A Simple Guide to

LMDI Decomposition Analysis

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IDA: Index Decomposition Analysis

LMDI: Logarithmic Mean Divisia Index

1. IDA is an analytical tool originated from energy studies in the late 1970s. It has since been extended to other areas including CO_2 emission analysis, environmental management, and sustainable use of natural resources.

2. Based on IDA, many specific decomposition methods can be developed, and LMDI is one of them.

3. LMDI has two versions:

   - Logarithmic Mean Divisia Index method I (LMDI-I)

   - Logarithmic Mean Divisia Index method II (LMDI-II)

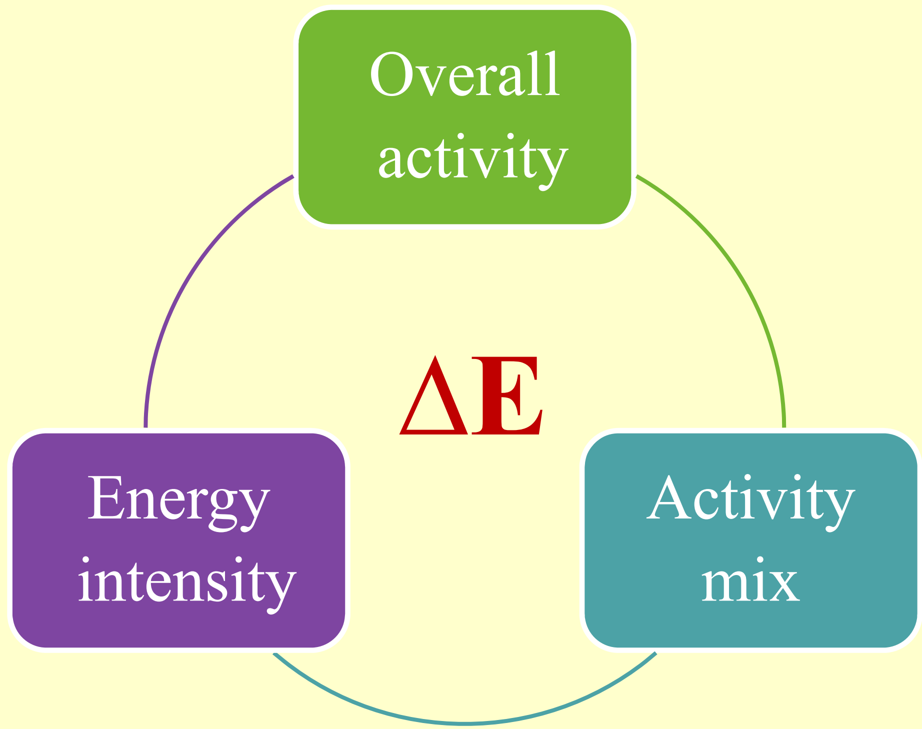
Decomposition of Energy Consumption Change

The conventional 3-factor case:

Overall activity
(Activity effect)

Activity mix
(Structure effect)

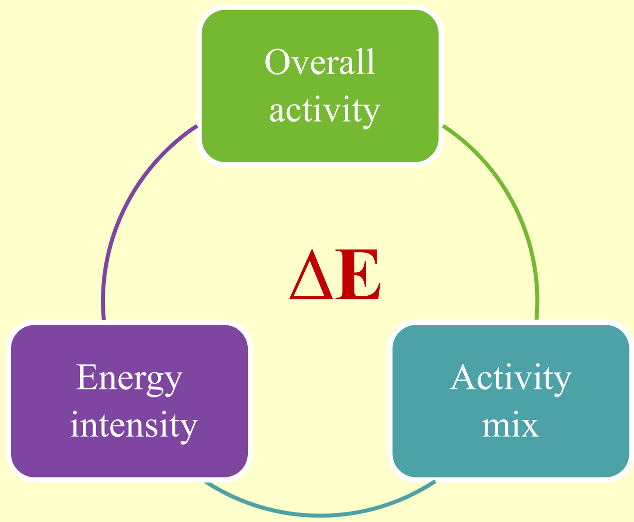
Sectoral energy intensity
(Intensity effect)



IDA Application: An Illustration

Observed aggregate energy consumption:

Consumption in 2010	140	Mtoe
Consumption in 2015	160	Mtoe
ΔE (Total)	20	Mtoe



Decomposition results:

ΔE (Overall activity)	50	Mtoe
ΔE (Activity structure)	-10	Mtoe
ΔE (Energy intensity)	- 20	Mtoe

Result interpretation:

Energy efficiency (EE) improvement contributed to a reduction of 20 Mtoe.

Activity structure change led to a reduction in energy consumption.

No EE improvement $\rightarrow$ Consumption would have been 180 Mtoe in 2015.

Index Decomposition Analysis (IDA)

Assume that V is an aggregate composed of n factors (x_1, …, x_n), i.e. $V = \sum_i V_i$ and $V_i = x_{1,i} x_{2,i} \cdots x_{n,i}$. Further assume that from period 0 to T the aggregate changes from V^0 to V^T. The objective is to derive the contributions of the n factors to the change in the aggregate which can be expressed as:

Additive form

$$\Delta V_{tot} = V^T - V^0 = \Delta V_{x_1} + \Delta V_{x_2} + \cdots + \Delta V_{x_n}$$

Multiplicative form

$$D_{tot} = V^T / V^0 = D_{x_1} D_{x_2} \cdots D_{x_n}$$

General Formulae of LMDI-I

$$\Delta V_{x_k} = \sum_i L(V_i^T, V_i^0) \ln\left(\frac{x_{k,i}^T}{x_{k,i}^0}\right)$$

$$D_{x_k} = \exp\left(\sum_i \frac{L(V_i^T, V_i^0)}{L(V^T, V^0)} \ln\left(\frac{x_{k,i}^T}{x_{k,i}^0}\right)\right)$$

where $L(a,b) = (a-b)/(\ln a - \ln b)$ is the logaritmic mean of a and b, and $L(a,a) = a$

Note: The simple guide focuses mainly on LMDI-I. LMDI-II is dealt with in the later part of this guide under the “Supplementary Material” section.

Decomposition of Energy Consumption Change

The IDA identify can be given by:

$$E = \sum_i E_i = \sum_i Q \frac{Q_i}{Q} \frac{E_i}{Q_i} = \sum_i Q S_i I_i$$

- E Total energy consumption (for all sectors)
- Q Overall activity level (for all sectors)
- E_i Energy consumption of sector i
- Q_i Activity level of sector i

- S_i Activity share of sector i
- I_i Energy intensity of sector i

Three Explanatory Effects

$$E^T - E^0 = \Delta E_{tot} = \Delta E_{act} + \Delta E_{str} + \Delta E_{int}$$

$$E^T / E^0 = D_{tot} = D_{act} D_{str} D_{int}$$

- **Activity effect**: The change in the aggregate associated with a change in the overall level of the activity.
- **Structure effect**: The change in the aggregate associated with a change in the mix of the activity by sub-category.
- **Intensity effect**: The change in the aggregate associated with changes in the sub-category energy intensities.

Formulae for LMDI-I

Additive

$$\Delta E_{act} = \sum_i w_i \ln\left(\frac{Q^T}{Q^0}\right)$$

$$\Delta E_{str} = \sum_i w_i \ln\left(\frac{S_i^T}{S_i^0}\right)$$

$$\Delta E_{int} = \sum_i w_i \ln\left(\frac{I_i^T}{I_i^0}\right)$$

$$w_i = \frac{E_i^T - E_i^0}{\ln E_i^T - \ln E_i^0}$$

Multiplicative

$$D_{act} = \exp\left(\sum_i \tilde{w}_i \ln\left(\frac{Q^T}{Q^0}\right)\right)$$

$$D_{str} = \exp\left(\sum_i \tilde{w}_i \ln\left(\frac{S_i^T}{S_i^0}\right)\right)$$

$$D_{int} = \exp\left(\sum_i \tilde{w}_i \ln\left(\frac{I_i^T}{I_i^0}\right)\right)$$

$$\tilde{w}_i = \frac{(E_i^T - E_i^0)/(\ln E_i^T - \ln E_i^0)}{(E^T - E^0)/(\ln E^T - \ln E^0)}$$

Example 1

Sector	Year *0*				Year *T*			
i	E_0	Q_0	S_0	I_0	E_T	Q_T	S_T	I_T
1	60	20	0.2	3.0	120	48	0.4	2.5
2	40	80	0.8	0.5	27	72	0.6	0.375
Total	100	100	1.0	1.0	147	120	1.0	1.225

Decomposition Results

ΔE_{tot}	ΔE_{act}	ΔE_{str}	ΔE_{int}
47	21.812	50.485	-25.297

D_{tot}	D_{act}	D_{str}	D_{int}
1.47	1.196	1.512	0.813

Calculation of the Intensity Effect (Additive)

$$\Delta E_{int} = \sum_i w_i \ln\left(\frac{I_i^T}{I_i^0}\right) \qquad\qquad w_i = \frac{E_i^T - E_i^0}{\ln E_i^T - \ln E_i^0}$$

The energy intensity effect

$= [(120\text{-}60)/\ln(120/60)] \times \ln(2.5/3.0)$

$\quad + [(27\text{-}40)/\ln(27/40)] \times \ln(0.375/0.5)$

$= 86.5617 \times (-0.1823) + 33.0753 \times (-0.2877)$

$= -15.782 - 9.515$

$= -25.297$

Example 2: Decomposition of manufacturing electricity consumption in Singapore

Source: Ang, B.W. , Zhang, F.Q., Choi, K-H (1998). Factorizing changes in energy and environmental indicators through decomposition. *Energy* 23, 489-495.

Sector classification, electricity consumption and production in Singapore industry

s/no	SIC Code	Sector	Electricity consumption		Production	
			1985	1990	1985	1990
1	371	Iron & Steel	309.65	294.85	13.15	16.37
2	372	No-ferrous metals	17.70	17.90	3.11	3.30
3	364	Cement	92.17	76.99	6.45	5.05
4	365	Cement & concrete products	17.45	10.52	14.47	12.20
5	361/362	Pottery & Glass products	6.75	29.35	0.78	0.59
6	363	Structural clay products	22.26	14.97	2.14	1.84
7	369	Non-metallic minerals products	12.84	19.57	1.61	3.73
8	353/354	Petroleum refining	879.43	1262.01	168.73	232.60
9	351	Industrial chemicals	564.91	803.75	38.97	60.94
10	357	Plastic products	143.39	237.29	16.83	21.46
11	352	Paints & chemical products	73.92	91.34	50.11	85.51
12	356	Rubber products	9.48	18.56	3.19	2.61
13	381	Fabricated metal products	184.02	306.11	75.19	114.21
14	382	Machinery	147.72	212.87	74.47	115.74
15	383/384	Electrical & electronic products	634.71	1349.88	243.72	611.09
16	385	Transport equipment	190.65	258.87	93.19	179.23
17	311/312	Food	131.97	173.69	31.68	38.56
18	313	Beverage	23.66	23.70	14.39	21.29
19	314	Tobacco products	5.88	10.90	9.47	23.96
20	321	Textiles	48.71	72.71	5.07	6.17
21	322	Apparel	51.22	66.85	38.41	53.18
22	323	Leather	1.04	1.02	1.12	0.98
23	324	Footwear	2.81	2.38	1.06	1.57
24	331	Timber	39.22	30.46	7.34	4.91
25	332	Furniture	31.18	30.38	11.05	12.10
26	341	Paper & paper products	43.99	83.87	7.51	13.06
27	342	Printing & publishing	61.76	93.04	49.31	76.60
28	355/386/390	Others	65.32	139.63	26.93	44.51
Total			3813.8	5733.5	1009.5	1763.4

Note: Electricity in GWh and production in index with base year 1982. Data Source: Department of Statistics (annually), *Report on Census of Industrial Production*, Singapore. Department of Statistics (annually), *Yearbook of Statistics, Singapore*, Singapore.

Example 2: Additive LMDI decomposition of manufacturing electricity consumption in Singapore

Manufacturing electricity consumption:

3,814 GWh in 1985

5,734 GWh in 1990

The change of 1920 GWh from 1985 to 1990 is to be decomposed

Manufacturing has 28 sectors

ΔE_{tot}	ΔE_{act}	ΔE_{str}	ΔE_{int}
1920	2612	-452	-240

Proof of Perfect in Decomposition (Additive)

$$\Delta E_{tot} = E^T - E^0 = \Delta E_{act} + \Delta E_{str} + \Delta E_{int}$$

$$= \sum_i \frac{E_i^T - E_i^0}{\ln E_i^T - \ln E_i^0} \ln\left(\frac{Q^T}{Q^0}\right) + \sum_i \frac{E_i^T - E_i^0}{\ln E_i^T - \ln E_i^0} \ln\left(\frac{S_i^T}{S_i^0}\right) + \sum_i \frac{E_i^T - E_i^0}{\ln E_i^T - \ln E_i^0} \ln\left(\frac{I_i^T}{I_i^0}\right)$$

$$= \sum_i \frac{E_i^T - E_i^0}{\ln E_i^T - \ln E_i^0} \left[\ln\left(\frac{Q^T}{Q^0}\right) + \ln\left(\frac{S_i^T}{S_i^0}\right) + \ln\left(\frac{I_i^T}{I_i^0}\right) \right]$$

$$= \sum_i \frac{E_i^T - E_i^0}{\ln E_i^T - \ln E_i^0} \ln\left(\frac{Q^T S_i^T I_i^T}{Q^0 S_i^0 I_i^0}\right)$$

$$= \sum_i \frac{E_i^T - E_i^0}{\ln E_i^T - \ln E_i^0} \ln\left(\frac{E_i^T}{E_i^0}\right)$$

$$= \sum_i (E_i^T - E_i^0) = \Delta E_{tot}$$

Proof of Perfect in Decomposition (Multiplicative)

$$D_{tot} = E^T / E^0 = D_{act} D_{str} D_{int}$$

$$= \exp\left(\sum_i \frac{(E_i^T - E_i^0)/(\ln E_i^T - \ln E_i^0)}{(E^T - E^0)/(\ln E^T - \ln E^0)} \ln\left(\frac{Q^T}{Q^0}\right)\right) + \exp\left(\sum_i \frac{(E_i^T - E_i^0)/(\ln E_i^T - \ln E_i^0)}{(E^T - E^0)/(\ln E^T - \ln E^0)} \ln\left(\frac{S_i^T}{S_i^0}\right)\right) +$$

$$\exp\left(\sum_i \frac{(E_i^T - E_i^0)/(\ln E_i^T - \ln E_i^0)}{(E^T - E^0)/(\ln E^T - \ln E^0)} \ln\left(\frac{I_i^T}{I_i^0}\right)\right)$$

$$= \exp\left(\sum_i \frac{(E_i^T - E_i^0)/(\ln E_i^T - \ln E_i^0)}{(E^T - E^0)/(\ln E^T - \ln E^0)} \left[\ln\left(\frac{Q^T}{Q^0}\right) + \ln\left(\frac{S_i^T}{S_i^0}\right) + \ln\left(\frac{I_i^T}{I_i^0}\right)\right]\right)$$

$$= \exp\left(\sum_i \frac{(E_i^T - E_i^0)/(\ln E_i^T - \ln E_i^0)}{(E^T - E^0)/(\ln E^T - \ln E^0)} \ln\left(\frac{E_i^T}{E_i^0}\right)\right)$$

$$= \exp\left(\sum_i \frac{E_i^T - E_i^0}{(E^T - E^0)/(\ln E^T - \ln E^0)} \right)$$

$$= \exp\left(\ln E^T - \ln E^0\right) = E^T / E^0$$

Energy-related Emission Studies

The conventional 5-factor case:

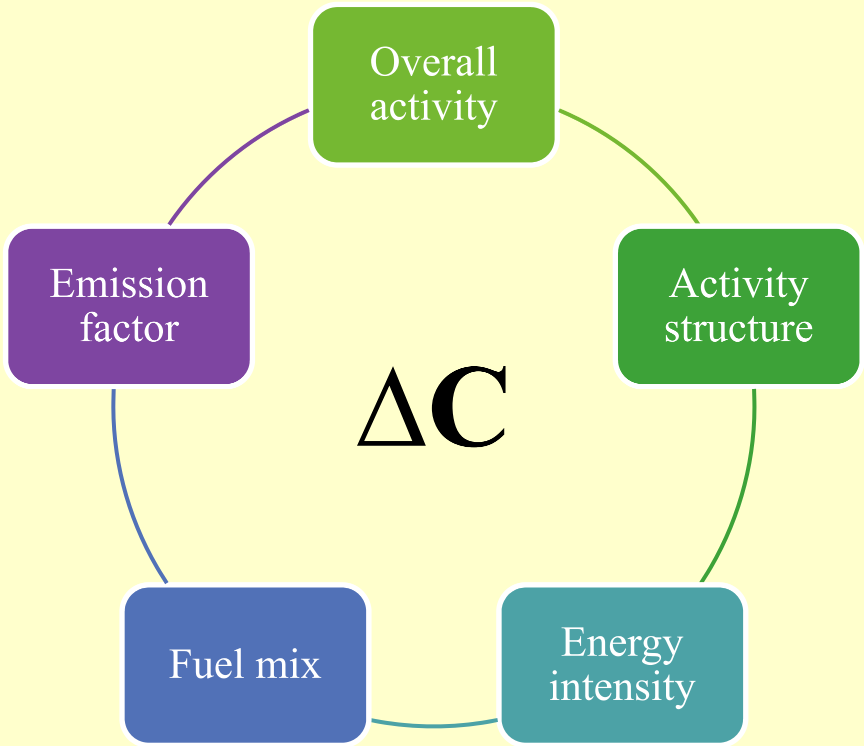
Overall activity
(Activity effect)

Activity mix
(Structure effect)

Sectoral energy intensity
(Intensity effect)

Sectoral energy mix
(Energy-mix effect)

Emission factors
(Emission-factor effect)



Five Explanatory Effects

The IDA identify can be given by:

$$C = \sum_{i,j} C_{ij} = \sum_{i,j} Q \frac{Q_i}{Q} \frac{E_i}{Q_i} \frac{E_{ij}}{E_i} \frac{C_{ij}}{E_{ij}} = \sum_{i,j} QS_i I_i M_{ij} U_{ij}$$

$$\Delta C_{tot} = C^T - C^0 = \Delta C_{act} + \Delta C_{str} + \Delta C_{int} + \Delta C_{mix} + \Delta C_{emf}$$

$$D_{tot} = C^T / C^0 = D_{act} D_{str} D_{int} D_{mix} D_{emf}$$

Formulae for LMDI-I: Additive and Multiplicative

$$\Delta C_{act} = \sum_{ij} w_{ij} \ln\left(\frac{Q^T}{Q^0}\right) \qquad D_{act} = \exp\left(\sum_{ij} \tilde{w}_{ij} \ln\left(\frac{Q^T}{Q^0}\right)\right)$$

$$\Delta C_{str} = \sum_{ij} w_{ij} \ln\left(\frac{S_i^T}{S_i^0}\right) \qquad D_{str} = \exp\left(\sum_{ij} \tilde{w}_{ij} \ln\left(\frac{S_i^T}{S_i^0}\right)\right)$$

$$\Delta C_{int} = \sum_{ij} w_{ij} \ln\left(\frac{I_i^T}{I_i^0}\right) \qquad D_{int} = \exp\left(\sum_{ij} \tilde{w}_{ij} \ln\left(\frac{I_i^T}{I_i^0}\right)\right)$$

$$\Delta C_{mix} = \sum_{ij} w_{ij} \ln\left(\frac{M_{ij}^T}{M_{ij}^0}\right) \qquad D_{mix} = \exp\left(\sum_{ij} \tilde{w}_{ij} \ln\left(\frac{M_{ij}^T}{M_{ij}^0}\right)\right)$$

$$\Delta C_{emf} = \sum_{ij} w_{ij} \ln\left(\frac{U_{ij}^T}{U_{ij}^0}\right) \qquad D_{emf} = \exp\left(\sum_{ij} \tilde{w}_{ij} \ln\left(\frac{U_{ij}^T}{U_{ij}^0}\right)\right)$$

$$w_{ij} = \frac{C_{ij}^T - C_{ij}^0}{\ln C_{ij}^T - \ln C_{ij}^0} \qquad \tilde{w}_{ij} = \frac{(C_{ij}^T - C_{ij}^0)/(\ln C_{ij}^T - \ln C_{ij}^0)}{(C^T - C^0)/(\ln C^T - \ln C^0)}$$

Example 3

	Year 0						
		C	*U*	*E*	*Q*	*S*	*I*
Sector 1	Coal	63	4.2	15	10	0.2	3.0
	Oil	46.5	3.1	15			
Sector 2	Coal	42	4.2	10	40	0.8	0.5
	Oil	31	3.1	10			
Industry		182.5		50	50		1.0

	Year *T*						
		C	*U*	*E*	*Q*	*S*	*I*
Sector 1	Coal	78	3.9	20	40	0.5	2.0
	Oil	180	3.0	60			
Sector 2	Coal	23.4	3.9	6	40	0.5	0.4
	Oil	30	3.0	10			
Industry		311.4		96	80		1.2

Decomposition Results

ΔC_{tot}	ΔC_{act}	ΔC_{str}	ΔC_{int}	ΔC_{mix}	ΔC_{emf}
128.9	108.6	125.5	-82.4	-11.0	-11.8

D_{tot}	D_{act}	D_{str}	D_{int}	D_{mix}	D_{emf}
1.706	1.569	1.682	0.711	0.955	0.952

Calculation of the Intensity Effect (Additive)

$$\Delta C_{int} = \sum_{ij} w_{ij} \ln\left(\frac{I_i^T}{I_i^0}\right) \qquad\qquad w_{ij} = \frac{C_{ij}^T - C_{ij}^0}{\ln C_{ij}^T - \ln C_{ij}^0}$$

The energy intensity effect

= [(78-63)/ln(78/63)] × ln(2/3) + [(180-46.5)/ln(180/46.5)] × ln(2/3)

+ [(23.4-42)/ln(23.4/42)] × ln(0.4/0.5) + [(30-31)/ln(30/31)] × ln(0.4/0.5)

= 15/0.21357 × (-0.40547) + 133.5/1.35350 × (-0.40547)

- 18.6/(-0.58493) × (-0.22314) - 1/(-0.03279) × (-0.22314)

= -28.478 - 39.993 - 7.096 - 6.805

= -82.4

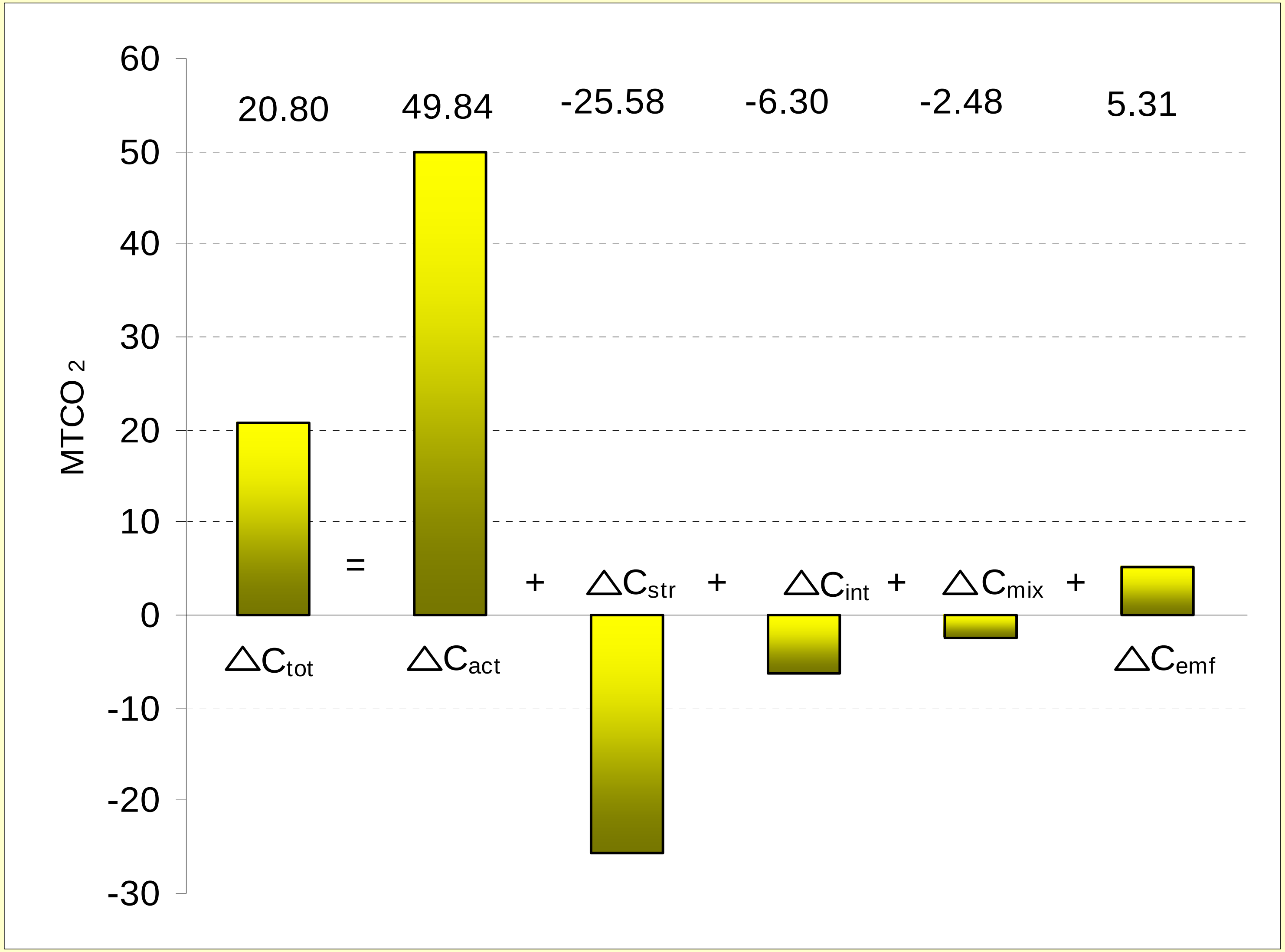
Example 4: Decomposition of CO_2 emissions of Canadian industry, 1990-2000, using LMDI

- Five-factor decomposition:

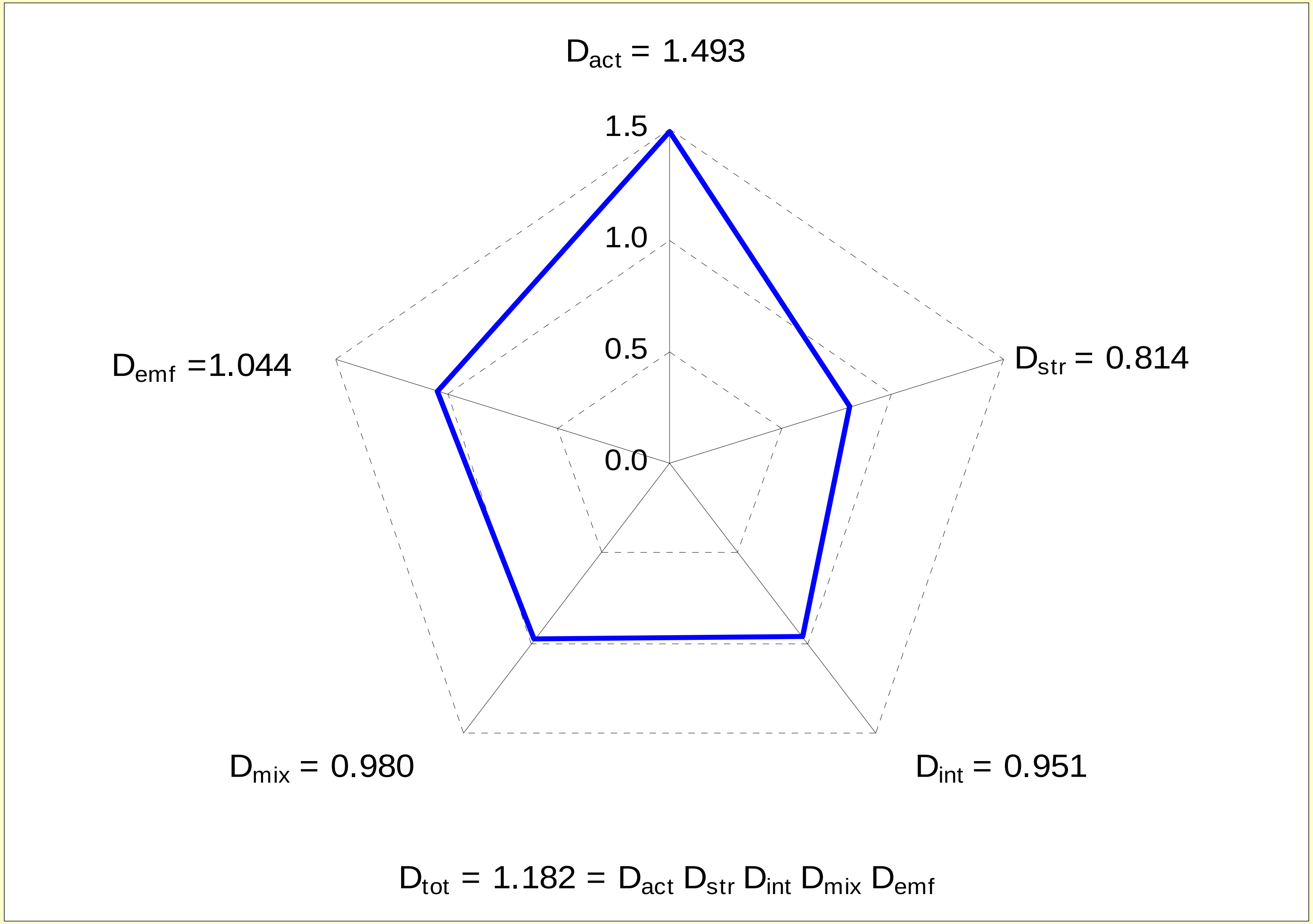
Overall industry activity	(Activity effect)
Industry activity mix	(Structure effect)
Sectoral energy intensity	(Intensity effect)
Sectoral energy mix	(Energy-mix effect)
CO_2 emission factors	(Emission-factor effect)

- 23 industrial sectors and 14 energy sources
- CO_2 emissions increased from 114.31 to 135.11 $MtCO_2$, or by 20.80 $MtCO_2$, from 1990 to 2000
- Source: Ang, B.W. (2005). The LMDI approach to decomposition analysis: A practical guide. *Energy Policy* 33, 867-871.

Results of Additive Decomposition



Results of Multiplicative Decomposition



Desirable Properties of LMDI

1. It passes a number of basic tests for a good index number.

2. Decomposition is perfect, i.e. no residual.

3. Easy to use as formulae take the same form irrespective of the number of explanatory factors.

4. Multiplicative and additive decomposition results are linked by a simple formula.

5. Multiplicative LMDI possesses the additive property in the log form.

Desirable Properties of LMDI

- Point 4: $$\frac{\Delta V_{tot}}{\ln D_{tot}} = \frac{\Delta V_{str}}{\ln D_{str}} = \frac{\Delta V_{int}}{\ln D_{int}}$$

- Point 5: $$\ln(D_{tot}) = \ln(D_{pdn}) + \ln(D_{str}) + \ln(D_{int})$$

Note: The above can be easily verified using the results of Example 1 and Example 3. Point 4 has practical significance, i.e. the choice between multiplicative form and additive form is inconsequential since the results of one form can be directly converted to the other.

Decomposing Aggregate Energy Intensity

$$I = \frac{E}{Q} = \sum_i \frac{E_i}{Q} = \sum_i \frac{Q_i}{Q}\frac{E_i}{Q_i} = \sum_i S_i I_i$$

$$\Delta I_{tot} = I^T - I^0 = \sum_i S_i^T I_i^T - \sum_i S_i^0 I_i^0 = \Delta I_{str} + \Delta I_{int}$$

$$D_{tot} = I^T / I^0 = \sum_i S_i^T I_i^T / \sum_i S_i^0 I_i^0 = D_{str} D_{int}$$

Decomposition of Aggregate Energy Intensity

LMDI-I (Additive)

$$\Delta I_{str} = \sum_i L\left(\frac{E_i^T}{Q^T}, \frac{E_i^0}{Q^0}\right) \ln\left(\frac{S_i^T}{S_i^0}\right) \qquad \Delta I_{int} = \sum_i L\left(\frac{E_i^T}{Q^T}, \frac{E_i^0}{Q^0}\right) \ln\left(\frac{I_i^T}{I_i^0}\right)$$

LMDI-I (Multiplicative)

$$D_{str} = \exp\left\{\sum_i \frac{L\left(\frac{E_i^T}{Q^T}, \frac{E_i^0}{Q^0}\right)}{L(I^T, I^0)} \ln\left(\frac{S_i^T}{S_i^0}\right)\right\} \qquad D_{int} = \exp\left\{\sum_i \frac{L\left(\frac{E_i^T}{Q^T}, \frac{E_i^0}{Q^0}\right)}{L(I^T, I^0)} \ln\left(\frac{I_i^T}{I_i^0}\right)\right\}$$

Note: See Choi, K.-H. and Ang, B.W. (2003). Decomposition of aggregate energy intensity changes in two measures: ratio and difference. *Energy Economics* 25, 615-624. For a numerical example, see Ang, B.W. (2004). Decomposition analysis for policymaking in energy: Which is the preferred method?. *Energy Policy* 32, 1131-1139.

More about LMDI

1. LMDI is a weighted sum of relative changes (growth rates) – thus it uses the Divisia index concept of Divisia (1925) .
2. The Divisia index approach was introduced to IDA by Boyd et al. (1987).
3. The logarithmic mean weight function was introduced to IDA by Ang and Choi (1997), giving the first of a family of LMDI decomposition methods.
4. The term "Logarithmic Mean Divisia Index" (LMDI) was coined in Ang, Zhang and Choi (1998). LMDI-I and LMDI-II were formalised in Ang and Liu (2001).
5. The term "Index Decomposition Analysis" (IDA) was first used in Ang and Zhang (2000) to differentiate the technique from "Structural Decomposition Analysis" (SDA).
6. The study by Ang (2004) consolidates IDA methodology and Ang (2005) provides a practical guide to LMDI.
7. Other desirable properties of LMDI (more specifically LMDI-I) are given in Ang, Huang and Mu (2009).
8. Formulae for eight basic LMDI models can be found in Ang (2015).

Useful References

1. Divisia, F. (1925). L'indice monétaire et la théorie de la monnaie. *Revue d'Economie Politique* 39, 980-1008.
2. Boyd, G.A., McDonald, J.F., Ross, M., Hanson, D.A. (1987). Separating the changing composition of US manufacturing production from energy efficiency improvement: a Divisia index approach. *Energy Journal* 8(2), 77-96.
3. Ang, B.W., Choi, K.H. (1997). Decomposition of aggregate energy and gas emission intensities for industry: A refined Divisia index method. *Energy Journal* 18(3), 59-73.
4. Ang, B.W., Zhang, F.Q., Choi, K.H. (1998). Factorizing changes in energy and environmental indicators through decomposition. *Energy* 23, 489-495.
5. Ang, B.W., Zhang, F.Q. (2000). A survey of index decomposition analysis in energy and environmental studies. *Energy* 25, 1149-1176.
6. Ang, B.W., Liu, F.L. (2001). A new energy decomposition method: Perfect in decomposition and consistent in aggregation. *Energy* 26, 537-547.
7. Ang, B.W., Liu, F.L., Chew, E.P. (2003). Perfect decomposition techniques in energy and environmental analysis. *Energy Policy* 31, 1561-1566.
8. Liu, F.L., Ang, B.W. (2003). Eight methods for decomposing the aggregate energy intensity for industry. *Applied Energy* 76, 15-23.

8. Choi, K.-H. and Ang, B.W. (2003). Decomposition no aggregate energy intensity changes in two measures: ratio and difference. *Energy Economics* 25, 615-624.
9. Ang, B.W. (2004). Decomposition analysis for policymaking in energy: Which is the preferred method?. *Energy Policy* 32, 1131-1139.
10. Ang, B.W. (2005). The LMDI approach to decomposition analysis: A practical guide. *Energy Policy* 33, 867-871.
11. Ang, B.W., Liu, N. (2007a). Handling zero values in the logarithmic mean Divisia index decomposition approach. *Energy Policy* 35, 238-246.
12. Ang, B.W., Liu, N. (2007b). Negative-value problems of the logarithmic mean Divisia index decomposition approach. *Energy Policy* 35, 739-742.
13. Ang, B.W., Liu, N. (2007c). Energy decomposition analysis: IEA method model versus other methods. *Energy Policy* 35, 71426-1432.
14. Ang, B.W., Huang, H.C., Mu, A.R. (2009). Properties and linkages of some index decomposition analysis methods. *Energy Policy* 37, 4624-4632.
15. Ang, B.W., Mu, A.R., Zhou, P. (2010). Accounting frameworks for tracking energy efficiency trends. *Energy Economics* 32, 1200-1219.
16. Ang, B.W., X.Y. Xu (2013). Tracking industrial energy efficiency trends using index decomposition analysis. *Energy Economics* 40, 1014-1021.
17. Ang, B.W. (2015). LMDI decomposition approach: A guide for implementation. *Energy Policy* 86, 233-238.

IDA and LMDI

Supplementary Material

(References cited are given in Slide #31 and Slide #32)

1. What is a logarithmic mean?
2. Why use the logarithmic mean in LMDI?
3. Where can I find more about the derivation of LMDI formulae?
4. How to handle zero values in LMDI?
5. How to handle negative values in LMDI?
6. How chaining and non-chaining decomposition differ?
7. Should I use the additive or multiplicative form of decomposition?
8. How to use IDA/LMDI to track economy-wide energy efficiency trends
9. How to compute and present energy efficiency index and energy saving (in absolute terms) in a consistent manner using IDA?
10. How to include both monetary and physical activity indicators in applying LMDI?

11. What are the IDA decomposition methods that can be found in the literature and how they are classified?
12. How can I find the formulae of the commonly used Laspeyres-based IDA methods?
13. How can I find the formulae of the commonly used Divisia-based IDA methods?
14. What are the properties and linkages of various IDA decomposition methods?
15. How are the results obtained differ when applying different IDA methods?
16. How LMDI-II differs from LMDI-I?
17. Where can I find the formulae for LMDI-II?
18. Should I use LMDI-II instead of LMDI-I?
19. IDA and LMDI: A timeline
20. Number of archival journal articles (in English) on IDA by year and the IDA method used

1. What is a logarithmic mean?

The logarithmic mean of two positive numbers *a* and *b* is given by

$$L(a,b) = (a-b)/(\ln a - \ln b) \text{ and } L(a,a) = a$$

It is smaller than the arithmetic mean but larger than the geometric mean (except when the two numbers are the same), i.e.

$$\sqrt{a \cdot b} \le L(a,b) \le \frac{a+b}{2}$$

Example: Assume $a = 2$ and $b = 4$.

Geometric mean = 2.828

Logarithmic mean = $L(2,4) = (2\text{-}4)/(ln\ 2 - ln\ 4) = 2.885$

Arithmetic mean = 3

2. Why use the logarithmic mean in LMDI?

Assume that the energy consumption of a sector increased from 10 units in year 0 to 20 units in year *T*.

The relative difference calculated in the ordinary percentage depends on which of the two years is used as the point of comparison, i.e. the consumption in year *T* is 100% higher than in year 0, or the consumption in year 0 is 50% lower than in year *T,* which is asymmetric.

In the case of the (natural) log change, the relative changes are, respectively, given by $\ln(20/10) = 0.693$ and $\ln(10/20) = -0.693$. The changes are symmetric.

3. Where can I find more about the derivation of LMDI formulae?

The derivation can be found in several papers. The study by Ang and Choi (1997) is the earliest and it deals with the decomposition of the aggregate energy intensity in the multiplicative form. Decomposing the aggregate energy intensity in both the multiplicative and additive forms can be found in Ang and Zhang (2000) and Choi and Ang (2003).

As to the decomposition of aggregate energy consumption or carbon emissions, derivation in the multiplicative form is given in Ang and Liu (2001) while that in the additive form in Ang et al. (2009).

4. How to handle zero values in LMDI?

Two strategies: "Small Value" (SV) strategy and "Analytical Limit" (AL) strategy.

- SV strategy: Replace all zero values in the data set by δ, where $\delta < 10^{-20}$ ($\delta = 10^{-100}$ gives almost perfect approximation).

- AL strategy: Superior to the SV strategy on theoretical ground.

Refer to Ang and Liu (2007a) for further details. Other useful references are Ang and Choi (1997), and Ang, Zhang and Choi (1998).

5. How to handle negative values in LMDI?

- Such cases rarely occur in IDA.
- Easy to deal with if one is familiar with IDA/LMDI. Refer to Ang and Liu (2007b) for the full details.
- Negative values may occur in the dataset in Structural Decomposition Analysis (SDA). LMDI has also been used in SDA. See: Su, B. and Ang, B.W. (2012). Structural decomposition analysis applied to energy and emissions: Some methodological developments. *Energy Economics* 34, 177-188.

6. How chaining and non-chaining decomposition differ?

Chaining and non-chaining are two different indexing procedures in IDA. When an analysis is over a period, say from year 0 to year T, with yearly data, decomposition can be conducted as follows:

Non-chaining analysis: Based only on the data for the starting year 0 and the ending year T without using the data in the intervening years.

Chaining analysis: Using the data for every two consecutive years, i.e. years 0 and 1, 1 and 2, and so on till T-1 and T. A total of T sets of decomposition results are obtained which can then be "chained" to give the results for the whole time period.

In chaining analysis, for each effect studied, yearly additive decomposition results are chained additively while multiplicative decomposition results are chained multiplicatively.

In IDA the chaining versus non-chaining decomposition issue was first studied in:

Ang, B.W. (1994). Decomposition of industrial energy consumption The energy intensity approach. *Energy Economics* 16, 163-174.

The study refers to the two approaches as "time-series decomposition" and "periodwise decomposition" respectively.

Numerical examples can be found in:

Ang, B.W., Liu, N (2007). Energy decomposition analysis: IEA model versus other methods. *Energy Policy* 35*, 1426-1432*

and in Ang, Mu and Zhou (2010).

Unless for some specific reasons, the results of chaining analysis are generally preferred to those of non-chaining analysis.

7. Should I use the additive or multiplicative form of decomposition?

The choice is largely a matter of personal preference. In result presentation, the additive form is simpler and they results can be more easily understood. When yearly decomposition results given in a time-series are to be presented, the multiplicative form may be used as they tend to be concise. A discussion is given an Ang (2015)

8. How to use IDA/LMDI to track economy-wide energy efficiency trends?

Refer to Ang, Mu and Zhou (2010), which provides a summary of the accounting frameworks used by national agencies and international organizations, numerical examples, and recommendations.

9. How to derive and present energy efficiency index and energy saving (in absolute terms) in a consistent manner using IDA?

LMDI can handle this since the results of multiplicative and additive decomposition are linked by a simple formula. As the relationship is unique, the results for one form can be readily converted to the other form. See Ang, Mu and Zhou (2010) for further details.

10. How to include both monetary and physical activity indicators in applying LMDI?

Refer to Ang and Xu (2013) for tracking industrial energy efficiency trends using both types of activity indicators at the same time.

11. What are the IDA decomposition methods that can be found in the literature and how they are classified?

See Ang (2004) and the next three slides.

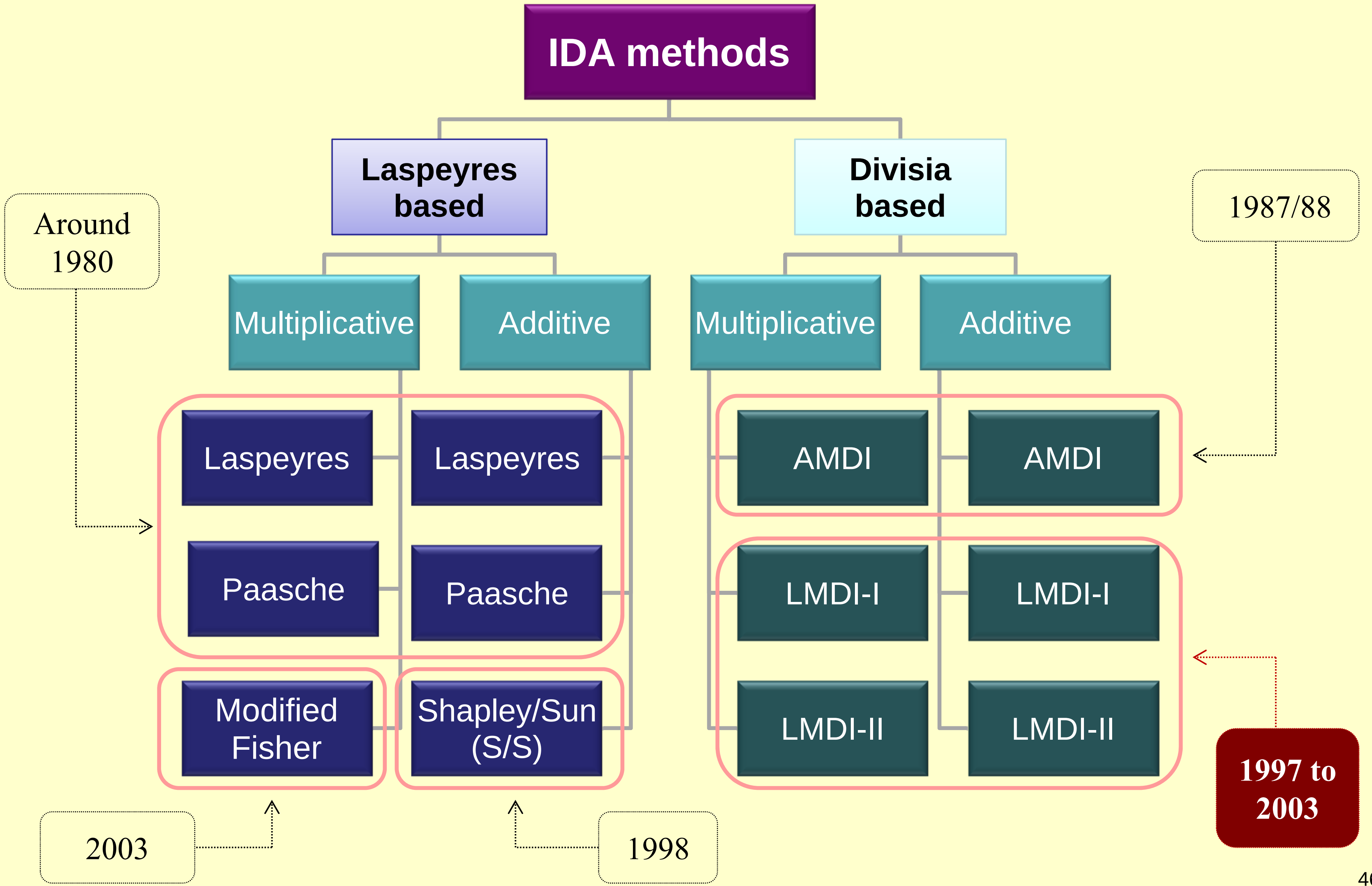
12. How can I find the formulae of the commonly used Laspeyres-based IDA methods?

Refer to the next three slides. See also Liu and Ang (2003), Ang (2004) and Ang, Huang and Mu (2009).

13. How can I find the formulae of the commonly used Divisia-based IDA methods?

Refer to the next three slides. See also Liu and Ang (2003), Ang (2004), Ang, Huang and Mu (2009), and Ang (2015). Generally the formulae take the same form irrespective of the number of factors in the IDA identity

IDA Methods (with Year of Introduction)



Formulae of IDA methods linked to Laspeyres index

Approach		Energy Consumption $E=\sum_{i=1}^{n}E_i=\sum_{i=1}^{n}Q\frac{Q_i}{Q}\frac{E_i}{Q_i}=\sum_{i=1}^{n}Q\,S_iI_i$		Aggregate Energy Intensity $V=E/Q=(\sum_{i=1}^{n}Q\,S_iI_i)/Q=\sum_{i=1}^{n}S_iI_i$	
		Additive $\Delta E_{tot}=E^T-E^0$	**Multiplicative** $D_{tot}=E^T/E^0$	**Additive** $\Delta V_{tot}=V^T-V^0$	**Multiplicative** $D_{tot}=V^T/V^0$
Method	Effect	$\Delta E_{act}+\Delta E_{str}+\Delta E_{int}+\Delta E_{rsd}$	$D_{act}D_{str}D_{int}D_{rsd}$	$\Delta V_{str}+\Delta V_{int}+\Delta V_{rsd}$	$D_{str}D_{int}D_{rsd}$
Laspeyres	Activity	$\sum_i Q^T S_i^0 I_i^0 - E^0$	$\frac{\sum_i Q^T S_i^0 I_i^0}{E^0}$	N.A	N.A
	Structure	$\sum_i Q^0 S_i^T I_i^0 - E^0$	$\frac{\sum_i Q^0 S_i^T I_i^0}{E^0}$	$\sum_i S_i^T I_i^0 - V^0$	$\frac{\sum_i S_i^T I_i^0}{V^0}$
	Intensity	$\sum_i Q^0 S_i^0 I_i^T - E^0$	$\frac{\sum_i Q^0 S_i^0 I_i^T}{E^0}$	$\sum_i S_i^0 I_i^T - V^0$	$\frac{\sum_i S_i^0 I_i^T}{V^0}$
Paasche	Activity	$E^T - \sum_i Q^0 S_i^T I_i^T$	$\frac{E^T}{\sum_i Q^0 S_i^T I_i^T}$	N.A	N.A
	Structure	$E^T - \sum_i Q^T S_i^0 I_i^T$	$\frac{E^T}{\sum_i Q^T S_i^0 I_i^T}$	$V^T - \sum_i S_i^0 I_i^T$	$\frac{V^T}{\sum_i S_i^0 I_i^T}$
	Intensity	$E^T - \sum_i Q^T S_i^T I_i^0$	$\frac{E^T}{\sum_i Q^T S_i^T I_i^0}$	$V^T - \sum_i S_i^T I_i^0$	$\frac{V^T}{\sum_i S_i^T I_i^0}$
Marshall-Edgeworth	Activity	$(\Delta E_{act}^L + \Delta E_{act}^P)/2$	$\frac{\sum_i(Q^T S_i^0 I_i^0 + Q^T S_i^T I_i^T)}{\sum_i(Q^0 S_i^0 I_i^0 + Q^0 S_i^T I_i^T)}$	N.A	N.A
	Structure	$(\Delta E_{str}^L + \Delta E_{str}^P)/2$	$\frac{\sum_i(Q^0 S_i^T I_i^0 + Q^T S_i^T I_i^T)}{\sum_i(Q^0 S_i^0 I_i^0 + Q^T S_i^0 I_i^T)}$	$(\Delta V_{str}^L + \Delta V_{str}^P)/2$	$\frac{\sum_i(S_i^T I_i^0 + S_i^T I_i^T)}{\sum_i(S_i^0 I_i^0 + S_i^0 I_i^T)}$
	Intensity	$(\Delta E_{int}^L + \Delta E_{int}^P)/2$	$\frac{\sum_i(Q^0 S_i^0 I_i^T + Q^T S_i^T I_i^T)}{\sum_i(Q^0 S_i^0 I_i^0 + Q^T S_i^T I_i^0)}$	$(\Delta V_{int}^L + \Delta V_{int}^P)/2$	$\frac{\sum_i((S_i^0 I_i^T + S_i^T I_i^T)}{\sum_i(S_i^0 I_i^0 + S_i^T I_i^0)}$
Fisher	Activity	$(\Delta E_{act}^L + \Delta E_{act}^P)/3 + \sum_i((Q^T - Q^0)S_i^T I_i^0 + (Q^T - Q^0)S_i^0 I_i^T)/6$	$(D_{act}^L \cdot D_{act}^P)^{\frac{1}{3}}\left(\frac{\sum_i Q^T S_i^0 I_i^T}{\sum_i Q^0 S_i^0 I_i^T}\frac{\sum_i Q^T S_i^T I_i^0}{\sum_i Q^0 S_i^T I_i^0}\right)^{\frac{1}{6}}$	*Same as Marshall-Edgeworth*	N.A
	Structure	$(\Delta E_{str}^L + \Delta E_{str}^P)/3 + \sum_i((S_i^T - S_i^0)Q^T I_i^0 + (S_i^T - S_i^0)Q^0 I_i^T)/6$	$(D_{str}^L \cdot D_{str}^P)^{\frac{1}{3}}\left(\frac{\sum_i Q^0 S_i^T I_i^T}{\sum_i Q^0 S_i^0 I_i^T}\frac{\sum_i Q^T S_i^T I_i^0}{\sum_i Q^T S_i^0 I_i^0}\right)^{\frac{1}{6}}$		$\sqrt{D_{str}^L \cdot D_{str}^P}$
	Intensity	$(\Delta E_{int}^L + \Delta E_{int}^P)/3 + \sum_i((I_i^T - I_i^0)Q^T S_i^0 + (I_i^T - I_i^0)Q^0 S_i^T)/6$	$(D_{int}^L \cdot D_{int}^P)^{\frac{1}{3}}\left(\frac{\sum_i Q^0 S_i^T I_i^T}{\sum_i Q^0 S_i^T I_i^0}\frac{\sum_i Q^T S_i^0 I_i^T}{\sum_i Q^T S_i^0 I_i^0}\right)^{\frac{1}{6}}$		$\sqrt{D_{int}^L \cdot D_{int}^P}$

Formulae of IDA methods linked to Divisia index

Approach		Energy Consumption $E=\sum_{i=1}^{n} E_i=\sum_{i=1}^{n} Q\frac{Q_i}{Q}\frac{E_i}{Q_i}=\sum_{i=1}^{n} Q S_i I_i$		Aggregate Energy Intensity $V=E/Q=(\sum_{i=1}^{n} Q S_i I_i)/Q=\sum_{i=1}^{n} S_i I_i$	
		Additive $\Delta E_{tot}=E^T-E^0$	**Multiplicative** $D_{tot}=E^T/E^0$	**Additive** $\Delta V_{tot}=V^T-V^0$	**Multiplicative** $D_{tot}=V^T/V^0$
Method	Effect	$\Delta E_{act}+\Delta E_{str}+\Delta E_{int}+\Delta E_{rsd}$	$D_{act}D_{str}D_{int}D_{rsd}$	$\Delta V_{str}+\Delta V_{int}+\Delta V_{rsd}$	$D_{str}D_{int}D_{rsd}$
AMDI Tornqvist	Activity	$\sum_i \frac{(E_i^T+E_i^0)}{2}\ln\left(\frac{Q^T}{Q^0}\right)$	$\exp\left(\sum_i \frac{(E_i^T/E^T+E_i^0/E^0)}{2}\ln\left(\frac{Q^T}{Q^0}\right)\right)$	N.A	N.A
	Structure	$\sum_i \frac{(E_i^T+E_i^0)}{2}\ln\left(\frac{S_i^T}{S_i^0}\right)$	$\exp\left(\sum_i \frac{(E_i^T/E^T+E_i^0/E^0)}{2}\ln\left(\frac{S_i^T}{S_i^0}\right)\right)$	$\sum_i \frac{(E_i^T/Q^T+E_i^0/Q^0)}{2}\ln\left(\frac{S_i^T}{S_i^0}\right)$	$\exp\left(\sum_i \frac{(E_i^T/E^T+E_i^0/E^0)}{2}\ln\left(\frac{S_i^T}{S_i^0}\right)\right)$
	Intensity	$\sum_i \frac{(E_i^T+E_i^0)}{2}\ln\left(\frac{I_i^T}{I_i^0}\right)$	$\exp\left(\sum_i \frac{(E_i^T/E^T+E_i^0/E^0)}{2}\ln\left(\frac{I_i^T}{I_i^0}\right)\right)$	$\sum_i \frac{(E_i^T/Q^T+E_i^0/Q^0)}{2}\ln\left(\frac{I_i^T}{I_i^0}\right)$	$\exp\left(\sum_i \frac{(E_i^T/E^T+E_i^0/E^0)}{2}\ln\left(\frac{I_i^T}{I_i^0}\right)\right)$
LMDI–I	Activity	$\sum_i L(E_i^T,E_i^0)\ln\left(\frac{Q^T}{Q^0}\right)$	$\exp\left(\sum_i \frac{L(E_i^T,E_i^0)}{L(E^T,E^0)}\ln\left(\frac{Q^T}{Q^0}\right)\right)$	N.A	N.A
	Structure	$\sum_i L(E_i^T,E_i^0)\ln\left(\frac{S_i^T}{S_i^0}\right)$	$\exp\left(\sum_i \frac{L(E_i^T,E_i^0)}{L(E^T,E^0)}\ln\left(\frac{S_i^T}{S_i^0}\right)\right)$	$\sum_i L\left(\frac{E_i^T}{Q^T},\frac{E_i^0}{Q^0}\right)\ln\left(\frac{S_i^T}{S_i^0}\right)$	$\exp\left(\sum_i \frac{L(E_i^T/Q^T,E_i^0/Q^0)}{L(V^T,V^0)}\ln\left(\frac{S_i^T}{S_i^0}\right)\right)$
	Intensity	$\sum_i L(E_i^T,E_i^0)\ln\left(\frac{I_i^T}{I_i^0}\right)$	$\exp\left(\sum_i \frac{L(E_i^T,E_i^0)}{L(E^T,E^0)}\ln\left(\frac{I_i^T}{I_i^0}\right)\right)$	$\sum_i L\left(\frac{E_i^T}{Q^T},\frac{E_i^0}{Q^0}\right)\ln\left(\frac{I_i^T}{I_i^0}\right)$	$\exp\left(\sum_i \frac{L(E_i^T/Q^T,E_i^0/Q^0)}{L(V^T,V^0)}\ln\left(\frac{I_i^T}{I_i^0}\right)\right)$
LMDI–II	Activity	$\sum_i \frac{L\left(\frac{E_i^T}{E^T},\frac{E_i^0}{E^0}\right)L(E^T,E^0)}{\sum_j L\left(\frac{E_j^T}{E^T},\frac{E_j^0}{E^0}\right)}\ln\left(\frac{Q^T}{Q^0}\right)$	$\exp\left(\sum_i \frac{L\left(\frac{E_i^T}{E^T},\frac{E_i^0}{E^0}\right)}{\sum_j L\left(\frac{E_j^T}{E^T},\frac{E_j^0}{E^0}\right)}\ln\left(\frac{Q^T}{Q^0}\right)\right)$	N.A	N.A
	Structure	$\sum_i \frac{L\left(\frac{E_i^T}{E^T},\frac{E_i^0}{E^0}\right)L(E^T,E^0)}{\sum_j L\left(\frac{E_j^T}{E^T},\frac{E_j^0}{E^0}\right)}\ln\left(\frac{S_i^T}{S_i^0}\right)$	$\exp\left(\sum_i \frac{L\left(\frac{E_i^T}{E^T},\frac{E_i^0}{E^0}\right)}{\sum_j L\left(\frac{E_j^T}{E^T},\frac{E_j^0}{E^0}\right)}\ln\left(\frac{S_i^T}{S_i^0}\right)\right)$	$\sum_i \frac{L\left(\frac{E_i^T}{E^T},\frac{E_i^0}{E^0}\right)L(V^T,V^0)}{\sum_j L\left(\frac{E_j^T}{E^T},\frac{E_j^0}{E^0}\right)}\ln\left(\frac{S_i^T}{S_i^0}\right)$	$\exp\left(\sum_i \frac{L\left(\frac{E_i^T}{E^T},\frac{E_i^0}{E^0}\right)}{\sum_j L\left(\frac{E_j^T}{E^T},\frac{E_j^0}{E^0}\right)}\ln\left(\frac{S_i^T}{S_i^0}\right)\right)$
	Intensity	$\sum_i \frac{L\left(\frac{E_i^T}{E^T},\frac{E_i^0}{E^0}\right)L(E^T,E^0)}{\sum_j L\left(\frac{E_j^T}{E^T},\frac{E_j^0}{E^0}\right)}\ln\left(\frac{I_i^T}{I_i^0}\right)$	$\exp\left(\sum_i \frac{L\left(\frac{E_i^T}{E^T},\frac{E_i^0}{E^0}\right)}{\sum_j L\left(\frac{E_j^T}{E^T},\frac{E_j^0}{E^0}\right)}\ln\left(\frac{I_i^T}{I_i^0}\right)\right)$	$\sum_i \frac{L\left(\frac{E_i^T}{E^T},\frac{E_i^0}{E^0}\right)L(V^T,V^0)}{\sum_j L\left(\frac{E_j^T}{E^T},\frac{E_j^0}{E^0}\right)}\ln\left(\frac{I_i^T}{I_i^0}\right)$	$\exp\left(\sum_i \frac{L\left(\frac{E_i^T}{E^T},\frac{E_i^0}{E^0}\right)}{\sum_j L\left(\frac{E_j^T}{E^T},\frac{E_j^0}{E^0}\right)}\ln\left(\frac{I_i^T}{I_i^0}\right)\right)$

L(x,y) is the logarithmic mean of two positive numbers x and y given by $L(x,y)=\frac{x-y}{\ln x-\ln y}$ $for\ x\neq y$
$= x$ $for\ x=y$

14. How are the results obtained differ among the various IDA methods?

Studies comparing the various methods and the results they give can be found in the literature. Some of these studies are:

Ang, B.W., Zhang, F.Q., Choi, K.H. (1998)
Liu, F.L., Ang, B.W. (2003)
Ang, B.W., Liu, N. (2007c)
Ang, B.W., Mu, A.R., Zhou, P. (2010)
Ang (2015)

The decomposition results obtained by applying the IDA methods in the previous two slides to the data in Example 1 (Slide #10) are shown in the next slide.

Ang, B.W. (2004) provides a detailed analysis and recommends the use of LMDI-I. Ang, B.W. (2015) compares eight basic LMDI models.

	$\Delta E_{tot}=E^T-E^0=\Delta E_{act}+\Delta E_{str}+\Delta E_{int}+\Delta E_{rsd}$					$D_{tot}=E^T/E^0=D_{act}\cdot D_{str}\cdot D_{int}\cdot D_{rsd}$				
	ΔE_{tot}	ΔE_{act}	ΔE_{str}	ΔE_{int}	ΔE_{rsd}	D_{tot}	D_{act}	D_{str}	D_{int}	D_{rsd}
Laspeyres	47	20.0000	50.0000	-20.0000	-3.0000	1.470	1.2000	1.5000	0.8000	1.0208
Paasche	47	24.5000	51.0000	-33.0000	4.5000	1.470	1.2000	1.5313	0.8167	0.9796
M-E	47	22.2500	50.5000	-26.5000	0.7500	1.470	1.2000	1.5153	0.8107	0.9972
S/S, Mod. Fisher	47	22.5000	50.7500	-26.5000	N/A	1.470	1.2000	1.5155	0.8083	N/A
AMDI	47	22.5167	52.7459	-26.0463	-2.2163	1.470	1.2000	1.5022	0.8081	1.0091
LMDI-I	47	21.8124	50.4848	-25.2972	N/A	1.470	1.1958	1.5126	0.8127	N/A
LMDI-II	47	22.2423	50.6434	-25.8856	N/A	1.470	1.2000	1.5146	0.8088	N/A

$V=E/Q$	$\Delta V_{tot}=V^T-V^0=\Delta V_{str}+\Delta V_{int}+\Delta V_{rsd}$					$D_{tot}=V^T/V^0=D_{str}\cdot D_{int}\cdot D_{rsd}$				
	ΔV_{tot}	–	ΔV_{str}	ΔV_{int}	ΔV_{rsd}	D_{tot}	–	D_{str}	D_{int}	D_{rsd}
Laspeyres	0.225	–	0.5000	-0.2000	-0.0750	1.225	–	1.5000	0.8000	1.0208
Paasche	0.225	–	0.4250	-0.2750	0.0750	1.225	–	1.5313	0.8167	0.9796
M-E	0.225	–	0.4625	-0.2375	N/A	–	–	–	–	–
Fisher	–	–	–	–	–	1.225	–	1.5155	0.8083	N/A
AMDI	0.225	–	0.4646	-0.2358	-0.0038	1.225	–	1.5022	0.8081	1.0091
LMDI-I	0.225	–	0.4553	-0.2303	N/A	1.225	–	1.5078	0.8125	N/A
LMDI-II	0.225	–	0.4603	-0.2353	N/A	1.225	–	1.5146	0.8088	N/A

Notes:

- N/A: Not applicable as the method gives perfect decomposition without a residual term.
- Residual term: The residual term denoted by the subscript "rsd" is a derived figure given by the discrepancy between the actual change of an aggregate and the sum or product of the estimated effects.
- The S/S method applies to the additive case why the Modified Fisher method applies to the multiplicative case. The latter is similar to the conventional Fisher index when there are only two effects or factors in the IDA identity.

15. What are the properties and linkages of these IDA decomposition methods?

Refer to Ang, Huang and Mu (2009) which gives a detailed treatment of methods linked to the Divisia index and those linked to the Laspeyres index in the additive form. The study proves that most methods linked to the Divisia index, includng AMDI and LMDI-II, collapse to LMDI-I after applying a principle called “proportionally distributed by sub-category”. For methods linked to the Laspeyres index, it is shown that the linkage can be established through defining the characteristic function in the Shapley value.

16. How LMDI-II differs from LMDI-I?

The differences are in the weight function. Useful references are: Ang and Liu (2001), Ang, Liu and Chew (2003), and Ang (2015)

Previous studies with formulae and numerical examples:

	Multiplicative	Additive
LMDI-I	Ang and Liu (2001) Choi and Ang (2003) Ang (2015)	Ang, Zhang and Choi (1998) Choi and Ang (2003) Ang (2015)
LMDI-II	Ang and Choi (1997) Ang (2015)	Ang, Liu and Chew (2003) Ang (2015)

17. Where can I find the formulae for LMDI-II?

The LMDI-II formulae corresponding to those of LMDI-I in Slide #9 and Slide #29 are reproduced in the next two slides. They are also given in Ang (2015).

Formulae for LMDI-II

Additive

$$\Delta E_{act} = \sum_i w_i \ln\left(\frac{Q^T}{Q^0}\right)$$

$$\Delta E_{str} = \sum_i w_i \ln\left(\frac{S_i^T}{S_i^0}\right)$$

$$\Delta E_{int} = \sum_i w_i \ln\left(\frac{I_i^T}{I_i^0}\right)$$

$$w_i = \frac{L\left(\frac{E_i^T}{E^T}, \frac{E_i^0}{E^0}\right)}{\sum_i L\left(\frac{E_i^T}{E^T}, \frac{E_i^0}{E^0}\right)} L(E^T, E^0)$$

Multiplicative

$$D_{act} = \exp\left(\sum_i \tilde{w}_i \ln\left(\frac{Q^T}{Q^0}\right)\right)$$

$$D_{str} = \exp\left(\sum_i \tilde{w}_i \ln\left(\frac{S_i^T}{S_i^0}\right)\right)$$

$$D_{int} = \exp\left(\sum_i \tilde{w}_i \ln\left(\frac{I_i^T}{I_i^0}\right)\right)$$

$$\tilde{w}_i = \frac{L\left(\frac{E_i^T}{E^T}, \frac{E_i^0}{E^0}\right)}{\sum_i L\left(\frac{E_i^T}{E^T}, \frac{E_i^0}{E^0}\right)}$$

Decomposition of Aggregate Energy Intensity

LMDI-II (Additive)

$$\Delta I_{str} = \sum_i \frac{L(\frac{E_i^T}{E^T}, \frac{E_i^0}{E^0})}{\sum_i L(\frac{E_i^T}{E^T}, \frac{E_i^0}{E^0})} L(I^T, I^0) \ln(\frac{S_i^T}{S_i^0}) \qquad \Delta I_{int} = \sum_i \frac{L(\frac{E_i^T}{E^T}, \frac{E_i^0}{E^0})}{\sum_i L(\frac{E_i^T}{E^T}, \frac{E_i^0}{E^0})} L(I^T, I^0) \ln(\frac{I_i^T}{I_i^0})$$

LMDI-II (Multiplicative)

$$D_{str} = \exp\{(\sum_i \frac{L(\frac{E_i^T}{E^T}, \frac{E_i^0}{E^0})}{\sum_i L(\frac{E_i^T}{E^T}, \frac{E_i^0}{E^0})} \ln(\frac{S_i^T}{S_i^0})\} \qquad D_{int} = \exp\{\sum_i \frac{L(\frac{E_i^T}{E^T}, \frac{E_i^0}{E^0})}{\sum_i L(\frac{E_i^T}{E^T}, \frac{E_i^0}{E^0})} \ln(\frac{I_i^T}{I_i^0})\}$$

18. Should I use LMDI-II instead of LMDI-I?

Application of LMDI-I and LMDI-II give practically the same numerical results. However, LMDI-I with simpler formulae is generally preferred (Ang, 2004).

More specifically, if decomposition is in the additive form, LMDI-I is preferred (Ang, Mu and Zhou, 2010). If decomposition is in the multiplicative form, both versions are equally applicable when the aggregate is absolute energy consumption or emissions, and LMDI-II may be chosen when the aggregate is aggregate energy or carbon intensity.

For further discussions, refer to Ang (2015).

19. IDA and LMDI: A timeline

1978/79	First reported studies using concepts similar to IDA on industrial electricity consumption in UK and USA.
1980s	Application extended to other sectors of energy use.
1991	First reported study on carbon emissions using IDA.
1990s	IDA adopted by national and international energy agencies to quantify key drivers of energy use. New developments in methodology.
1997	First IDA method that is perfect in decomposition proposed (later referred to as LMDI-II).
1998	The term LMDI is coined.
2000	The term IDA is introduced. A literature survey lists 117 journal papers.
2004	Study classifies IDA methods and recommends LMDI.
2000s	Rapid growth in the use of LMDI to analyse drivers of changes in energy consumption, carbon emissions and other aggregates.
2015	More than 500 archival journal articles (in English) on IDA, with LMDI as the most widely used IDA method in these publications since 2009.

20. Number of archival journal articles (in English) on IDA by year and the IDA method used