Optimal Interpolation and Prediction in Pulsar Timing

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Who I am and what I am doing.

- **Not** standard pulsar astronomer.
- Doing PhD with J. H. Zheng on **spacecraft engineering**.
- Co-supervised by George to work on **pulsar**-based **navigation** methods.
- Trying to find a job on **pulsar astronomy** or **spacecraft engineering**.
- Pulsar-based navigation requires dealing with pulsar **timing noise**.
- IPTA research is also affected by pulsar **timing noise**.
- Therefore giving a talk at this conference.

Why is this important for the IPTA?

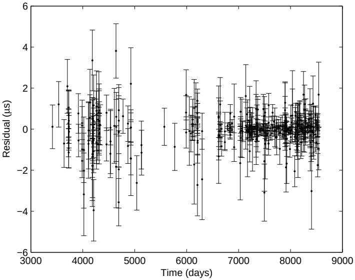
- We need to characterise the timing noise seen in the IPTA data sets in order to predict when we'll be able to detect gravitational waves.
- Need the best quality observations possible => folding pulsar data online requires an optimal extrapolation procedure.
- Need to obtain measurements of the astrometric and orbital parameters of the IPTA pulsars. Need to deal with the red noise when making these measurements.

Simulated white and red timing residuals

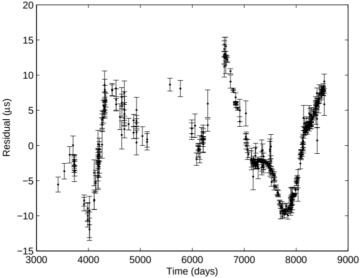
with the real sampling and ToA uncertainties of PSR J1713+0747

We define **timing noise** as **red noise** that has a well defined spectrum model $P(f) = A \cdot f^{-\alpha}$.

White timing residuals
(ToA uncertainties)



Red timing residuals
(ToA uncertainties + Timing noise)



FITWAVES ($\times$)

FITWAVES is a commonly used algorithm to model **timing noise** by fitting a sequence of harmonically related sinusoids to **pulsar timing residuals**.

- It can **not** be extrapolated **past** the end of real observations (or back-extrapolated **before** the start of real observations).
- It requires an **arbitrary** choice of the number of **harmonics** to include in the fitting procedure.
  - **Too few harmonics** implies that **not all the features** in the timing residuals are completely modelled.
  - **Too many harmonics** leads to an **un-physical model** for the timing residuals (particularly when large **gaps** exist in the data).

Maximum likelihood estimator ($\surd$)

Given observations:

$\mathbf{o} = \mathbf{s} + \mathbf{n}$

$\mathbf{o}$: Timing residuals

$\mathbf{s}$: Timing noise

$\mathbf{n}$: ToA uncertainties

The likelihood function:

$\mathbf{s}^{\mathrm{T}}\mathbf{C}_{\mathrm{s}}^{-1}\mathbf{s} + \mathbf{n}^{\mathrm{T}}\mathbf{C}_{\mathrm{n}}^{-1}\mathbf{n} = \mathbf{s}^{\mathrm{T}}\mathbf{C}_{\mathrm{s}}^{-1}\mathbf{s} + (\mathbf{o} - \mathbf{s})^{\mathrm{T}}\mathbf{C}_{\mathrm{n}}^{-1}(\mathbf{o} - \mathbf{s})$

The gradient of the likehood function with respect to $\mathbf{s}$ to zero:

$(\mathbf{C}_{\mathrm{s}}^{-1} + \mathbf{C}_{\mathrm{n}}^{-1})\mathbf{s} = \mathbf{C}_{\mathrm{n}}^{-1}\mathbf{o}$

$\mathbf{C}_{\mathrm{n}}$: Covariance matrix of ToA uncertainties

$\mathbf{C}_{\mathrm{s}}$: Covariance matrix of timing noise

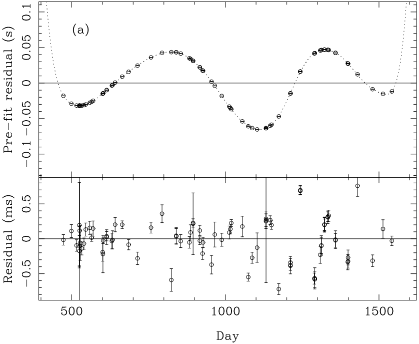
The advantages of the MLE algorithm

- It can deal with **variable sampling error**.
- It can deal with **irregular sampling**.
- It **can** be extrapolated **past** the end of real observations(and back-extrapolated **before** the start of real observations).
- It can give reasonable interpolation in **gaps**.
- It can deal with **weakly non-stationary** observations (e. g. small **glitch** events).

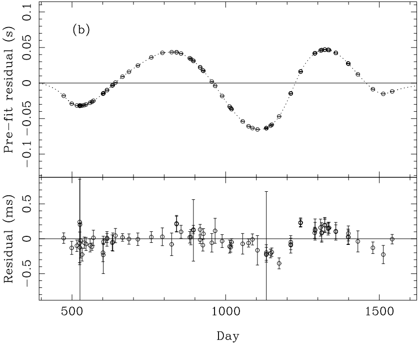
Modelling the timing noise for the Vela pulsar

Note : The lower panel of each figure shows the **difference** between the **real residuals** and the **timing noise model**.

FITWAVES



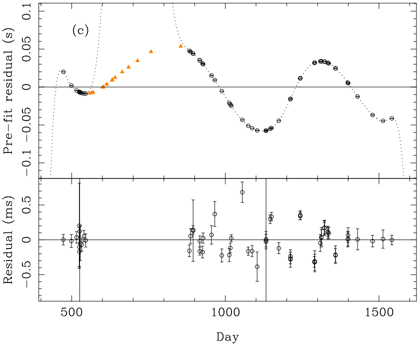
MLE



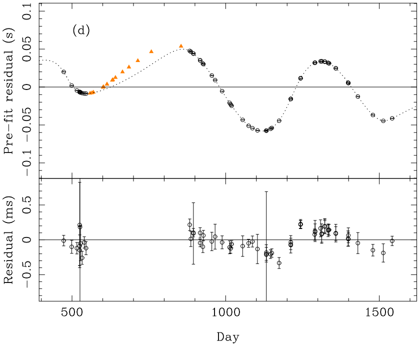
Modelling the timing noise for the Vela pulsar (with **gap** created by deleting observations)

Note : The lower panel for each figure shows the **difference** between the **real residuals** and the **timing noise model**.

FITWAVES



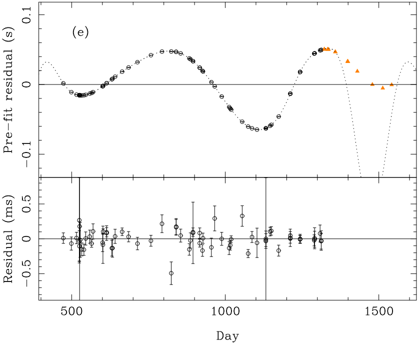
MLE



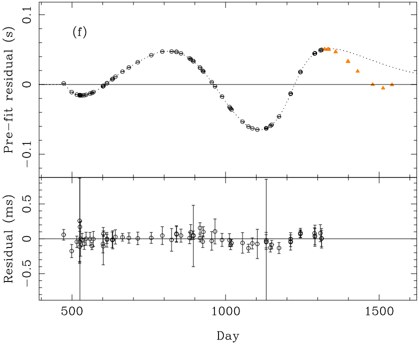
Timing noise prediction for the Vela pulsar (with 220 days observations deleted)

Note : The lower panel for each figure shows the **difference** between the **real residuals** and the **timing noise model**.

FITWAVES

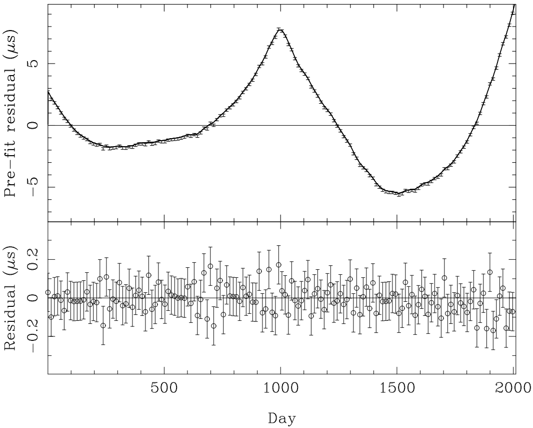


MLE



Modelling simulated timing noise with a glitch event

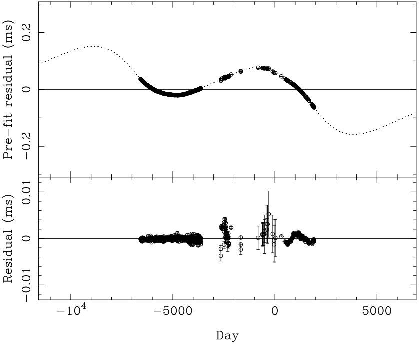
Glitch: Frequency change of 1×10^{-12} Hz.
ToA uncertainties: 100 ns.



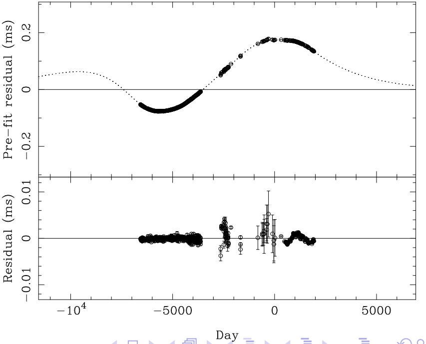
The effect of quadratic removal in the timing noise modeling for PSR J1939+2134

Note : The lower panel for each figure shows the **difference** between the **real residuals** and the **timing noise model**.

Timing noise modeling **without Cholesky** procedure.

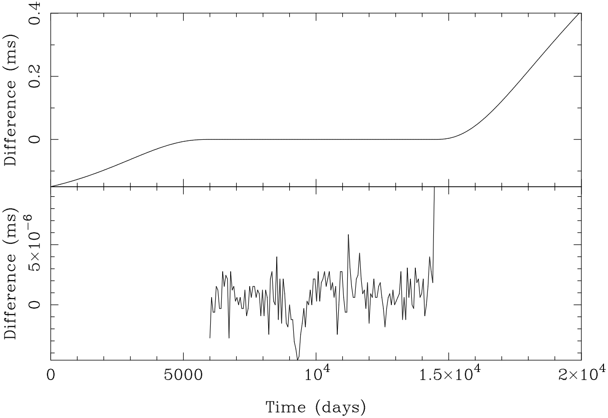


Timing noise modeling **with Cholesky** procedure.



The effect of quadratic removal in the absolute arrival time determination for PSR J1939+2134

Timing model + Timing noise model



Conclusion

We recommend that our new method is **always** used to model, interpolate or extrapolate pulsar timing noise **because**:

- MLE method is applicable even under extreme conditions such as:
  - very steep spectra timing noise;
  - very large gaps;
  - highly variable ToA uncertainties.
- MLE method is close to optimal.

Thank you!