```
carbon:~$ c2
carbon:17_Fall_2041$ cd carbon-repos/repo-score100/Hwk_04
carbon:Hwk_04$ utop
           Welcome to utop version 1.14 (using OCaml version 4.01.0)!
Type #utop_help for help about using utop.
-(18:00:00) -< command 0 > ---
                                                            ------{ counter: 0 }-
utop # #use "eval.ml" ;;
type expr =
    Add of expr * expr
   Sub of expr * expr
  | Mul of expr * expr
  | Div of expr * expr
  | Lt of expr * expr
  | Eq of expr * expr
  | And of expr * expr
  | If of expr * expr * expr
   Id of string
  | Let of string * expr * expr
  | LetRec of string * expr * expr
  | App of expr * expr
  | Lambda of string * expr
  | Value of value
and value =
    Int of int
    Bool of bool
  | Closure of string * expr * environment
  | Ref of value ref
and environment = (string * value) list
val lookup : string -> environment -> value = <fun>
val freevars : expr -> string list = <fun>
val eval : environment -> expr -> value = <fun>
val evaluate : expr -> value = <fun>
val i0 : expr = Value (Int 0)
val i1 : expr = Value (Int 1)
val i2 : expr = Value (Int 2)
val i3 : expr = Value (Int 3)
val i4 : expr = Value (Int 4)
val a1 : expr = Add (Value (Int 2), Value (Int 4))
val m1 : expr = Mul (Add (Value (Int 2), Value (Int 4)), Value (Int 3))
val e1 : expr = Add (Add (Value (Int 2), Value (Int 4)), Id "x")
val e2 : expr =
  Mul (Add (Add (Value (Int 2), Value (Int 4)), Id "x"),
   Add (Add (Value (Int 2), Value (Int 4)), Id "x"))
val inc : expr = Lambda ("n", Add (Id "n", Value (Int 1)))
val two : expr = App (Lambda ("n", Add (Id "n", Value (Int 1))), Value (Int 1))
val sumToBody : expr =
  If (Eq (Id "n", Value (Int 0)), Value (Int 0),
```

Add (Id "n", App (Id "sumTo", Sub (Id "n", Value (Int 1)))))

val sumTo : expr =

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```
LetRec ("sumTo",
   Lambda ("n",
  If (Eq (Id "n", Value (Int 0)), Value (Int 0),
     Add (Id "n", App (Id "sumTo", Sub (Id "n", Value (Int 1))))),
   Id "sumTo")
val dummy : value = Int 999
val sumToRef : value ref = {contents = Int 999}
val sumToV : value =
  Closure ("n",
   If (Eq (Id "n", Value (Int 0)), Value (Int 0),
   Add (Id "n", App (Id "sumTo", Sub (Id "n", Value (Int 1))))),
   [("sumTo", Ref {contents = Int 999})])
val sumTo4 : expr =
  App
   (Value
     (Closure ("n",
       If (Eq (Id "n", Value (Int 0)), Value (Int 0),
        Add (Id "n", App (Id "sumTo", Sub (Id "n", Value (Int 1))))),
       [("sumTo",
         Ref
          {contents =
            Closure ("n",
             If (Eq (Id "n", Value (Int 0)), Value (Int 0),
              Add (Id "n", App (Id "sumTo", Sub (Id "n", Value (Int 1))))),
             [("sumTo",
               Ref
                {contents =
                  Closure ("n",
                   If (Eq (Id "n", Value (Int 0)), Value (Int 0),
                    Add (Id "n", App (Id "sumTo", Sub (Id "n", Value (Int 1)))))
,
                    [("sumTo",
                     Ref
                       {contents =
                         Closure ("n",
                          If (Eq (Id "n", Value (Int 0)), Value (Int 0),
                           Add (Id "n",
                            App (Id "sumTo", Sub (Id "n", Value (Int 1)))),
                          [("sumTo".
                           Ref
                             {contents =
                               Closure ("n",
                                If (Eq (Id "n", Value (Int 0)), Value (Int 0),
                                 Add (Id "n",
                                  App (Id "sumTo", Sub (Id "n", Value (Int 1)))))
                                [("sumTo",
                                  Ref
                                   {contents =
                                     Closure ("n",
                                      If (Eq (Id "n", Value (Int 0)),
                                       Value (Int 0),
                                       Add (Id "n",
                                        App (Id "sumTo",
```

```
[("sumTo",
                                         Ref
                                          {contents =
                                            Closure ("n",
                                             If (Eq (Id "n", Value (Int 0)),
                                              Value (Int 0),
                                              Add (Id "n",
                                               App (Id "sumTo",
                                                Sub (Id "n", Value (Int 1)))),
                                             [("sumTo",
                                               Ref
                                                {contents =
                                                  Closure ("n",
                                                   Ιf
                                                    (Eq (Id "n", Value (Int 0)),
                                                    Value (Int 0),
                                                    Add (Id "n",
                                                     App (Id "sumTo",
                                                      Sub (Id "n", Value (Int 1)))
)),
                                                    [("sumTo",
                                                     Ref
                                                       {contents =
                                                         Closure ("n",
                                                          Ιf
                                                           (Eq (Id "n",
                                                             Value (Int 0)),
                                                           Value (Int 0),
                                                           Add (Id "n",
                                                            App (Id "sumTo",
                                                             Sub (Id "n",
                                                             Value (Int 1)))),
                                                          [("sumTo",
                                                            Ref
                                                             {contents =
                                                               Closure ("n",
                                                                Ιf
                                                                 (Eq (Id "n",
                                                                   Value (Int 0)),
                                                                 Value (Int 0),
                                                                 Add (Id "n",
                                                                  App (Id "sumTo",
                                                                   Sub (
                                                                    Id "n",
                                                                    Value (Int 1)))
)),
                                                                [("sumTo", ...);
                                                                 ...])});
                                                           ...])});
                                                     ...])});
                                              ...])});
                                        ...])});
                                 ...])});
```

Sub (Id "n", Value (Int 1)))),

```
...])});
                  ...])});
            ...])});
       ...])),
   ...)
val sumToWith : expr =
  Lambda ("i",
  LetRec ("sTW",
   Lambda ("n",
    If (Eq (Id "n", Value (Int 0)), Id "i",
     Add (Id "n", App (Id "sTW", Sub (Id "n", Value (Int 1))))),
   Id "sTW"))
val sT100 : expr =
  App
   (Lambda ("i",
    LetRec ("sTW",
     Lambda ("n",
      If (Eq (Id "n", Value (Int 0)), Id "i",
       Add (Id "n", App (Id "sTW", Sub (Id "n", Value (Int 1))))),
     Id "sTW")),
  Value (Int 1000))
val st4 : expr =
  App
   (App
    (Lambda ("i",
      LetRec ("sTW",
       Lambda ("n",
        If (Eq (Id "n", Value (Int 0)), Id "i",
         Add (Id "n", App (Id "sTW", Sub (Id "n", Value (Int 1))))),
       Id "sTW")),
    Value (Int 1000)),
  Value (Int 4))
val add : expr = Lambda ("x", Lambda ("y", Add (Id "x", Id "y")))
val inc' : expr =
  App (Lambda ("x", Lambda ("y", Add (Id "x", Id "y"))), Value (Int 1))
val five : expr =
 App (App (Lambda ("x", Lambda ("y", Add (Id "x", Id "y"))), Value (Int 1)),
  Value (Int 4))
                                                   _____{ counter: 0 }-
-( 13:22:37 )-< command 1 >---
utop # evaluate (App (sumTo, Value (Int 3))) ;;
- : value = Int 6
utop # evaluate (App (sumTo, Value (Int 6))) ;;
- : value = Int 21
utop # #quit ;;
carbon: Hwk 04$ c
```

```
carbon:Hwk_04$ utop
```

Welcome to utop version 1.14 (using OCaml version 4.01.0)!

Type #utop_help for help about using utop.

```
-(18:00:00)-< command 0>-
                                                           ------{ counter: 0 }-
utop # #use "eval.ml" ;;
type expr =
    Add of expr * expr
   Sub of expr * expr
   Mul of expr * expr
  | Div of expr * expr
  | Lt of expr * expr
  | Eq of expr * expr
  | And of expr * expr
   If of expr * expr * expr
  | Id of string
  | Let of string * expr * expr
  | LetRec of string * expr * expr
   App of expr * expr
  | Lambda of string * expr
  | Value of value
and value =
    Int of int
   Bool of bool
   Closure of string * expr * environment
  | Ref of value ref
and environment = (string * value) list
val lookup : string -> environment -> value = <fun>
val freevars : expr -> string list = <fun>
val eval : environment -> expr -> value = <fun>
val evaluate : expr -> value = <fun>
val i0 : expr = Value (Int 0)
val i1 : expr = Value (Int 1)
```

```
val i2 : expr = Value (Int 2)
val i3 : expr = Value (Int 3)
val i4 : expr = Value (Int 4)
val a1 : expr = Add (Value (Int 2), Value (Int 4))
val m1 : expr = Mul (Add (Value (Int 2), Value (Int 4)), Value (Int 3))
val e1 : expr = Add (Add (Value (Int 2), Value (Int 4)), Id "x")
val e2 : expr =
  Mul (Add (Add (Value (Int 2), Value (Int 4)), Id "x"),
   Add (Add (Value (Int 2), Value (Int 4)), Id "x"))
val inc : expr = Lambda ("n", Add (Id "n", Value (Int 1)))
val two : expr = App (Lambda ("n", Add (Id "n", Value (Int 1))), Value (Int 1))
val sumToBody : expr =
  If (Eq (Id "n", Value (Int 0)), Value (Int 0),
   Add (Id "n", App (Id "sumTo", Sub (Id "n", Value (Int 1)))))
val sumTo : expr =
  LetRec ("sumTo",
   Lambda ("n",
    If (Eq (Id "n", Value (Int 0)), Value (Int 0),
     Add (Id "n", App (Id "sumTo", Sub (Id "n", Value (Int 1))))),
   Id "sumTo")
val dummy : value = Int 999
val sumToRef : value ref = {contents = Int 999}
val sumToV : value =
  Closure ("n",
   If (Eq (Id "n", Value (Int 0)), Value (Int 0),
    Add (Id "n", App (Id "sumTo", Sub (Id "n", Value (Int 1))))),
   [("sumTo", Ref {contents = Int 999})])
val sumTo4 : expr =
  App
   (Value
     (Closure ("n",
       If (Eq (Id "n", Value (Int 0)), Value (Int 0),
        Add (Id "n", App (Id "sumTo", Sub (Id "n", Value (Int 1))))),
       [("sumTo",
         Ref
          {contents =
            Closure ("n",
             If (Eq (Id "n", Value (Int 0)), Value (Int 0),
              Add (Id "n", App (Id "sumTo", Sub (Id "n", Value (Int 1))))),
             [("sumTo",
               Ref
                {contents =
                  Closure ("n",
                   If (Eq (Id "n", Value (Int 0)), Value (Int 0),
                    Add (Id "n", App (Id "sumTo", Sub (Id "n", Value (Int 1)))))
                   [("sumTo",
                     Ref
                      {contents =
                        Closure ("n",
                         If (Eq (Id "n", Value (Int 0)), Value (Int 0),
                          Add (Id "n",
                           App (Id "sumTo", Sub (Id "n", Value (Int 1)))),
                         [("sumTo",
```

```
Ref
 {contents =
   Closure ("n",
    If (Eq (Id "n", Value (Int 0)), Value (Int 0),
     Add (Id "n",
      App (Id "sumTo", Sub (Id "n", Value (Int 1)))))
    [("sumTo",
      Ref
       {contents =
         Closure ("n",
          If (Eq (Id "n", Value (Int 0)),
           Value (Int 0),
           Add (Id "n",
            App (Id "sumTo",
             Sub (Id "n", Value (Int 1)))),
          [("sumTo",
            Ref
             {contents =
               Closure ("n",
                If (Eq (Id "n", Value (Int 0)),
                 Value (Int 0),
                 Add (Id "n",
                  App (Id "sumTo",
                   Sub (Id "n", Value (Int 1)))),
                 [("sumTo",
                  Ref
                    {contents =
                      Closure ("n",
                       Ιf
                        (Eq (Id "n", Value (Int 0)),
                        Value (Int 0),
                        Add (Id "n",
                         App (Id "sumTo",
                          Sub (Id "n", Value (Int 1)))
                       [("sumTo",
                         Ref
                          {contents =
                            Closure ("n",
                             Ιf
                              (Eq (Id "n",
                                Value (Int 0)),
                              Value (Int 0).
                              Add (Id "n",
                               App (Id "sumTo",
                                Sub (Id "n",
                                 Value (Int 1)))),
                             [("sumTo",
                               Ref
                                {contents =
                                  Closure ("n",
                                   Ιf
                                    (Eq (Id "n",
```

)),

```
Value (Int 0)),
                                                                Value (Int 0),
                                                                Add (Id "n",
                                                                 App (Id "sumTo",
                                                                  Sub (
                                                                   Id "n",
                                                                   Value (Int 1)))
)),
                                                               [("sumTo", ...);
                                                                ...])});
                                                          ...])});
                                                    ...])});
                                             ...])});
                                       ...])});
                                 ...])});
                           ...])});
                    ...])});
              ...])});
        ...])),
   ...)
val sumToWith : expr =
  Lambda ("i",
   LetRec ("sTW".
    Lambda ("n",
     If (Eq (Id "n", Value (Int 0)), Id "i",
      Add (Id "n", App (Id "sTW", Sub (Id "n", Value (Int 1))))),
    Id "sTW"))
val sT100 : expr =
  App
   (Lambda ("i",
     LetRec ("sTW",
      Lambda ("n",
       If (Eq (Id "n", Value (Int 0)), Id "i",
        Add (Id "n", App (Id "sTW", Sub (Id "n", Value (Int 1))))),
      Id "sTW")),
   Value (Int 1000))
val st4 : expr =
  App
   (App
     (Lambda ("i",
       LetRec ("sTW",
        Lambda ("n",
         If (Eq (Id "n", Value (Int 0)), Id "i",
          Add (Id "n", App (Id "sTW", Sub (Id "n", Value (Int 1))))),
        Id "sTW")),
     Value (Int 1000)),
   Value (Int 4))
val add : expr = Lambda ("x", Lambda ("y", Add (Id "x", Id "y")))
val inc' : expr =
  App (Lambda ("x", Lambda ("y", Add (Id "x", Id "y"))), Value (Int 1))
val five : expr =
  App (App (Lambda ("x", Lambda ("y", Add (Id "x", Id "y"))), Value (Int 1)),
   Value (Int 4))
-(13:28:29) -< command 1> -
                                                                   -{ counter: 0 }-
```

```
utop # evalute (App (sumTo, Value (Int 6))) ;;
Error: Unbound value evalute
Did vou mean evaluate?
                                -(13:28:34) -< command 2 >--
utop # evaluate (App (sumTo, Value (Int 6)));;
- : value = Int 21
utop # #quit ::
carbon:Hwk 04$ c
carbon:Hwk_04$ utop
          Welcome to utop version 1.14 (using OCaml version 4.01.0)!
Type #utop_help for help about using utop.
-( 18:00:00 )-< command 0 >----
                                                     _____{ counter: 0 }-
utop # #use "eval.ml";;
type expr =
   Add of expr * expr
   Sub of expr * expr
  | Mul of expr * expr
  | Div of expr * expr
  | Lt of expr * expr
  | Eq of expr * expr
  | And of expr * expr
  | If of expr * expr * expr
  | Id of string
  | Let of string * expr * expr
  | LetRec of string * expr * expr
  | App of expr * expr
  | Lambda of string * expr
  l Value of value
and value =
   Int of int
   Bool of bool
  | Closure of string * expr * environment
  | Ref of value ref
and environment = (string * value) list
val lookup : string -> environment -> value = <fun>
val freevars : expr -> string list = <fun>
val eval : environment -> expr -> value = <fun>
val evaluate : expr -> value = <fun>
val i0 : expr = Value (Int 0)
val i1 : expr = Value (Int 1)
val i2 : expr = Value (Int 2)
val i3 : expr = Value (Int 3)
val i4 : expr = Value (Int 4)
val a1 : expr = Add (Value (Int 2), Value (Int 4))
val m1 : expr = Mul (Add (Value (Int 2), Value (Int 4)), Value (Int 3))
val e1 : expr = Add (Add (Value (Int 2), Value (Int 4)), Id "x")
val e2 : expr =
```

```
Mul (Add (Add (Value (Int 2), Value (Int 4)), Id "x"),
   Add (Add (Value (Int 2), Value (Int 4)), Id "x"))
val inc : expr = Lambda ("n", Add (Id "n", Value (Int 1)))
val two : expr = App (Lambda ("n", Add (Id "n", Value (Int 1))), Value (Int 1))
val sumToBody : expr =
  If (Eq (Id "n", Value (Int 0)), Value (Int 0),
   Add (Id "n", App (Id "sumTo", Sub (Id "n", Value (Int 1)))))
val sumTo : expr =
  LetRec ("sumTo",
   Lambda ("n",
    If (Eq (Id "n", Value (Int 0)), Value (Int 0),
     Add (Id "n", App (Id "sumTo", Sub (Id "n", Value (Int 1))))),
   Id "sumTo")
val sumToN_expr : expr =
  LetRec ("sumToN",
   Lambda ("n",
    If (Eq (Id "n", Value (Int 0)), Value (Int 0),
     Add (Id "n", App (Id "sumToN", Sub (Id "n", Value (Int 1))))),
   Id "sumToN")
val dummy : value = Int 999
val sumToRef : value ref = {contents = Int 999}
val sumToV : value =
  Closure ("n",
   If (Eq (Id "n", Value (Int 0)), Value (Int 0),
    Add (Id "n", App (Id "sumTo", Sub (Id "n", Value (Int 1))))),
   [("sumTo", Ref {contents = Int 999})])
val sumTo4 : expr =
  App
   (Value
     (Closure ("n"
       If (Eq (Id "n", Value (Int 0)), Value (Int 0),
        Add (Id "n", App (Id "sumTo", Sub (Id "n", Value (Int 1))))),
       [("sumTo",
         Ref
          {contents =
            Closure ("n",
             If (Eq (Id "n", Value (Int 0)), Value (Int 0),
              Add (Id "n", App (Id "sumTo", Sub (Id "n", Value (Int 1))))),
             [("sumTo",
               Ref
                {contents =
                  Closure ("n",
                   If (Eq (Id "n", Value (Int 0)), Value (Int 0),
                    Add (Id "n", App (Id "sumTo", Sub (Id "n", Value (Int 1)))))
                   [("sumTo",
                     Ref
                      {contents =
                        Closure ("n",
                         If (Eq (Id "n", Value (Int 0)), Value (Int 0),
                          Add (Id "n",
                           App (Id "sumTo", Sub (Id "n", Value (Int 1))))),
                         [("sumTo",
                           Ref
```

```
{contents =
 Closure ("n",
   If (Eq (Id "n", Value (Int 0)), Value (Int 0),
   Add (Id "n",
     App (Id "sumTo", Sub (Id "n", Value (Int 1)))))
   [("sumTo",
     Ref
      {contents =
        Closure ("n",
         If (Eq (Id "n", Value (Int 0)),
          Value (Int 0),
          Add (Id "n",
           App (Id "sumTo",
            Sub (Id "n", Value (Int 1)))),
         [("sumTo",
           Ref
            {contents =
              Closure ("n",
               If (Eq (Id "n", Value (Int 0)),
                Value (Int 0),
                Add (Id "n",
                 App (Id "sumTo",
                  Sub (Id "n", Value (Int 1)))),
               [("sumTo",
                 Ref
                  {contents =
                    Closure ("n",
                      Ιf
                       (Eq (Id "n", Value (Int 0)),
                      Value (Int 0),
                      Add (Id "n",
                       App (Id "sumTo",
                        Sub (Id "n", Value (Int 1)))
                      [("sumTo",
                       Ref
                         {contents =
                           Closure ("n",
                            Ιf
                             (Eq (Id "n",
                               Value (Int 0)),
                             Value (Int 0),
                             Add (Id "n",
                              App (Id "sumTo",
                               Sub (Id "n",
                                Value (Int 1)))),
                            [("sumTo",
                              Ref
                               {contents =
                                 Closure ("n",
                                  Ιf
                                   (Eq (Id "n",
                                     Value (Int 0)),
```

))**,**

```
Value (Int 0),
                                                                 Add (Id "n",
                                                                  App (Id "sumTo",
                                                                   Sub (
                                                                    Id "n",
                                                                    Value (Int 1)))
)),
                                                                [("sumTo", ...);
                                                                 ...])});
                                                           ...])});
                                                    ...])});
                                              ...])});
                                        ...])});
                                 ...])});
                           ...])});
                     ...])});
              ...])});
        ...])),
   ...)
val sumToWith : expr =
  Lambda ("i",
   LetRec ("sTW",
    Lambda ("n", If (Eq (Id "n", Value (Int 0)), Id "i",
      Add (Id "n", App (Id "sTW", Sub (Id "n", Value (Int 1))))),
    Id "sTW"))
val sT100 : expr =
  App
   (Lambda ("i",
     LetRec ("sTW",
      Lambda ("n", If (Eq (Id "n", Value (Int 0)), Id "i",
        Add (Id "n", App (Id "sTW", Sub (Id "n", Value (Int 1))))),
      Id "sTW")),
   Value (Int 1000))
val st4 : expr =
  App
   (App
     (Lambda ("i",
       LetRec ("sTW",
        Lambda ("n",
         If (Eq (Id "n", Value (Int 0)), Id "i",
          Add (Id "n", App (Id "sTW", Sub (Id "n", Value (Int 1))))),
        Id "sTW")),
     Value (Int 1000)),
   Value (Int 4))
val add : expr = Lambda ("x", Lambda ("y", Add (Id "x", Id "y")))
val inc' : expr =
  App (Lambda ("x", Lambda ("y", Add (Id "x", Id "y"))), Value (Int 1))
val five : expr =
  App (App (Lambda ("x", Lambda ("y", Add (Id "x", Id "y"))), Value (Int 1)),
   Value (Int 4))
-( 15:38:34 )-< command 1 >----
                                                             -----{ counter: 0 }-
utop # evaluate ;;
```

```
- : expr -> value = <fun>
                                               _____{ counter: 0 }-
-(15:38:39) -< command 2> --
utop # evaluate (App (sumToN, Value (Int 6)));;
Error: Unbound value sumToN
Did you mean sumTo, sumTo4 or sumToV?
-( 15:38:46 )-< command 3 >----
                                                      -{ counter: 0 }-
utop # #quit;;
carbon:Hwk 04$ pwd
/project/evw/Teaching/17_Fall_2041/carbon-repos/repo-score100/Hwk_04
carbon: Hwk 04$ c
carbon:Hwk_04$ utop
         Welcome to utop version 1.14 (using OCaml version 4.01.0)!
Type #utop_help for help about using utop.
-( 18:00:00 )-< command 0 >---
-( 15:39:41 )-< command 0 >---
                                                  _____{ counter: 0 }-
utop #
 Arg|Arith_status|Array|ArrayLabels|Assert_failure|Big_int|Bigarray|Buffer|Call
```