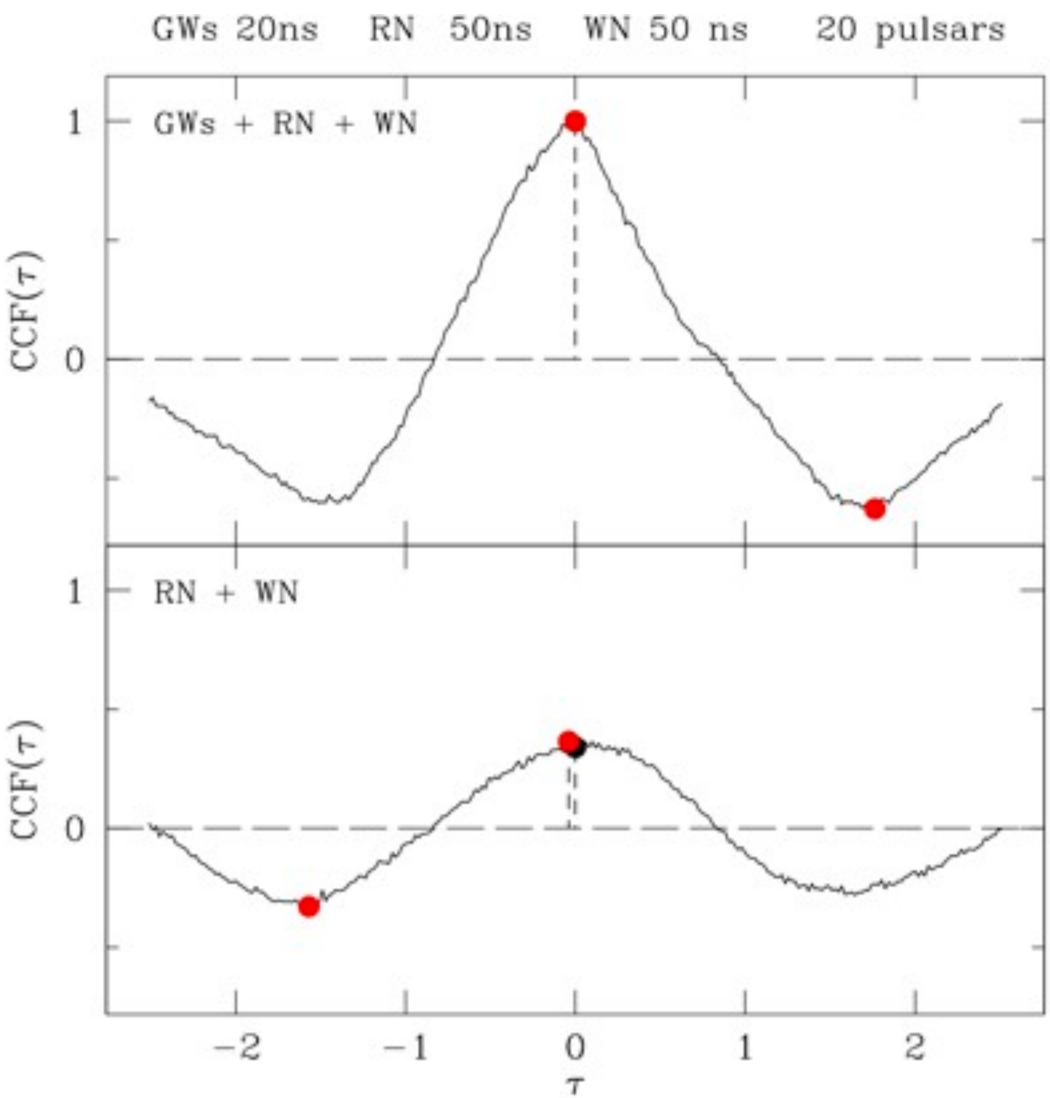
Each CCF is the sum of all CCF pairs in a 20-MSP PTA

More difficult: RN: 50ns WN: 50ns



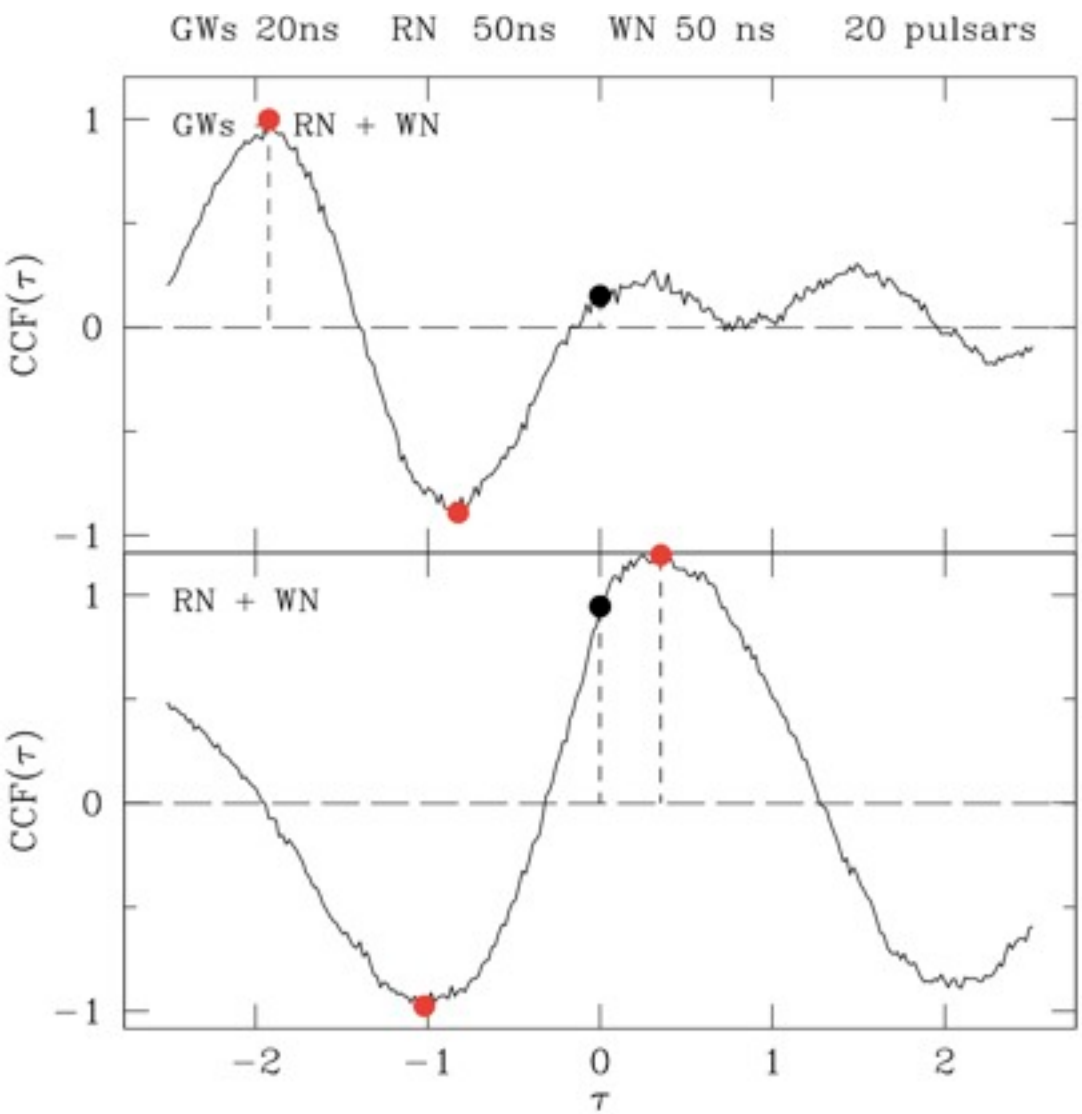
Four realizations shown

More difficult: RN: 50ns WN: 50ns



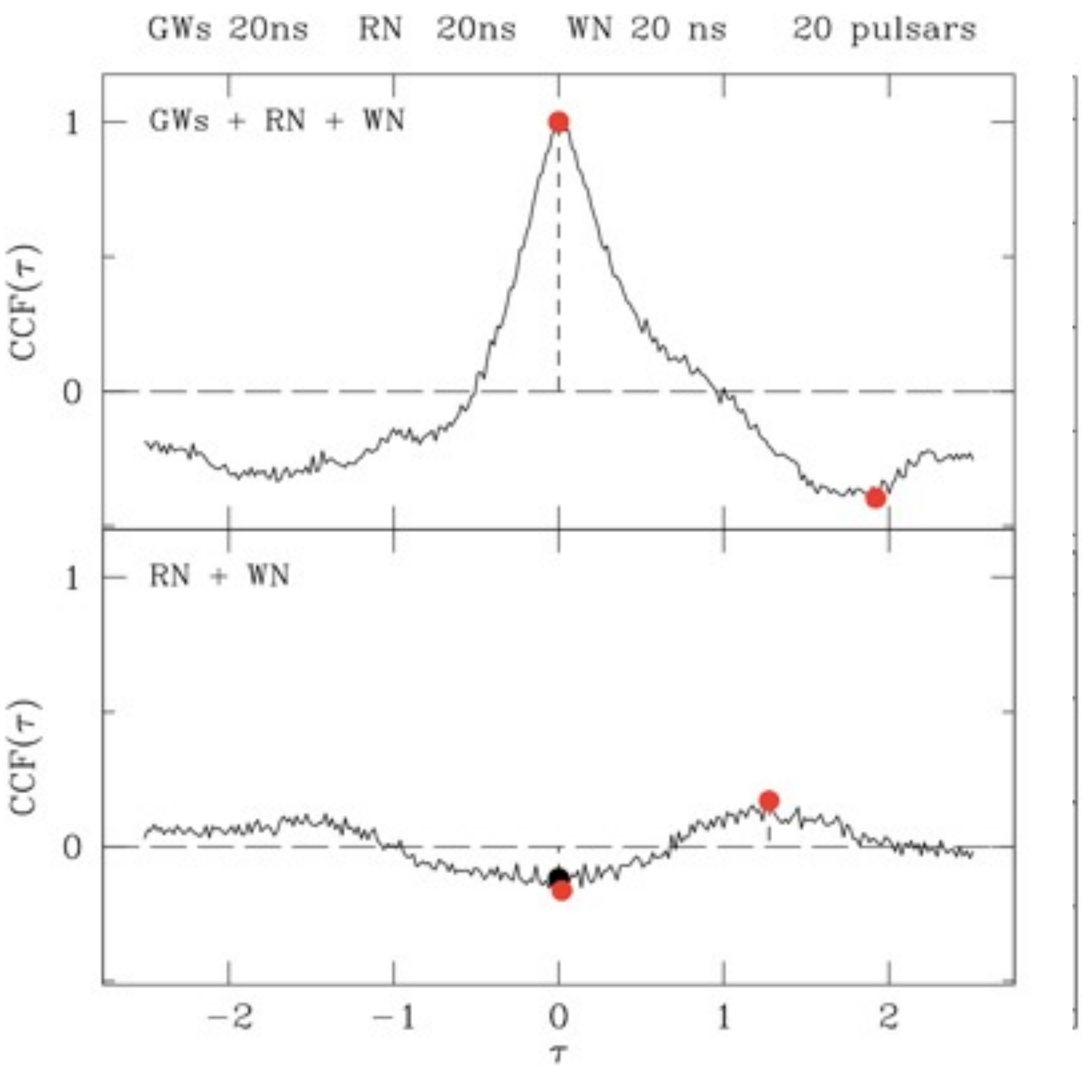
Four realizations shown

More difficult: RN: 50ns WN: 50ns



Four realizations shown

More difficult: RN: 50ns WN: 50ns



Four realizations shown

Non-Gaussian Correlation Statistics

The zero-lag CCF C_{00} is used to construct an HD estimator
The distribution of C_{00} is skewed even if no GWs contribute
➔ Strong effect on false-alarm rate as well as detection rate

An interesting case of the CLT not applying.

Skewness is larger for redder spectra and more MSPs in the PTA

Metrics used for Detection

$m_1 = C_{00}/C_{minimum}$ m_2 = lag of max correlation/max calculated lag

Detection criterion: $m_1 > 1$ and $|m_2| < 0.1$

detections
(include some
false alarms)

non-detections
(include some
false-positives)

Detections vs. False Alarms

ROC curves for different PTAs

good

bad

Implications:

~20 hyperstable MSPs (rms red noise < 20 ns over 5 yr) are sufficient

OR

Many more MSPs needed to increase significance as $\sqrt{N_p}$ e.g. $N_p \sim 50$ to 100

Detection Fraction vs. S

S = signal to noise ratio of C_{00}

red + white noise

white noise only

Analytical Expression for

$$S = \frac{\sqrt{\psi N_p M}}{2}\left\{ w_{gg} + \xi_M w_{rr} + \frac{(w_{gg} + \xi_M^2 w_{rr} + 2\xi_M w_{gr})}{2\psi(N_p - 1)} + \frac{\eta_M M}{N_t}\left[1 + \frac{(\eta_M / N_s + 2 + 2\xi_M)}{2\psi(N_p - 1)}\right]\right\}^{-1/2} \quad (12)$$

Agrees with simulations

Takes into account:

- smoothing of residuals
- analysis in blocks (~ pre-whitening)
- strength of red noise relative to GWs
- strength of white noise rel to GWs
- characteristic correlation times of red noise + GWs ~ T/M = length of data set

S vs Blocking

Optimal blocking is M ~ 3 to 5

i.e. the total time span T is divided into M blocks of length T/M, each of which is processed independently and then combined into an average CCF

An Alternative Approach: Coherent Sum

- Pairwise correlation:

$$\hat{C}(\theta,\tau) = \frac{1}{N_X(\theta)} \sum_{i,j:\theta} \frac{1}{T} \int_0^T dt\, x_i(t) x_j(t+\tau),$$

- This is: correlate | sum
- Alternative: sum | correlate:
- Sum time series of N_p pulsars with weights ~ HD(θ_{ref}) for some reference direction
- Cross correlate with another sum for a different reference direction
- Can vary reference directions to get C(θ)
- Advantage: full statistical signficance for each θ and can visually inspect a curve that is equivalent to the HD function

Climbing Mount Significance

- Need to increase S to be in a good place in the ROC curve
- Given red noise and white noise, the primary means for increasing S are:
    - best possible TOAs
    - use blocking (which requires a large cadence)
    - more pulsars, more pulsars, more pulsars
- Further discussion in CS11 (submitted)

Summary and Other Points

- DM removal requires multi-frequency observations simultaneous to < a few days
- Better TOA precision requires attention to non-DM ISM effects + appropriate fitting
- Red noise is likely present in all MSPs to varying degrees and drastically alters ROC curves (detection/false-alarms)
- Inspection of the temporal CCF is an important under-the-hood diagnostic as well as providing a detection method
- Many more high-quality MSPs needed for a convincing detection

Extra Slides