Using Cyclic Spectroscopy

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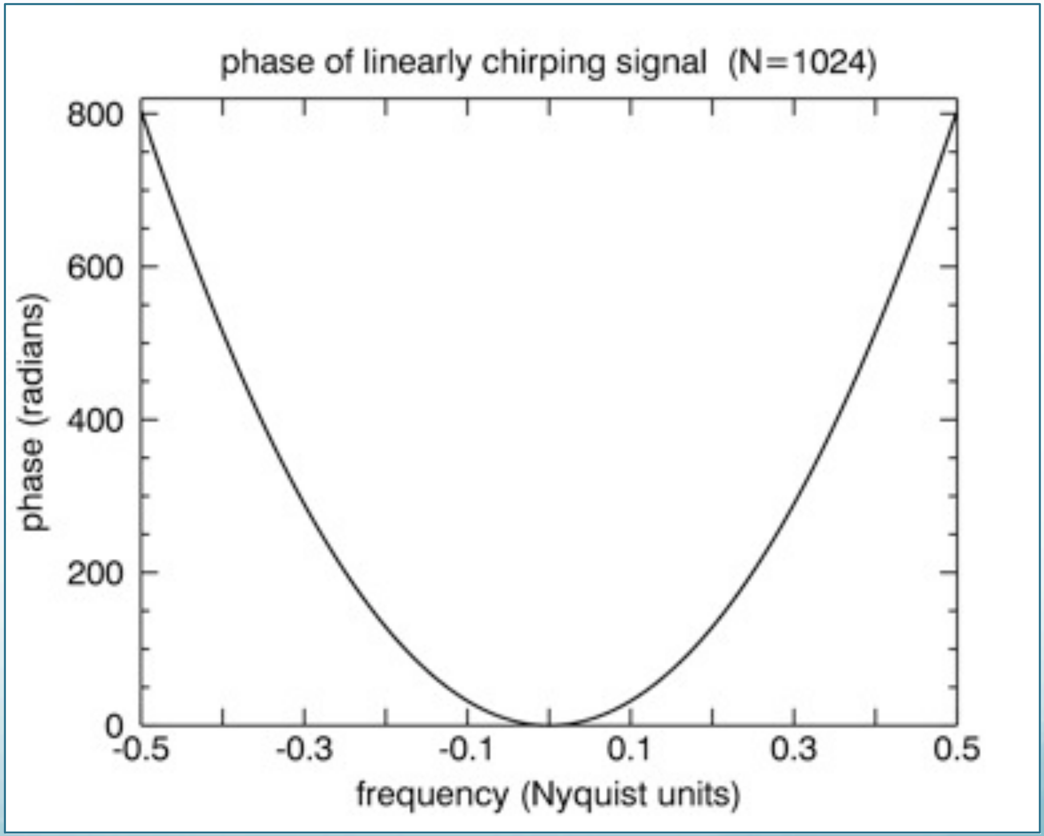
Thanks to:

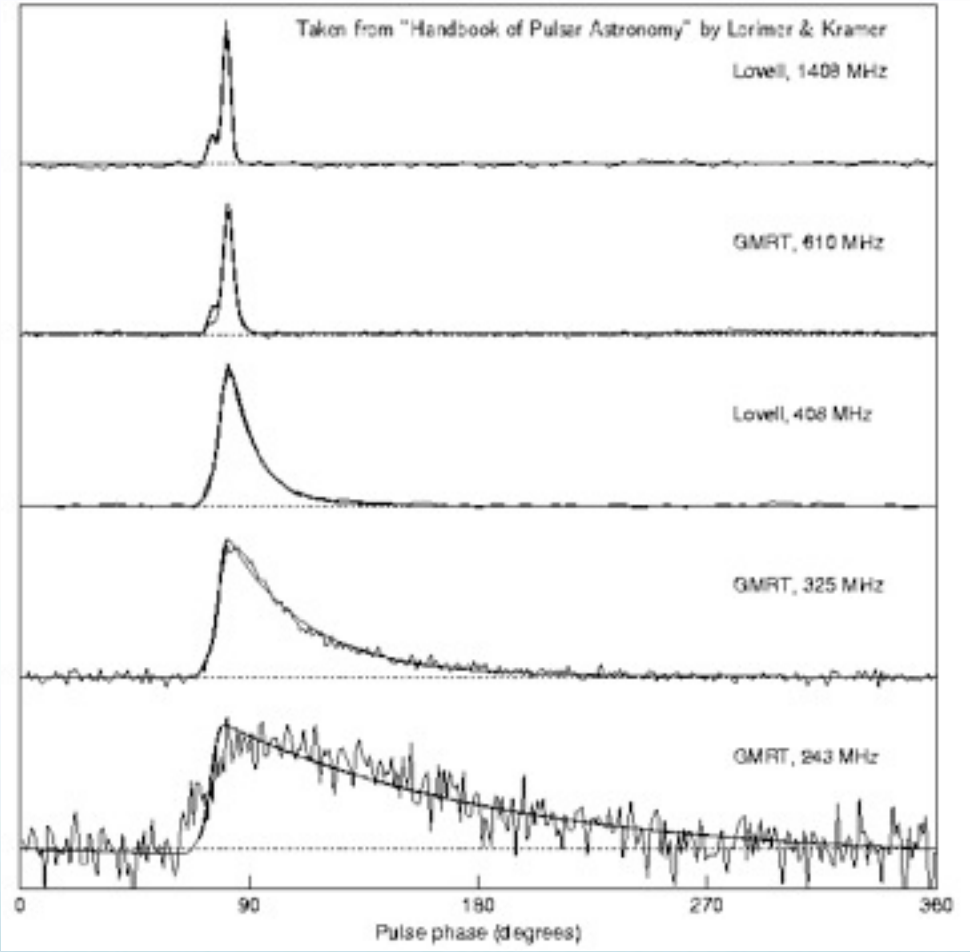
Jim Cordes
Paul Demorest
Maura McLaughlin
Nipuni Palliyaguru
Mark Walker

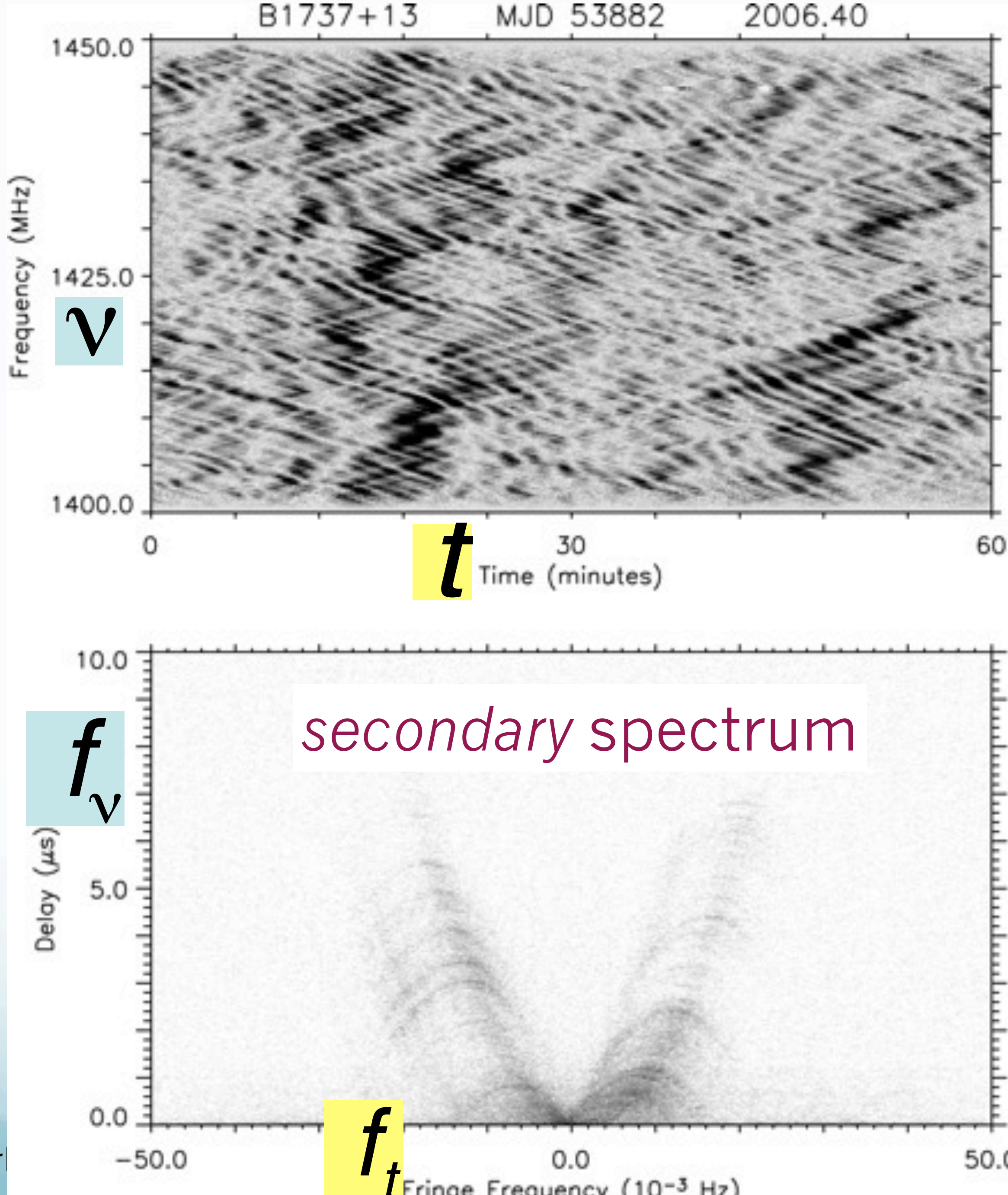
Coherent Dedispersion

- Developed in 1971 by Tim Hankins. Use the phase information of the ISM transfer function, $B(\nu)$, to perfectly remove dispersion.



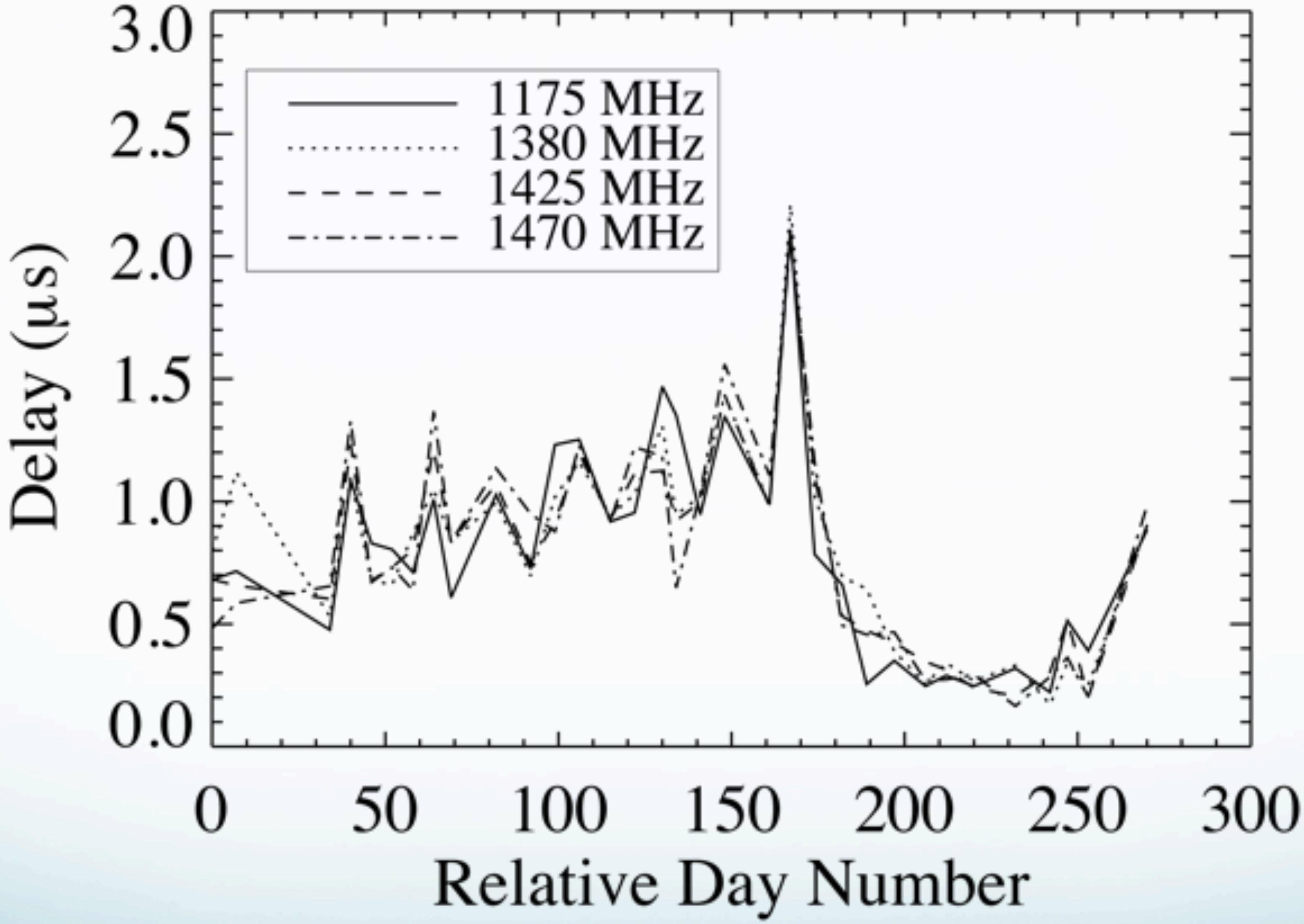
Scatter Broadening





linear grayscale

logarithmic grayscale



Hemberger & Stinebring 2008, ApJ, 674, L37

AMN model

- The pulsar signal is consistent with an amplitude modulated noise process (Rickett 1976, ...)

$$\epsilon_i(t) = a(t)m(t)$$

- Including the pulse **b**roadening of the ISM and additive noise, we have

$$\epsilon(t) = b(t) * \epsilon_i(t) = b(t) * [a(t)m(t)] + n(t)$$

AMN model 2

- In the frequency domain

$$\tilde{\epsilon}(\nu) = B(\nu)\ [\tilde{a}(\nu) * \tilde{m}(\nu)] + \tilde{n}(\nu)$$

- $B(\nu)$ is the **transfer function** of the ISM, the Fourier transform of the pulse broadening function $b(t)$.

Cyclic Spectral Analysis of Radio Pulsars

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Cyclic Spectrum

- The standard spectrum of the source is

$$S_i(\nu) = < |\tilde{\epsilon}_i(\nu)|^2 >$$

- But, the spectrum is more informative for a periodic source (Demorest 2011)

$$S_i(\nu;\alpha_k) = < \tilde{\epsilon}_i(\nu + \alpha_k/2) \;\; \tilde{\epsilon}_i^*(\nu - \alpha_k/2) >$$

Cyclic Spectrum

- The standard spectrum of the source is

$$k$$

$$\alpha_k =$$

$$k = 0, 1, 2, \ldots$$

$$S_i(\nu) = < |\tilde{\epsilon}_i(\nu)|^2 >$$

- But, the spectrum is more informative for a periodic source (Demorest 2011)

$$S_i(\nu; \alpha_k) = < \tilde{\epsilon}_i(\nu + \alpha_k/2) \;\; \tilde{\epsilon}_i^*(\nu - \alpha_k/2) >$$

Cyclic Spectroscopy

Take the complex voltage spectrum,
amplitude and phase, and ...

the conjugate of the
voltage spectrum ...

frequency →

Cyclic Spectroscopy

shift one sample to the right

shift one sample to the left

(note – we have to oversample by a factor of two in order to get a symmetric shift – net shift is 1/P)

frequency →

Cyclic Spectroscopy

X

Multiply the arrays together

(the end effects are minor in practice)

This is a complex, intensity-like array **that can be averaged over many pulses!**

frequency →

CS of (AMN + ISM)

- When amplitude modulated noise propagates through the ISM this becomes (no additive noise)

$$S(\nu; \alpha_k) = \langle B(\nu + \quad \alpha_k$$

CS of (AMN + ISM)

- When amplitude modulated noise propagates through the ISM this becomes (no additive noise)

pulsar noise power

ISM transfer function

FT of pulse profile

$$S(\nu;\alpha_k) = \langle B(\nu + \quad \alpha_k$$

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$$S(\nu;\alpha_k) = \langle B(\nu + \quad \alpha_k$$

- Comment 1:
  There is one such spectrum for each of the k values. But, in practice, k_{max} will be set by the sharpness of the pulse. A duty cycle of d will yield $k_{max} \sim 1/d$.

CS of (AMN + ISM)

- When amplitude modulated noise propagates through the ISM this becomes (no additive noise)

pulsar noise power

ISM transfer function

FT of pulse profile

$$S(\nu;\alpha_k) = \langle B(\nu + \alpha_k$$

- Comment 2:
  Each of the k_{max} spectra contains similar information. They can be combined in a weighted manner to estimate quantities of interest, e.g. $B(\nu)$

CS of (AMN + ISM)

pulsar noise power

- When amplitude modulated noise propagates through the ISM this becomes (no additive noise)

ISM transfer function

FT of pulse profile

$$S(\nu;\alpha_k) = \langle B(\nu + \quad {}^{\alpha_k}$$

- Comment 3:
  The amplitude and phase of $S(\nu;\alpha_k) = S(\nu;k/P)$ depends critically on $A = \mathrm{F.T.}\ [a^2(t)]$. If you change the profile you change the CS. Later ... comment about pulse jitter.

CS of (AMN + ISM)

pulsar noise power

- When amplitude modulated noise propagates through the ISM this becomes (no additive noise)

ISM transfer function

FT of pulse profile

$$S(\nu;\alpha_k) = \langle B(\nu + \quad {}^{\alpha_k}$$

- Comment 4:
  With P = 4 ms, e.g., the frequency shift represented by $\alpha_1 = 1/P = 250$ Hz. This is a *small* shift, particularly compared to large bandwidths of 10 – 100 MHz (baseband sample intervals of 100 – 10 ns).

CS of (AMN + ISM)

- When amplitude modulated noise propagates through the ISM this becomes (no additive noise)

pulsar noise power

ISM transfer function

FT of pulse profile

$$S(\nu;\alpha_k) = \langle B(\nu + \ \alpha_k$$

- Comment 5:
  Additive noise simply adds a power N_0 to the RHS of this equation.

Determining the phase

- Use the CS to determine $B(\nu)$ to within an additive constant. Represent $B(\nu)$ as an amplitude and a phase. Then substitute into the CS formula:

$$B(\nu) = B_a(\nu)\, \Phi_B(\nu)$$

$$\Phi_S(\nu) = \Phi_B(\nu + \alpha_k/2) - \Phi_B(\nu - \alpha_k/2)$$

  - The phase of the CS is the *change* in phase of the transfer function over $\Delta\nu = \alpha_k = k/P$.

Reconstructing $B(\nu)$

- The amplitude is easy: the standard spectrum provides a good measurement of it.
- The phase is harder, but that's where the payoff comes from. Some approaches:

1. Direct Phase Integration

$$\Phi_B(\nu;\alpha_k) = \int_{-B/2}^{\nu} \Phi_S(\nu';\alpha_k)d\nu'$$

2. Demorest-Walker least-squares optimization. Adjust $B(\nu)$ to minimize:

$$\left|S_{\rm obs}(\nu;\alpha_k) - S_{\rm model}(\nu;\alpha_k)\right|^2$$

Signal-to-Noise

- How big is the signal that we're trying to estimate with Cyclic Spectroscopy?

- The CS – corrected for the profile $A(\alpha_k)$ – is almost completely Real. You are typically estimating a very small Φ_B in the presence of noise (see next slides).

An example – linear shift

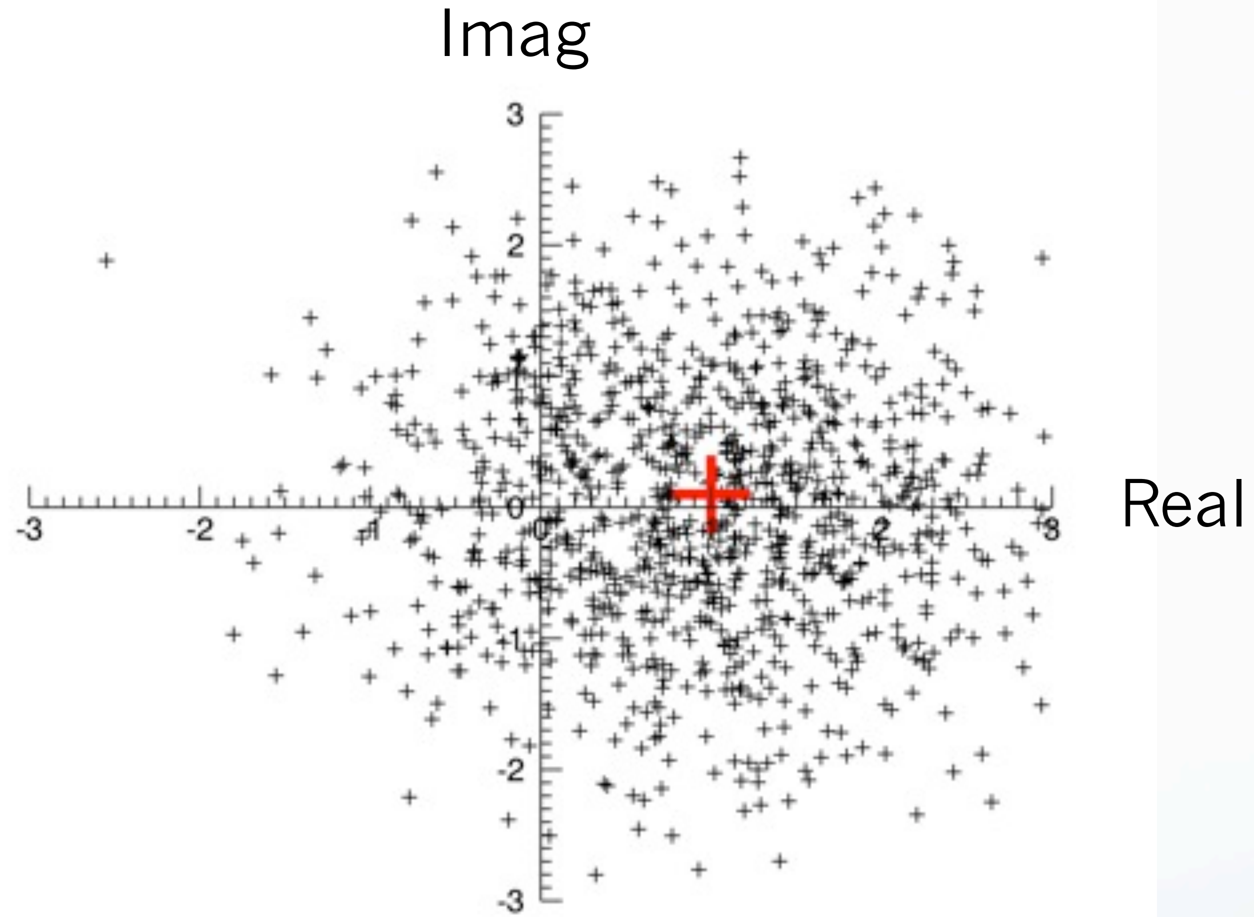
- If the profile is shifted by 1 (Nyquist) sample there is a 2π phase change across the band for $B(\nu)$.

$$\Delta\Phi_B = \Phi_S = 2\pi/N$$

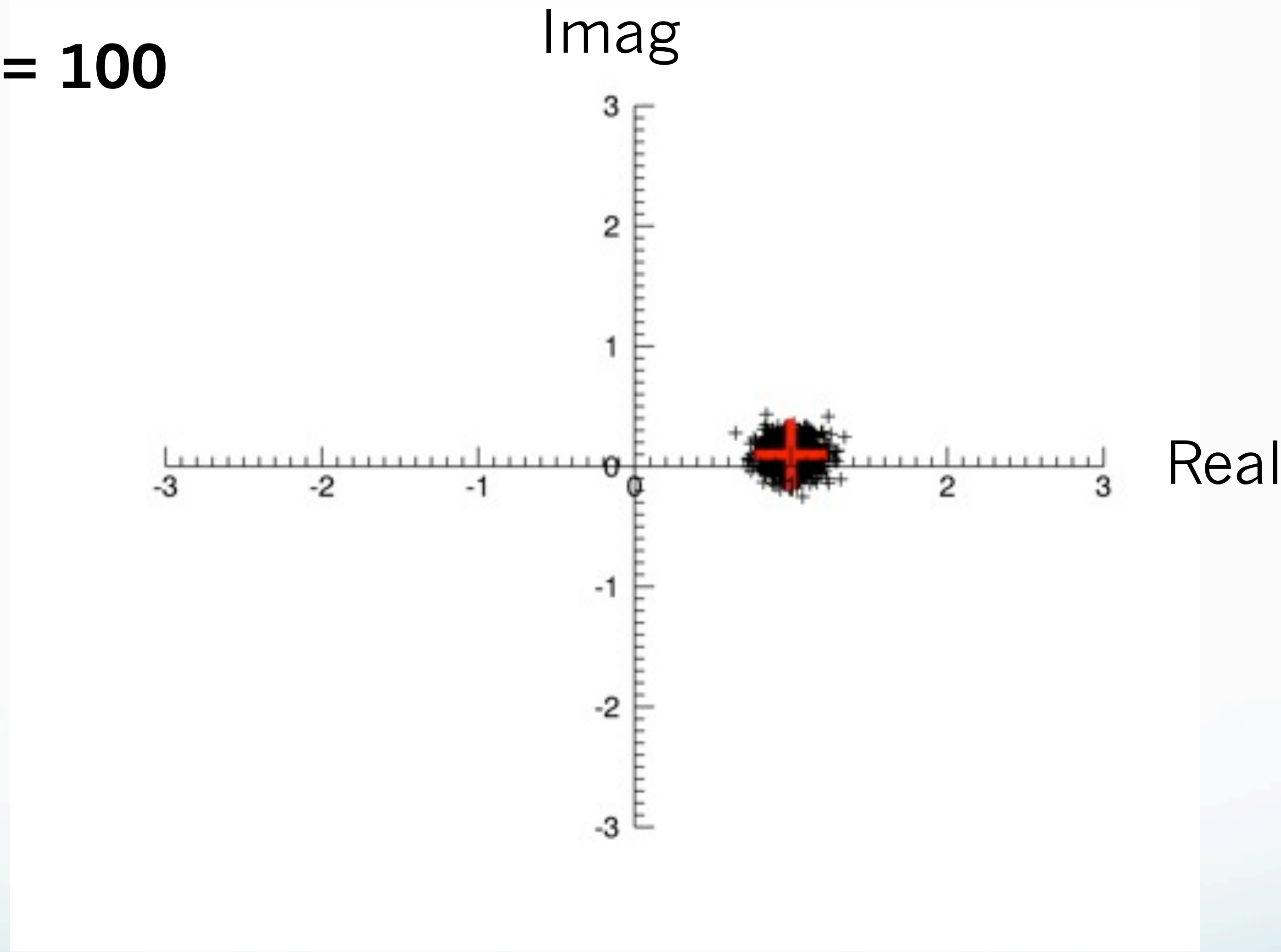
- For P = 4 ms, BW = 10 MHz, N = 4 x 10^4. This yields a CS signal of

$$\Delta\Phi_B = \Phi_S = 1.6 \times 10^{-4}$$

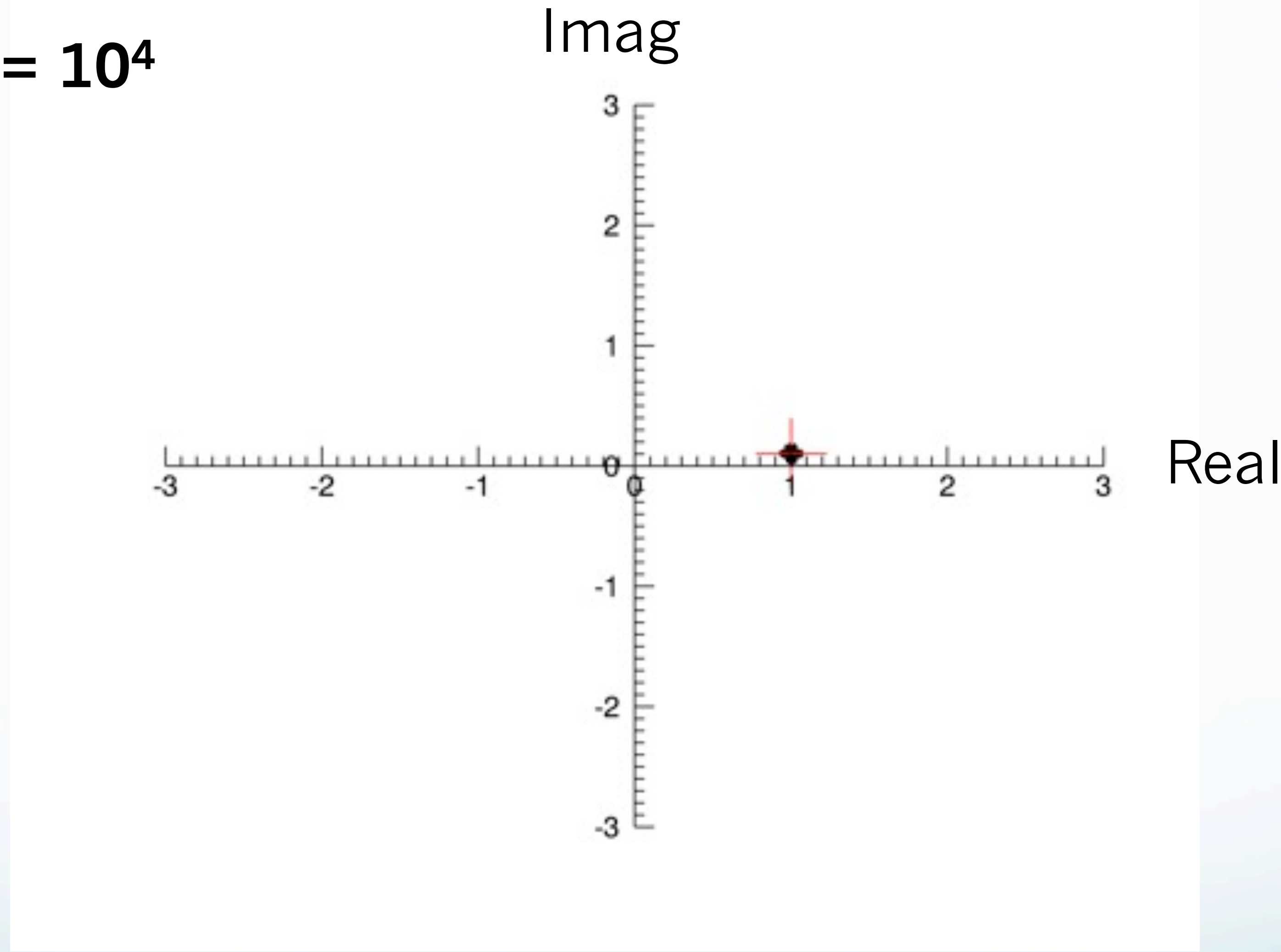
N = 1



N = 100

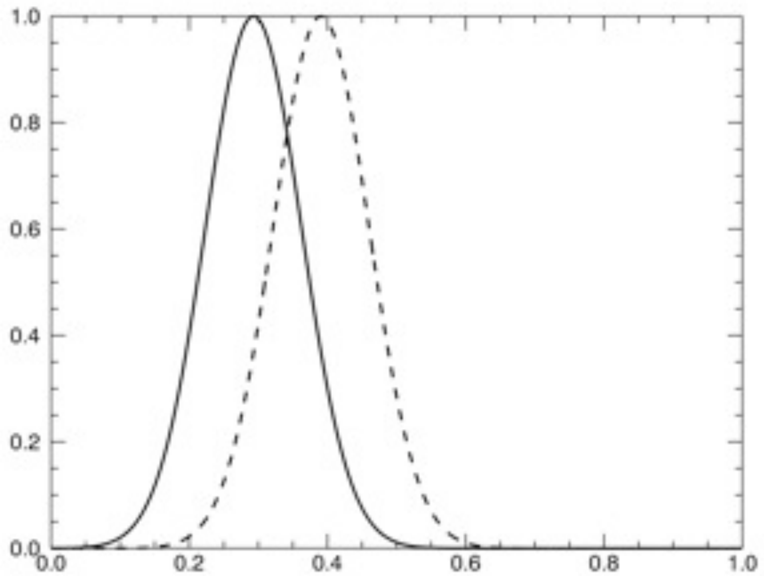


N = 10^4



Pulse Jitter

- A shift in the pulse is a phase ramp in *B(ν)*.



If you shift by a fraction ε of the period, it will produce a phase error of

$$\epsilon(2\pi)$$

- This is an offset (either positive or negative) in the phase of the Cyclic Spectrum, $\Phi_S(\nu)$. This will be another form of phase noise (i.e. similar to the N=1, N=100, N= 10^4 plots).

Closing Comments

- CS produces a *function* (the pulse broadening function) instead of only a single number, the TOA. This is useful for **detecting and mitigating scattering noise.**

- The signal-to-noise ratio of the TOA estimate is the same in both cases, but the CS has additional information about the nature of the time delay: scattering and profile changes.

- We are still very much in the **Research and Development** phase. Next crucial step: demonstrate that CS correction of real data improves arrival times (Nipuni Palliyaguru's PhD work).