



# There and back again. A scientists tale.

(An overview of single-source detection algorithms)

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# Outline

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- Brief overview of signal model.
- Current prospects (?).
- Current detection algorithms and results.
- Looking towards the future.

# Signal Model

- For most single source detection algorithms (continuous) we assume a signal that has the form

$$s(t; \vec{\lambda}) = F^+(\hat{\Omega}) \Delta s_+(t; \vec{\lambda}) + F^\times(\hat{\Omega}) \Delta s_\times(t; \vec{\lambda})$$

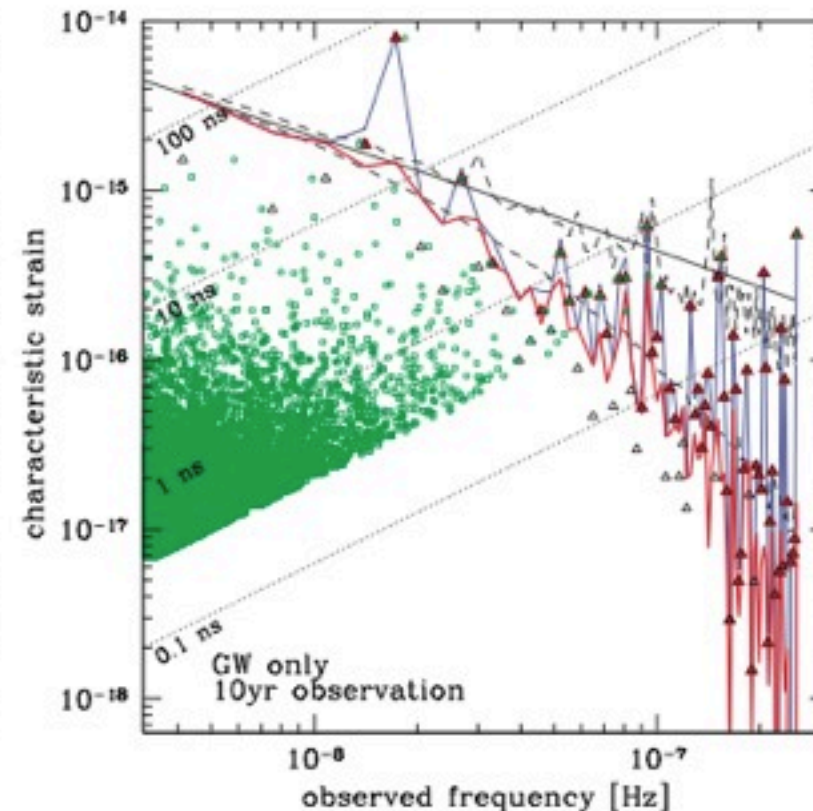
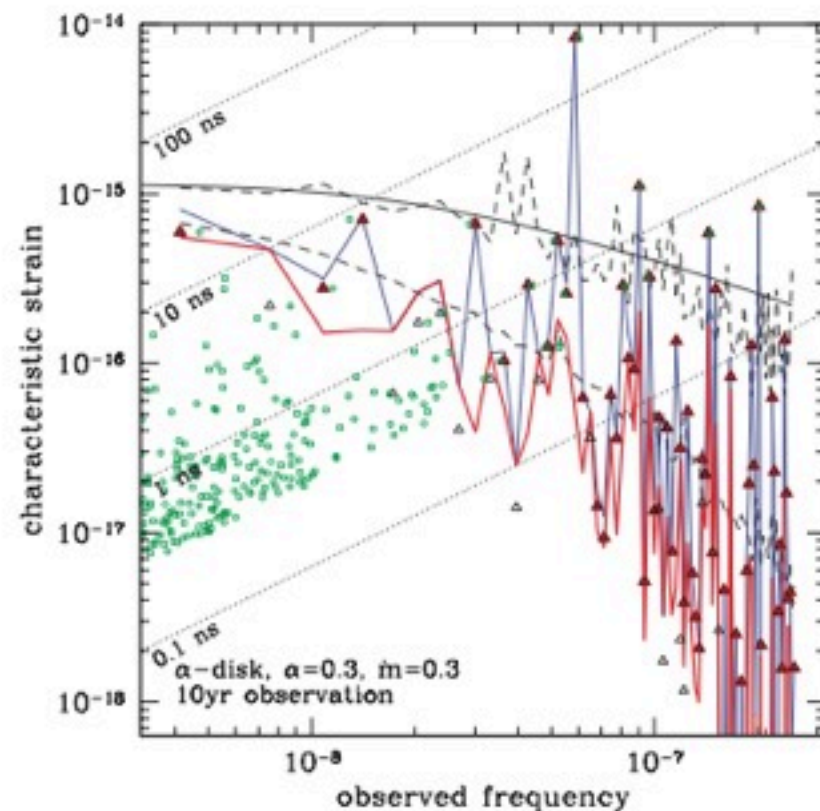
Diagram illustrating the components of the signal model:

- Antenna Pattern Functions** (pink box) points to  $F^+(\hat{\Omega})$  and  $F^\times(\hat{\Omega})$ .
- Sinusoids** (green box) points to  $\Delta s_+(t; \vec{\lambda})$  and  $\Delta s_\times(t; \vec{\lambda})$ .

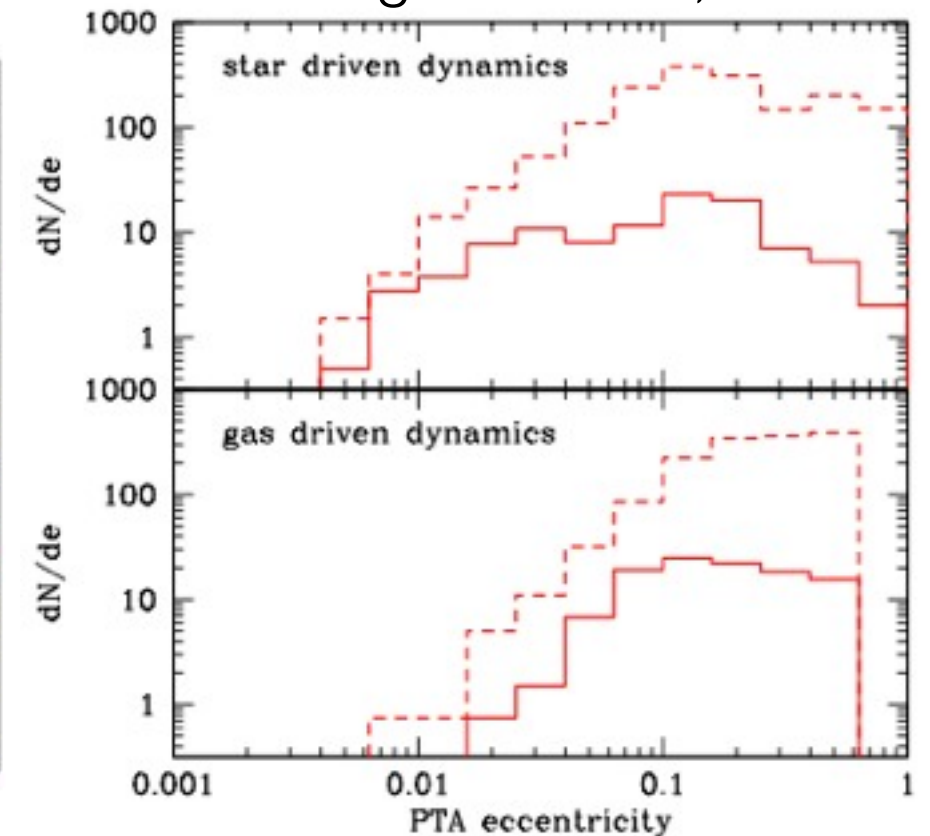
- Function of 8 parameters:  $\vec{\lambda} = \{\theta, \varphi, \iota, \mathcal{M}, D, f, \psi, \phi_0\}$
- Earth and Pulsar Terms:  $\Delta s_A(t; \vec{\lambda}) = s_A(t_p; \vec{\lambda}) - s_A(t_e; \vec{\lambda})$

# Current Prospects (?)

Kocsis & Sesana, 2011



Roedig & Sesana, 2011



Overall we will be dominated by the stochastic GW background. However, single “bright” resolvable sources may stand out above the background.

Expected that many sources will have non-negligible eccentricity. (However algorithms that assume circular orbits should still be ok.)



# Legacy papers

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## The Doppler Response to Gravitational Waves from a Binary Star Source

Hugo Wahlquist<sup>1</sup>

*Received February 20, 1987*

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## USING PULSARS TO DETECT MASSIVE BLACK HOLE BINARIES VIA GRAVITATIONAL RADIATION: SAGITTARIUS A\* AND NEARBY GALAXIES

A. N. LOMMEN AND D. C. BACKER

Astronomy Department and Radio Astronomy Laboratory, University of California, Berkeley, CA 94720-3411; [alommen@astro.berkeley.edu](mailto:alommen@astro.berkeley.edu),  
[dbacker@astro.berkeley.edu](mailto:dbacker@astro.berkeley.edu)

*Received 2001 April 20; accepted 2001 July 25*

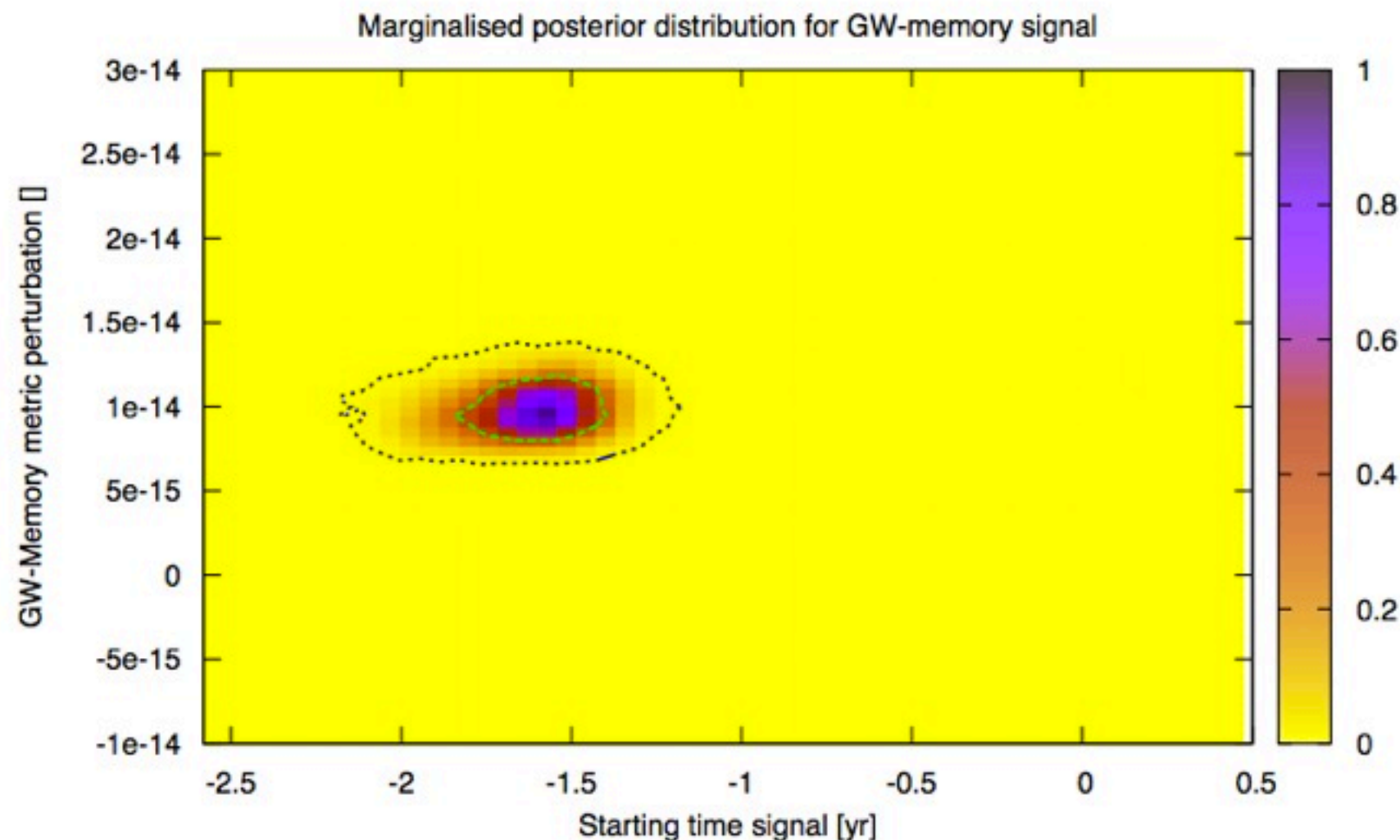
## CONSTRAINING THE PROPERTIES OF SUPERMASSIVE BLACK HOLE SYSTEMS USING PULSAR TIMING: APPLICATION TO 3C 66B

FREDRICK A. JENET,<sup>1</sup> ANDREA LOMMEN,<sup>2</sup> SHANE L. LARSON,<sup>3</sup> AND LINQING WEN<sup>4</sup>

*Received 2003 October 10; accepted 2004 January 20*

## van Haasteren & Levin (2009)

- Bayesian method for detecting single sources (memory) that fully takes account of timing model by marginalizing over all timing parameters. Basis for current EPTA single source algorithms.



Conclusion: GW memory effects unlikely to be detected with current PTAs

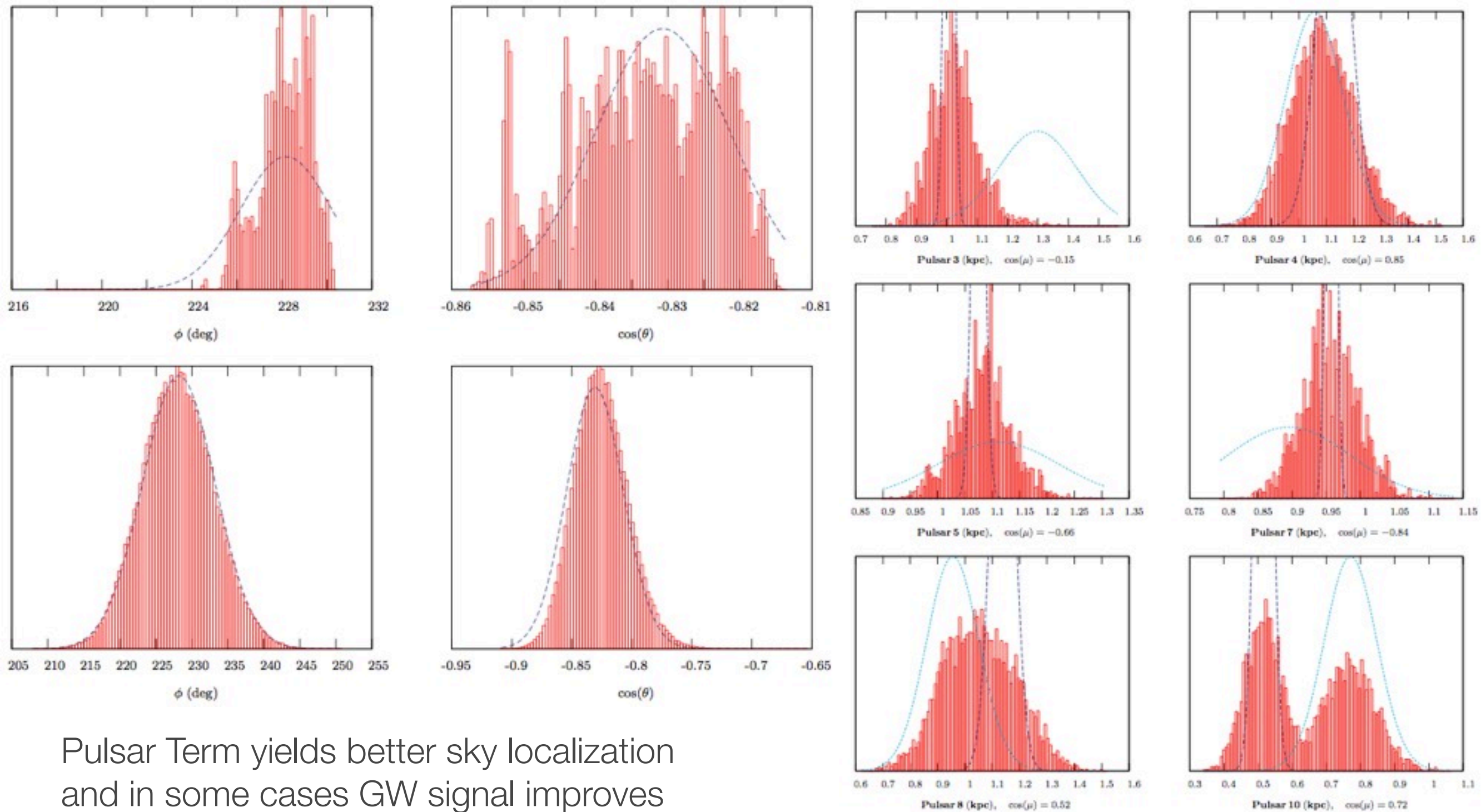
## Corbin & Cornish (2010)

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- LISA-like MCMC pipeline applied to simulated PTA data.
- Makes use of pulsar term to increase SNR by factor of 2 and to break degeneracy between chirp mass and distance. Inclusion of pulsar term also leads to better sky localization ( $2.6 \text{ deg}^2$ ).
- Focuses on parameter estimation and not so much on detection (uses  $\text{SNR}=20$ ).
- Does not use timing model. Models data as signal+noise.



# Corbin and Cornish (2010) cont.



Pulsar Term yields better sky localization  
and in some cases GW signal improves  
pulsar distance measurements

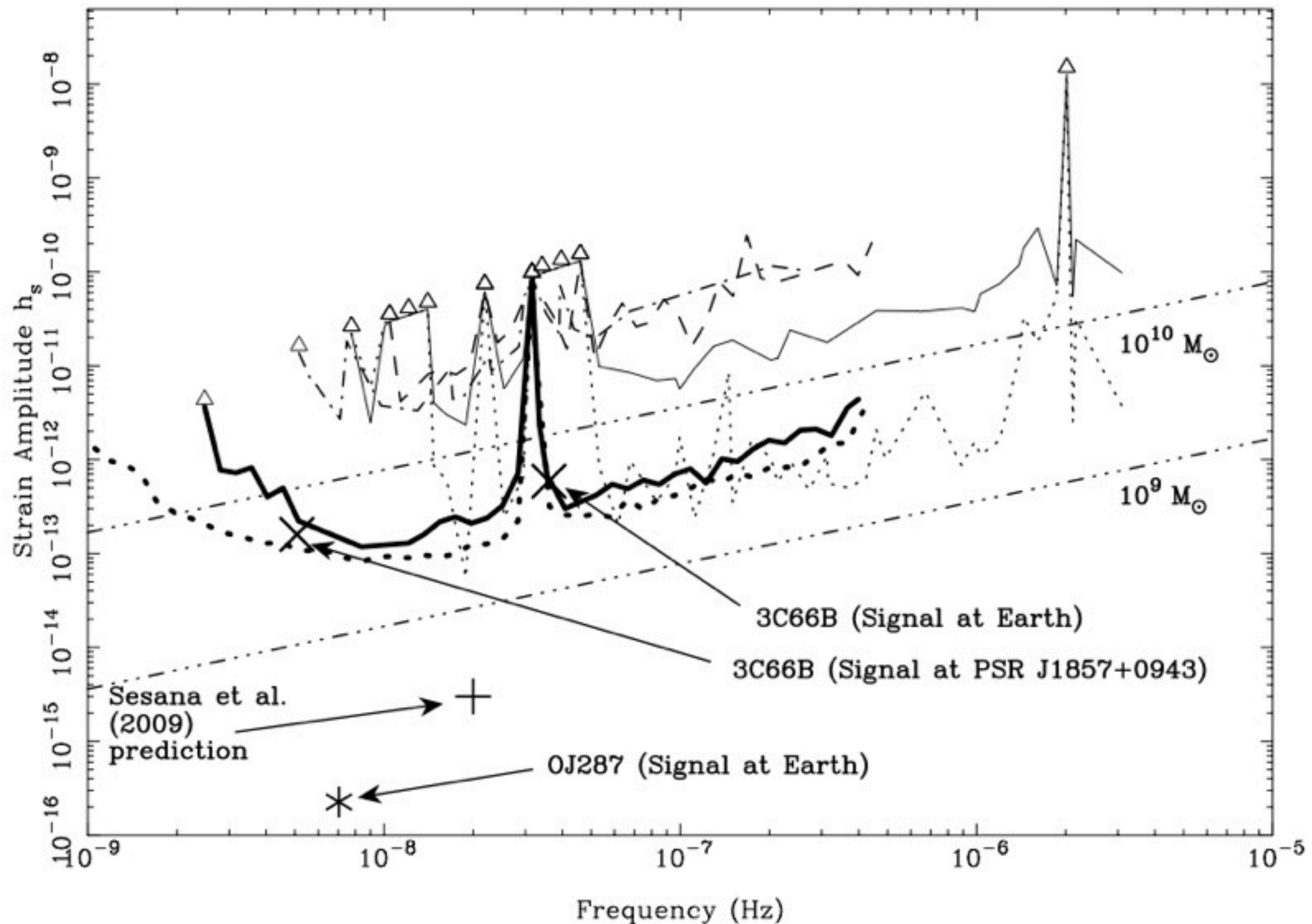


## Yardley et al. (2010)

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- First and only published limit on single sources using real PTA data.
- Uses Lomb-Scargle based power spectrum and many injections to produce upper limits and sensitivity curves for PPTA data set.
- Results can be used to place an upper bound on the number of coalescing binary systems of a given chirp mass as a function of redshift. Current observations do not yet rule out any likely GW sources.

# Yardley et al. (2010) cont.



## Finn & Lommen (2010)

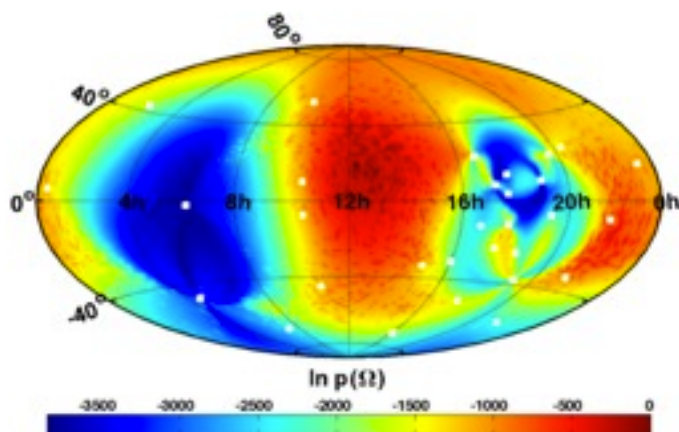
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- General Bayesian method to detect GW bursts from a variety of sources.
- Breaks problem up into three distinct categories: detection, localization and characterization.
- The full data analysis pipeline based on the methods in this paper is available in the BayesGWDA Matlab package at <http://sourceforge.net/projects/nanosoft/files/BayesGWDA/>

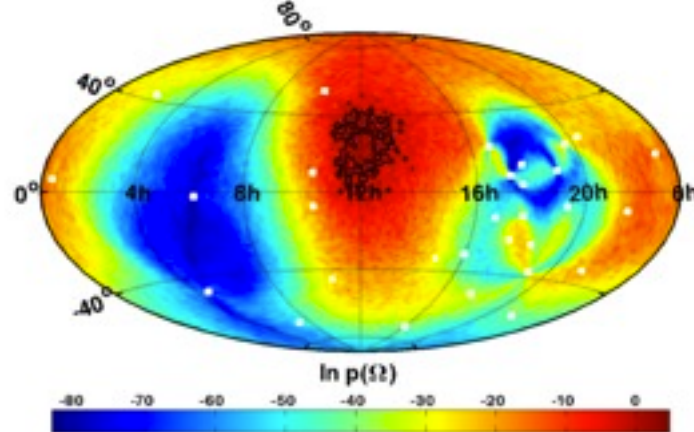


# Finn & Lommen (2012) cont.

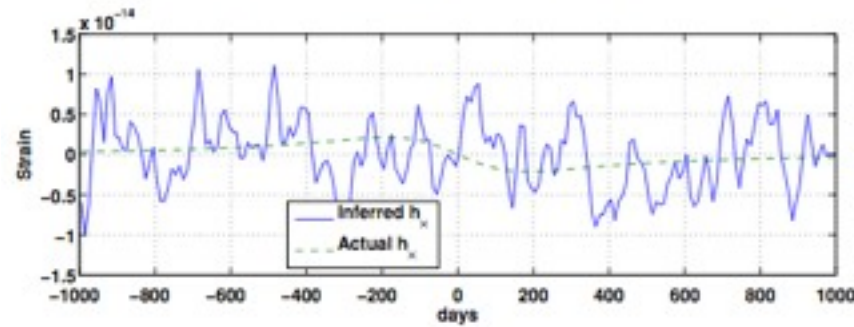
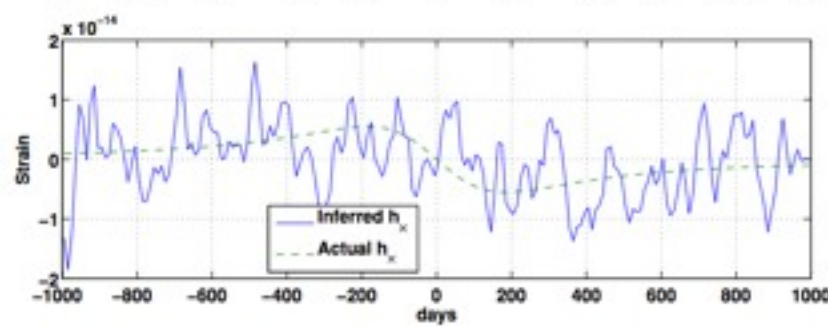
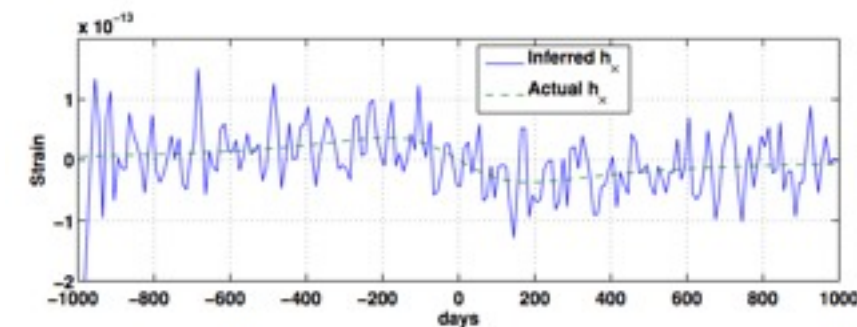
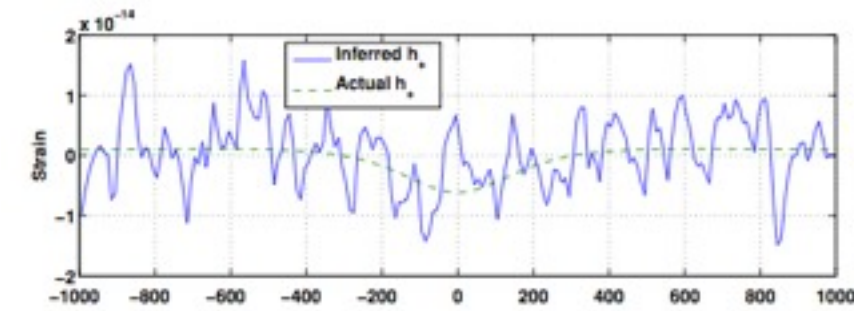
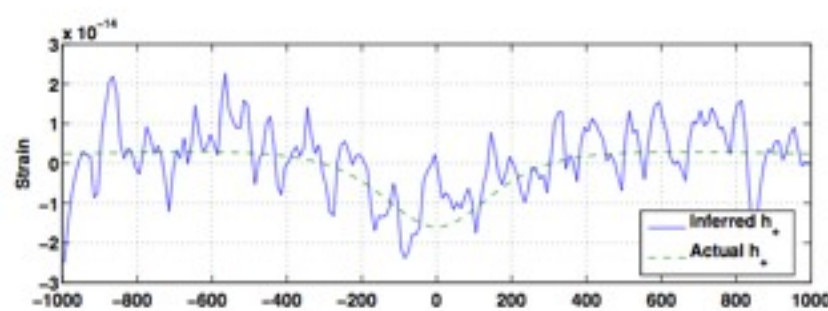
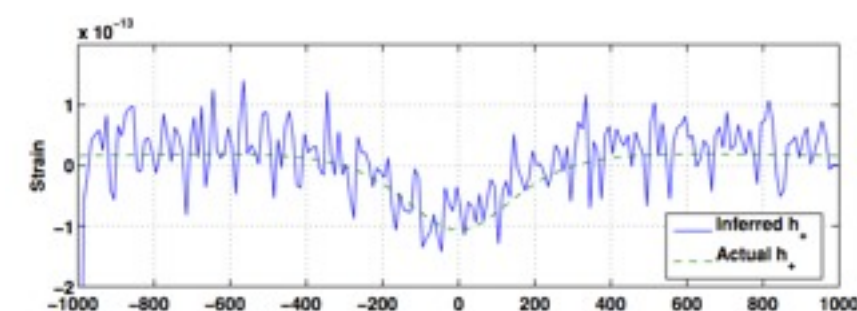
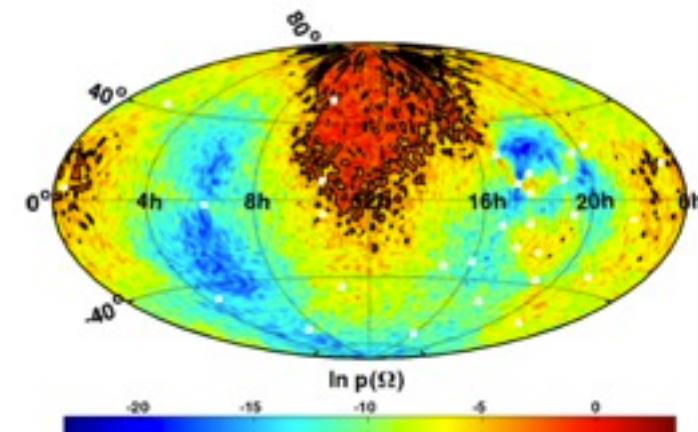
## Characterization



## Localization



## Detection

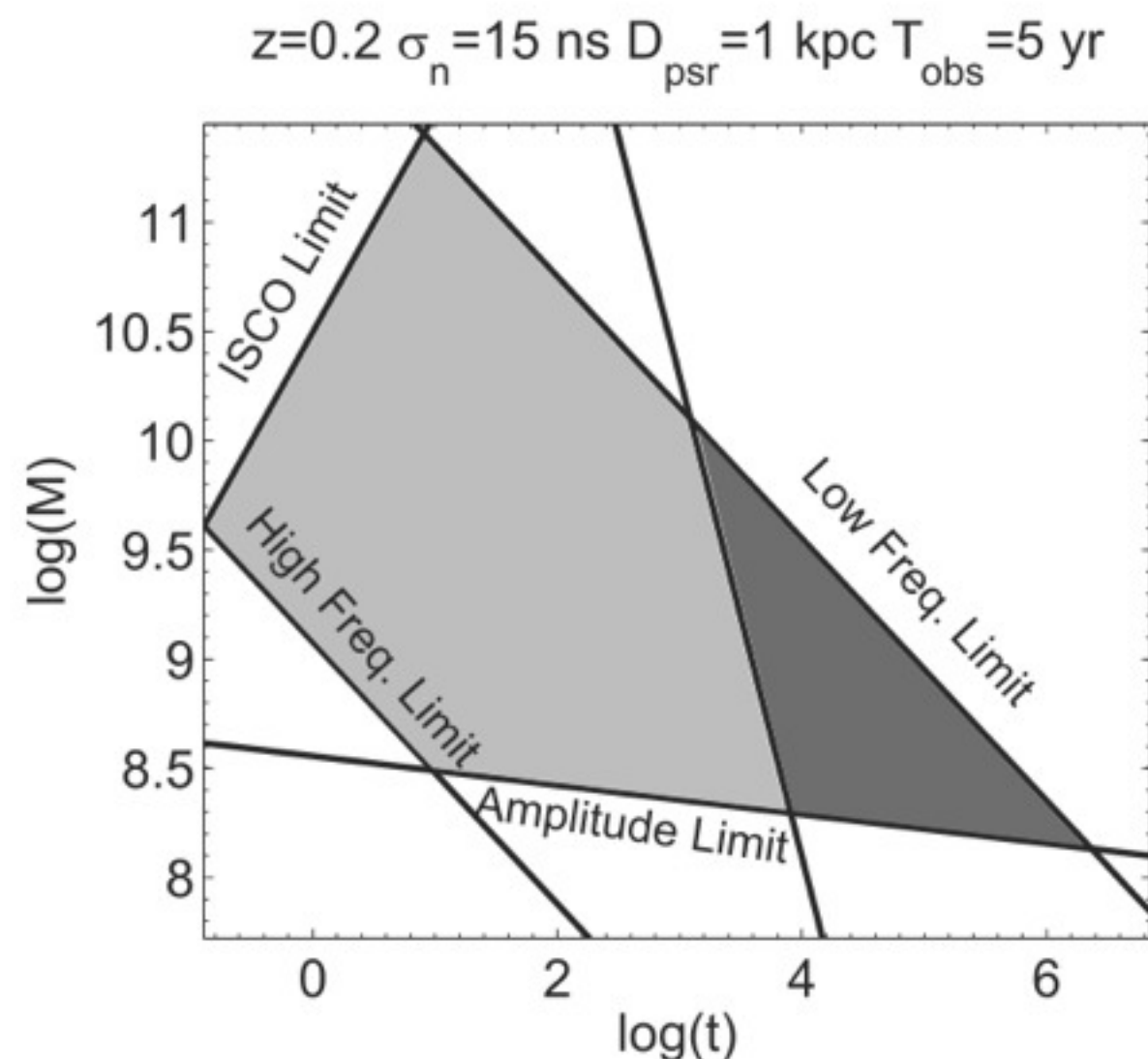


Signal	$\ln B(d)$	$\Delta\Omega_{90\%} (\text{deg}^2)$	$\rho^2$
Strong	$3.8 \times 10^3$	$\ll 1$	$3.8 \times 10^{+1}$
Moderate	$6.6 \times 10^1$	$5.8 \times 10^2$	$8.7 \times 10^{-1}$
Weak	$2.2 \times 10^0$	$4.2 \times 10^3$	$2.1 \times 10^{-1}$
Absent	$-8.4 \times 10^0$	$1.2 \times 10^4$	$7.9 \times 10^{-2}$



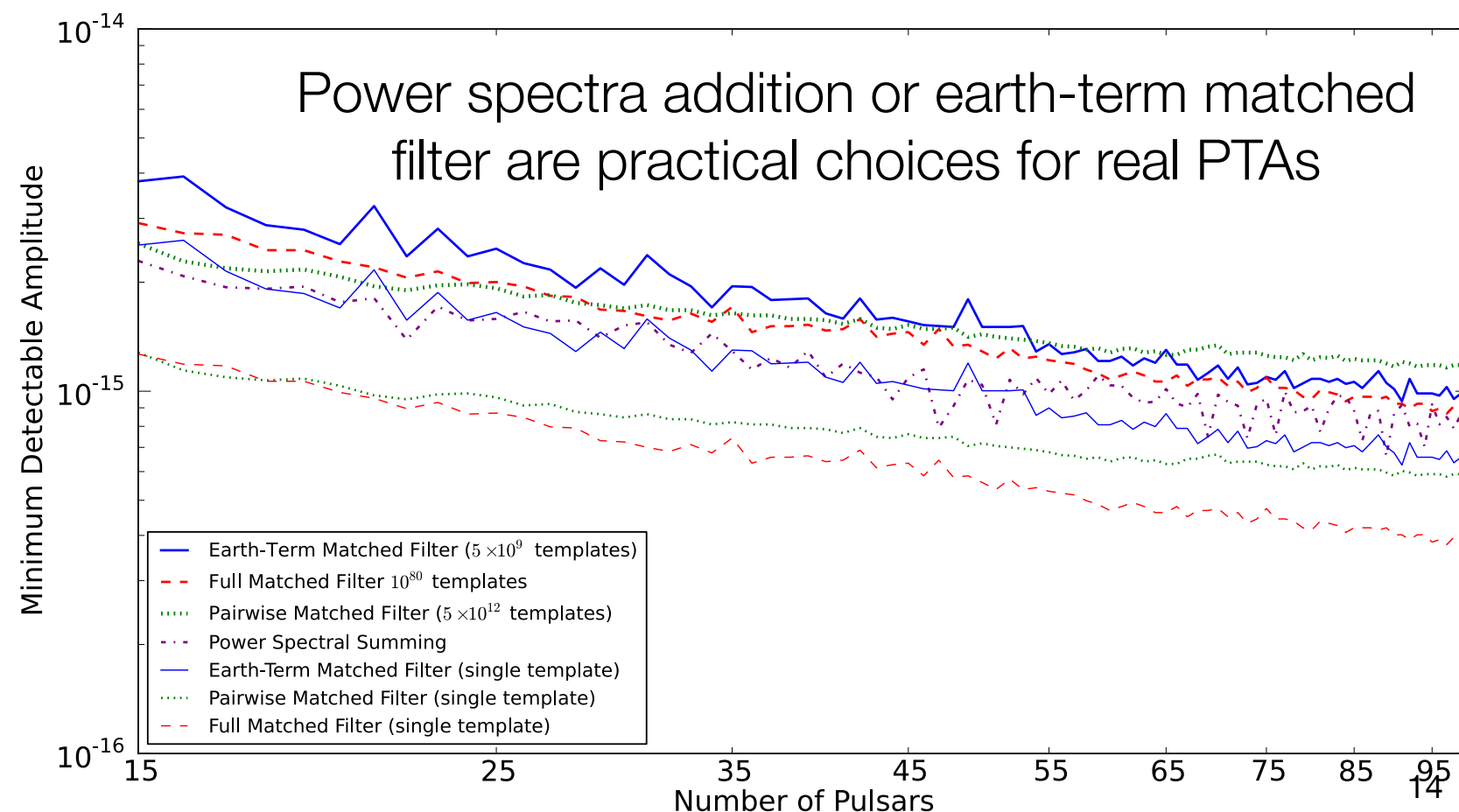
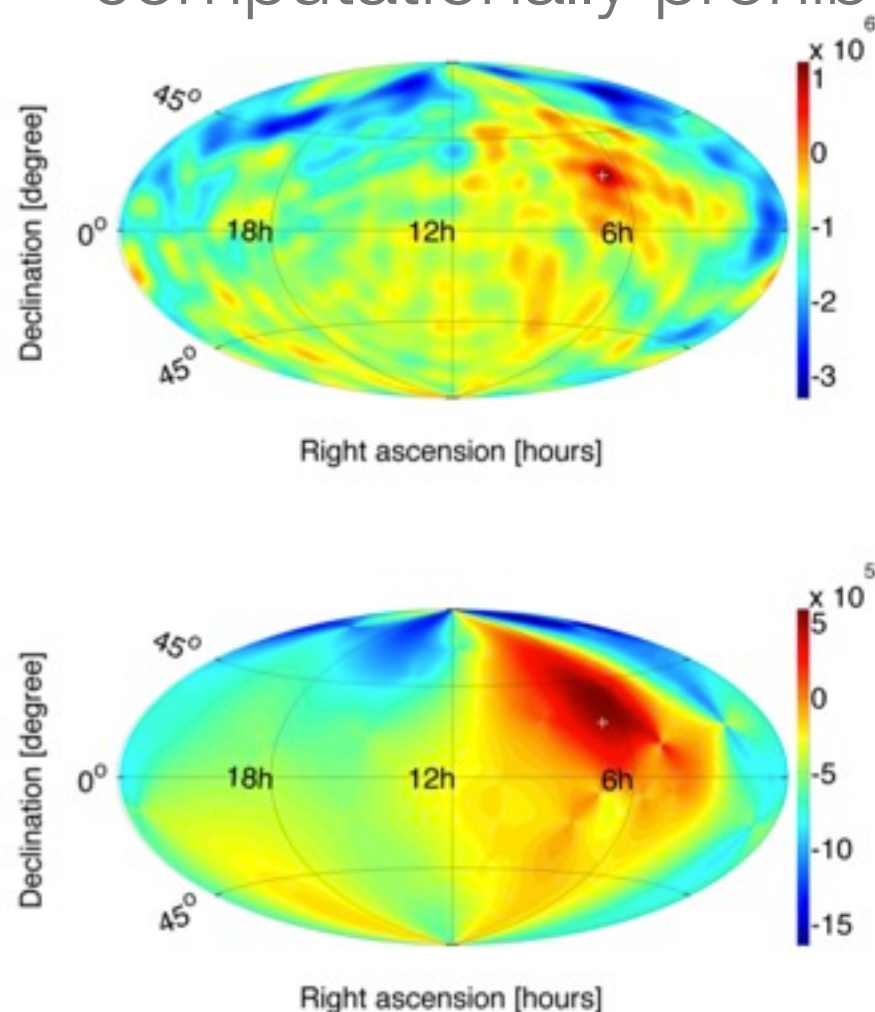
# Lee et al. (2011)

- Explore parameter space of SMBHBs.
- Uses Ziv-Zakai bounds to calculate measurability of SMBHB parameters.
- Show that along with parallax measurements can measure pulsar distance with GW observations.
- Assume SKA-like PTA.



# Ellis et al. (2012a)

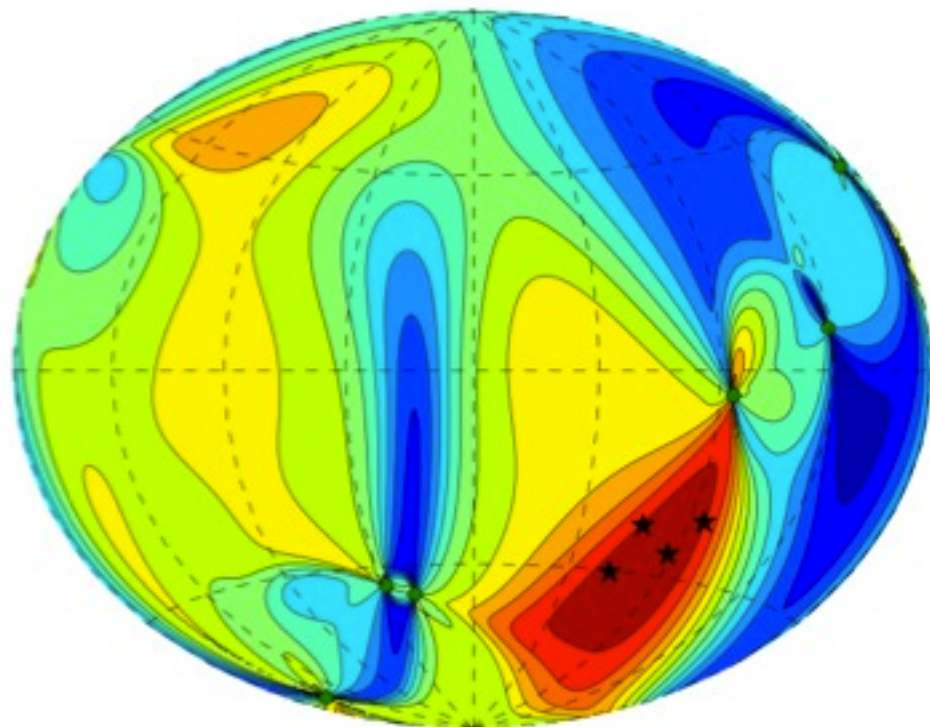
- Compare various detection statistics for completely monochromatic signals through monte-carlo simulations.
- Show that matched filter explicitly including the pulsar distance is computationally prohibitive (at least for a grid based search)



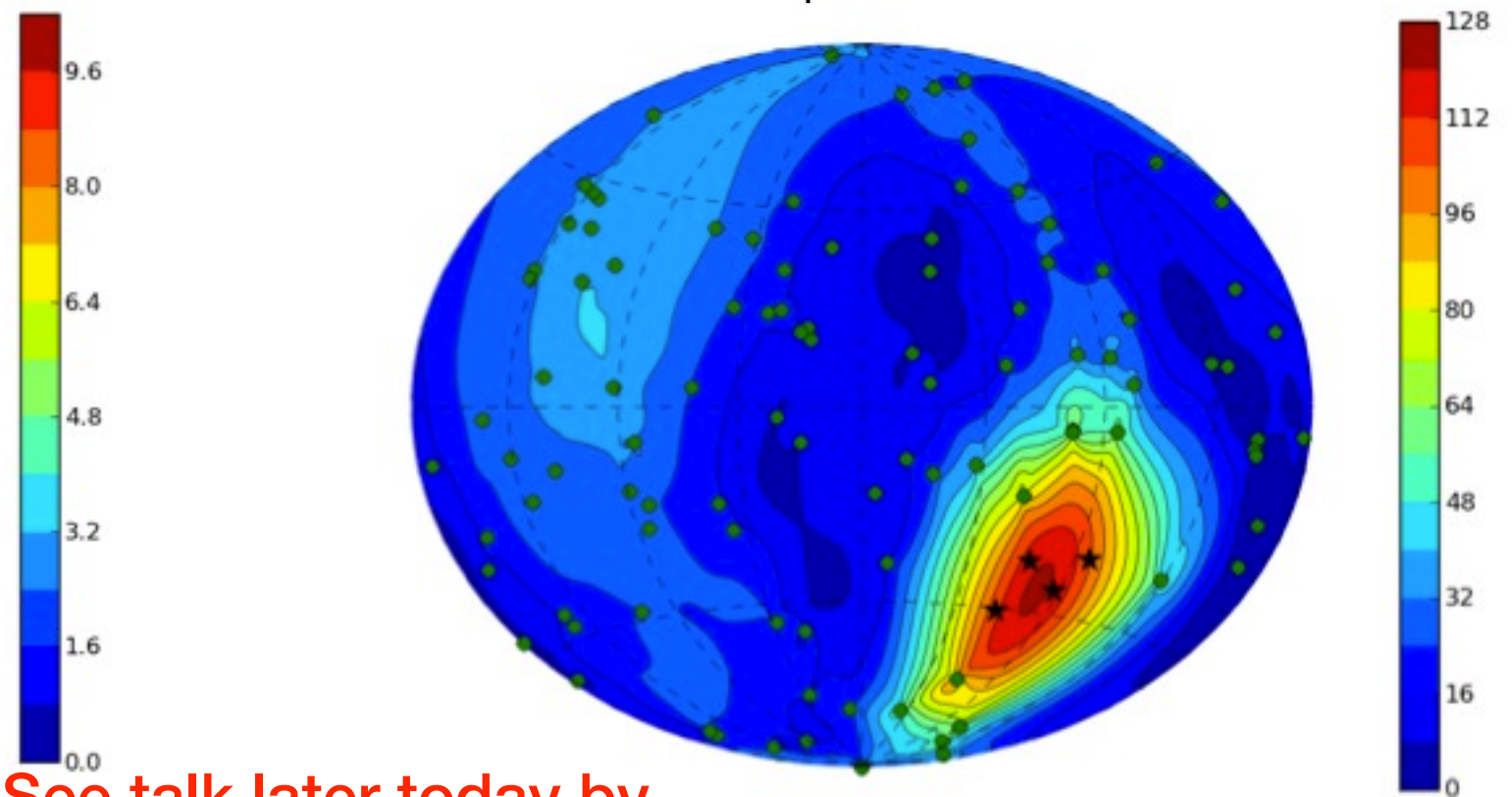
## Babak & Sesana (2012)

- First paper to deal with multiple single-sources. Show that many individual sources can be resolved with PTAs.
- Takes advantage of the fact that at higher frequencies there will not be a stochastic background but instead several semi-resolvable sources.

6 pulsars



100 pulsars



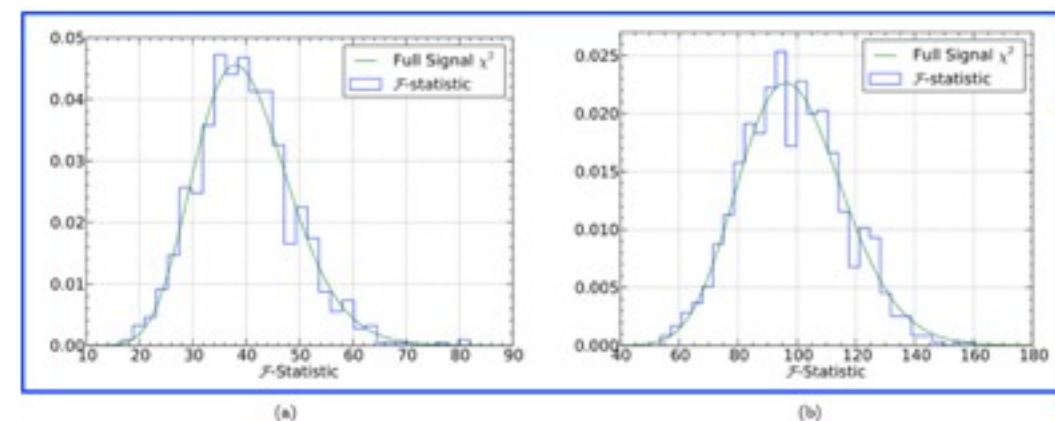
See talk later today by  
Stanislav Babak



# Ellis et al. (2012b)

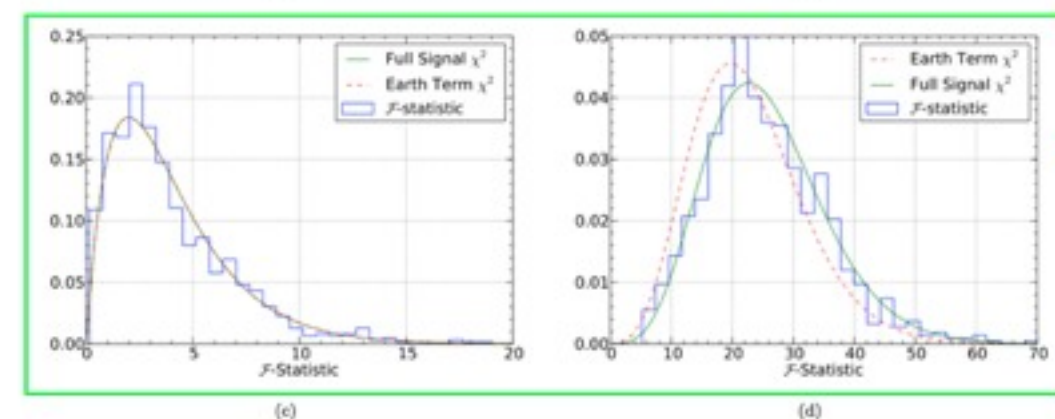
- Build on Babak and Sesana (2012) to construct realistic detection statistic that takes timing model, irregular sampling and correlated colored noise into account.
- Treat pulsar term as a noise source and derive a coherent and incoherent detection statistic that maximizes over many GW parameters.

Chi square  
with 2M d.o.f



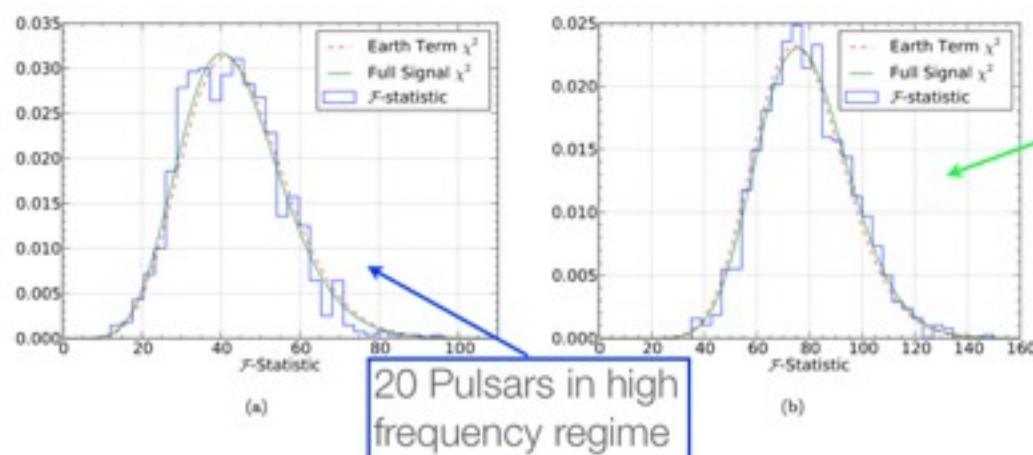
Incoherent  
F-statistic

Chi square  
with 4 d.o.f



Coherent  
F-statistic

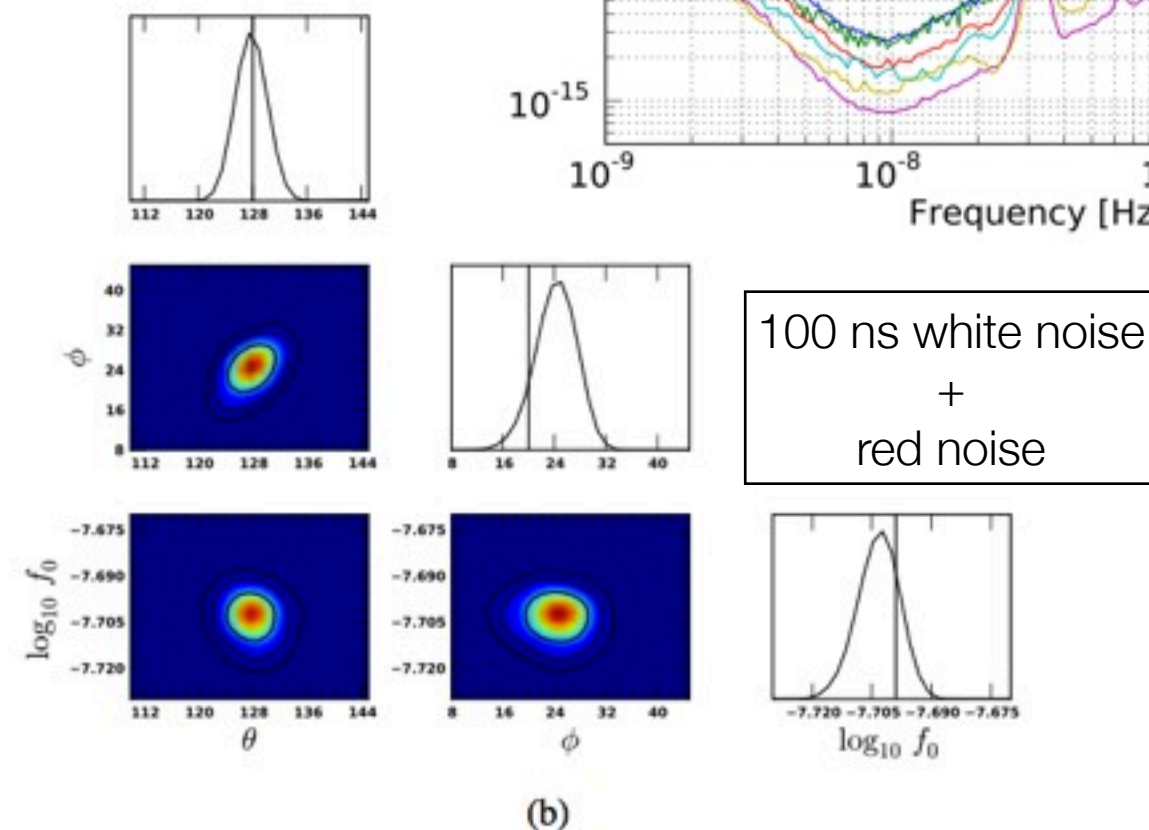
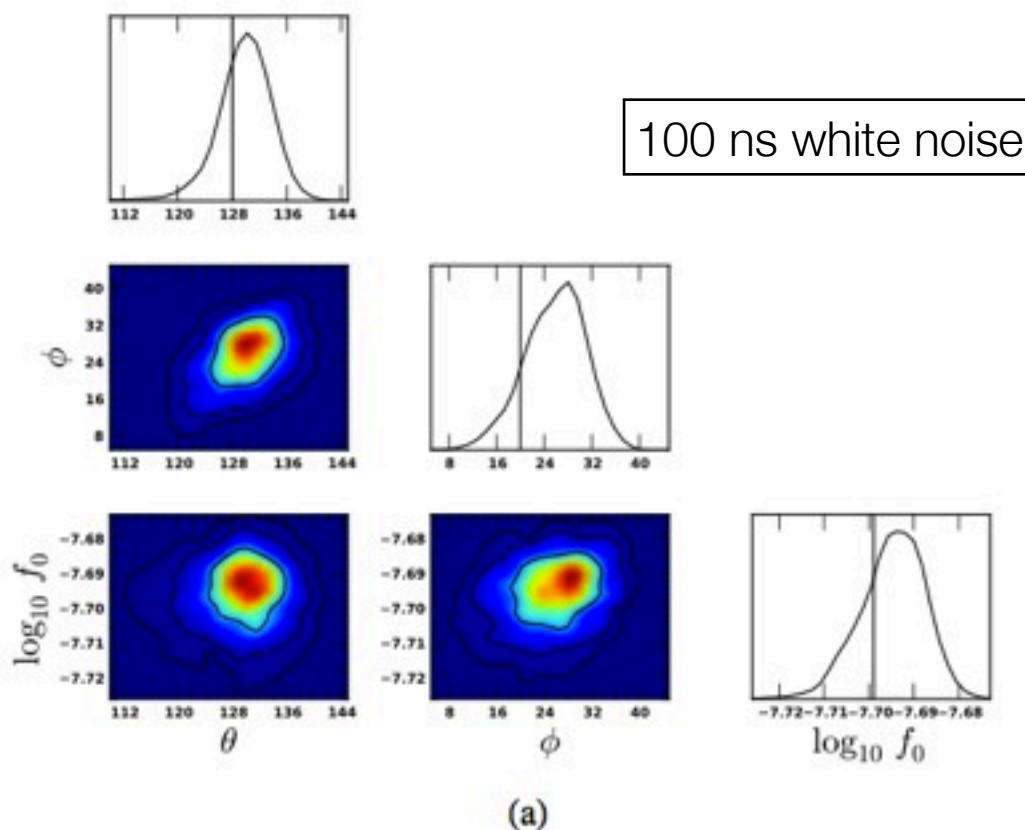
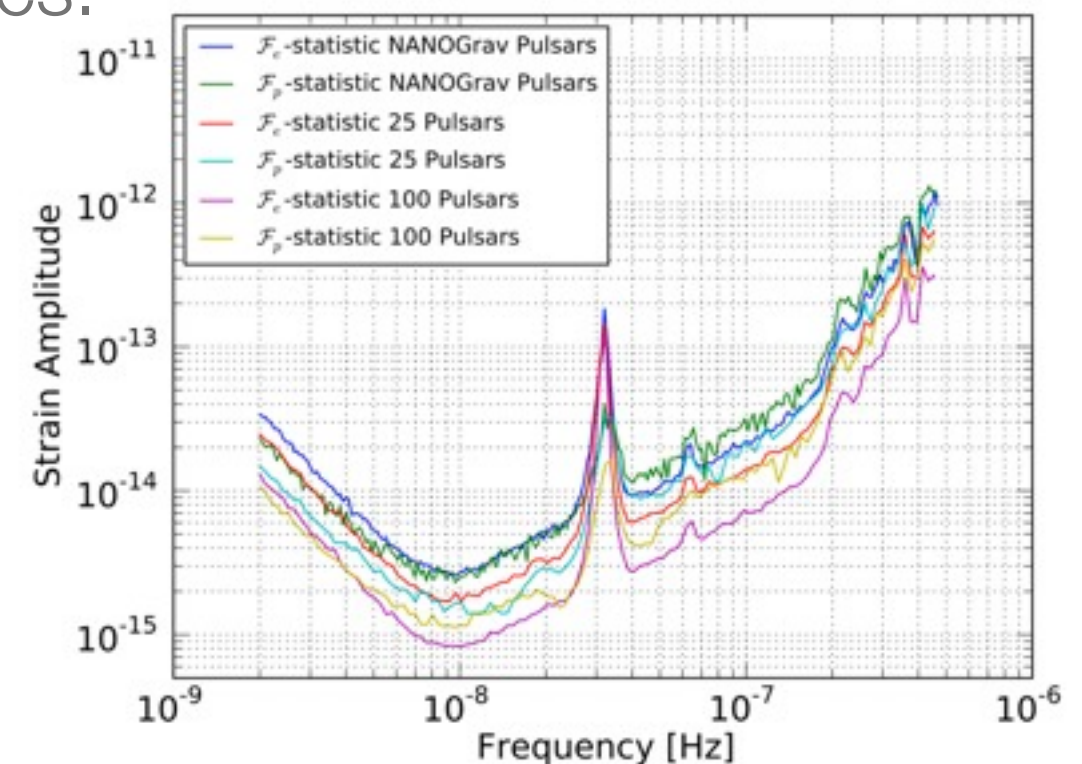
50 pulsars in low  
frequency regime





# Ellis et al. (2012b) cont.

- Outline detection pipeline and methods for producing sensitivity curves and upper limits with both statistics.
- Test efficacy on simulated data sets with and without red noise.



# Maximization of GW phases at pulsar

In low frequency limit, the full signal is:  $s_\alpha(t) = \sum_{i=0}^M [(\cos \Phi_\alpha - 1)\delta_{ij} + \sin \Phi_\alpha \varepsilon_{ij}] a^j A^i$

where,  $\varepsilon = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

The likelihood function is then:

$$\ln \Lambda = \sum_{\alpha=1}^M [b(\cos^2 \Phi_\alpha - \sin^2 \Phi_\alpha) + c \cos \Phi_\alpha + d \sin \Phi_\alpha + f \sin \Phi_\alpha \cos \Phi_\alpha]$$

$$b = -\frac{1}{2} M_\alpha^{ij} a_i a_j$$

$$c = N_\alpha^i a_i + M_\alpha^{ij} a_i a_j$$

$$d = N_\alpha^i \varepsilon_{ij} a^j$$

$$f = -M_\alpha^{ij} \varepsilon_{\ell j} a_i a^\ell$$

Maximizing the likelihood w.r.t the phase, we get the quartic equation

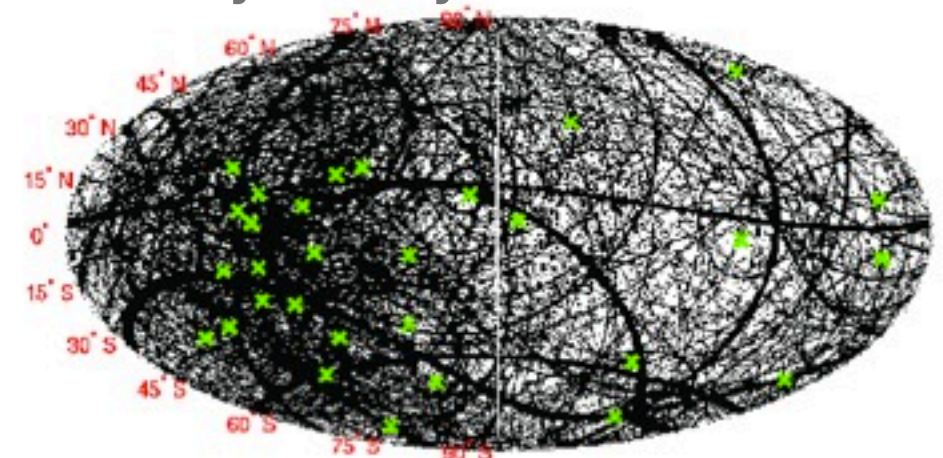
$$0 = (4f^2 + 16b^2)x^4 + (4fd + 8cb)x^3 + (c^2 - 4f^2 - 16b^2)x^2 + (-2fd - 8cb)x + f^2 - c^2$$

where,  $x = \cos \Phi_\beta$

This equation is guaranteed 1 unique solution. We can use the physical solutions to solve for the phase and plug back into the likelihood.

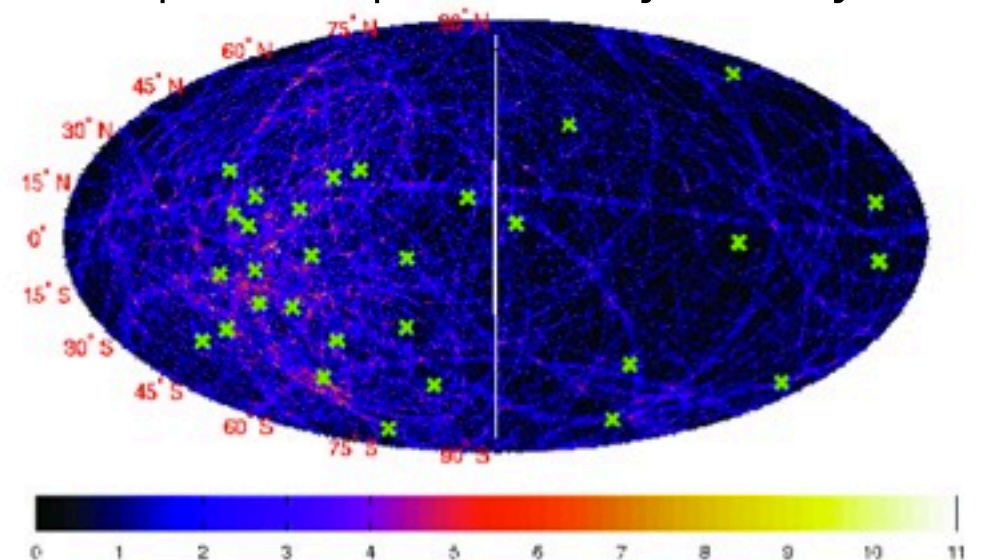
# Pitkin (2012)

- Extends burst search to include pulsar terms. Usually pulsar term is ignored because the signal may be separated by delays on the order of the pulsar distance.
- May be lucky enough for a GW burst to be in a part of the sky where the signal will be observed in at least one pulsar pair.
- Future work will apply to realistic datasets.
- Will greatly increase temporal coverage



at least 1 source separated by  $< 10$  yrs

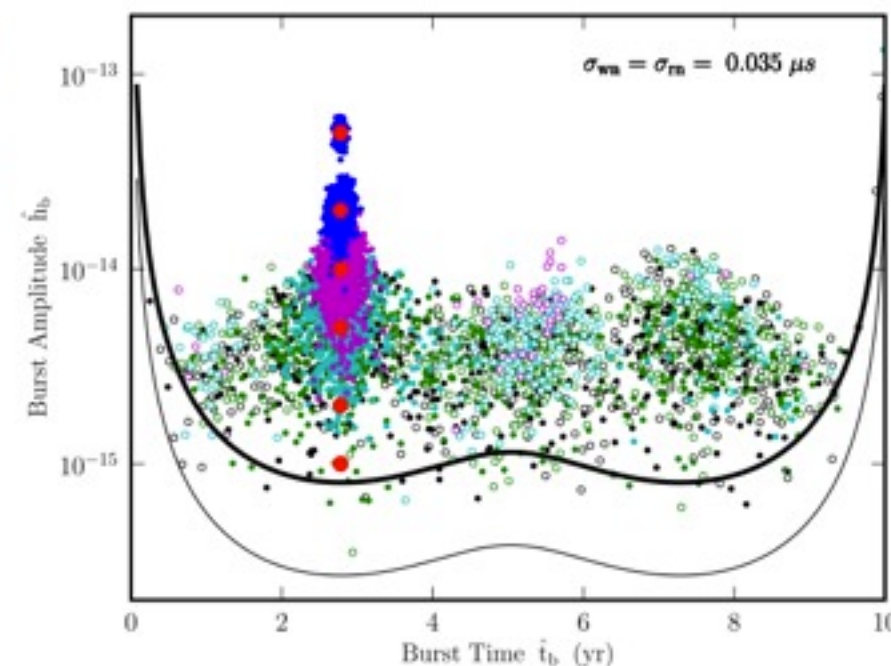
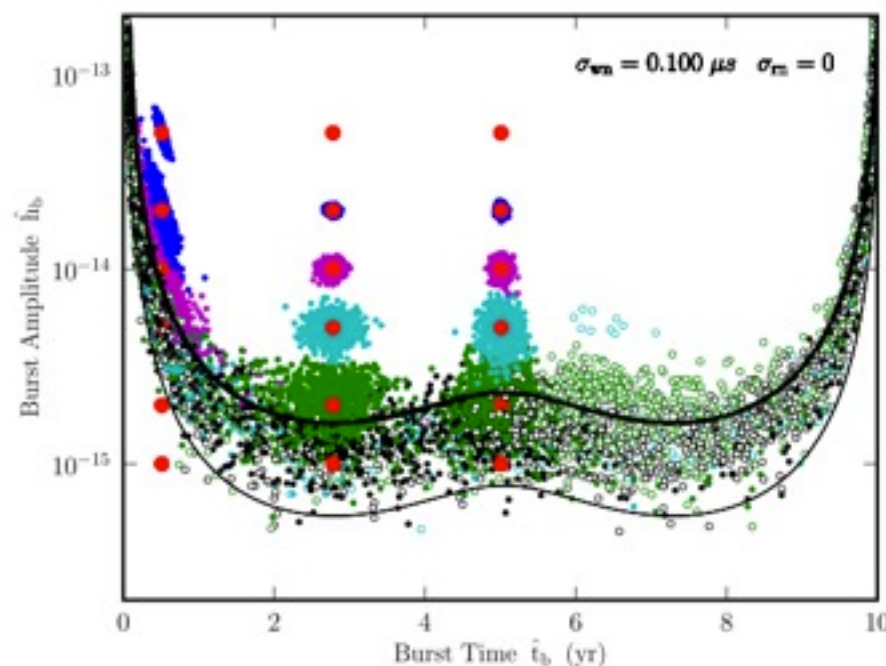
# of pairs separated by  $< 10$  yrs





# Cordes & Jenet (2012)

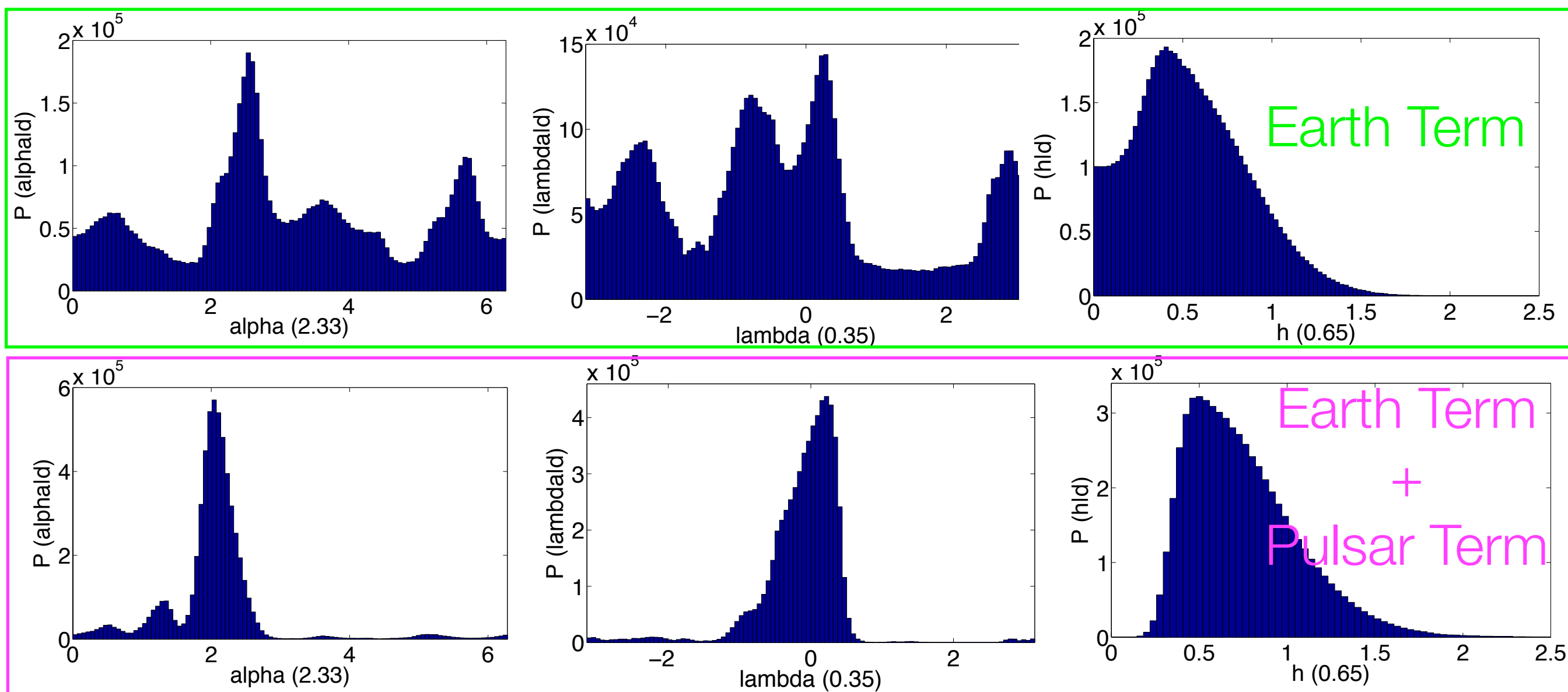
- Looks at detectability of GW bursts in Earth terms and/or pulsar terms.
- Show that burst rate and amplitude distribution may favor detection in pulsar terms rather than earth terms.
- Perform least squares fit for burst epoch and estimate amplitude.





# Xihao Deng (see talk later today)

- Bayesian method that includes pulsar term as unknown phase for each pulsar.
- Uses MCMC method to search the full  $7+N_{\text{psr}}$  dimensional search space.



# tempo2 global fitting

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- Uses tempo2 global fitting to fit GW parameters simultaneously with timing model parameters.
- Fits for 4 amplitudes that depend on sky location for every frequency.

# Where we need to go in the future

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- Develop/finish end-to-end data analysis pipelines and make publicly available (verification).
- Produce sensitivity curves/upper limits for all datasets (and combined datasets!) for different detection methods and compare!
- Be sure to include effect of pulsar timing model in data analysis methods.
- Begin to explore other types of sources (eccentric binaries/ Post-Newtonian effects etc.)