Auto
Regressive
Conditional
Heteroscedasticity

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Decoding the ARCH accronym

Auto
Regressive — dependent on its lag
Conditional — different across time, conditional to something
Heteroscedasticity — volatility

ARCH : the volatility at some particular time is depend on (i.e. conditional to) its lag

ipsh.

Mean modeling and conditional variance modeling

- **AR, MA, ARMA/ARIMA:** Model the mean (average level) of the series.
- **ARCH, GARCH:** Model the conditional variance (volatility) of the series, **typically applied to the residuals of an ARMA model.**

Mean modeling — the case of stock return

Model	Equation	Explanation
$AR(p)$ Autoregressive	$Y_t = c + \sum_{i=1}^{p} \phi_i \, Y_{t-i} + \varepsilon_t$	The current return Y_t depends on its p past returns Y_{t-i}. ϕ_i are the parameters that quantify this dependence. ε_t is the error term (or shock) at time t.
$MA(q)$ Moving Average	$Y_t = \mu + \sum_{j=1}^{q} \theta_j \, \varepsilon_{t-j} + \varepsilon_t$	The current return Y_t depends on its q past error terms (i.e. past forecast inaccuracy) ε_{t-j}. θ_j are the parameters that quantify this dependence. μ is the overall mean return. ε_t is the error term (or shock) at time t.
$ARMA(p, q)$	$Y_t = c + \sum_{i=1}^{p} \phi_i \, Y_{t-i} + \sum_{j=1}^{q} \theta_j \, \varepsilon_{t-j} + \varepsilon_t$	Combines the AR and MA components. It models the return based on both past returns and past shocks.
$ARIMA(p, d, q)$		Adds the "Integrated" (I) component, d, which stands for the number of times the series must be differenced to become stationary. For stock returns, the series is usually already stationary, so $d = 0$ (meaning ARIMA is equivalent to ARMA).

Conditional volatility modeling — the case of stock return

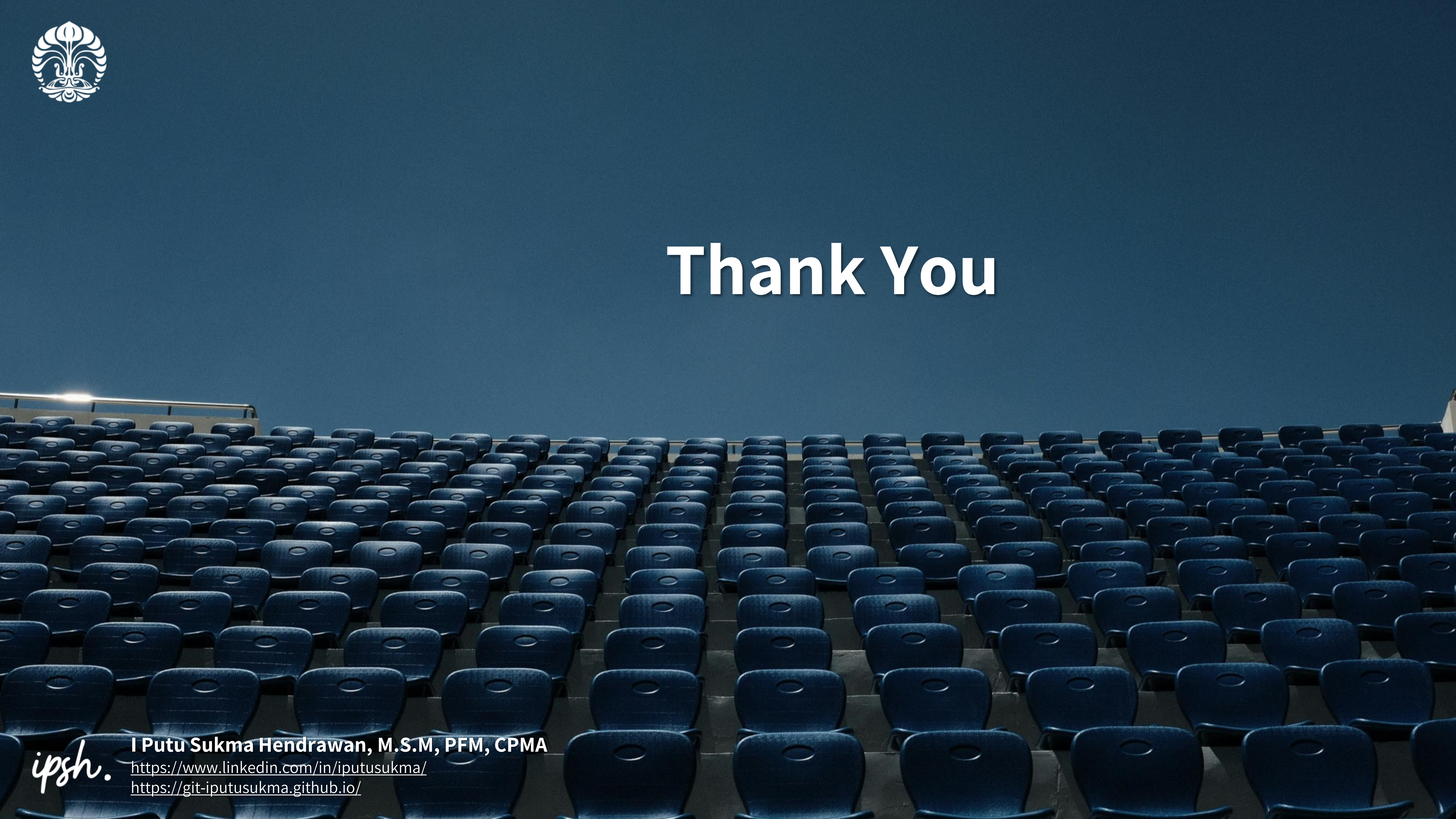
- These models focus on **the conditional variance of the error term** ε_t , denoted as σ_t^2 . In finance, this is known as volatility. They are crucial because stock returns exhibit volatility clustering (large changes tend to follow large changes, and small changes follow small changes).
- We assume the total stock return Y_t has a mean component μ_t — often modeled using mean modeling i.e. ARMA model — and error term ε_t. The equation would be $Y_t = \mu_t + \varepsilon_t$ where the error term ε_t has a conditional variance σ_t^2.
- The error term ε_t is the the unobservable random error or shock in a regression or time series at time t. In financial modeling, this often represents the unexpected component of the return.
- $\boldsymbol{\varepsilon_t = z_t\, \sigma_t}$ — **The error term $\boldsymbol{\varepsilon_t}$** is the product of two distinct components:
- **standardized random variable / standardized shock $\boldsymbol{z_t}$:**
    - A white noise process, it is the standardized and independent part of the shock
    - Independent and identically distributed with mean of zero $E(z_t) = 0$ and variance of one $Var(z_t) = 1$
    - z_t captures the randomness of the shock, while its unit variance ensures that all the time-varying characteristics of the shock are captured by σ_t
- **the time-varying standard deviation $\boldsymbol{\sigma_t}$**
    - The conditional standard deviation or volatility at time t. This is the key element that makes models like ARCH/GARCH different from standard models.
    - σ_t is a function of the past information (shocks and/or past volatilities, depending on whether the $ARCH(q)$ or $GARCH(p, q)$ model applied).
    - It acts as a scaling factor. Since σ_t changes over time, it means the magnitude of the errors ε_t is not constant, thus modeling heteroskedasticity (non-constant variance).

Conditional volatility modeling — the case of stock return

Model	Equation	Explanation
$ARCH(q)$ Conditional heterorscedasticity	$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \, \varepsilon_{t-i}^2$	The current variance σ_t^2 depends on a baseline α_0 and the q most recent squared residuals ε_{t-i}^2. α_i are parameters quantifying the shock's impact. An $ARCH(1)$ model suggests that if yesterday's stock return had a large shock (i.e., a large positive or large negative ε_{t-1} , resulting in a large ε_{t-1}^2 then today's market is likely to be more volatile (σ_t^2 is higher).
$GARCH(p,q)$ Generalized $ARCH$	$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \, \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$	The GARCH model extends ARCH by adding an autoregressive term for the variance itself, meaning volatility also depends on its own past values. The current conditional variance σ_t^2 depends on a long-run average baseline ω, the impact α_i of past shocks ε_{t-i}^2, and the persistence β_j of past volatility σ_{t-j}^2

ARCH Model

1. Fit the best possible model to your data (using mean modelling like AR and MA model)
2. Consider residuals of your model ε_t , any indication of different variance of ε_t across time would be a great candidate of ARCH model.



Thank You

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