Pulsar Timing and Instrumental Polarization

International Pulsar Timing Array 2012 Science Meeting

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Pulsar Polarization

- variation with pulse phase
  - Rotating Vector Model
  - Orthogonally Polarized Modes
- variation with radio frequency
  - Faraday rotation
- variation with time
  - e.g. mode-changing
  - instrumental distortion

Electromagnetic Radiation

$$\boldsymbol{e}(t) = \begin{pmatrix} e_0(t) \\ e_1(t) \end{pmatrix}$$

Coherency Matrix

$$\boldsymbol{\rho} \equiv \langle \boldsymbol{e}\,\boldsymbol{e}^{\dagger} \rangle = \begin{pmatrix} \langle e_0 e_0^* \rangle & \langle e_0 e_1^* \rangle \\ \langle e_1 e_0^* \rangle & \langle e_1 e_1^* \rangle \end{pmatrix}$$

$$\boldsymbol{\rho} = \boldsymbol{\rho}^{\dagger}$$

Stokes Parameters

$$\boldsymbol{\rho} = S_k \, \boldsymbol{\sigma}_k / 2$$

$$S_k = \mathrm{Tr}(\boldsymbol{\sigma}_k \boldsymbol{\rho})$$

$$\boldsymbol{\sigma}_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \boldsymbol{\sigma}_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\boldsymbol{\sigma}_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \boldsymbol{\sigma}_3 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

Stokes 4-vector

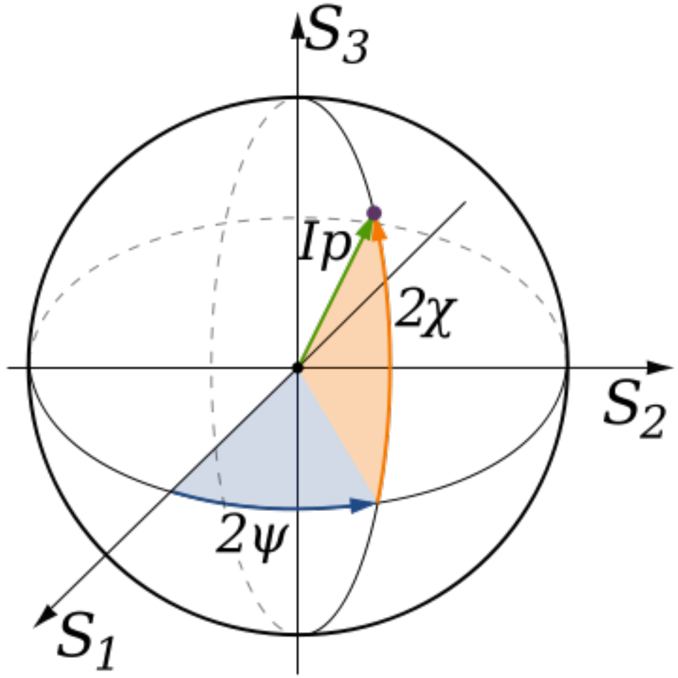
- Total intensity (time-like)

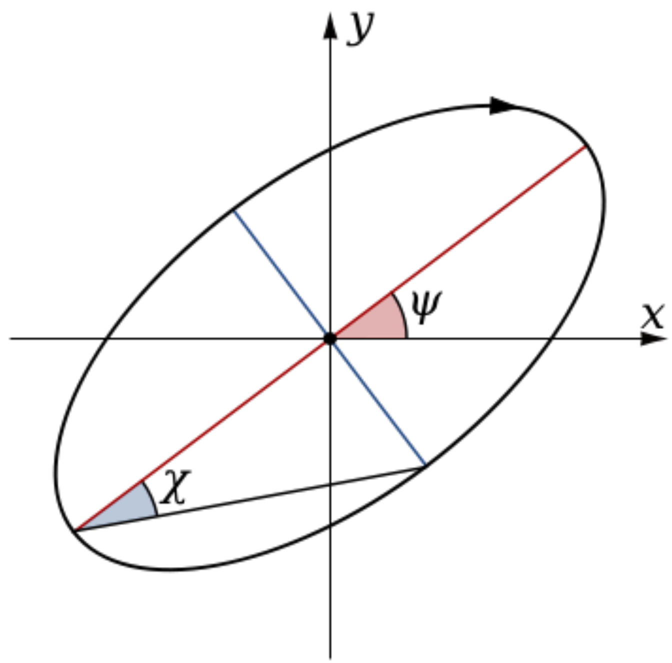
$$S_0 = I$$

- Polarization vector (space-like)

$$\boldsymbol{S} = (S_1, S_2, S_3)$$

Stokes Vector – Polarisation Ellipse





Transformations

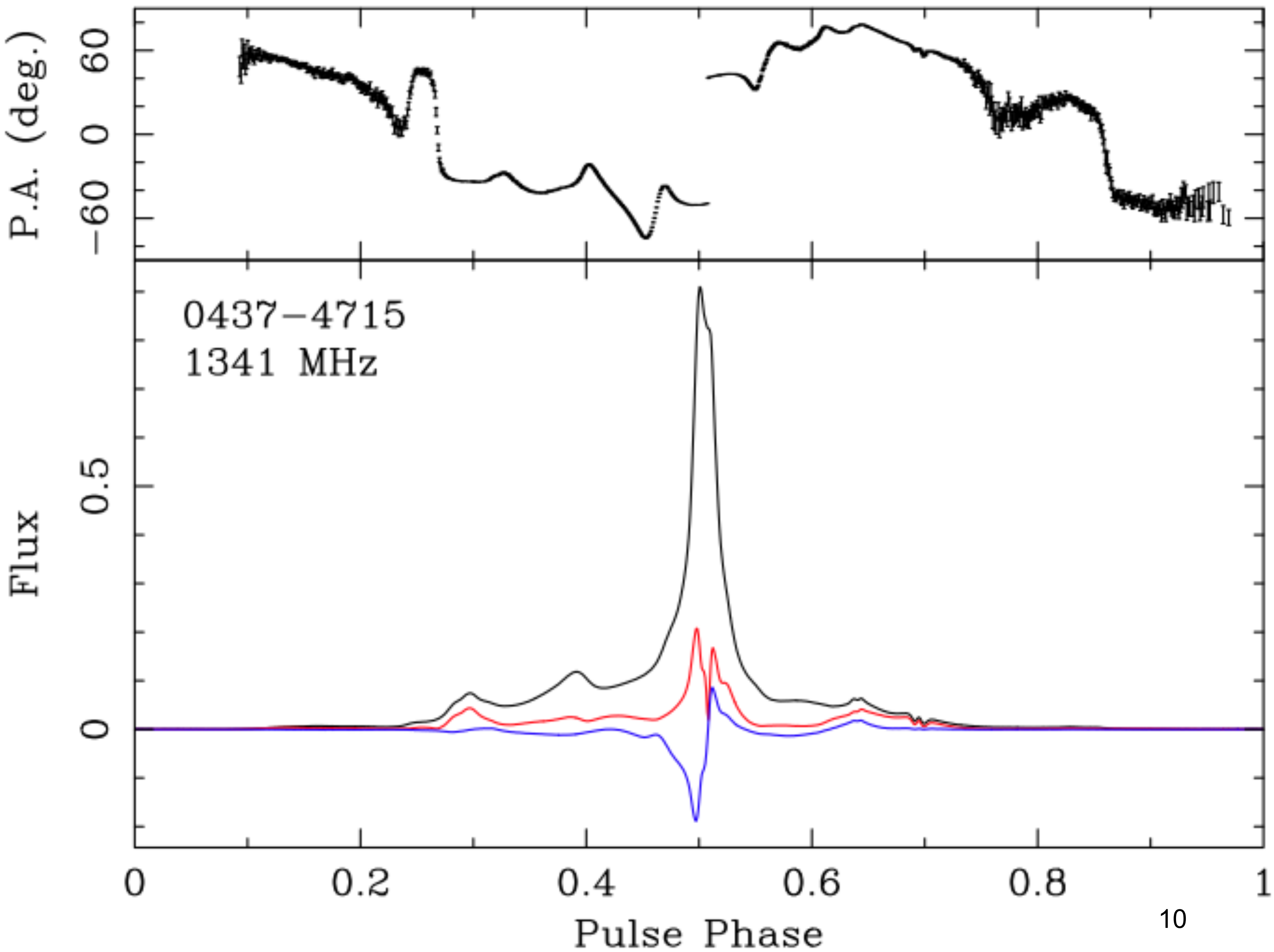
- Linear transformations of electric field
- Rotations:
  - rotate $\boldsymbol{S}$ about an axis by an angle
  - leave S_0 unchanged
- Boosts:
  - boost along an axis by an impact factor
  - modify both S_0 and $\boldsymbol{S}$

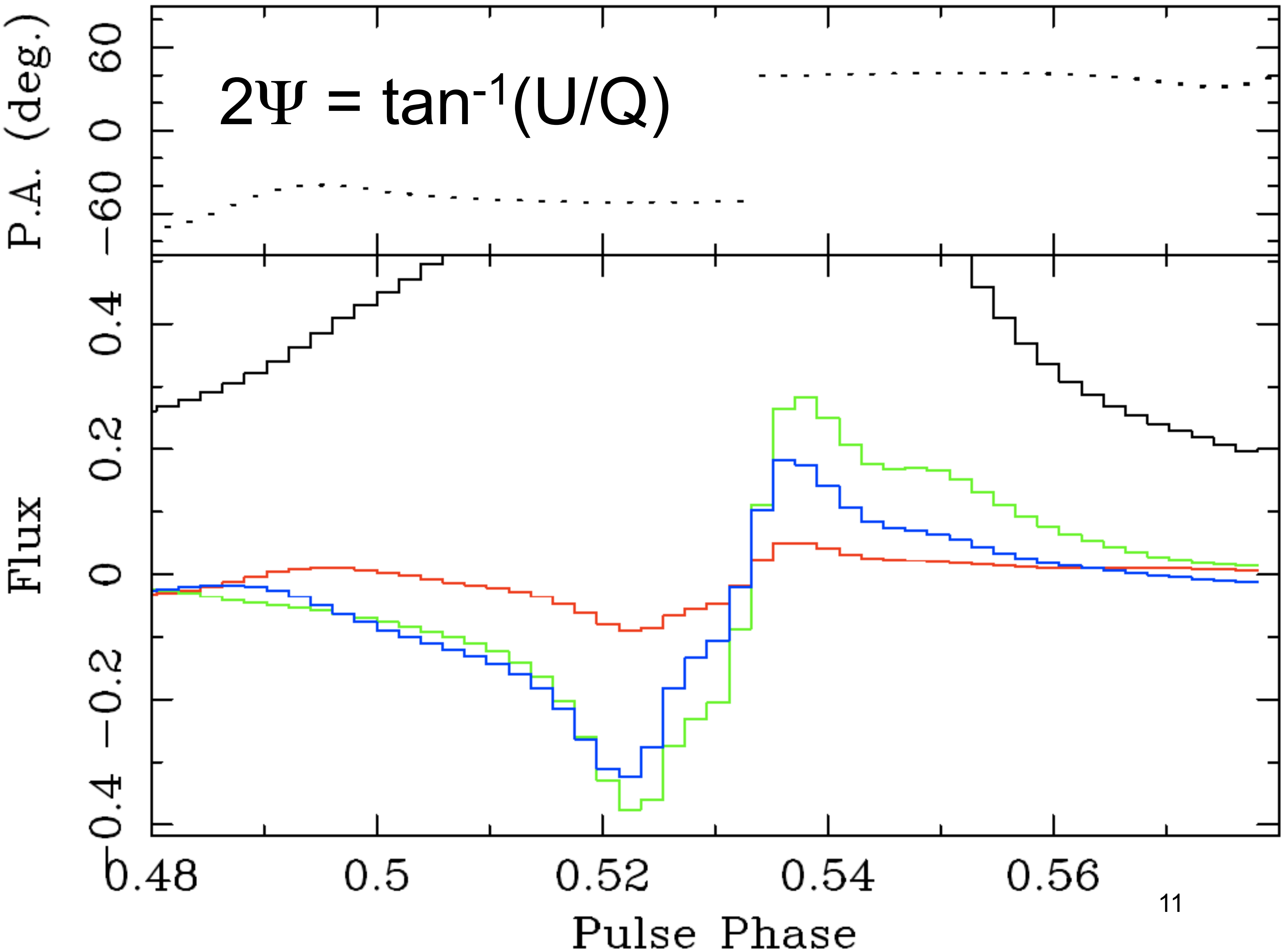
Instrumental Distortion

- Non-orthonormality of receptors:
  - differential gain (along S_1 axis)
  - cross-coupling (along S_2 & S_3 axes)

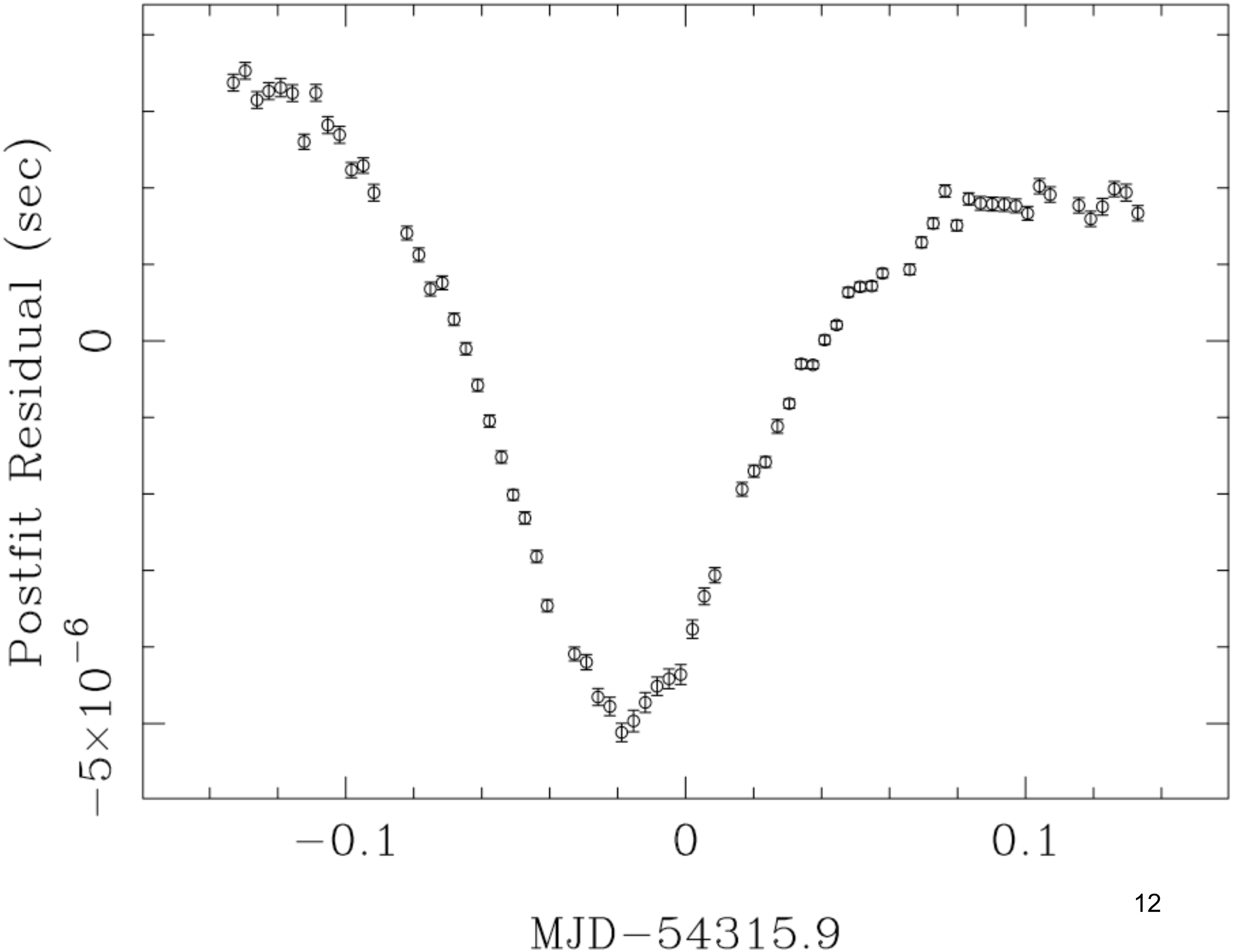
- Mixing of S_0 and $\boldsymbol{S}$ depends on $\boldsymbol{S} \bullet \boldsymbol{\beta}$

- Produce pulse phase dependent distortions of total intensity profile


P.A. (deg.)
60
0
−60
0437−4715
1341 MHz
Flux
0.5
0
0
0.2
0.4
0.6
0.8
1
Pulse Phase



$2\Psi = \tan^{-1}(U/Q)$
P.A. (deg.)
60
0
−60
Flux
0.4
0.2
0
−0.2
−0.4
0.48
0.5
0.52
0.54
0.56
Pulse Phase



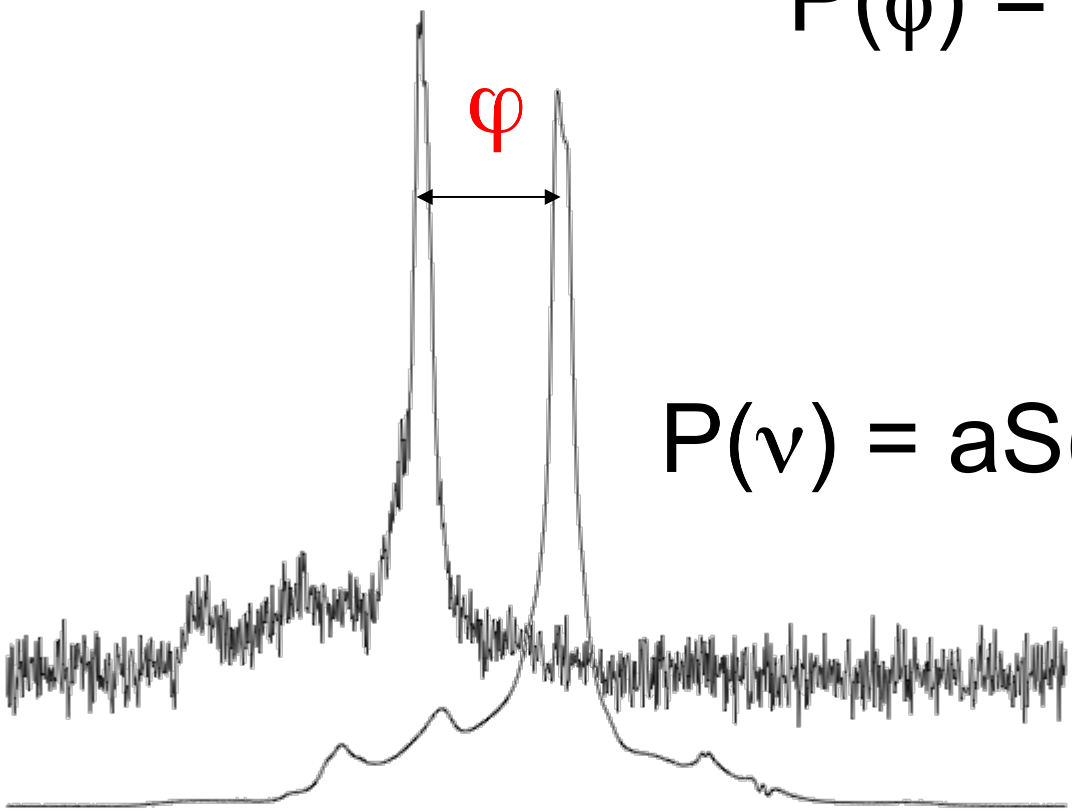
Postfit Residual (sec)
0
-5×10^{-6}
-0.1
0
0.1
MJD$-$54315.9

Template Matching

$$P(\phi) = aS(\phi + \varphi) + b$$

FFT

$$P(\nu) = aS(\nu)e^{-i2\pi\varphi\nu} + \delta(\nu)b$$



$$\dot{\varphi}_k \equiv \left.\frac{\partial \varphi}{\partial b_k}\right|_{\beta=0} = \frac{\sum \mathrm{Im}[S_k^*(\nu_m)S_0(\nu_m)]\nu_m}{\pi \sum |S_0(\nu_m)|^2 \nu_m^2}$$

$$\dot{\varphi}_\beta \equiv |\dot{\boldsymbol{\varphi}}|$$

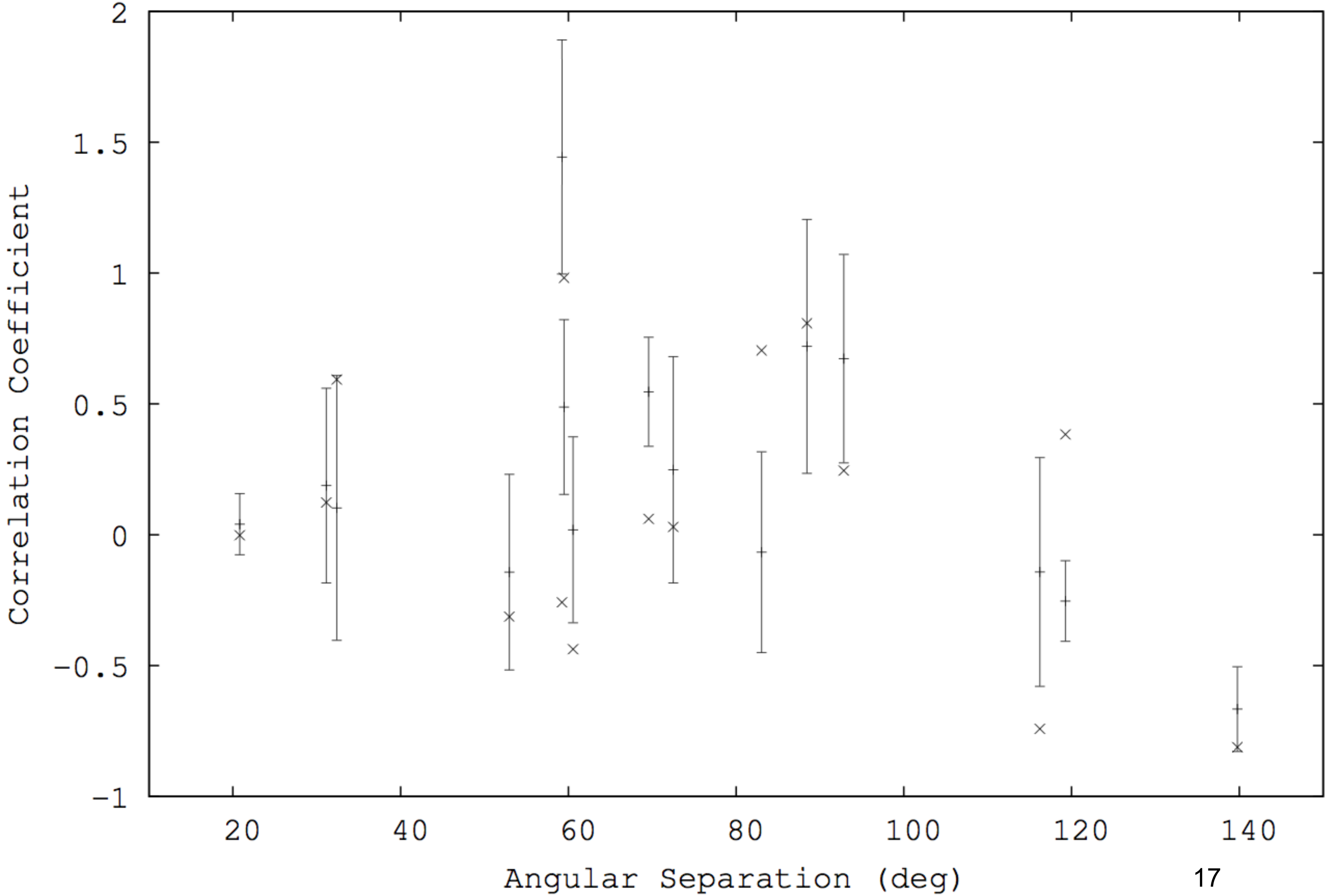
$$\tau_\beta \equiv 5 \times 10^{-3} \dot{\varphi}_\beta P$$

Pulsar	$\hat{\sigma}_{\varphi}$	$\hat{\sigma}_{\tilde{\varphi}}$	τ_{β} (ns)
J0437−4715	0.85	1.43	207
J0613−0200	0.92	1.46	59
J0711−6830	0.88	1.54	81
J1022+1001	0.68	1.65	282
J1024−0719	0.74	2.11	34
J1045−4509	0.88	1.48	338
J1600−3053	0.90	1.39	115
J1603−7202	0.85	1.55	142
J1643−1224	0.91	1.40	266
J1713+0747	0.85	1.58	6
J1730−2304	0.71	1.70	198
J1732−5049	0.96	1.38	185
J1744−1134	1.56	6.43	105
J1824−2452A	0.88	2.56	18
J1857+0943	0.89	1.43	124
J1909−3744	1.02	1.51	22
J1939+2134	0.95	1.49	44
J2124−3358	0.85	1.45	127
J2129−5721	1.15	1.61	211
J2145−0750	0.95	1.44	147

$$\mathbf{C}_\tau(f) = \boldsymbol{\Upsilon}\mathbf{C}_\beta(f)\boldsymbol{\Upsilon}^T$$

$$\Upsilon_j^k \equiv \frac{\partial \tau_j}{\partial b_k} = P_j \frac{\partial \varphi_j}{\partial b_k}$$

$$c_{AB} = \frac{\dot{\boldsymbol{\varphi}}_A \cdot \dot{\boldsymbol{\varphi}}_B}{|\dot{\boldsymbol{\varphi}}_A||\dot{\boldsymbol{\varphi}}_B|}$$


Correlation Coefficient
Angular Separation (deg)
2
1.5
1
0.5
0
-0.5
-1
20
40
60
80
100
120
140

Calibration Methods

- Measurement Equation Modeling (MEM; van Straten 2004)
  - unknown source and calibrators observed over a range of parallactic angles

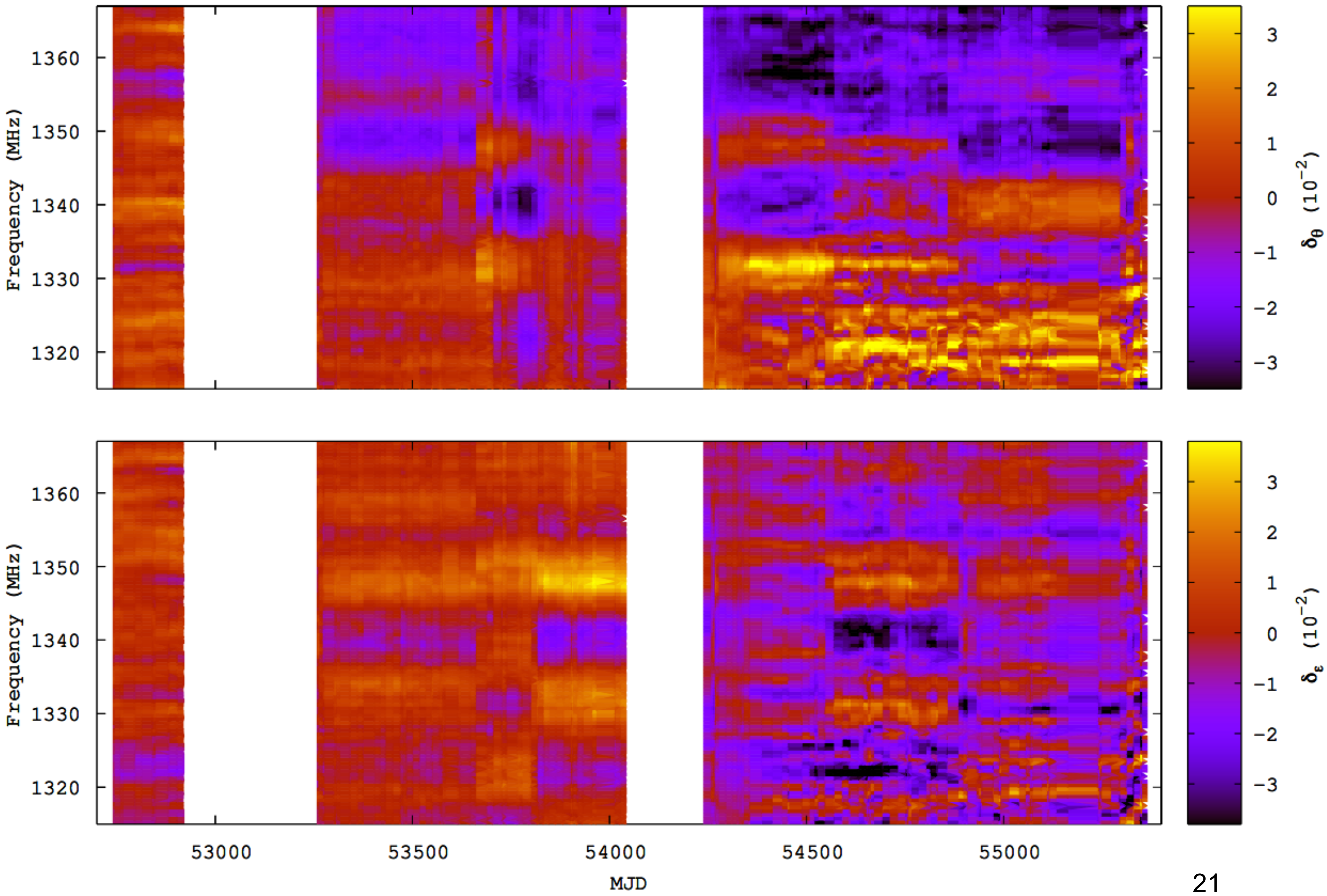
- Matrix Template Matching (MTM; van Straten 2006)
  - known source matched to a single (short) observation; no calibrator input

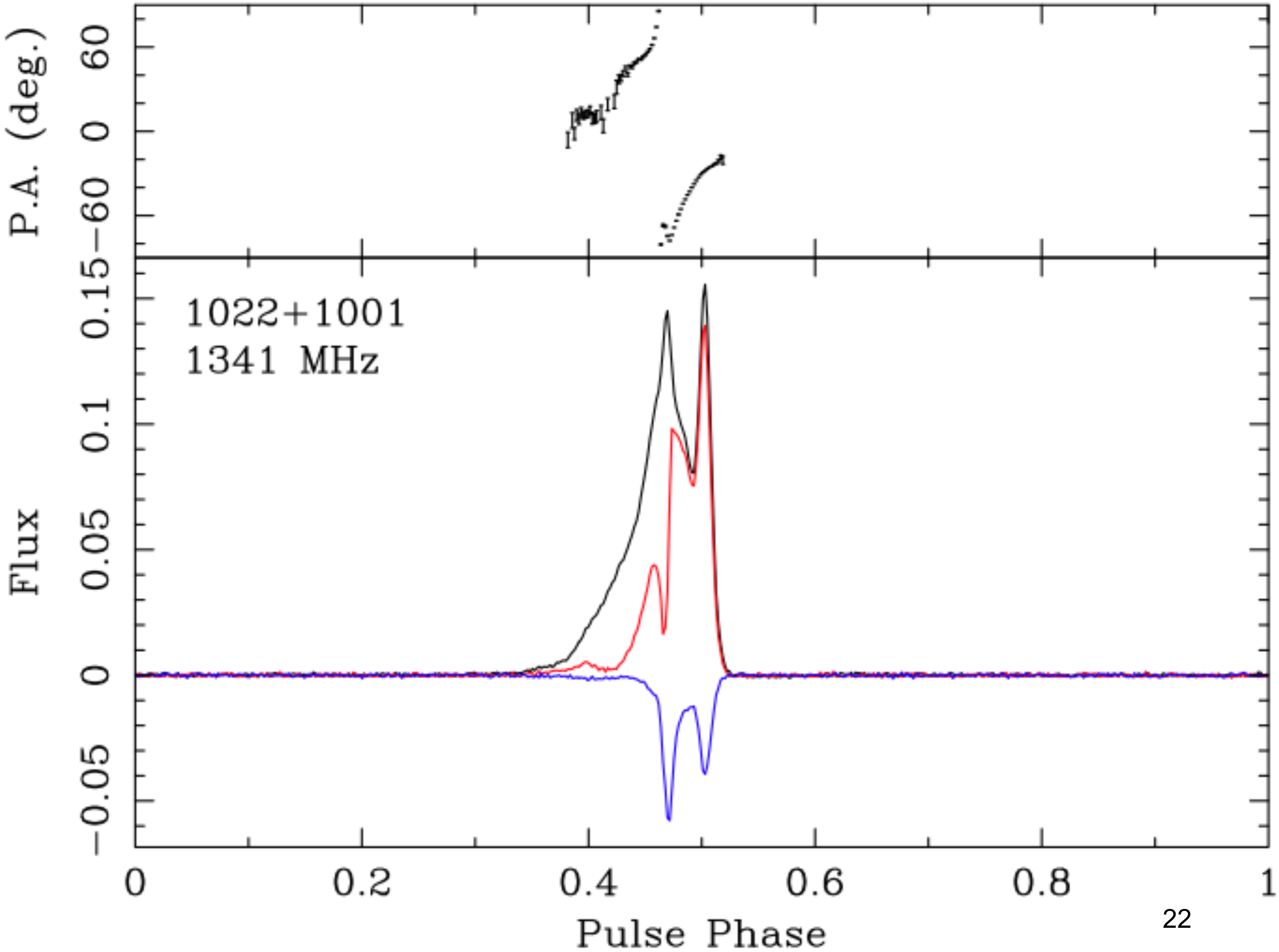
METM

- Use 0437-4715 as polarized reference
- Merge MEM and MTM methods
  - match template to multiple observations
  - incorporate and constrain calibrator
    - polarization of noise diode not assumed
  - model time-variations
    - parallactic angle, drifting gains and phases, etc.

METM Workflow

1) Derive 50 MEM solutions (~8hr each)
   – J0437-4715 & wide range in parallactic angle
2) MTM Integrate into standard (~150hr)
   – reduced χ^2 within 4% of unity in each fit
3) Derive 350 MEM/MTM solutions (~3.5hr)
   – both receiver and noise diode estimated
4) Apply solutions to another pulsar


Frequency (MHz)
1320
1330
1340
1350
1360
3
2
1
0
−1
−2
−3
δ_θ (10^{-2})
δ_ϵ (10^{-2})
53000
53500
54000
54500
55000
MJD



P.A. (deg.)
60
0
−60
0.15
0.1
0.05
0
−0.05
Flux
1022+1001
1341 MHz
0
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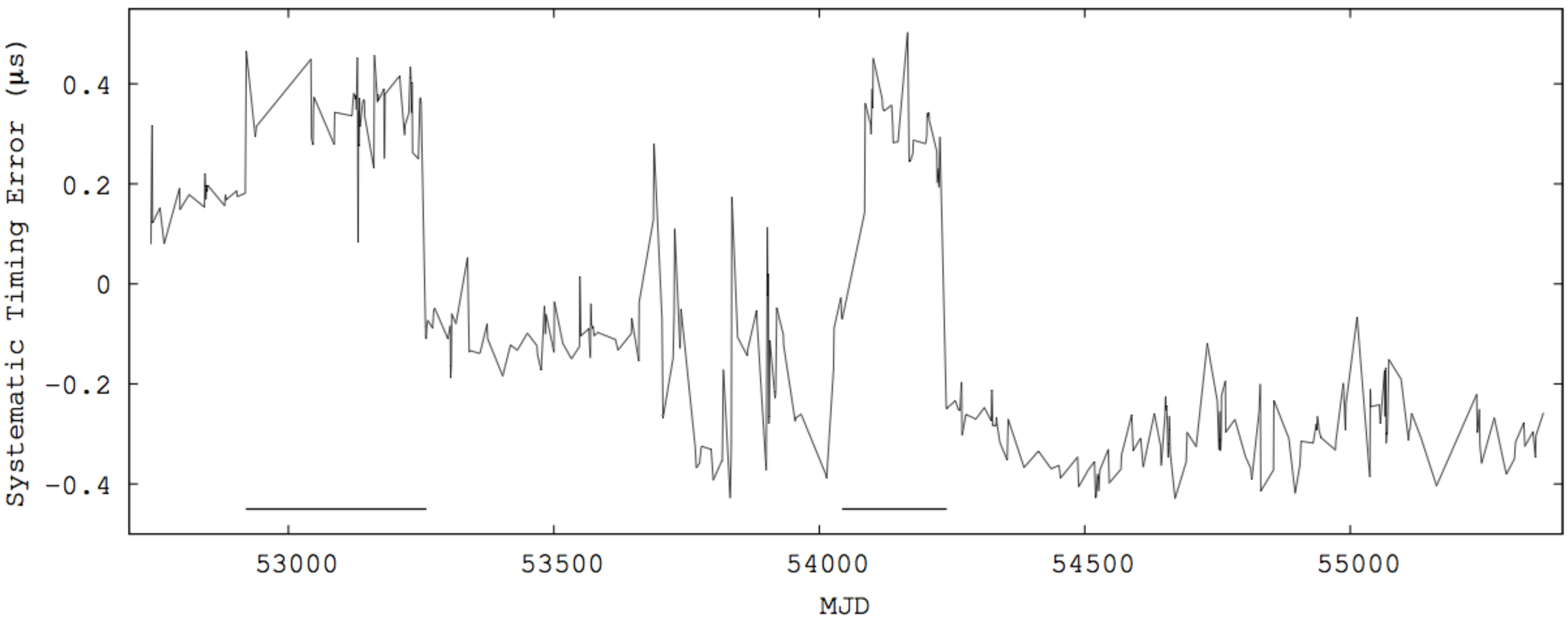
Systematic Timing Error (μs)
0.4
0.2
0
−0.2
−0.4
53000
53500
54000
54500
55000
MJD

Arrival Time Comparison

Method	σ_τ (μs)	$\chi^2/N_{\rm free}$
IFA–STI	1.9	2.2
METM–STI	1.4	2.1
IFA–MTM	0.92	1.1
METM–MTM	0.89	1.0

- 278 TOAS spanning 6.9 years

Conclusions

- Instrumental polarization introduces correlated systematic timing error
- Long-term stability of MSP polarization can be used to calibrate instrumentation
- In some cases, Matrix Template Matching is better than doubling integration length

Conclusions

- METM and MTM can potentially reduce the cost of the SKA (for timing)

- PSR J1022+1001 is a great timer and should be observed by every PTA