Measuring the evolution of a super-massive black hole binary using Pulsar Timing Arrays

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Pulsar Timing Arrays

- PTAs are only avenue for direct study of $10^8 - 10^9 M_\odot$ SMBHB.
- Observations around the world (and future SKA) expected to yield timing accuracy to observe stochastic GW background from SMBHB.
- These observations may resolve a few individual sources which are sufficiently close, massive and high f that their GWs rise above background levels.

Earth term and pulsar term

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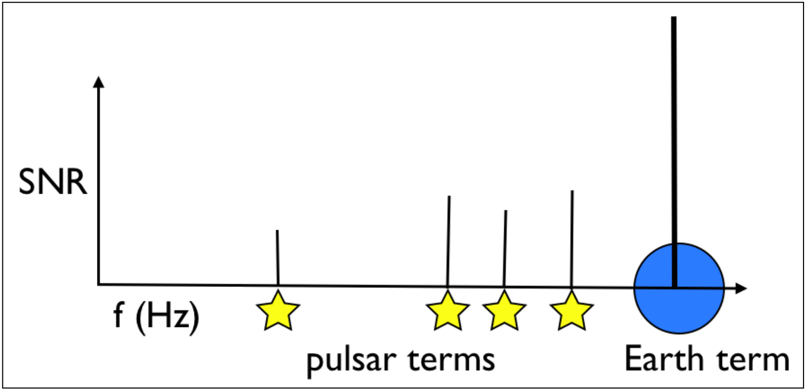
- If pulsar term is detected at SNR ~ 8, the Earth term is at SNR $\sim 36\sqrt{N/20}$

Connecting all the pulsar terms



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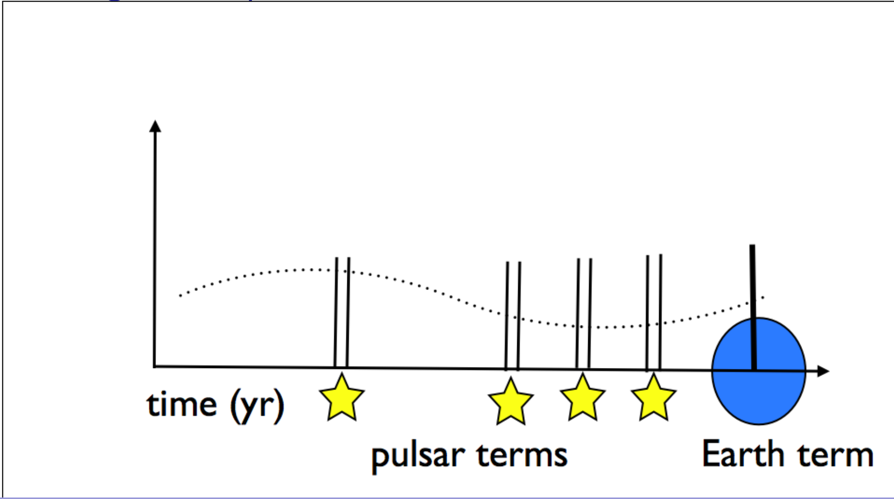

Typical parameters

- Expect to detect SMBHBs that are still in the weak field adiabatic inspiral regime, with an orbital velocity $v = 1.7 \times 10^{-2} \, (M/10^9 \, M_\odot)^{2/3} \, (f/50 \, \mathrm{nHz})^{2/3}$, can do pN expansion.

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- Extended baseline is now comparable to orbital timescale.

Detectability of sources: 1 kpc

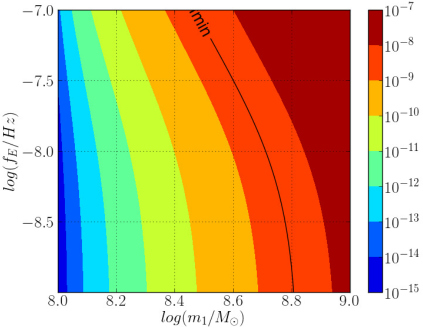
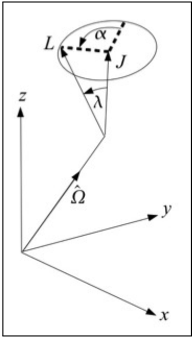


Figure: angular resolution of $\lesssim 3(100\,\mathrm{nHz}/f)(1\,\mathrm{kpc}/L_p)$ arcsec, $\Delta L_p < 0.01(f/100\mathrm{nHz})^{-1}$ pc (SKA, R. Smits et al., 2011)

Simple precession

- Use simple precession approximation to model spin-orbit coupling: $m_1 = m_2$
- total spin $\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$ and $\mathbf{L}$, precesses around the (essentially) constant direction of the total angular momentum, $\hat{\mathbf{J}}$
- precesses at the same rate $d\alpha/dt = (2 + 3m_2/(2m_1))\,(|\mathbf{L} + \mathbf{S}|)/r(t)^3$, while preserving the angle of the precision cone, λ_L. In this case, $|S/L| \sim \mathcal{O}(0.1)$



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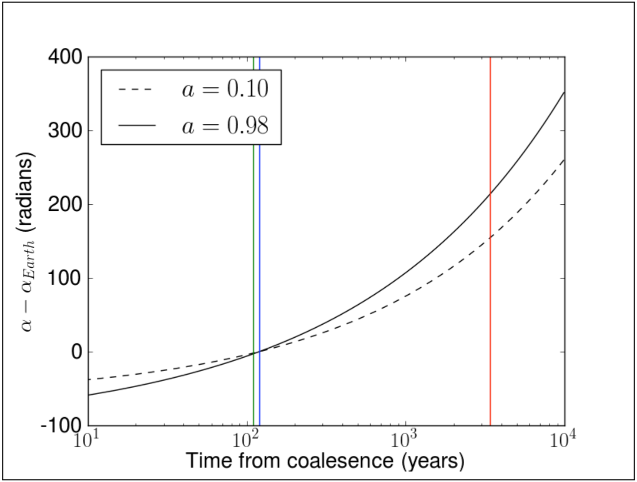


Signals from precessing black holes

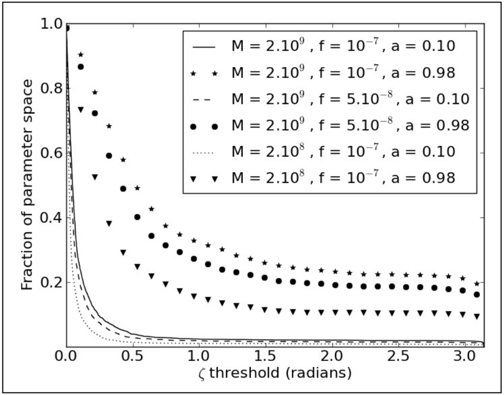
- model GW from a SMBHB using “restricted” pN approximation: amplitude is taken at leading Newtonian order, but we include the modulation effects produced by spin-orbit coupling and pN corrections are included only in the phase.
- strain for a single source given by $h(t) = -A_{\rm gw}(t)A_{\rm p}(t)\cos[\Phi(t) + \zeta(t) + \varphi(t)]$, where $A_{\rm gw}(t)$ is the lowest order Newtonian GW amplitude.
- The physical parameters leave different observational signatures in $h(t)$ and therefore in the TOAs.

Spins: 3 distinctive imprints in waveform

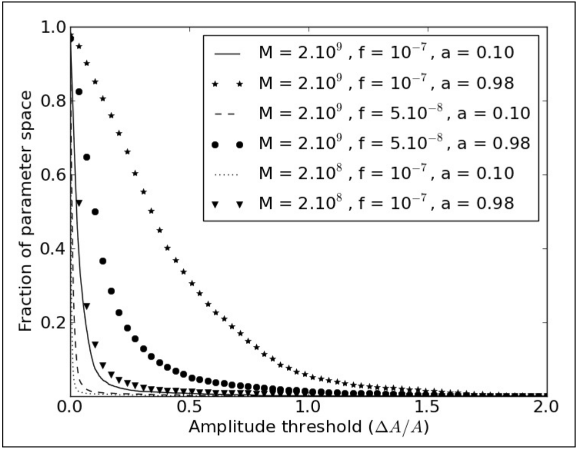
1. Alter the phase evolution through spin-orbit coupling (at $p^{1.5}$N order, proportional to the parameter $\beta = (1/12)\sum_{i=1}^{2}\left[113(m_i/M)^2 + 75\eta\right]\hat{\mathbf{L}}\cdot\hat{\mathbf{S}}_i)$ and at p^2N via spin-spin coupling $\sigma = (\eta/48)[721(\hat{\mathbf{L}}\cdot\hat{\mathbf{S}}_1)(\hat{\mathbf{L}}\cdot\hat{\mathbf{S}}_2) - 247(\hat{\mathbf{S}}_1\cdot\hat{\mathbf{S}}_2])$
2. they cause the orbital plane to precess due to (at lowest order) spin-orbit coupling and therefore induce amplitude and phase modulations in the waveform through $A_p(t)$ and $\zeta(t)$
3. through spin-orbit precession they introduce $\varphi(t)$, analogous to Thomas precession, to the waveform phase.

phase modulations

Imprint of precession is in $A_p(t)$, $\zeta(t)$ and $\varphi(t)$ whose size depends on λ_L, (maximised) $\hat{\mathbf{p}}$ and $\hat{\mathbf{\Omega}}$



Amplitude modulations



Comparing timescales and importance of spin

- For $m_{1,2} = 10^9\, M_\odot$ and $f = 10^{-7}$ Hz at the Earth, there it a total of 4305 (32.1) GW cycles over 1 kpc (10yr) evolution, with 4267 (32) of them driven by the leading order Newtonian term, which provides information about $\mathcal{M}$.

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- Over 1 kpc, 10s of cycles are contributed by the p^{1}N and p$^{1.5}$N contribution with 3.6β. This provides information about, η, and 2 wavecycles are contributed by the p^{2}N term with $\sigma \sim 0$.

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- the resolvable SMBHB is sufficiently massive and high frequency
- need to use the full precession equations, not just simple precession.

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- This work has now been submitted as C.M.F. Mingarelli, K. Grover, T. Sidery, R. Smith, A. Vecchio (2012).

Questions?