

Supplementary Materials for  
“A Decomposition Method Based on a Model of Continuous Change”  
by Shiro Horiuchi, John R. Wilmoth and Scott D. Pletcher  
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## Introduction

This manuscript comprises a few sections and appendices that were included in some early versions but not in the final version of the article entitled “A Decomposition Method Based on a Model of Continuous Change” by Shiro Horiuchi, John R. Wilmoth and Scott D. Pletcher. The article has been published in *Demography*, Vol. 45, No. 4 (November 2008), pp. 785-801. Because the article is cited for a number of times in this manuscript, we refer to it simply as HWP.

It is assumed that readers of this manuscript have already read HWP, which presents a new, general method (the line integral method) of decomposition analysis. Two application examples of the method (Example 1 and Example 3) are given in HWP. Additional two examples (Example 3 and Example 4) are shown in Sections 1 and 2 of this manuscript. The four examples, together with the previous studies using the method (Glei and Horiuchi 2007; Pletcher et al., 2000; Wilmoth and Horiuchi 1999; Wilmoth et al., 2000), demonstrate the flexibility and wide applicability of the method.

Section 3 examines the sensitivity of the method with respect to departures from the assumption of proportionality. Appendix A explains an indicator used in Section 3, which has been devised for measuring discrepancies between two vectors, and Appendix B displays a MATLAB program for the method.

### 1. Example 3: Demographic transition in Sweden

In economically developed countries, mortality and fertility changed markedly during the last two centuries, thereby affecting the rate of population growth. However, levels of mortality and fertility by age are not directly reflected in the *observed* rate of population growth, which is affected as well by the age structure of the population and by migratory flows. The *intrinsic* rate of population growth, on the other hand, is determined solely by current regimes of mortality and fertility, as expressed by Lotka's well-known equation:

$$\int_0^{\infty} e^{-rx} \ell(x) m(x) dx = 1, \quad (1)$$

where  $r$  is the intrinsic growth rate,  $\ell(x)$  is the proportion of survivors from birth to exact age  $x$  in the current life table, and  $m(x)$  is the current birth rate at exact age  $x$ . Because we are interested in assessing effects of the mortality function rather than those of the survivor function, the following expression represents the decomposition problem more clearly:

$$\int_0^{\infty} e^{-rx} e^{-\int_0^x \mu(y) dy} m(x) dx = 1, \quad (2)$$

where  $\mu(y)$  is the force of mortality, or the instantaneous death rate, at exact age  $y$ .

In this example, we investigate the effects of changing mortality and fertility on trends in the intrinsic growth rate for the female population of Sweden during 1778-2002 using data from a previous study (Horiuchi 1995) supplemented with information on recent fertility (Statistics Sweden 2007) and mortality (HMD 2007). Changes in the intrinsic growth rate were decomposed into effects of death rates for ten 5-year age groups (0-4, 5-9, ..., 45-49) and birth rates for seven 5-year age groups (15-19, ..., 45-49). Fertility outside this age range is negligible, and mortality at post-reproductive ages has no effect on the intrinsic growth rate. Between two consecutive 5-year periods, all changes in age-specific birth rates and logarithms of age-specific death rates were assumed proportional to each other.

Thus, the line integral method of decomposition analysis was applied to each of the 44 pairs of successive 5-year time periods. The number of intervals,  $N$ , used for numerical integration was set at four. Thus, the entire period was divided into 176 (4x44) intervals. In this analysis,  $N=4$  turned out to be large enough to produce sufficiently accurate results. The absolute proportional error ( $\varepsilon$ ) was less than one percent except for seven of the 44 pairs (in all seven cases, the intrinsic growth rate remained virtually unchanged between two successive periods, making the denominator of the fraction in Eq. (8) of HWP almost zero).

Effects of the ten age-specific death rates and of the seven age-specific birth rates can be aggregated to obtain the total effects of mortality and fertility, respectively. Cumulative effects of mortality and fertility changes since 1778-1782 are shown in Figure 1A. In each 5-year time period, the sum of cumulative mortality (dash-dot line) and fertility (dashed line) effects equals the *change* in the intrinsic growth rate between 1778-1782 and the period in question (i.e., the *difference* between the solid curve and the horizontal dotted line). Overall, the intrinsic growth rate (per annum) decreased from 0.59 percent in 1778-1782 to -0.96 percent in 1998-2002 because the negative effects of fertility decline (-3.41) exceeded the positive effects of mortality reduction (1.86). Figure 1B displays the effects of changing *age-specific* birth (dashed line) and death rates (solid line) on the overall change in the intrinsic growth rate between 1778-1782 and 1998-2002. The greatest positive effect is found for the death rate at ages 0-4, and pronounced negative effects are seen for birth rates in the 20's and 30's of age.

These decomposition results suggest that population dynamics of Sweden during this 225-year time period can be divided into four distinct phases, as summarized in Table 1. In the first phase, mortality reduction, especially at young ages, resulted in a substantial increase in the intrinsic rate of population growth (per annum) from 0.59 percent in 1778-1782 to 1.13 percent in 1873-1877. In the

second phase, however, fertility decline led to an enormous reduction in the intrinsic growth rate, which fell to  $-0.12$  percent in 1923-1927.

The third phase, 1923-1927 to 1948-1952, is characterized by two idiosyncratic historical events, the Great Depression and World War II. This special period witnessed a downturn and an upturn due to strong temporary effects of these two events, rather than long-term trends. The economic crisis brought even lower fertility, which reduced the intrinsic growth rate further to  $-1.00$  percent in 1933-1937, although the end of the war and the post-war Baby Boom raised fertility again, yielding positive values for the intrinsic growth rate of  $0.48$  percent in 1943-1947 and  $0.27$  percent in 1948-1952.

The final phase is after World War II. The intrinsic growth rate, after being slightly positive during most of the 1950s and 1960s, became negative again due to below-replacement fertility in the following decades. To anyone familiar with the population history of Sweden or Western Europe in general, these conclusions are hardly surprising, but they are now supported by simple quantitative results linking changes in age-specific mortality and fertility rates to changes in the intrinsic rate of population growth.<sup>1</sup>

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<sup>1</sup> Keyfitz (1968:189) partitioned the change in the intrinsic growth rate in Australia into three components: fertility, mortality, and their interaction. For fertility and mortality, only total effects (not age-specific effects) were computed, because interaction terms would be complicated if the procedure had been used for deriving age-specific mortality and fertility effects. The present method, however, makes it possible to derive age-specific effects without producing interaction terms.

## 2. Example 4: Sex difference in the longevity of bean beetles

Females tend to live longer than males. This has been observed for many human populations as well as a number of other species. Is this mainly because (a) males have consistently higher mortality than females throughout most phases of the life, (b) males have higher mortality risks than females particularly at old ages, or (c) mortality rates rise faster with age in males than in females? In this discussion, we separate old ages from other ages because the age pattern of mortality in old ages differs noticeably from the pattern at younger ages: the age-related mortality rise tends to level off at old ages in virtually every species whose mortality patterns have been investigated with a large number of old individuals (Vaupel et al. 1998).<sup>2</sup>

The data produced by Tatar and Carey (1994) provide an opportunity to investigate this question for bean beetles. These authors recorded deaths in two generations of bean beetles raised under controlled conditions. In both cohorts, females lived significantly longer, on average, than males. In the older generation, for example, the average length of life was 24.1 days for females and 14.6 days for males.

We investigated this sex differential using a logistic model of mortality. It has been shown that logistic models fit mortality patterns of various species well, including humans at adult ages (Horiuchi and Coale 1990; Thatcher, Kannisto and Vaupel 1998). In particular, a three-parameter version of the logistic equation accurately represents age variations of mortality in two species of flies: the common

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<sup>2</sup> Although the distinction between childhood mortality and adult mortality is important for analyzing sex differentials in the life expectancy of humans, it is not necessarily so for a number of other species. For some egg-laying species, it is very difficult to distinguish defective eggs from deaths after hatching out of eggs, and mortality on the first day is thus ignored. For some species that go through metamorphosis, mortality immediately after eclosion from pupa is negligibly low in laboratory settings.

fruit fly, *Drosophila melanogaster* (Fukui et al. 1996; Pletcher et al. 2000; Promislow et al. 1996), and Mediterranean fruit flies (Wilson 1994, using data from Carey et al. 1992). We adopted this model because the age patterns of mortality (Figure 2D) in the bean beetle populations of Tatar and Carey, and the trajectories of their life table aging rate<sup>3</sup> (not shown here), appeared highly consistent with the logistic equation.<sup>4</sup>

In the logistic model, the force of mortality, or instantaneous death rate, at exact age  $x$  is given by

$$\mu(x) = \frac{\alpha e^{\theta x}}{1 + \alpha \beta (e^{\theta x} - 1)} . \quad (3)$$

The three parameters of the model can be interpreted as follows:  $\alpha$  is the force of mortality at age zero and represents the overall level of mortality throughout most of the life span (except for very old ages);  $1/\beta$  is the level of the mortality plateau at very old ages; and  $\theta$  indicates the pace of (exponential) mortality increase with age. Hypothetical mortality curves illustrating the effects of these parameters are shown in Figures 2A, 2B and 2C. Thus,  $\alpha$ ,  $\beta$ , and  $\theta$  correspond to the possible

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<sup>3</sup> The concept and characteristics of the life table aging rate are discussed by Horiuchi and Wilmoth (1997) and its use for assessing mortality models has been demonstrated by Horiuchi and Coale (1990).

<sup>4</sup> The fitness of the logistic model to these data can be compared with those of other models using the likelihood ratio test (Pletcher 1999). The test was not conducted here because the two pieces of visual evidence (i.e., age trajectories of logarithmic mortality and that of the life table aging rate) suggested strongly that the logistic model should be more appropriate than some other widely used models such as Gompertz and Makeham.



explanations for sex differentials in mortality that were mentioned earlier (i.e., (a), (b), and (c), respectively).<sup>5</sup>

Maximum likelihood estimates of the three parameters were obtained for the female and male cohorts of bean beetles by fitting the age distribution of deaths implied by the logistic model to the observed age distribution. Often, mathematical models are fitted directly to age-specific death rates using ordinary or weighted least squares. However, the maximum likelihood approach is more appropriate than these conventional methods if the age distribution of deaths for the cohort is available (Promislow et al. 1999; Pletcher 1999). Both the estimated logistic curves and the observed death rates for females and males are shown in Figure 2D.

The sex difference in the life expectancy of bean beetles was decomposed into four elements, consisting of the effects of differences in the three parameters and of disparities between observed life expectancies and those implied by the fitted logistic models. For these calculations, the number of intervals,  $N$ , was set at 100, and the proportional error of the decomposition,  $\varepsilon$ , was 0.0006 percent. The results in Table 2 suggest that about 55 percent of the sex difference is due to  $\theta$  (the pace of age-related mortality increase), 40 percent is attributable to  $\alpha$  (the overall mortality level), and only 5 percent results from  $\beta$  (the mortality plateau level). Thus, male bean beetles may senesce significantly faster than females, and this sex difference in the rate of aging is the primary cause of the sex difference in life expectancy. The decomposition method enables us to quantify and compare the effects of different aspects of the mortality trajectory on the sex differential in life expectancy.

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<sup>5</sup> Several researchers have specified the three-parameter logistic model in different ways. Those specifications are mathematically equivalent but the parameters are differently interpreted. Equation (3) is a modified version of the formulation by Vaupel (1990).

### 3. Sensitivity Analysis

In principle, the line-integral method of decomposition analysis can be used in combination with any assumption about relationships among changes of covariates. However, in practice, it is convenient to adopt the assumption that changes in all covariates are in fixed proportions to one another throughout the interval, which seems to be the simplest and most neutral assumption. Because the method is based on the very general relation,  $y(t) = f(\mathbf{x}(t))$ , it is difficult to examine analytically the sensitivity of decomposition results with respect to deviations from the proportionality assumption. Therefore, in this section, we attempt to evaluate the sensitivity using Examples 1 and 3.<sup>6</sup>

Each example covers a number of consecutive periods (55 calendar years in Example 1, and 45 quinquennia in Example 3). In each case, it is possible to apply the decomposition method in two distinct manners: either by examining changes between the first and last sets of observations only, or by examining changes in each pair of successive 1- or 5-year periods and then aggregating effects across the entire period. While an assumption of proportional change among all covariates seems reasonably plausible over short time periods, the same assumption appears less well justified when applied over a period of several decades or even centuries. Thus, by comparing these two means of applying the decomposition technique, we can evaluate the sensitivity of the results to departures from the assumption of proportional increments. Table 1 of HWP and table 1 of this manuscript showed the second kind of results for Examples 1 and 3 (aggregated effects from multiple short-period decompositions), which are now compared with the first kind (overall effects from a single long-period decomposition).

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<sup>6</sup> Example 2 or 4 cannot be used for this purpose, because in those examples,  $t$  is a hypothetical dimension of change between the two populations.

To measure discrepancies between the two sets of decomposition results, we compute the sum of the absolute differences between the covariate effects (the  $c_i$ 's), scaled by the sum of absolute values of those effects:

$$\lambda(A, B) = \frac{\sum_{i=1}^n |\hat{c}_{iA} - \hat{c}_{iB}|}{\sum_{i=1}^n |\hat{c}_{iA}| + \sum_{i=1}^n |\hat{c}_{iB}|}, \quad (4)$$

where  $\hat{c}_{iA}$  and  $\hat{c}_{iB}$  are the estimated effects of the  $i$ -th covariate based on the two methods, denoted by  $A$  and  $B$ , of applying the decomposition, and  $n$  is the number of covariates.  $\lambda$  ranges from zero (complete agreement) to one (complete dissonance). It is a generalized form of the index of dissimilarity.<sup>7</sup> (More discussion on  $\lambda$  is given in Appendix A.) In Example 1,  $\lambda$  was 0.069 for the median age, 0.025 for the mean age at death, and 0.063 for the standard deviation.<sup>8</sup> In Example 3,  $\lambda$  was 0.011. These  $\lambda$  values indicate that differences between the aggregated effects and overall-change effects are small relative to their total variations. The close agreement between the two sets of effects can also be observed in figure 3 (Example 1) and figure 1B (Example 3).

These findings do not prove that the results of this decomposition technique are always insensitive to departures from the assumption of proportional increments. The robustness of results undoubtedly depends as well on the functional relationship and the data. Nevertheless, it appears that the method is fairly reliable in this respect, since we know that actual trends in the covariates of Examples 1 and 3 deviate considerably from a pattern of proportional change. Age-specific death rates for women in

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<sup>7</sup> This new measure was suggested by Joel E. Cohen through personal communication.

<sup>8</sup> For calculating  $\lambda$  using the results of Example 1, effects of single-year ages are aggregated for five-year age groups (including age groups 0, 1-4 and 100+) and differences between the two kinds of effect for those five-year age groups are used.

Japan followed markedly different trajectories between 1950 and 2004: the decline of old-age mortality accelerated, while the reduction of mortality at other ages slowed down. Likewise, mortality and fertility in Sweden changed through entirely different paths between 1778-1782 and 1998-2002: mortality declined through most of the periods, whereas fertility decreased, increased, or remained nearly constant in different periods.

Another issue about the assumption of proportional increments is that some mathematical transformations of the covariates ( $h_i(x_i)$ 's) may meet the condition better than the covariates in their original scale ( $x_i$ 's). For example, changes in the logarithm of age-specific death rates may be more closely proportional to each other than changes in the original death rates. Thus, a more general form of the assumption of proportional increments given in Eq. (7) of HWP is

$$\frac{h_i(x_i(t)) - h_i(x_i(t_1))}{h_i(x_i(t_2)) - h_i(x_i(t_1))} = g(t) , \text{ for any } i. \quad (5)$$

Note that the function  $h$  has a subscript  $i$  because different transformations may be applied to each covariate.

We arbitrarily selected five transformations –  $\ln(x)$ ,  $e^x$ ,  $x^3$ ,  $1/x^3$ , and  $x$  itself – and obtained covariate effects for Examples 1 and 3. As described earlier, data for each of these examples comprise observed values of the dependent variable and its covariates for a number of consecutive periods. In this experimental calculation, however, only the first and last sets of observations were used as inputs to the decomposition analysis. Then the long-term change in the dependent variable between the two periods was decomposed assuming proportional changes in the selected functional form of the covariates. The five sets of decomposition results ( $\hat{c}_i$ 's) produce ten  $\lambda$  values, because ten different combinations of two functions can be chosen from the five transformations.

The decomposition results did not differ in a substantial or meaningful way among these transformations. In Example 1,  $\lambda$  was below 0.05 for 26 of the 30 pairs (10 pairs for each of median, mean, and standard deviation), with a maximum of 0.066 for the median age at death. In Example 3,  $\lambda$  was below 0.05 in all 10 cases. Thus, differences in decomposition results associated with alternative transformations of the covariates do not appear to have practical significance, at least for the data and functions considered here.

#### 4. Appendix A: Measuring Discrepancies between Two Vectors

Discrepancies between two sets of decomposition results (two real-numbered vectors of the same size) cannot be summarized by the correlation coefficient or the index of dissimilarity. The correlation coefficient indicates the extent of linear relationship, but not the extent of exact agreement, between two real-numbered vectors. The index of dissimilarity (Duncan and Duncan 1955) compares two probability distributions but does not deal with vectors that have negative elements.

A more appropriate measure is given by:

$$\lambda(A, B) = \frac{\sum_{i=1}^n |\hat{c}_{iA} - \hat{c}_{iB}|}{\sum_{i=1}^n |\hat{c}_{iA}| + \sum_{i=1}^n |\hat{c}_{iB}|} . \quad (B1)$$

where  $\hat{c}_{iA}$ 's and  $\hat{c}_{iB}$ 's are elements of the two real-number vectors,  $A$  and  $B$ .  $\lambda(A, B)$  ranges from zero to one.  $\lambda(A, B)$  is zero if the two vectors are identical.  $\lambda(A, B)$  is one if the two vectors are completely dissonant in the sense that the sign (+, 0, -) of  $\hat{c}_{iA}$  and the sign of  $\hat{c}_{iB}$  are different for every  $i$ .

$\lambda(A, B)$  is a generalization of Dbrushin's coefficient, which measures the maximum difference among multiple probability distributions and is computed as  $\frac{1}{2} \max_{A, B} \sum_{i=1}^n |p_{iA} - p_{iB}|$ , where  $p_{iA} \geq 0$  for any  $(i, A)$  and  $\sum_{i=1}^n p_{iA} = 1$  for any  $A$  (Cohen, Kemperman, and Zbaganu 1998). When only two distributions are compared, Dbrushin's coefficient becomes the index of dissimilarity.  $\lambda(A, B)$  is an extension of the index of dissimilarity from probability distributions to real-numbered vectors.

## 5. Appendix B: MATLAB program

A short program of this decomposition method, written in MATLAB 6.1, is attached below. The algorithm is explained in the appendix of HWP. In writing this program, priority was given to clarity and understandability, rather than computational efficiency. The user has to provide a function with the dependent variable as its output and the covariates as its input. The name of the function is contained in a string variable `fun`. Two column vectors, `x1` and `x2`, contain the covariates for the two sets of observations, and `y1` and `y2` are corresponding values of the dependent variable. Other variables in the program are as follows: `n` is the number of covariates, `N` is the number of intervals for numerical integration, `w` contains parameters that the function may need in addition to the covariates, and `delta` is a column vector containing  $\Delta x_i$ 's. The estimated contributions of the covariates ( $\hat{c}_i$ 's) will be returned as a row vector, called `c`.

```
function c=decom(fun,x1,x2,N,w)
eval(['y2=',fun,'(x2,w);'])
eval(['y1=',fun,'(x1,w);'])
d=x2-x1;n=length(d);delta=d/N;
x=x1*ones(1,N)+(d*ones(1,N).*(ones(n,1)*[0.5:N-0.5]/N));
cc=zeros(n,N);
for j=1:N,for i=1:n
    z=zeros(n,1);z(i)=delta(i)/2;
    eval(['cc(i,j)=',fun,'(x(:,j)+z,w)-',fun,'(x(:,j)-z,w);'])
end,end
disp('Error of decomposition (in %) ')
e=100*(sum(sum(cc))/(y2-y1)-1)
disp('Effects of covariates '), c=sum(cc')
return
```

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**Table 1.** Summary results of decomposition of changes in the intrinsic rate of population growth (in percent per year), Swedish females, from 1778-1782 to 1998-2002.

	1778-1782 to 1873-1877	1873-1877 to 1923-1927	1923-1927 to 1948-1952	1948-1952 to 1998-2002
(1) r at the beginning	0.587	1.133	-0.118	0.271
(2) r at the end	1.133	-0.118	0.271	-0.962
(3) increase in r	0.545	-1.251	0.389	-1.232
(4) mortality effect <sup>a</sup>	0.572	0.832	0.347	0.107
(5) fertility effect <sup>a</sup>	-0.030	-2.082	0.041	-1.339

Note: Rows of the table have the following relationships: ( 3 ) = ( 2 ) - ( 1 ) = ( 4 ) + ( 5 ) (with rounding errors).

Source: Human Mortality Database (2007); Horiuchi (1995, appendix); Statistics Sweden (2007).

a. Sum of the effects of all age groups.

**Table 2.** Decomposition analysis of sex differentials  
in the life expectancy of bean beetles.

	Females	Males	Effect <sup>a</sup> (in %)
<u>Expectation of life at eclosion (in days)</u>			
Observed	24.12	14.63	9.49 (100.0)
Estimated <sup>b</sup>	24.11	14.64	9.48 ( 99.9)
Error	0.01	-0.01	0.01 ( 0.1)
<u>Estimated values of the logistic parameters <sup>c</sup></u>			
$\alpha$ (x10,000)	0.2649 (0.1398)	1.5315 (0.9742)	3.80 ( 40.0)
$\beta$	2.9397 (0.2118)	2.2252 (0.0474)	0.48 ( 5.1)
$\theta$	0.4047 (0.0271)	0.5751 (0.0488)	5.20 ( 54.8)
Number of cases	843	829	

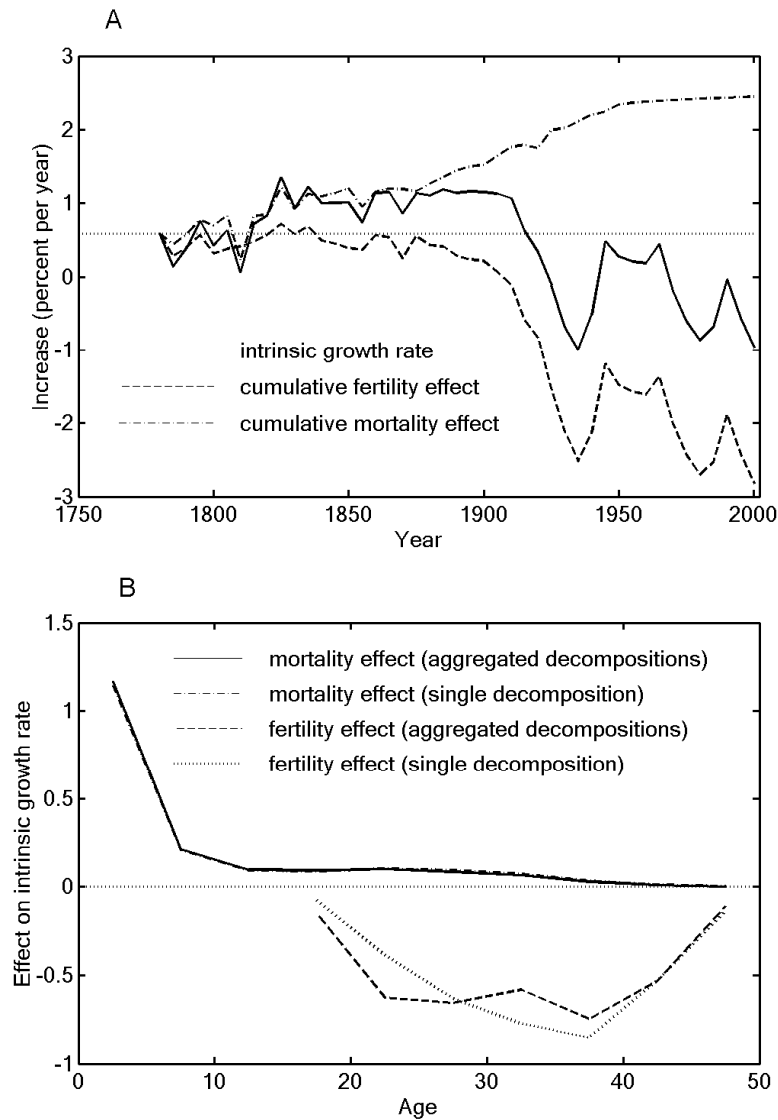
Source: Tatar and Carey 1994, table 1.

a. Contribution to the observed sex difference in the expectation of life at eclosion.

b. The expectation of life was estimated using the logistic model.

c. Figures in parentheses under the estimated parameter values are their standard errors, computed by asymptotic approximation (Searle et al., 1992).

Figure 1. (A) Intrinsic rate of population growth, Swedish females, from 1778-1782 to 1998-2002, and cumulative effects of changes due to mortality and fertility.<sup>a</sup> (B) Aggregate and overall-change effects of age-specific mortality and fertility rates on the change in the intrinsic growth rate.<sup>b</sup>



- a. The cumulative mortality and fertility effects were added to the value of the intrinsic growth rate in 1778-1782, which is indicated by the horizontal dotted line (Figure 1A).
- b. The two sets of mortality effects overlap with each other almost completely (Figure 1B).

Figure 2. (A, B, and C) Hypothetical logistic mortality schedules illustrating effects of the parameters  $\alpha$ ,  $\theta$ , and  $\beta$ , respectively. (D) Observed mortality rates and estimated logistic models for female and male bean beetles: observed female (\*), estimated female (solid), observed male (o), and estimated male (dashed) mortality patterns.

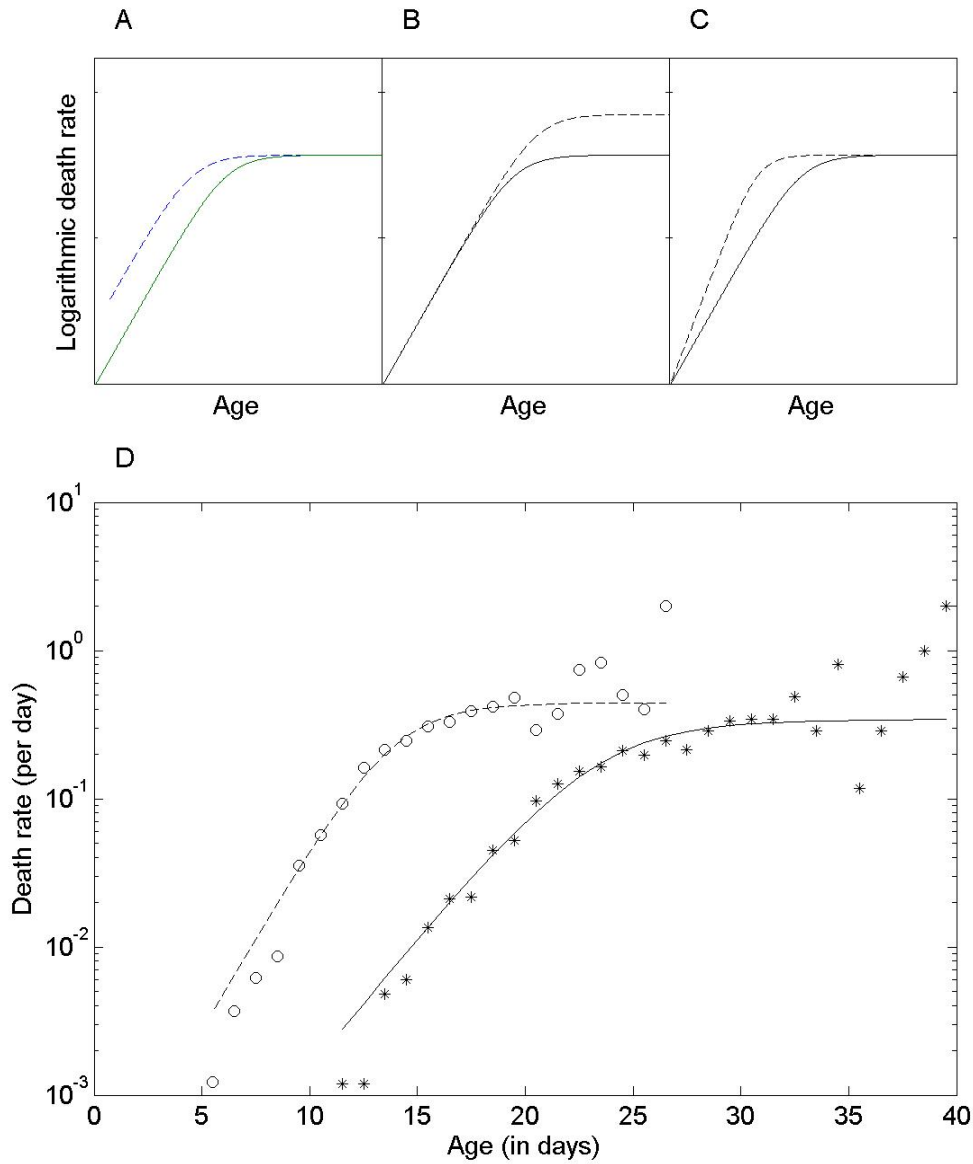
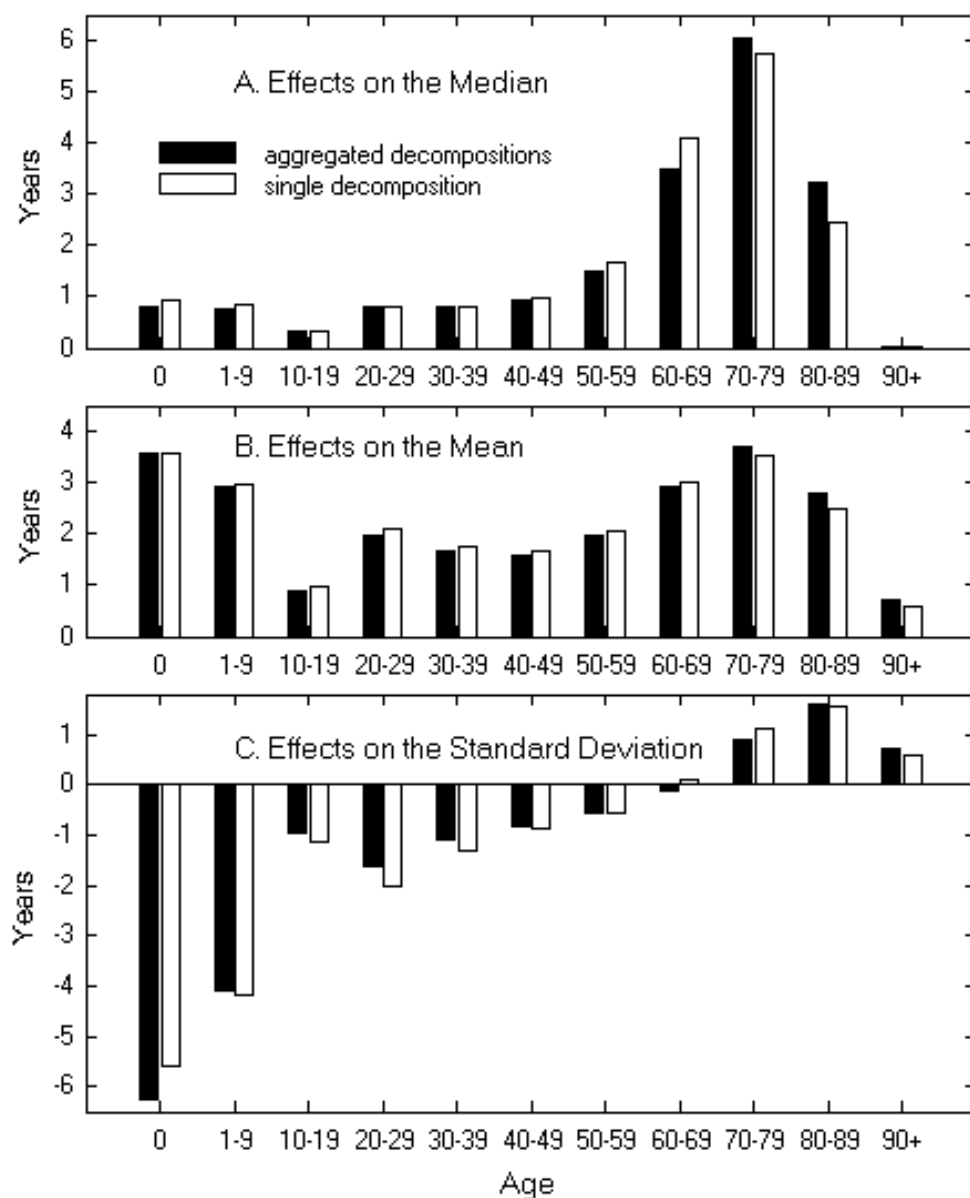


Figure 3. Aggregate and overall-change effects of age-specific death rates on changes in the median, mean, and standard deviation of the age distribution of life-table deaths, Japanese females, 1950-2004.<sup>a</sup>



a. Effects of single-year ages are summed for broader age groups.