Low-Frequency Noise

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Why do we care?

PTAs are designed to study low-frequency phenomena.

We think that the GWB will add a random process to the timing residuals with a power spectrum of the form $P_G = K f^{-13/3}$.

In addition the residuals of different pulsars (i and j) will be correlated with a correlation coefficient $\zeta(\theta_{ij})$ that depends only on the angle between them θ_{ij}.

We need to estimate P_G and $\zeta(\theta_{ij})$ with sufficient accuracy to claim a detection or, failing that, to establish an upper bound on K.

I will give a quick overview of the detection/bounding process to show how the low-frequency noise enters.

Spectral and Cross-Spectral Estimation

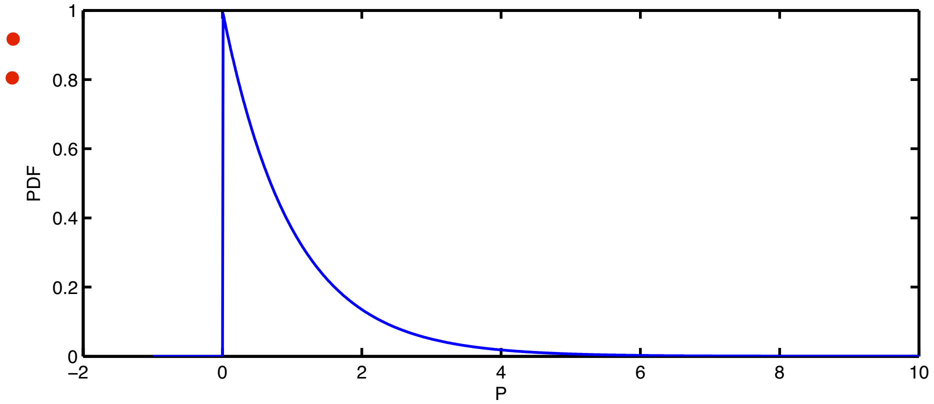
The timing residuals are $R_i(t_k)$. The power spectral estimates are: $P_i(f_k) = |DFT(R_i(t_k))|^2 \, 2\, T_{iOBS} / N_{iOBS}^2$ where $f_k = k/T_{iOBS}$ are the harmonics of $1/T_{iOBS}$

The cross power spectral estimates are:
$C_{ij}(f_k) = DFT(R_i(t_k)) \, DFT^C(R_j(t_k)) \, 2\, T_{ijOBS} / N_{iOBS} N_{jOBS}$.
at $f_k = k/T_{ijOBS}$.

The DFT $(R_i(t_k))$ are weighted sums of residuals so the CLT applies and we can easily find the statistical moments of the $P_i(f_k)$ and $C_{ij}(f_k)$ estimators.

(in real life how we take that DFT is important)

The estimator $P_i(f_k)$ is exponential:



$\mathrm{pdf}(P) = a^{-1}\exp(-P/a)$ for $P, a > 0$.

$\mathrm{Mean}(P) = a$ and $\mathrm{RMS}(P) = a$

$\mathrm{Prob}(P < 0.05\ a) = 5\%$; and $\mathrm{Prob}(P > 3\ a) = 5\%$.

Given a sample P^* the MLE for a is $\tilde{a} = P^*$.

Upper bound at 95% confidence is $a < P^*/0.05$, and

Lower bound at 95% confidence is $a > P^*/3$.

However $P_{obs}(f) = P_{gwb}(f) + P_{noise}(f) > P_{gwb}(f)$.

So the lower bound (detection) is not useful unless $P_{noise}(f)$ is well known (not presently true).

The upper bound is useful, but loose at $a < 20\ P^*$!

Tighten the upper-bound with Averaging/Prewhitening:

$K_{est\_ik} = P^*_i(f_k)\ f_k^{+13/3}$ if $P^*_i(f_k) > P_{noise}(f_k)$

and all estimates have the same variance, so

$K_{est} = Sum_{ik}(K_{est\_ik})/N_{ik}$

This estimator has X^2 pdf with 2 N_{ik} degrees of freedom.

It can be biased upwards, but remains an upper bound.

So the name of the game is to maximize the number of harmonics for which $P^*_i(f_k) > P_{noise}(f_k)$ and the S/N ratio of the estimator will go like sqrt(N_{dof})

(you can remove the known $P_{noise}(f_k)$ and weight the Sum)

What about the correlation between pulsars?

Using $C_{ij}(f_k) = \zeta(\theta_{ij})\ P_{gwb}(f_k)$

$K_{est\_ijk} = (\ C^*_{ij}(f_k)/\zeta(\theta_{ij}))\ f_k^{\ +13/3}$ with variance $\propto\ 1/\zeta_{ij}^2$

so:

$K_{est} = \mathrm{Sum}(\zeta_{ij}^2\ K_{est\_ijk})/\mathrm{Sum}(\zeta_{ij}^2)$

$N_{ijk} \gg N_{ik}$ but $\mathrm{mean}(\zeta_{ij}^2) \ll 1$

K_{est} is not X^2 but we can work out all its moments.

This K_{est} works for both detection and bounding

because the correlated noise is well understood.

It can be estimated and removed so K_{est} is unbiased.

The cross-power K_{est} is more sensitive than the auto power estimator if the PTA has > 13 equal pulsars.

This is not true for the PPTA!

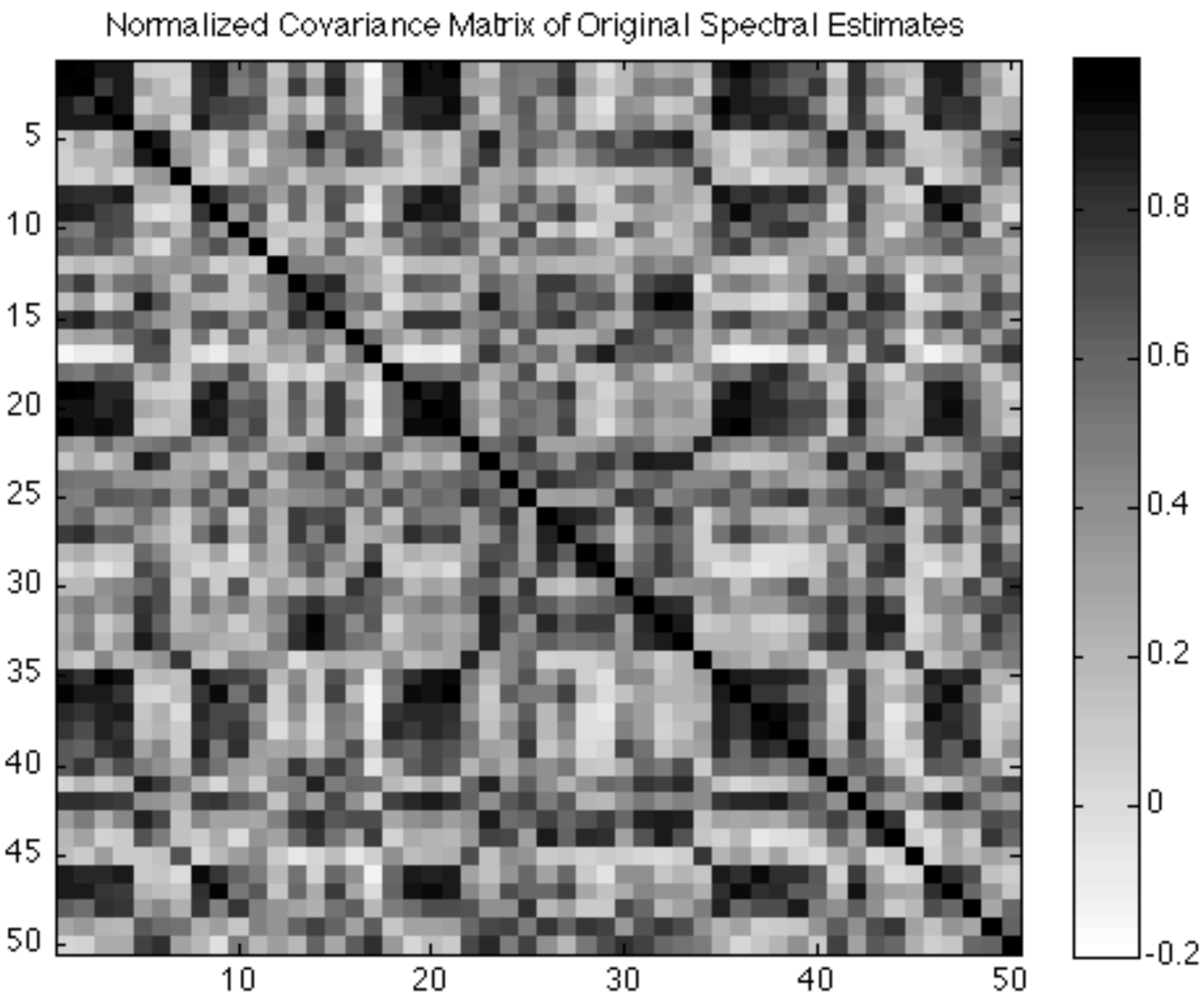
Which is why Ryan uses the spectral estimator for bounding rather than the cross spectral estimator.

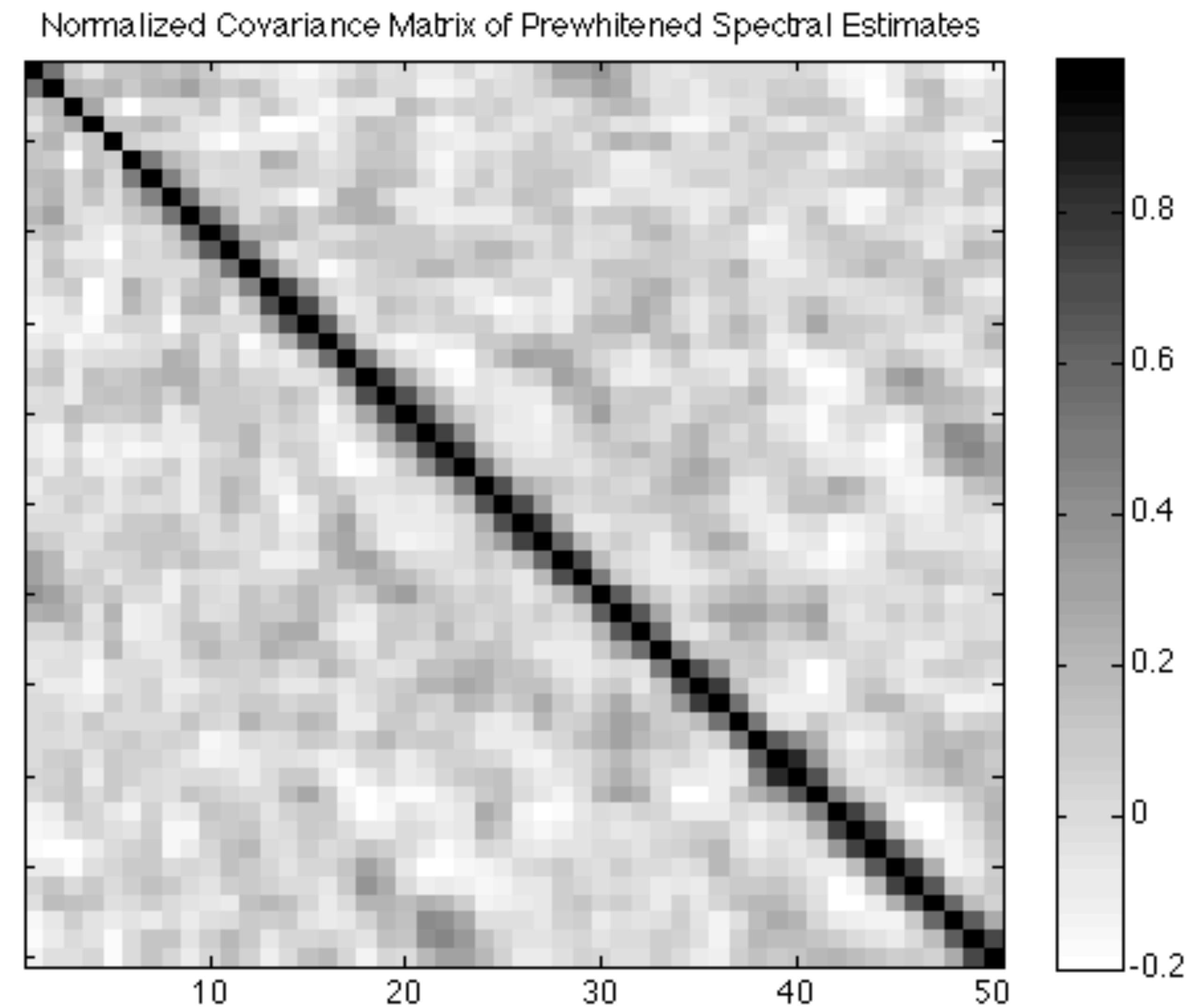
To take advantage of Averaging/Prewhitening the spectral harmonics must be independent!

So much for the lightning tour of detection/bounding!
What do we need to do in practice?

1. Make sure the spectral estimates are independent.

-this requires prewhitening the residuals and works best when the sampling is regular.





2. Need the PDF of the detection statistics. Why?

Detection is a binary problem: either

Hypothesis H_0 :no signal is present; or

Hypothesis H_1 :a signal is present, must be true.

The test is: H_1 is true if statistic $S >$ Th; else H_0 is true.

We need the error probabilities:

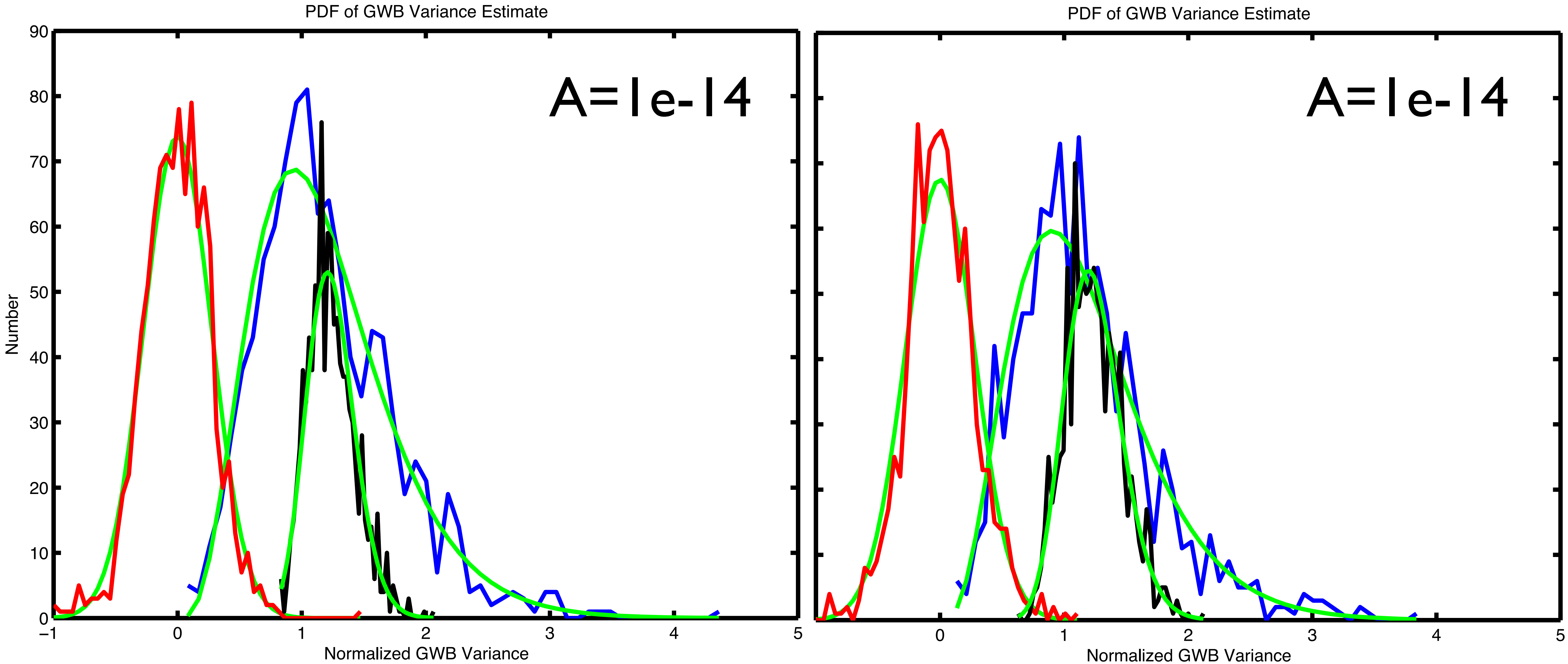
(1) failure to detect: i.e. $S <$ Th under H_1 ; and

(2) false detection: i.e. $S >$ Th under H_0 .

To compute them we need the PDF of S under H_0 and H_1.

The first is easy. Generally we set Th so the probability of a false detection is small.

This requires simulation:



New GWB generator

GWB generator in TEMPO2

-black = power spectral detection statistic
-blue = cross spectral detection statistic (real)
-red = cross spectral detection statistic (imaginary)

Simulations are needed in many aspects of PTA analysis, so it is important to confirm that the results are not dependent on the simulation engine.

3. We need to know the "Best Possible Estimator":

-this would be one where the data sampling is regular and you know the statistics of the signal and the noise

-this gives you a target for dealing with Real Data.

4. Learn as much as possible about the noise:

-so it can be subtracted from the power spectra

-so you have a better estimate of correct weighting and can get closer to the optimal estimator.

-the more you know the better off you are.

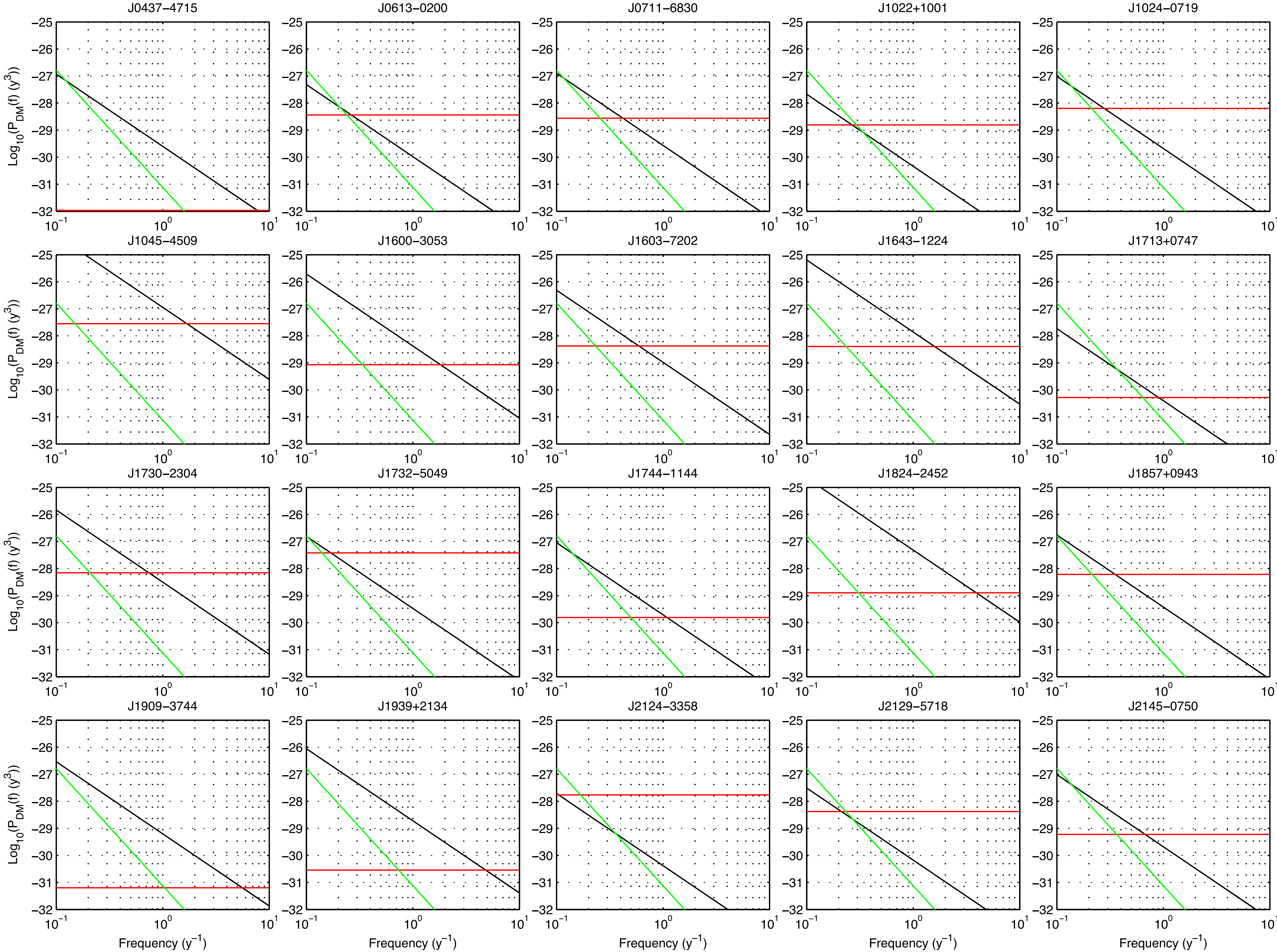
Low Frequency Noise Components:

The most important noise is DM(t) variation. Its power spectrum (almost) always exceeds that of the GWB. Estimation and removal has been discussed already today and the PPTA scheme in a recent Keith et al submission.

The next is Red Noise. It is not (yet) removable. Jim will discuss it in detail after tea. I'll outline its effects.

Probably the next is Scattering Noise. It has not been thoroughly investigated yet. It limits the use of some pulsars, and prevents the use of longer λ in estimating DM(t) for some pulsars.

P_{GWB}, P_W and P_{DM} for PPTA



DM Estimation and Correction

The DM(t) fluctuations can be estimated using multi-wavelength observations but one must also estimate the common-mode residual at the same time.

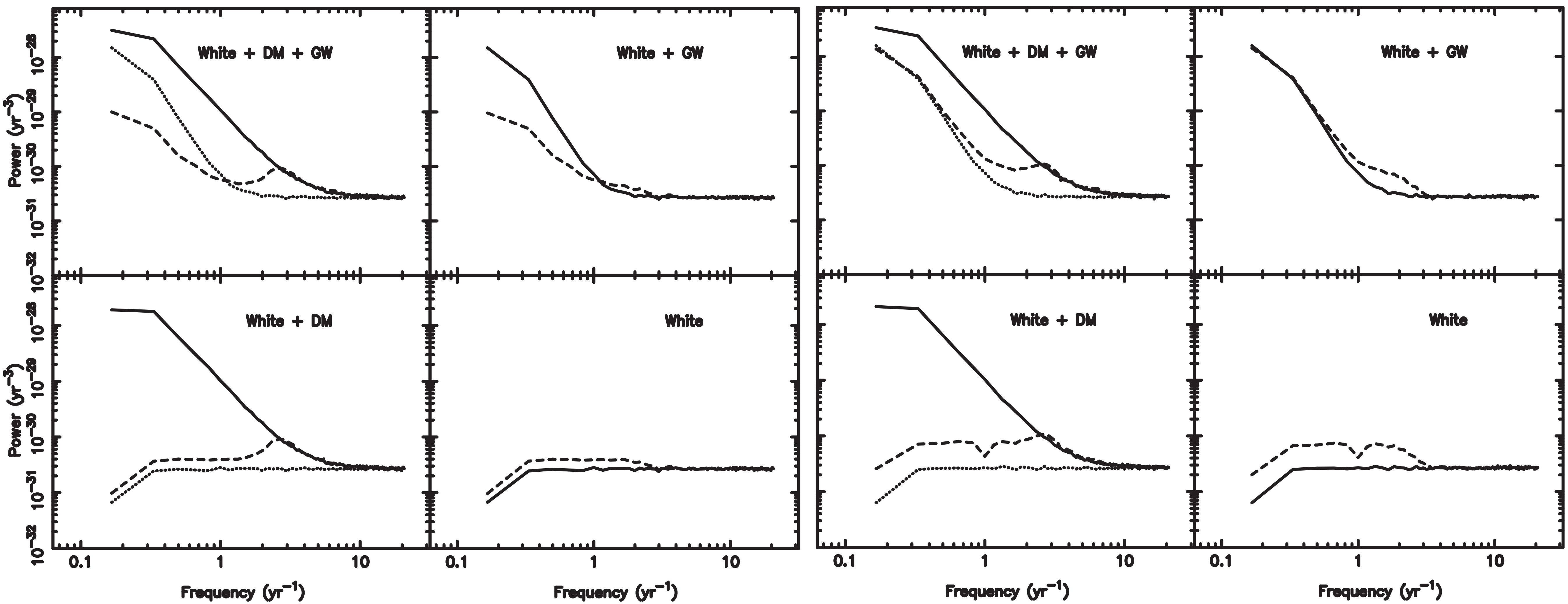
DM(t) estimation attracts white noise and that white noise will be transmitted to the corrected residuals.

This correction is absolutely essential and very expensive.

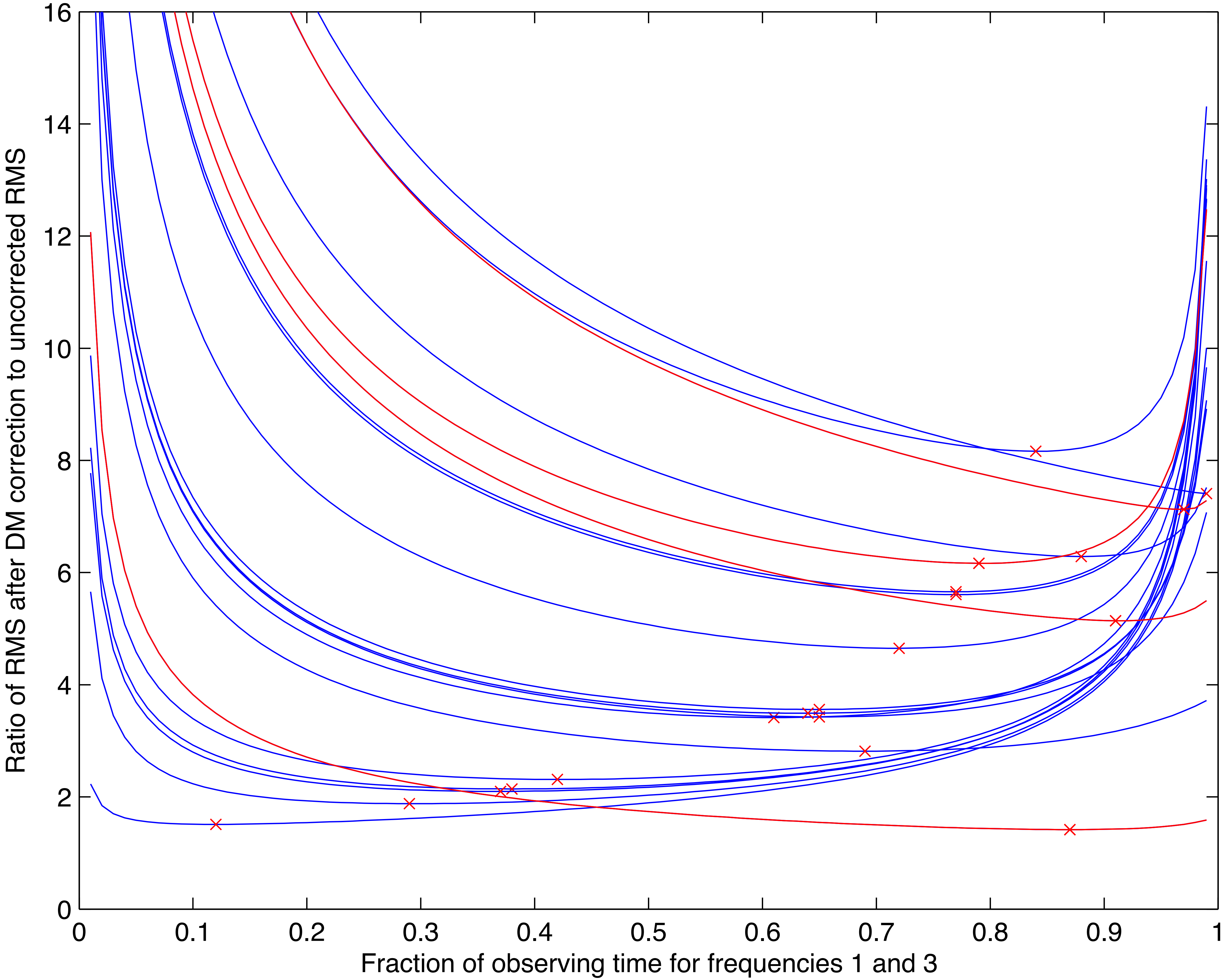
DM(t) correction: Power spectra with and without a common-mode

without C-M

with C-M



Cost of DM(t) correction at the PPTA



Red Noise:

The red noise observed in J1939+2134 and J1824-2452 is steeper than the GWB

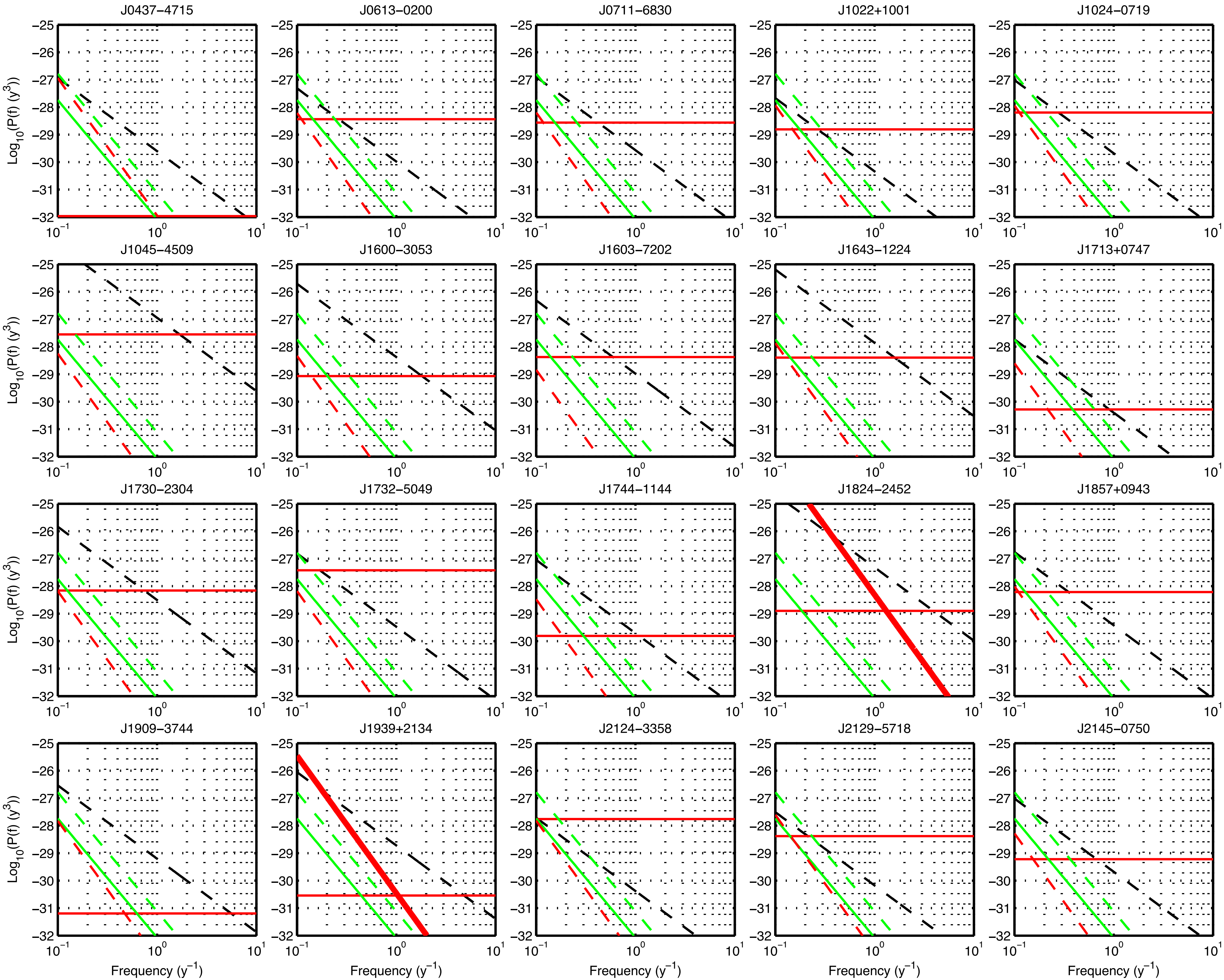
- Jim and Ryan estimate |exp| > 5
- similar red timing noise in young pulsars has |exp| > 6

So $P_{gwb}(f) > P_{red}(f)$ for $f > f_{cred}$

and $P_{gwb}(f) > P_{w}(f)$ for $f < f_{cw}$

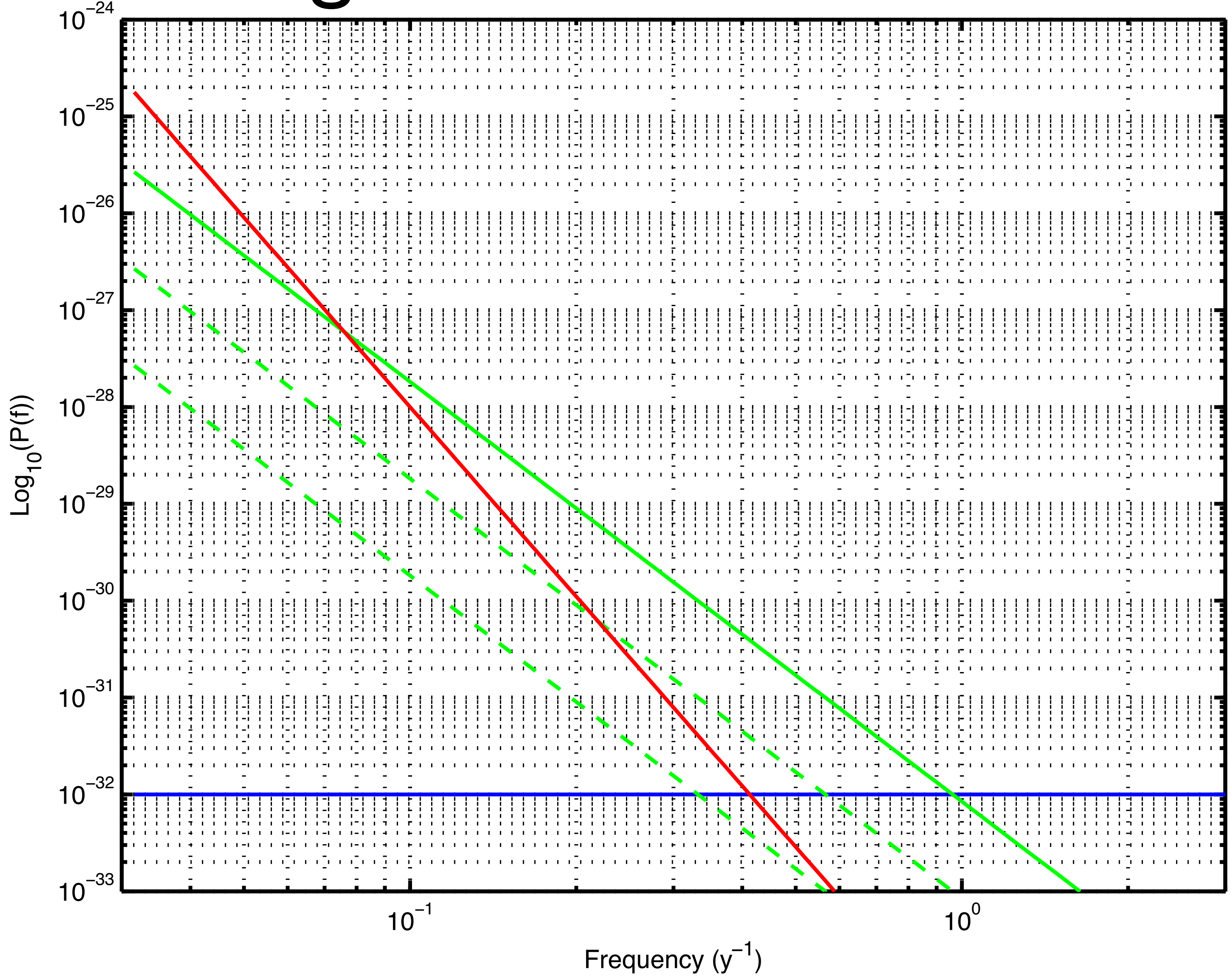
require a bandpass weighting or "Wiener" filter.

P_{GWB}, P_{W}, P_{DM} and P_{RED} for PPTA



This appears to be a catastrophe because the low f's are where the GWB is strongest. Not quite correct!

e.g. if you have a marginal detection at, A=1e-15 you can still double N_{DOF} by doubling T_{OBS}, but at a fixed P_W it sets a lower limit on the detectable A. This can only be reduced by reducing P_W.



Scattering Noise:

This contribution is from time variation in the scattered pulse width t_0. In J1939+2134 the rms is about 30% of t_0 and one must assume that it is similar for other pulsars. It has a time scale of weeks, i.e. $\approx$ white.
It is comparable with P_W at λ=20cm for J1045, J1600, J1643, and J1824 and is much larger at longer λ.

It enters the DM-corrected residuals for less strongly scattered pulsars through the longer λ observations used to estimate DM. It makes use of LOFAR for estimating DM(t) impossible for 10 of the PPTA pulsars.