UNIVERSITAS INDONESIA

Finance Theory

Modern Portfolio Theory

From Markowitz to Factor Model

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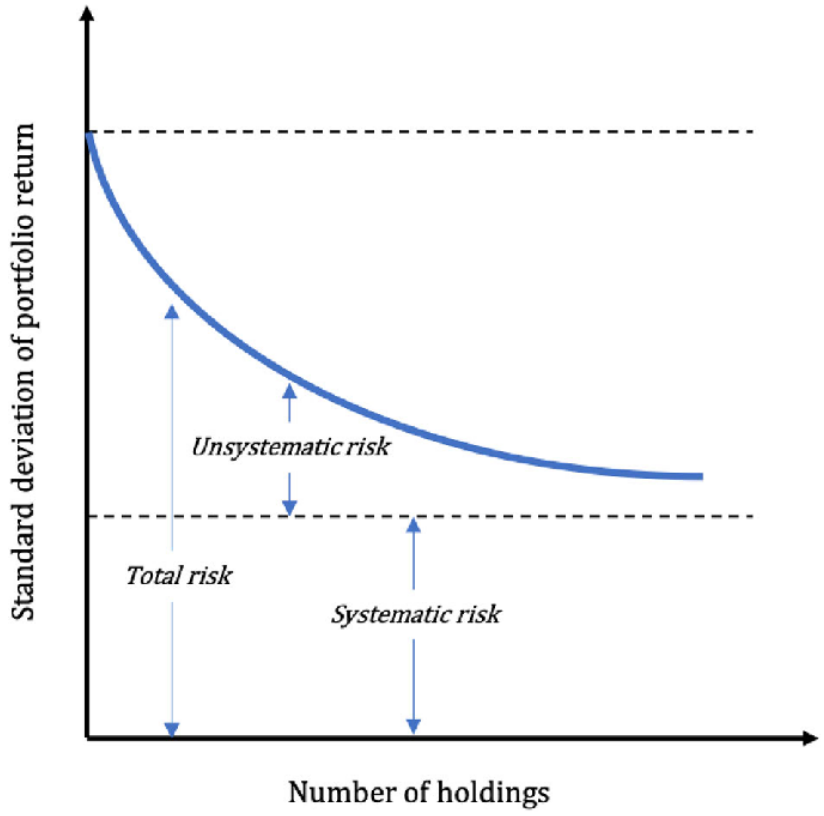
Modern Portfolio Theory (MPT)

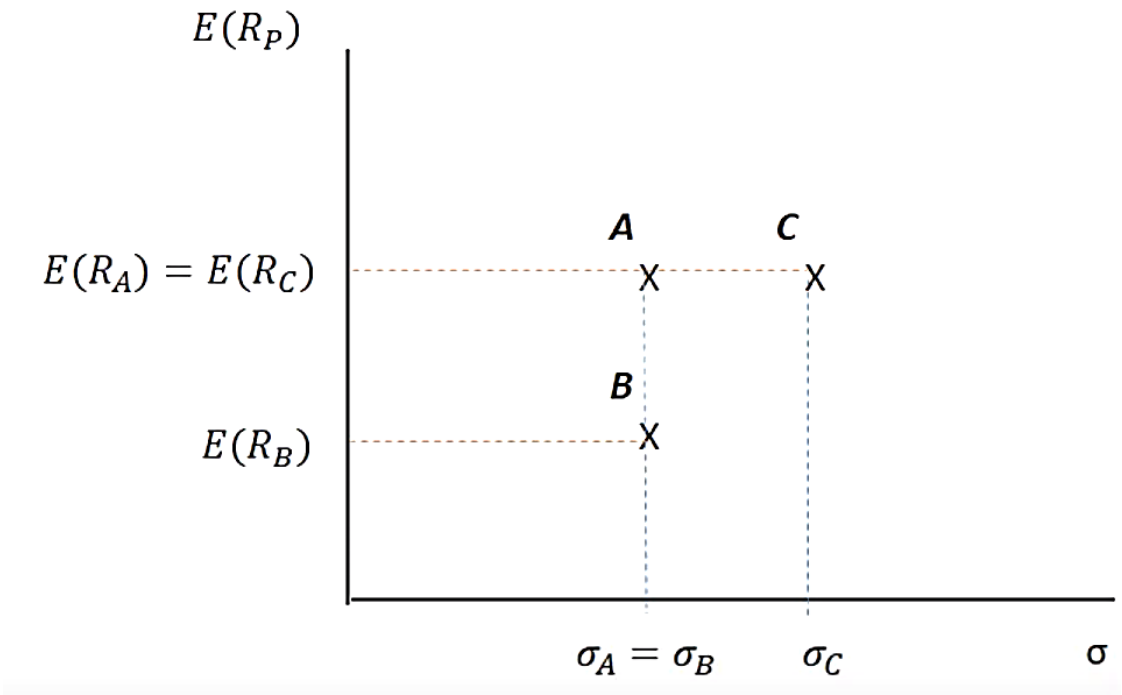
- Introduced by Harry Markowitz in 1952
- Shows mathematically how to create a portfolio with the optimum risk-return characteristics i.e., mean-variance model
- Quantification of previously qualitative concept of “not putting allof your eggs in onbe basket”
- The expected return of portfolio (mean) is the weighted average of expected returns of the securities constituting the portfolio $E(R_p) = \sum_{i=1}^{N} W_i . E(R_i)$
- The risk of portfolio (variance) is the riskiness of individual securities in the portfolio plus the diversification effect. For portfolio comprised of 2 securities, the variance of the portfolio is as follows:

$$\mathrm{Var}(R_p) = W_i^2 . \sigma_i^2 + W_j^2 . \sigma_j^2 + 2 . W_i . W_j . \mathrm{Cov}(R_i, R_j)$$

- As we add more securities to the portfolio, the number of covariance terms will increase faster than the number of variance terms. The number of covariance $\frac{N^2 - N}{2}$



Dominant Portfolio



Portfolio A is better than B because for the same level of risk $\sigma_A = \sigma_B$ we have higher return i.e., portfolio A dominates portfolio B.

Portfolio A is better than C because for the same level of return, risk σ_A is lower than σ_C i.e., portfolio A dominates portfolio C.

However, which one is better between A and C could not be determined

Thus **"Efficient Frontier"**

- The portfolios with highest expected return for a given level of risk.
- The portfolios with lowest risk for a gioven expected return.

Efficient Frontier in Markowitz (1952)

This is what we see in Markowitz (1952), with Expected Return (E) in the x-axis while Variance (V) – the proxy of risk in the y-axis

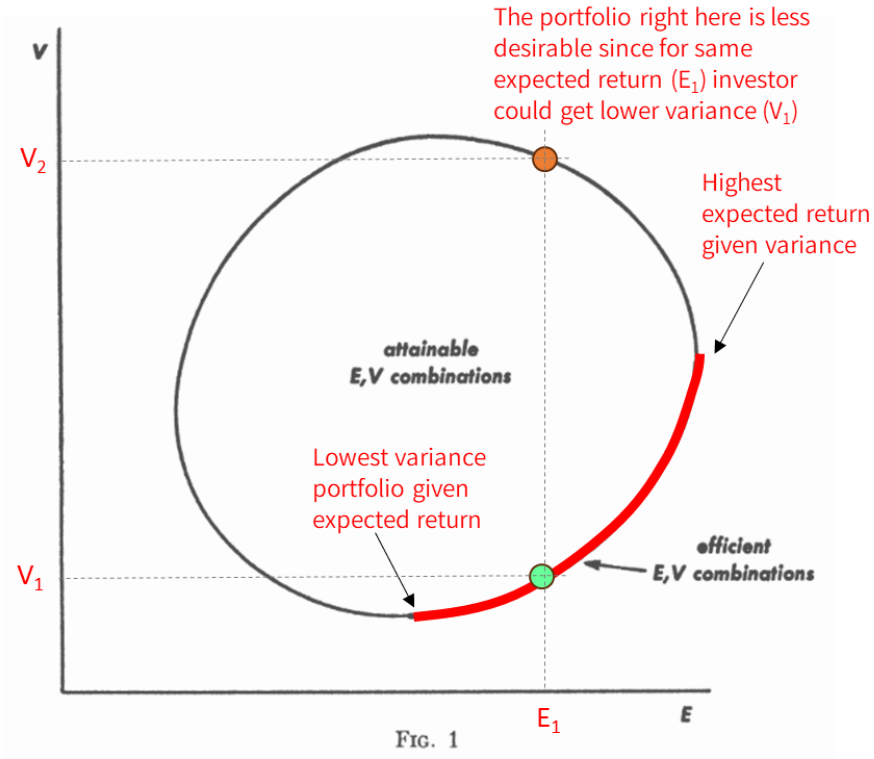
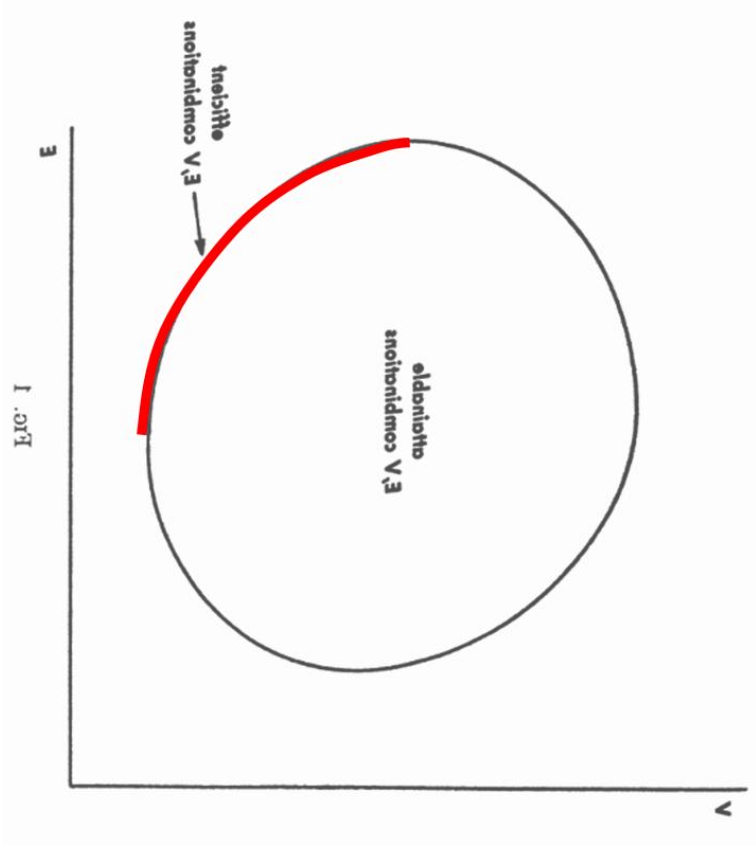
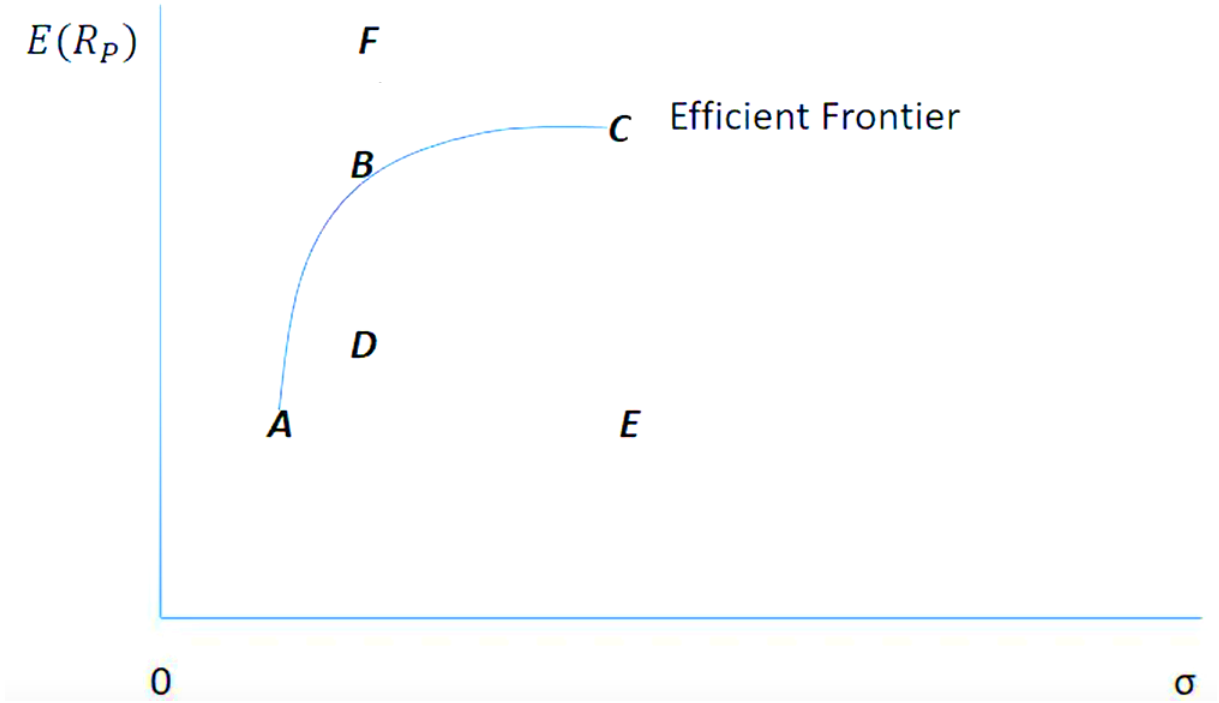


FIG. 1

This is what commonly see in literature/textbook, with Variance (V) – the proxy of risk in the x-axis while Expected Return (E) in the y-axis



Efficient Frontier Graphically

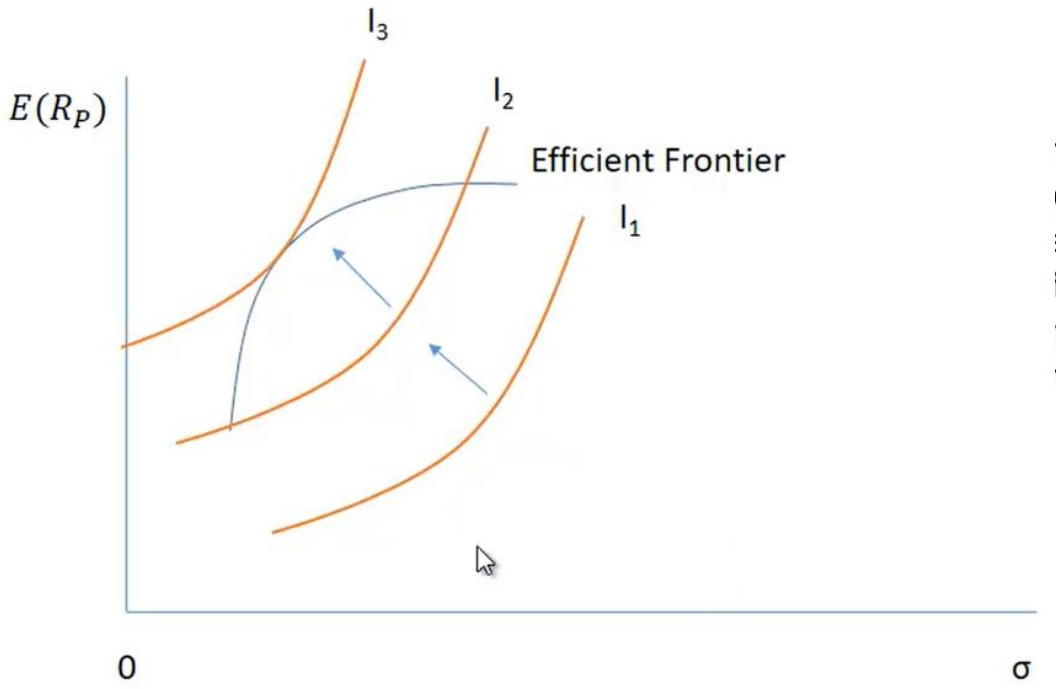


Portfolio A, B, and C all lied on the efficient frontier.

Portfolio D and E are dominated thus less desirable compare to other portfolio in the efficient frontier.

Portfolio F, albeit naturally most desirable, is unattainable

Efficient Frontier and Utility Function (Indifference Curve)

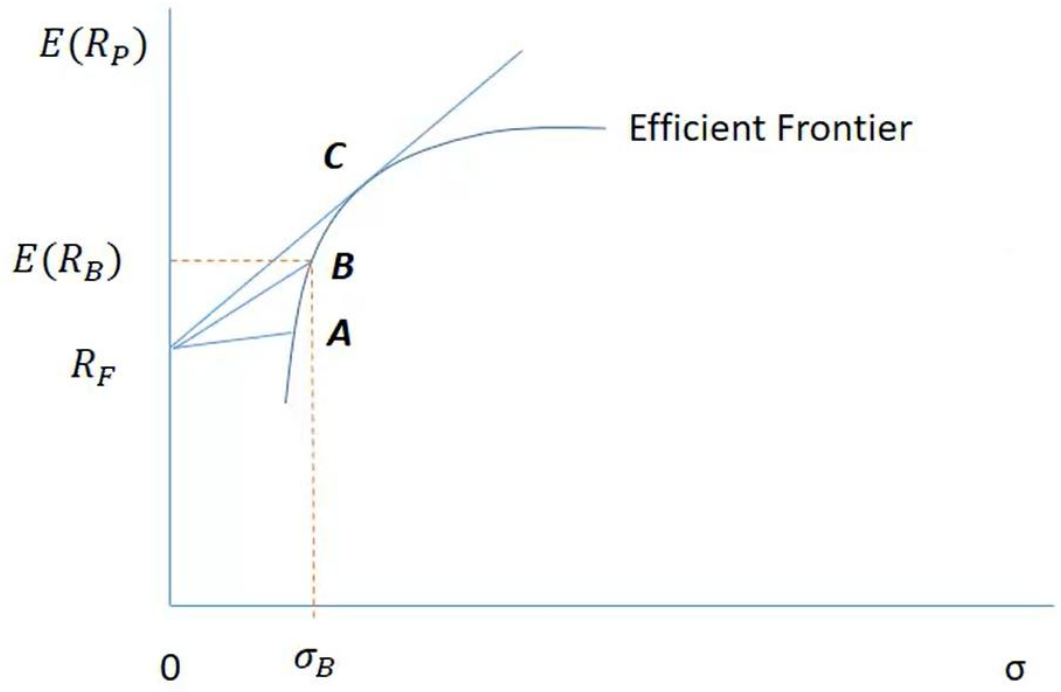


The indifference curve represent the investor preference on the combination of two “goods” which are expected return and risk that give same utility or satisfaction.

The slope of the indifference curve is upward sloping hinting the “high risk high return” adagium.

Each investor could have different indifference curve thus different tangency point i.e., different level of optimum portfolio.

Capital Allocation Line (CAL)



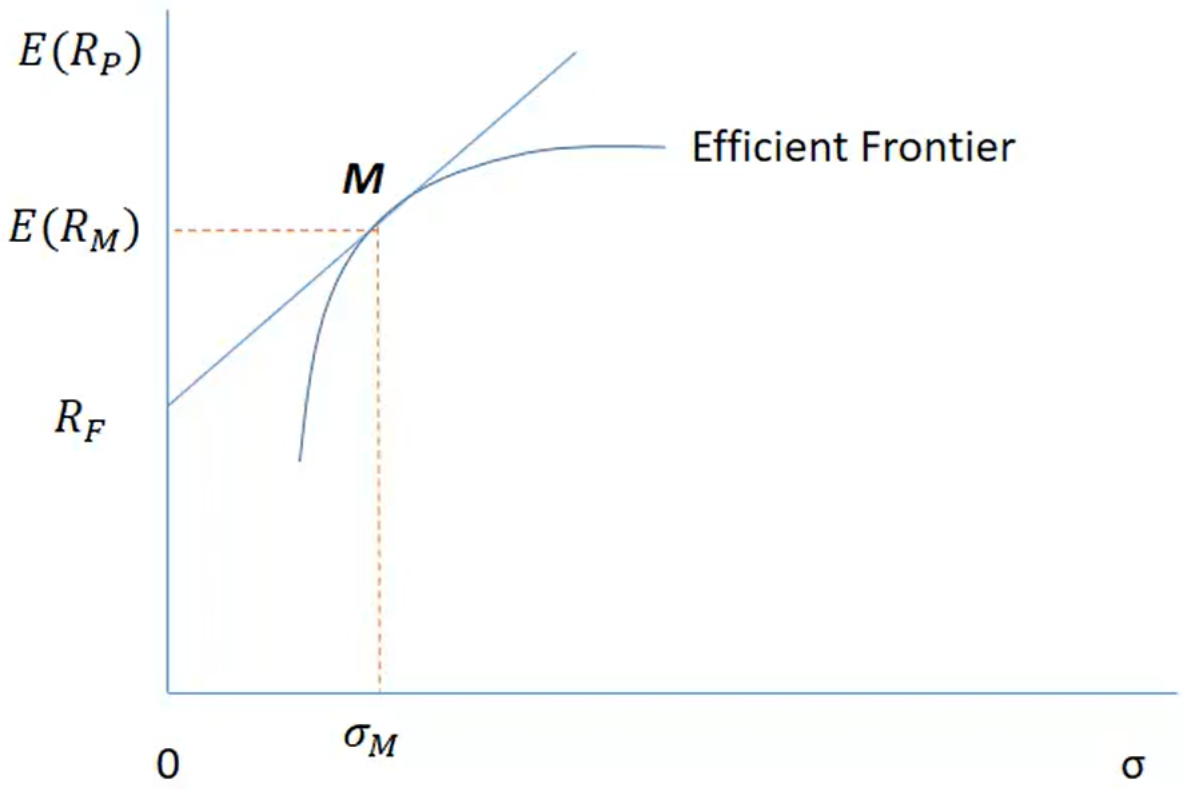
If there is a risk free asset, any combination of the risk free asset and risky portfolio will have a straight line relationship between risk and return. The expected return for portfolio B is:

$$E(R_p) = R_f + \sigma_P \left[\frac{E(R_B) - R_F}{\sigma_B}\right]$$

The best portfolio is the one where a line from risk free asset is tangent to the efficient frontier. In this case, portfolio C.

The slope of CAL is **the Sharpe ratio**. The steeper the slope represented by by higher Sharpe ratio implying a higher marginal return for same level of risk exposure.

Capital Market Line (CML)

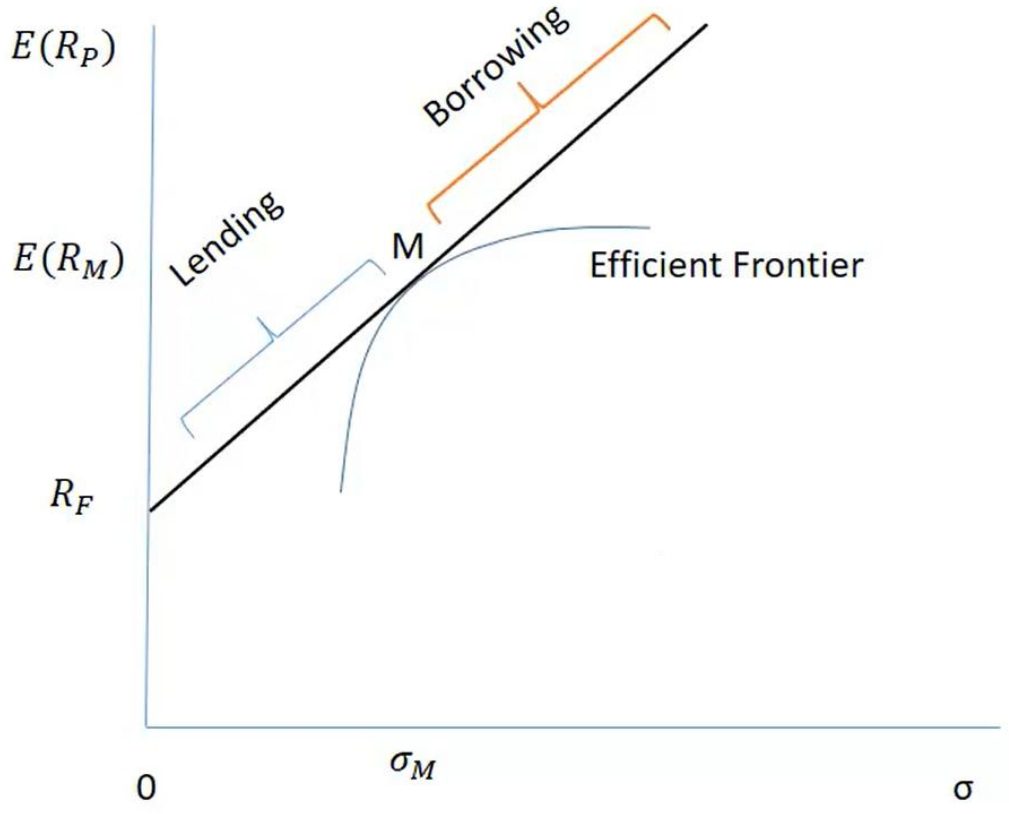


Capital market line (CML) is a special case for capital allocation line. This special case assumed that investors in the market all have same expectation thus same utility function.

$$E(R_p) = R_f + \sigma_P \left[\frac{E(R_M) - R_F}{\sigma_M}\right]$$

The best portfolio is the one where a line from risk free asset is tangent to the efficient frontier. In this case, portfolio M or commonly known as the market portfolio.

Capital Market Line (CML)



Investor can adjust the risk of their portfolio by changin the proportion of risk free asset and the market portfolio.

More conservative investors will lend by buying treasury securities (long position). Earned lower return but with lower risk as weill

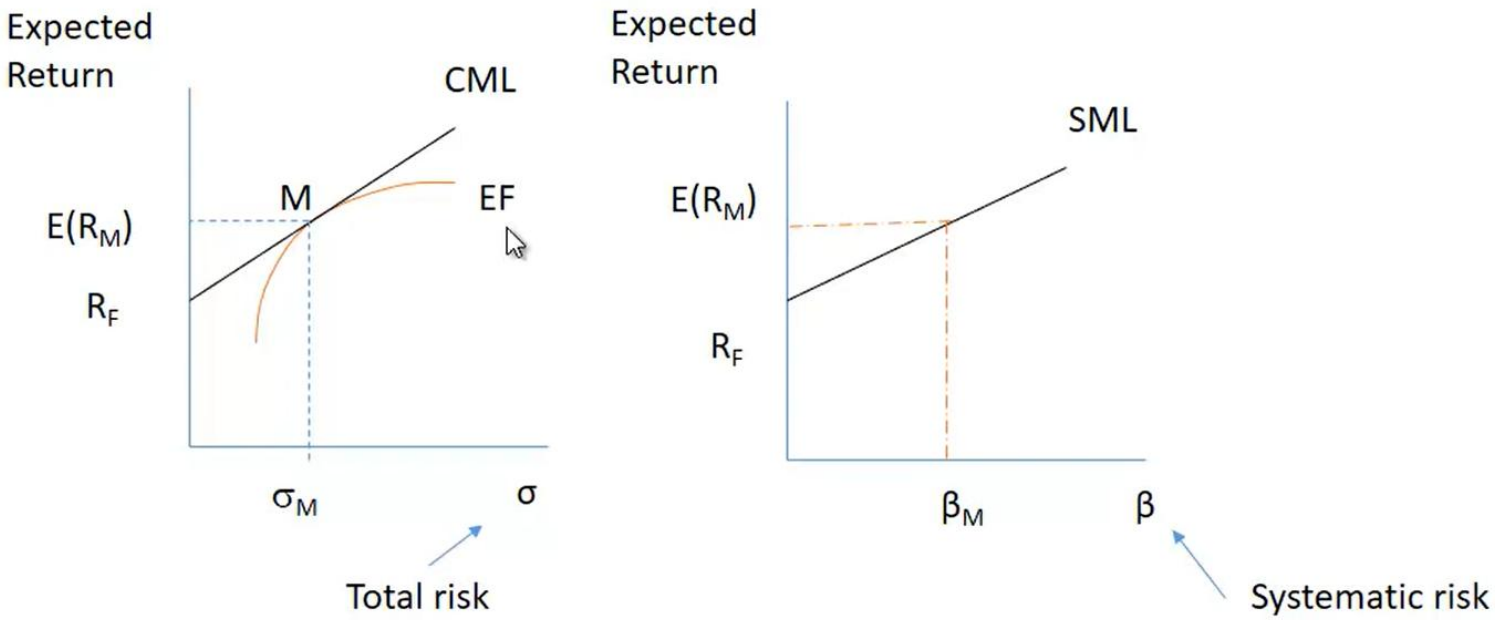
More aggressive investors will borrow at the risk free rate and use the money to buy more of the market portfolio

Quick Overview

- **What rule is imposed in Markowitz (1952) paper?**
  - expected return – variance of return or mean variance model
- **What is implied as the main objective of the Markowitz (1952) paper?**
  - To illustrate geometric relation between beliefs and choice of portfolio, according to the rules of "expected return – variance of return" within 2 dimensional cartesian
- **Mention geometric figures that used by Markowitz (1952)?**
  - Attainable set of E-V combination portfolio
  - Isomean line
  - Isovariance curve
  - Efficient portfolio/efficient frontier
- **What are the main assumptions of Sharpe (1964) paper?**
  - common pure of interest rate, that investor could lend and borrow at the same rate of interest
  - homogeneity of expectation
- **What are the main similiarity and difference between Capital market Line (CML) and Security Market Lina (SML)?**
  - Both are showing tradeoff between risk and return
  - CML shows tradeoff between risk and return for portfolio that consist of Rf and market (diversified) portfolio. CML is tangent to the efficient frontier
  - SML shows tradeof between systematic risk and return for individual asset or portfolio.
- **Which one of them is the graphical depiction of Capital Asset Pricing Model (CAPM?** The SML

SML vs CML – the variables

- The Capital Market Line (CML) depict the relation between total risk (variance) and return.
- The Security Market Line (SML) depict the relation between systematic risk (beta) and return



CML vs SML: The slope

- The slope of CML is the the expected market premium scale to the standard deviation the market which is the Sharpe Ratio

$$E(R_p) = R_f + \sigma_P \left[\frac{E(R_M) - R_F}{\sigma_M}\right]$$

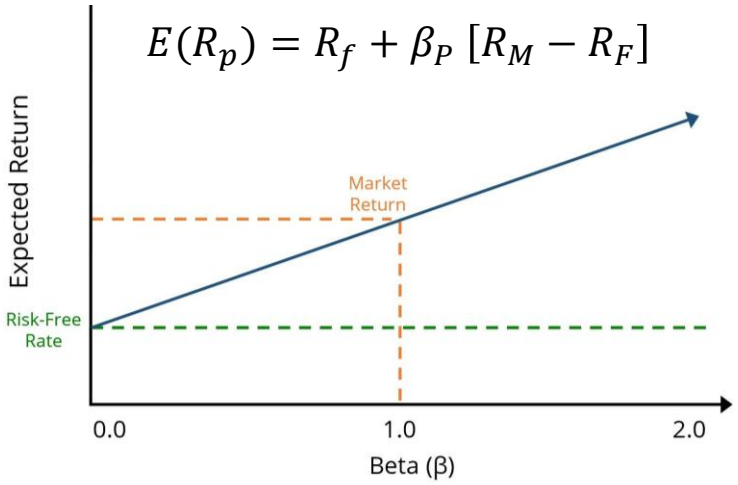
- The slope of SML is the market risk premium.

$$E(R_p) = R_f + \beta_P \left[R_M - R_F\right]$$

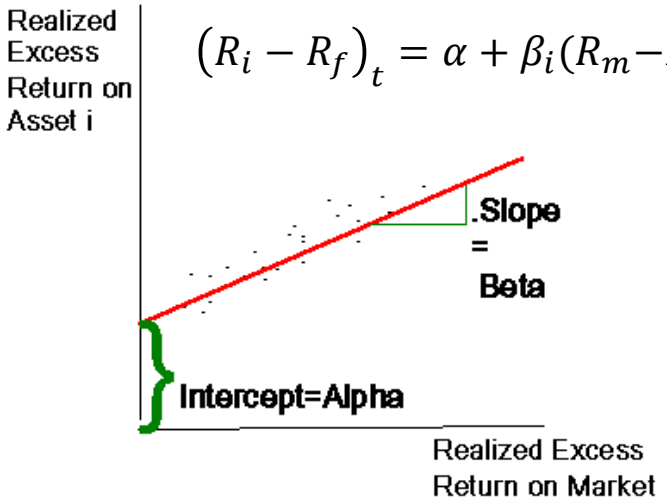
SML vs beta regression

- Do not mistaken between SML equation and beta regression. While both equation put R_P in the left hand side of the equation, the variable of interest of SML and CAPM are different:
- SML equation has β_P as it variable of interest, thus positioned at the X-axis in the SML graphical representation) and put
- Beta regression aims to estimate the parameter of $[R_M - R_F]$ thus the $[R_M - R_F]$ is placed it in the X-axis
- The intercept of SML equation is the risk free rate while the intercept in the beta regression is alpha.

$$E(R_p) = R_f + \beta_P\,[R_M - R_F]$$

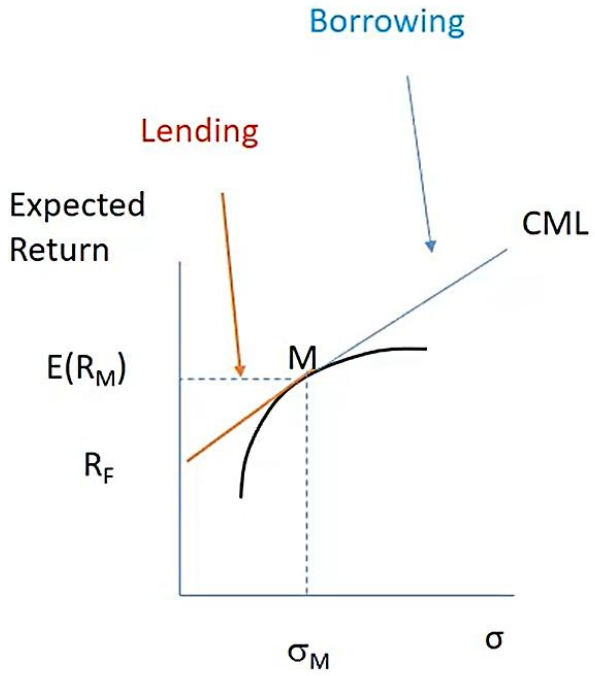


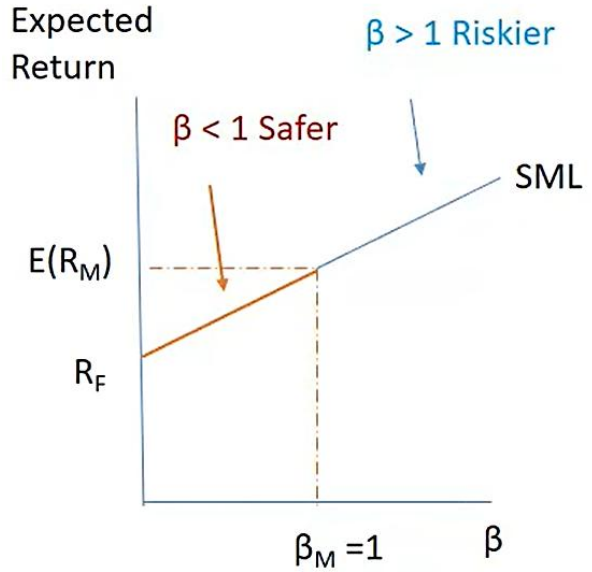
$$\left(R_i - R_f\right)_t = \alpha + \beta_i(R_m - R_f)_t + \varepsilon_{it}$$



Moving Along CML and SML

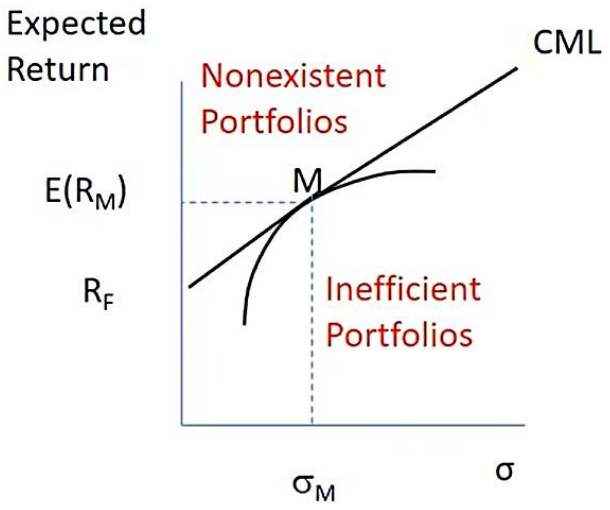
- The CML shows that the tangency point between CML and the efficient frontier is the market portfolio (M). The portfolio on the left of M means that investor in lending position/invest in the risk free and/or risky asset. The portfolio on the right of M means that investor borrow at the risk free and invest more in risky asset
- In the SML, investment on the left hand side $\beta_M = 1$ (the beta of the market) considered safer investments while on the right hand side considered as the riskier investments

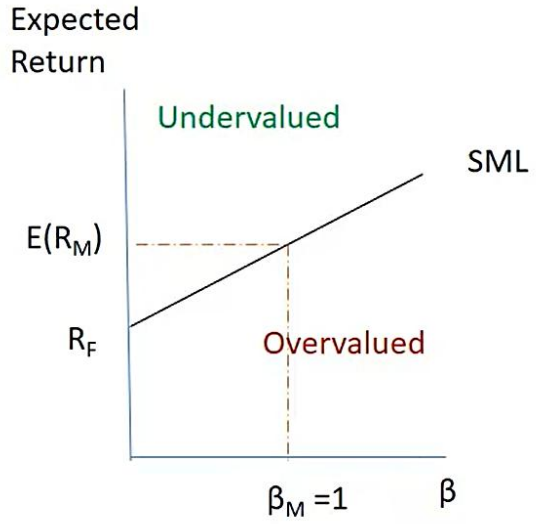




Points not on the CML and SML

- Point above the CML considered as non-existent portfolio since no issuer would issue such low-risk high-return investment.
- Point below the CML considered inefficient portfolio, since the expected return is lower given same risk or the risk is higher givent same expected return compare to the CML.
- Point above the SML considered as undervalued stock, give higher return given same amount of risk or lower risk given same return.
- Point below the SML considered as overvalued stock, give lower return given same amount of risk or higher risk given same return





Asset Pricing Model

Model	Econometric Equation of Common Risk Factors
CAPM (Sharpe, 1964; Lintner, 1965)	$R_{i,t} - R_{f,t} = \beta_i(R_{m,t} - R_f) + \epsilon_{i,t}$
3-Factors (Fama & French, 1992)	$R_{i,t} - R_{f,t} = \beta_i(R_{m,t} - R_f) + s_i SMB_t + h_i HML_t + \epsilon_{i,t}$
4-Factors (Carhart, 1997)	$R_{i,t} - R_{f,t} = \beta_i(R_{m,t} - R_f) + s_i SMB_t + h_i HML_t + w_i WML_t + \epsilon_{i,t}$
5-Factors (Fama & French, 2014)	$R_{i,t} - R_{f,t} = \beta_i(R_{m,t} - R_f) + s_i SMB_t + h_i HML_t + r_i RMW_t + c_i CMA_t + \epsilon_{i,t}$

- Rm - Rf → market risk premium i.e., market factor
- SMB → small minus big i.e., size factor
- HML → high (book to market ratio) to low i.e., growth opportunity factor
- WML → winner minus loser i.e., momentum factor
- RMW → robust (operating profitability) minus weak i.e., profitability factor
- CMA → conservative (portfolio) minus aggressive i.e., investment policy factor

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THANK YOU

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