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The Method of Extreme Representations and its applications

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The work is devoted to the development and study of model functional and their corresponding model spaces, which have numerous applications in various fields of science and technology. For this purpose, a nonlinear set is taken as the domain of definition of the functional under consideration — a partition of units of the analogue a partition of units of the linear case from functional analysis. It is shown that the introduction of functional with a norm of type max (sup) form normalized spaces. In addition, the functional themselves are the norms of some functions from function spaces. For these model spaces, algebraic representations of the corresponding elements and objects and processes are constructed and obtained. And they are presented in the form of some polynomials that depend on the points of some specially constructed hyper plane. Conversion groups are found that translate points of spaces from two-dimensional to the corresponding points of three-dimensional, and then to four-dimensional, etc. and vice versa. The results are applied to various objects and processes. In the two-dimensional case, these functional are shown to be production functions and at the maximum of which, fairly well-known equations are obtained. It is shown that some known physical laws coincide qualitatively. Examples for different field science and technological are given.

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The Brochure is devoted by questions modeling a tree of numbers arising at the analysis of the numerical and corresponding text information and on the base it constructed the method of extreme representation.

§1. The Model of Numbers Tree

Definition of numbers tree**.** *Let* N *- some natural number. We shall tell, that the number* N *forms a tree of numbers if there will be natural numbers* $n, m \geq 2$ *and integers* $a_1, a_2, ..., a_m$ *for which*

$$N^n = a_1^n + a_2^n + \ldots + a_m^n \ , \qquad (1.1)$$

and in turn some a_j *(or all) from (1.1) represented as*

$$a_j^n = a_{1j}^n + a_{2j}^n + \ldots + a_{m_1 j}^n \ , \ m_1 \leq m \qquad (1.2)$$

and some a_{ij} *of (1.2) also can be submitted as*

$$a_{ij}^n = a_{1ij}^n + \ldots + a_{m_2 ij}^n \ , \ \ m_2 \leq m_1, \ldots,$$

and at last decomposition takes place

$$a_{ijj_1 \ldots j_{m_k}}^n = a_{1ijj_1 \ldots j_{m_k}}^n + a_{2ijj_1 \ldots j_k}^n \ , \qquad (1.3)$$

in which members of the right part (1.3) can not beat are submitted as the final sum composed n-th degrees of some integers so-called by a basis (basis) of a tree.

Conceptual Model of Numbers Tree in general case is given in fig. 1.

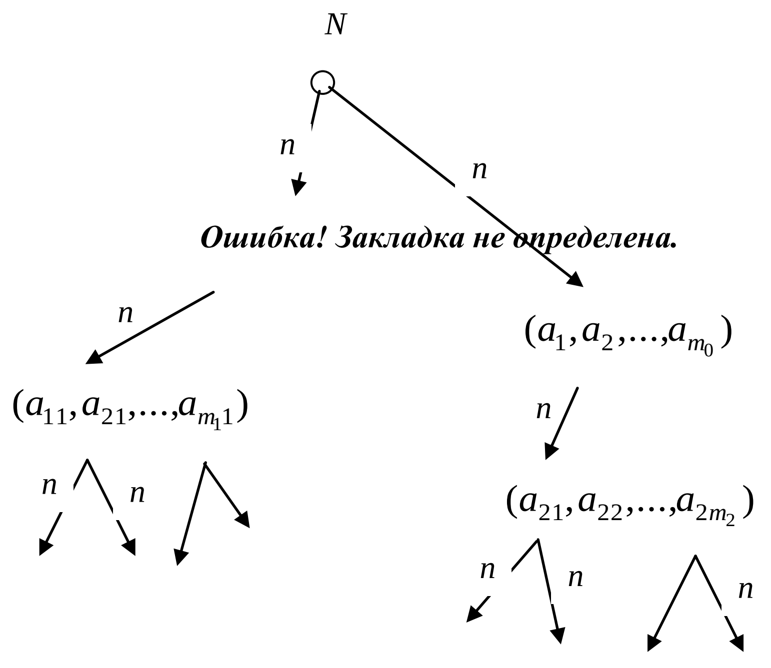


Fig.1. Conceptual Model of Numbers Tree in general case

So last a level the tree consists of the sum such as (1.3). base elements can enter into each level a tree, therefore from each level we take only those elements, which not представимы as (1.2), (1.3). Then in result the number N *is uniquely represented* as

$$N^n = \sum_{j_q} k_{j_q} a^n_{ijj_1 \ldots j_q}, \tag{1.4}$$

where k_{j_q} - number of occurrence of a basic element $a_{ijj_1\cdots j_q}$ in a tree of numbers.

We shall consider a problem for number N and a vector $a = (a_1, \ldots a_k),\ k \geq 2$ from the equation $N = \max_{\alpha \in M} (\alpha, a),$ where

$M = \left\{ (\alpha_1, \ldots, \alpha_k) = \alpha;\ \sum_{j=1}^{k} \alpha_j^{\overline{\frac{h}{n-s}}} = 1,\ n > s > 0,\ 0 < \alpha_j < 1 \right\}$. The set M *nd* represents a measured curvilinear spheroid and at $s = 1,\ n = 2$ turns in usual m-a measured spheroid. Using theorems 1 from work [2] we shall receive:

$$\begin{cases} N^n = a_1^n + \ldots + a_k^n, \\ N_k^n = a_{1k}^n + \ldots + a_{kk}^n, \quad k \geq 2 \end{cases} \tag{4'}$$

Thus, this equation is optimum in sense (1.4), and the tree of numbers appropriate by this equation represented on fig. 1 also is an optimum tree.

§2. Models of the best representation reducing to problem of modeling number tree

Now we consider questions of the best representation some character of any objects (elements) with the help of some properties (elements)given object and its applications in some case where distributions processes of grows processes, heats and waves processes, diffusion processes, at some parameters values take place in the maximal regimes. Extreme regimes of such physical processes are arising in the case when the values of their parameters are chaining in some given set. They can be an accumulation of the warmth, particles, wave energy in some areas where the considered physical processes are arising. Series computational experiments also carried out with models data for some considering heat transfer processes under complex conditions and in extreme regime. Let *H* and *L* - some normalized spaces and the set of *M* is given in the following way:

$$M = \left\{ \alpha = (\alpha_1 \ldots \alpha_m) : \sum_{j=1}^{m} \alpha_j^{\overline{\frac{n}{n-s}}} = 1,\ \alpha_j \geq 0,\ n > s > 0,\ j = \overline{1, m} \right\}$$

Let's assume, that for any objects (elements) $z \in H$ and any properties of objects $x_j \in L,\ j = \overline{1, m}$ norms $P = \|z\|_H,\quad h_j = \left\| x_j \right\|_L,\quad j = \overline{1, m},\ \ m > 1$ and also ways of display of properties of the considered objects are determined.

For example, we can define their norms in the appropriate spaces, i.e.

$$\mu(\alpha)=\left(\int\limits_T \left(\sum_{j=1}^{m}\alpha_j\left|x_j\right|^s\right)^{\frac{n}{s}} dt\right)^{1/n} \text{ и } \left\|x_j\right\|_{L_m^n(T)}=\left(\int\limits_T \sum_{j=1}^{m}\left|x_j\right|^n dt\right)^{1/n}<\infty .$$

Definition. We shall tell, that any object (element) $z\in H$ is in the best way submitted with the help of some properties (elements) $x_j \in L,\ j=\overline{1,m}$, if for some element $\alpha^0\in M$ fairly a parity $P=\max\limits_{\alpha\in M}(\alpha,h^s)^{1/s}=\left(\alpha_1^0 h_1^s+...+\alpha_m^0 h_m^s\right)^{1/s}$
For anyone $s>0$ and $s<n<\infty$.

The Theorem 1. That the object (element) $z\in H$ the best is submitted (maximal) image through properties $x_j\in L,\ j=\overline{1,m}$, it was necessary and enough that had places about a parity

$$\alpha_j^{\frac{n}{n-s}}=\frac{h_j^n}{\sum\limits_{j=1}^{m}h_j^n},\ j=\overline{1,m}\qquad ,\ n>s>0,\quad P^n=\sum_{j=1}^{m}h_j^n$$

Example 1. *Distribution problem of resources*. As an example it is possible to result a problem of representation of a monetary stream, or a word at coding etc. as the sum m numbers $\left(\sum\limits_{j=1}^{m}\alpha_j h_j^s\right)^{\frac{1}{s}}$ then $P=\max\left(\alpha,h^s\right)^{1/s}$ we shall receive the equation $P^n=\sum\limits_{j=1}^{m}h_j^n$. Last equation for anyone m > 2, n > 1 has accounting number of integer decisions $h_1, h_2, ...h_m$ and P.

Example 2. *Grows of plant*. Let us consider model of growth of plants with *m, m≥2* parts (for example, roots, trunk, leaves, buds). Let first part makes means of production biomass x_1, which can be spent for development of all other parts of plant. Let $x_j(t),\ j=\overline{2,m}$ are biomass of part now of time t. Development of plant we shall set to the following model:

$$\dot{x}_1=\alpha_1 f(x_1,l),\ \dot{x}_j=\alpha_j x_j,\ x_1(0)=x_1^0,\ x_j(0)=x_j^0,\ j=\overline{2,m}\ ,$$

$$I(\alpha)=\varphi(x(t_{t_k}),t_k)\qquad -\max,\quad \alpha=(\alpha_1,...,\alpha_m)\in M,\ \ 0\le t\le t_k ,$$

where $x_j^0 \geq 0$, $j=1, \ldots, m$ are given numbers, $f = f(.)$ the law of formation of a new biomass, l - size of photosynthesis parameters. On the base of our theorem Bellman function
correspondents to optimal control problem satisfy next equation

$$-\frac{\partial \mu}{\partial t} = \max_{\alpha \in M} \left\{ \left(\alpha, x_1 \frac{\partial \mu}{\partial x} \right) \right\} \text{ i.e. } \left(\frac{\partial \mu}{\partial t} \right)^n = \sum_{j=1}^{m} \left(x_1 \frac{\partial \mu}{\partial x_j} \right)^n$$

or $\mu(x_1, x_2, \ldots, x_m, t) = \mu_0 + Ct + \ln \left| \left(\frac{x_1}{x_1^0} \right)^{C_1} \left(\frac{x_2}{x_2^0} \right)^{C_1} \ldots \left(\frac{x_m}{x_m^0} \right)^{C_m} \right|$, $C_1^n + C_2^n + \ldots + C_m^n = C^n$

Example 3. *Differential equations with extreme properties*. Many processes (distributions processes of heats and waves, diffusion processes) belong to so-called model equations with extreme properties. In the general case such equations may be represented in the form of:

$Lu = \max_{\alpha \in M} \left\{ \sum_{j=1}^{m} \alpha_j \left(L_j u \right)^s \right\}^{1/s}$, or $(Lu)^n = \sum_{j=1}^{m} \left(L_j u \right)^n$, $n > s > o,$ here L, L_j are given operators, which characterize the considered physical processes.
Let it is given area $G \subseteq E^m$ with border Γ: $\Gamma = \cup \Gamma_j$, and then it is known, that the temperature mode inside area G and wave process satisfy to next equations[2]:

$\frac{\partial u}{\partial t} = \sum_{j=1}^{m} \frac{\partial}{\partial x_j} \left(k_j \frac{\partial u}{\partial x_j} \right)$, $\begin{matrix} x \in G \\ o < \leq t \leq t_k \end{matrix}$, $\frac{\partial^2 u}{\partial t^2} = \sum_{j=1}^{m} \frac{\partial}{\partial x_j} \left(k_j \frac{\partial u}{\partial x_j} \right)$, $x \in G$, $o < t \leq t_k$. Here $k_j = k_j(\cdot)$, $j = \overline{1, m}$ are coefficients of heat conductivity (or wave velocity) characterizing a version of environment. We shall make the following assumption. Let the environment of heat conductivity is those, that

$\frac{\partial}{\partial x_j} \left(k_j \frac{\partial u}{\partial x_j} \right) = \alpha_j \frac{\partial}{\partial x_j} \left(k \frac{\partial u}{\partial x_j} \right)$, $j = \overline{1, m}$, where $\alpha_j(\cdot) \geq o$, $\sum_{j=1}^{m} \alpha_j^{\frac{n}{n-s}} = 1$, $n > s > o$, $m > 1$,

$\alpha = (\alpha_1 \ldots \alpha_m) \in M$. The equation of heat conductivity corresponds a case when s=1, i.e. heat exchange occurs under the usual law of Newton. Therefore at a choice of a set of parameters $\alpha = (\alpha_1 \ldots \alpha_m)$ from M at $s=1$. Then we have the equations with extreme properties

$\frac{\partial u}{\partial t} = \max_{\alpha \in M} \Big|_{s=1} \sum_{j=1}^{m} \alpha_j \frac{\partial}{\partial x_j} \left(K \cdot \frac{\partial u}{\partial x_j} \right)$, and $\frac{\partial^2 u}{\partial t^2} = \max_{\alpha \in M} \Big|_{s=1} \sum_{j=1}^{m} \alpha_j \frac{\partial}{\partial x_j} \left(K \frac{\partial u}{\partial x_j} \right)$, $x \in G$, $o < t \leq t_k$

And therefore, on the basis of the basic theorem we shall receive the nonlinear equations of type

$$\left(\frac{\partial u}{\partial t} \right)^n = \sum_{j=1}^{m} \left[\frac{\partial}{\partial x_j} \left(K \frac{\partial u}{\partial x_j} \right) \right]^n, \quad \begin{matrix} x \in G \\ o < t \leq t_k \end{matrix} \quad \left(\frac{\partial^2 u}{\partial t^2} \right)^n = \sum_{j=1}^{m} \left[\frac{\partial}{\partial x_j} \left(K \frac{\partial u}{\partial x_j} \right) \right]^n, \quad x \in G, \ o \leq t \leq t_k$$

The right parts of last equations corresponds to a maximum quantity of heat and a wave in area G, formed as a result of maximization of the previous equations on a set $\alpha \in M\Big|_{s=1}$. For the solutions of last equations (for example, the first equation) is necessary to set a class of possible solutions or simple type, $\frac{\partial u}{\partial t} = c$, $\frac{\partial}{\partial x_j}\left(K\frac{\partial u}{\partial x_j}\right) = c_j$, $j = \overline{1, m}$, or exponent type $\frac{\partial u}{\partial t} = cu$, $\frac{\partial}{\partial x_j}\left(K\frac{\partial u}{\partial x_j}\right) = c_j u$, $j = \overline{1, m}$,

where c_j, c are solutions of the coordination equation: $\sum_{j=1}^{m} c_j^n = c^n$.

Example 4. *Extreme integrals***.** Let numbers P_k, $k = -\infty, \dots + \infty$ are given which satisfying next inequalities: $\dots < P_{-k} < \dots < P_{-1} < P_o < P_1 < \dots < P_k < \dots,$ and $P_k \to \infty,\ \ P_{-k} \to -\infty$ at $k \to \infty$ beside let measurable set D is given also and the function $f(x),\ x \in D$ is determined. Let's define sets

$D_k = \left\{\begin{matrix} x: \ x \in D,\ P_k \le f(x) \le P_{k+1} \\ k = 0, \pm 1, \pm 2, \end{matrix}\right\}$. Using these sets we form the sum $\sum_k P_k \, meas\, D_k$

Splitting $\delta_л = meas\, D_k$ we shall define from any set $\left\{\delta_k:\ \sum_k \delta_k^{\frac{n}{n-s}} = \Delta,\quad n > s,\ \ s > 0\right\}$

It is easy to see, that on set the integral sum accepts the maximal value$\left(\Delta \sum_k P_k^n\right)^{\frac{1}{n}}$. If at $\Delta \to 0$ there is a limit we shall name it in integral from function $f(x)$ on the set D in extreme sense and shall write.

$\left(\Delta \sum_k P_k^n\right)^{\frac{1}{n}} \to \left(\int_D f^n(x)dx\right)^{\frac{1}{n}}$. The integral $I_n = \left(\int_D f^n(x)dx\right)^{\frac{1}{n}}$ forms a sequence of integrals and at $n=1$ coincides with usual integral.

Example 5. *The function* ψ. Let there is a bunch of particles with the certain pulse and some energy. It is known, that such bunch of particles is described by wave function ψ, i.e. amplitude of probability. This wave falls on the screen in a crack, and is farther from these cracks leaves as a spherical wave and on the following screen, these waves are unreferenced. Summing wave from the top and bottom cracks of the screen is organized as a wave $\psi = \sum_{j=1}^{k} \alpha_{j,k} \psi_{jk}$,

where $k = 2$ for one particle and $k = 2^m$ for m particles, α_{jk} accordingly shares of waves leaving cracks in general summing waves. We shall assume, that $\sum_{j=1}^{k} \alpha_{jk}^{\frac{n}{n-s}} = 1,$ where $\alpha^n{}_{jk}$ the appropriate probabilities, n, s parameters of the environment of distribution (for example $n = 2,\ s = 1$). Then considering integrated approach wave function, i.e. $\psi = P + iP^*,\ \psi_{jk} = h_{jk} + ih_{jk}$ on the basis of a principle of optimum representation we have: $extrP_k^{\,n} = \sum_{j=1}^{k} h_{jk}^{n}$, $extrP_k^{*s} \cdot extrP_k^{\,n-s} = \sum_{j=1}^{k} h_{jk}^{*s}\, h_{jk}^{n-s}$,where $extr$ =(max at n>s, and min at n<s). The first formula is characterized probability of particles detection, and the second is interference picture. Using μ-transformation $h'_{i\,k+1} = x\, h_{i\,k}$, $h'_{k+1k+1} = y\; extr\, P_k$, $P'_{k+1} = z\, extr\, P_k$ we are easily established connections by conditions for various particles. Here *(x, y, z)* is some solutions of the equation $x^n + y^n = z^n$.

<u>Example 6</u>. *Some properties of* $\mu-$ *function and it application*. Let's consider so-called $\mu-$function:

$$\mu(\alpha) = \left(\alpha\, x^s + \left(1 - \alpha^{\frac{n}{n-s}}\right)^{\frac{n-s}{n}} y^s\right)^{1/s}, \quad s > 0,$$

where *x*, *y* - positive numbers, *n*, *s* - natural numbers, $0 < \alpha < 1$.

<u>Properties 1.</u> Function $\mu(\alpha), 0 < \alpha < 1$ in a point $\alpha^* = \left[\frac{x^n}{x^n + y^n}\right]^{\frac{n-s}{n}}$ at $s < n$ has maximal, and at $s > n$ minimal values equal $z = \left(x^n + y^n\right)^{1/n}$,

i.e. $x^n + y^n = z^n$, where $z = \max\limits_{0<\alpha<1} \mu(x)$ at $s < n$ and $z = \min\limits_{0<\alpha<1} \mu(x)$ at $s > n$.

Really, as

$$\frac{d\mu}{d\alpha} = \frac{1}{s}\left[\alpha x^s + \left(1 - \alpha^{\frac{n}{n-s}}\right)^{\frac{n-s}{n}} y^s\right]^{\frac{1-s}{s}} \cdot \left[x^s - \alpha^{\frac{s}{n-s}}\left(1 - \alpha^{\frac{n}{n-s}}\right)^{-\frac{s}{n}} \cdot y^s\right]$$

and

$$\frac{d^2\mu}{d\alpha^2} = \frac{1}{s}\left(\frac{1}{s} - 1\right)\left[\alpha x^s + \left(1 - \alpha^{\frac{n}{n-s}}\right)^{\frac{n-s}{n}} \cdot y^s\right]^{\frac{1}{s}-2}\left[x^s - \alpha^{\frac{s}{n-s}}\left(1 - \alpha^{\frac{n}{n-s}}\right)^{-\frac{s}{n}} \cdot y^s\right]^2 +$$

$$-\frac{1}{n-s}\left[\alpha^{\frac{s}{n-s}-1}\left(1-\alpha^{\frac{n}{n-s}}\right)^{-\frac{s}{n}}+\left(1-\alpha^{\frac{n}{n-s}}\right)^{-\frac{s}{n}-1}\cdot\alpha^{\frac{2s}{n-s}}\right]\cdot y^{s}$$

$$\left[\alpha x^{s}+\left(1-\alpha^{\frac{n}{n-s}}\right)^{\frac{n-s}{n}}\cdot y^{s}\right]^{\frac{1-s}{s}},$$

that from a condition $\left.\frac{d\mu}{d\alpha}\right|_{\alpha*}=0$ we have $\alpha*=\left[\frac{x^{n}}{x^{n}+y^{n}}\right]^{\frac{n-s}{n}}, 0<\alpha^{*}<1$. It is easy to see, that $\left.\frac{d^{2}\mu}{d\alpha^{2}}\right|_{\alpha*}<0,$ at $s<n$ and $\left.\frac{d^{2}\mu}{d\alpha^{2}}\right|_{\alpha*}>0,$ at $s>n$. Hence, the point $\alpha*, 0<\alpha*<1$ at $s<n$ is unique point of a maximum of function μ: $z=\max\limits_{0<\alpha<1}\mu(x)=\mu(\alpha*)$ and at $s<n$ is a unique point of a minimum of function μ: $z=\min\limits_{0<\alpha<1}\mu(x)=\mu(\alpha*)$. Besides $z=\left(x^{n}+y^{n}\right)^{1/n}$ and $x^{n}+y^{n}=z^{n}.$

<u>*Properties 2.*</u> If *(x, y, z)* is the decision of the equation $x^{n}+y^{n}=z^{n}$ at some n the point $\alpha=\left(\frac{x^{n}}{x^{n}+y^{n}}\right)^{\frac{n-s}{n}}$ is a unique point extreme of the functions $\mu(\alpha), 0<\alpha<1.$

Really, let (*x, y, z)* is the decision of the equation $x^{n}+y^{n}=z^{n}$. As $x^{n-s}\cdot x^{s}+y^{n-s}y^{s}-z^{n-s}z^{s}$, having entered designations $\alpha=\left(\frac{x}{z}\right)^{n-s}, \beta=\left(\frac{y}{z}\right)^{n-s}$, we shall receive system of the equation be relative (x, y, z):

$$\begin{cases}\alpha x^{s}+\beta y^{s}=z^{s}\quad, \ \alpha+\beta>1, \ \alpha+\beta<2\\ x^{s}-\alpha^{\frac{s}{n-s}}z^{s}=0\\ y^{s}-\beta^{\frac{s}{n-s}}z^{s}=0\end{cases}$$

Last system be relative (x^{s}, y^{s}, z^{s}) has the unique decision (it is a positive) as the determinant of system is equal to zero $\det = \alpha^{\frac{n}{n-s}} + \beta^{\frac{n}{n-s}} - 1 = 0.$. From here $\beta = \left(1 - \alpha^{\frac{n}{n-s}}\right)^{\frac{n-s}{n}}$ and from 1-st equation of system we shall receive value $z = \left(\alpha x^s + \left(1 - \alpha^{\frac{n}{n-s}}\right)^{\frac{n-s}{n}} y^s\right)^{1/s}$ and function $\mu = \left(\alpha x^s + \left(1 - \alpha^{\frac{n-s}{n}}\right)^{\frac{n-s}{n}} y^s\right)^{1/s}$.

<u>*Properties 3*</u>. All decisions of the equation $x^n + y^n = z^n$ are represented in the following parametrical formulas: $x = z\, t^{\frac{1}{n}},\ y = z(1-t)^{\frac{1}{n}},\ 0 < t < 1$, (*)

where t and z any positive numbers, and $t = \alpha^{\frac{s}{n-s}}$, $0 < \alpha < 1$, $z = k^{\frac{1}{n}}$. Really, using presentations $\alpha^{\frac{n}{n-s}} = \dfrac{x^n}{x^n + y^n}$, $\beta^{\frac{n}{n-s}} = \dfrac{y^n}{x^n + y^n}$ we have homogeneous system of the algebraic equations be relative (x^n, y^n):

$$\begin{cases} \left(1 - \alpha^{\frac{n}{n-s}}\right) x^n + \alpha^{\frac{n}{n-s}} y^n = 0 \\ -\beta^{\frac{n}{n-s}} x^n + \left(1 - \beta^{\frac{n}{n-s}}\right) y^n = 0 \end{cases} \quad \text{or} \quad \begin{cases} \left(1 - \alpha^{\frac{n}{n-s}}\right) x^n + \alpha^{\frac{n}{n-s}} y^n = 0 \\ -\left(1 - \alpha^{\frac{n}{n-s}}\right) x^n + \alpha^{\frac{n}{n-s}} y^n = 0 \end{cases}$$

As the determinant of system is equal to zero it has the not trivial decision $x^n = \alpha^{\frac{n}{n-s}},\ y^n = \left(1 - \alpha^{\frac{n}{n-s}}\right),\ 0 < \alpha < 1$. By virtue of uniformity of system, its decision are represented as $x^n = k\,\alpha^{\frac{n}{n-s}},\ y^n = k\left(1 - \alpha^{\frac{n}{n-s}}\right),\ k = const.$. From here $x = k^{1/n}\, \alpha^{\frac{1}{n-s}},\ y = k^{1/n}\left(1 - \alpha^{\frac{n}{n-s}}\right)^{1/n},\ z = k^{1/n}$ and having entered designations $t = \alpha^{\frac{s}{n-s}}$ all decisions of the equation $x^n + y^n = z^n$ we shall copy as (*).

The remark. *For any decisions (*) are fair estimations*

1). 2 $z^s < x^s + y^s < 2^{1-\frac{s}{n}} z^s,\quad s < n,$ *2).* $2^{1-\frac{s}{n}} z^s < x^s + y^s < z^s,\quad n > s,$ *and also for them takes place formulas:*

$$\int_0^1 \frac{x^s}{y^s}\,dt = \frac{\left(\frac{s}{n}\right)\pi}{\sin\left(\frac{s}{n}\right)\pi}, \qquad \int_0^1 \frac{y^s}{x^s}\,dt = \frac{\left(\frac{s}{n}\right)\pi}{\sin\left(\frac{s}{n}\right)\pi}.$$

Corollary. *At n > 2 decisions (*) are not the whole positive numbers.*

It is necessary to note, that at n=2 decisions (*) can be integers and no integers. For example, all decisions of type (*) at $t=\left(\frac{2j}{j^2+1}\right)^2$ *are integer's numbers and they are represented as:* $x=j$, $y=\frac{j^2-1}{2}$, $z=\frac{j^2+1}{2}$ *at odd* j *and* $x=2j$ $y=\left(j^2-1\right)$, $z=\left(j^2+1\right)$ *at even j, j=1,2,3,4, ….*

The method of extreme representation consist in describing stats of objects and real processes with help of extreme equation and reducing it to algebraic representations.

For everything $\sum_{i=1}^{k}\omega_i^{\frac{n}{n-s}}=1$, $n > s > 0$ $\omega_i \ge 0,\ i=1,\dots,k$ right part of the equation (2) has the maximal value that there corresponds the worst condition of system, i.e. the initial equation has represented in next form

$$Y_k = \max_{\omega\in M}\left(\sum_{i=1}^{k}\omega_i Y_{ik}^{\ s}\right)^{1/s} \tag{2.5}$$

where $M=\left\{\omega: 0\le\omega_j\le 1,\ \sum_{j=1}^{k}\omega_j^{\frac{n}{n-s}}=1,\ n>s,\ s>0,\ k>1\right\}$.

The Theorem 2. The equation (1.4) and the equation

$$Y_k^{\ n} = \sum_{i=1}^{k} Y^n{}_{ik} \tag{2.5'}$$

are equivalent. Representations (1.1), (1.2), (1.3) or (1.5),(1.5') and trees of numbers appropriate to them are optimum.

The Proof: *The necessity***.** We shall introduce a designation $Z=Y_k$, $X_j=Y_{jk}$, $j=\overline{1,k}$. Let the condition (1.5') takes place then

$$Z^n = \sum_{j=1}^{m} X_j^{\ n} \tag{2.6}$$

Let's show, validity (1.4) i.e.

$$Z = \max_{\omega\in M}\left(\sum_{j=1}^{k}\omega_j X_j^{\ s}\right)^{1/s} \tag{2.7}$$

Let $\left(X_1, X_2, \ldots, X_k, Z\right)$ is the decision of the equation (1.6), then having entered a designation

$$\omega_j = \left(\frac{X_j^n}{Z^n}\right)^{\frac{n-s}{n}},$$

from (6.1) we have the following system:

$$\omega_1 X_1^s + \ldots + \omega_k X_k^s - Z^s = 0,\ X_j^s - \alpha_j^{\frac{s}{n-s}} Z^s = 0\ , \tag{2.8}$$

As $\left(X_1, X_2, \ldots, X_k, Z\right)$ is the decision (2.6), i.e. (2.4), it is easy to see $\sum_{j=1}^{k} \omega_j^{\frac{n}{n-s}} = \dfrac{\sum_{j=1}^{k} X_j^n}{Z^n} = 1$, as, hence, determinant of system (2.8) is equal to zero $\sum_{j=1}^{k} \omega_j^{\frac{n}{n-s}} - 1 = 0.$ Really, we apply a method of a mathematical induction, and as

$$\Delta_2 = \omega_1^{\frac{n}{n-s}} - 1,\ \Delta_3 = \omega_1^{\frac{n}{n-s}} + \omega_2^{\frac{n}{n-s}} - 1\ .$$

It can be assumed, that

$$\Delta_k = \sum_{j=1}^{k-1} \omega_j^{\frac{n}{n-s}} - 1,\ k=2,3,4\ldots.$$

Let us show it's validity at $k+1$, is valid, decomposing determinant on *k+1* elements of a line is received:

$$\Delta_{k+1} = \begin{vmatrix} -1 & \omega_1 & \omega_2 & \cdots & \omega_{km-1} & \omega_m \\ -\omega_1^{\frac{s}{n-s}} & 1 & 0 \ldots & 0 & & 0 \\ -\omega_2^{\frac{s}{n-s}} & 0 & 1 \ldots & 0 & & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ -\omega_{m-1}^{\frac{n}{n-s}} & 0 & 0 & \ldots & 1 & 0 \\ -\omega_m^{\frac{s}{n-s}} & 0 & 0 \ldots & 0 & & 1 \end{vmatrix} =$$

$$= (-1)^{k+2} \cdot \left(-\alpha_k^{\frac{s}{n-s}} \right) \begin{vmatrix} \omega_1 & \omega_2 & \dots & \omega_{k-1} & \omega_k \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 & 0 \end{vmatrix} +$$

$$+ (-1)^{2m+2} \Delta_m = (-1)^{k+3} \cdot \alpha_m^{\frac{s}{n-s}} \cdot (-1)^{m+1} \cdot \alpha_m \begin{vmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{vmatrix} +$$

$$+ \sum_{j=1}^{k-1} \omega_j^{\frac{s}{n-s}} - 1 = (-1)^{2k+4} \cdot \omega_k^{\frac{n}{n-s}} + \sum_{j=1}^{k-1} \omega_j^{\frac{n}{n-s}} - 1 = \sum_{j=1}^{k} \omega_j^{\frac{n}{n-s}} - 1 \; .$$

As it was shown and as $\omega \in M$, where $\sum_{j=1}^{k} \omega_j^{\frac{n}{n-s}} = 1$, i.e. it means $\Delta_{k+1} = 0$. From 1st equation (1.6) we have:

$Z^s = \left(\sum_{j=1}^{k} \omega_j X_j^s \right)$ и $\omega_j = \left(\dfrac{X_j^n}{Z^n} \right)^{\frac{n-s}{n}}$. Hence, as,

$\sum_{j=1}^{k} \omega_j X^s \le \left(\sum_{j=1}^{k} \omega_j^0 X_j^s \right) = \sum_{j=1}^{k} \dfrac{X_j^n}{Z^{n-s}}$ т.е. $Z^s \cdot Z^{n-s} = \sum_{j=1}^{k} X_j^n$, from here for any $\omega \in M$, $Z^n = \sum_{j=1}^{k} X_j^n$, $\omega_j^0 = \left(\dfrac{X_j^n}{\sum_{j=1}^{k} X_j^n} \right)^{\frac{n-s}{n}}$. And in equality (1.4) the maximum is reached. Thus $Z = \max\limits_{\omega \in M} \left(\sum_{j=1}^{k} \omega_j X^s \right)^{1/s}$.

The sufficiency: The equation (1.4) let takes place. Let's prove validity (2.6).

Let's designate $Z = \mu(\omega) = \left(\sum_{j=1}^{k} \omega_j X_j^s \right)^{1/s}$, $\omega \in M$. It is easy to see, that from a condition $\frac{\partial \mu}{\partial \omega_j} = 0$ the system of the equations follow

$X_j^s - \omega_k^{-\frac{s}{n-s}} \cdot \omega_j^{\frac{s}{n-s}} \cdot X_k^s = 0,\ j = \overline{1,k}$ and from here $\omega_k^{-\frac{s}{n-s}} \cdot \omega_j^{\frac{s}{n-s}} = \frac{X_j^s}{X_k^s}$ or $\omega_k^{-\frac{n}{n-s}} \cdot \omega_j^{\frac{n}{n-s}} = \frac{X_j^n}{X_k^n}$. To sum last equality on *j* from up *1* to *k*, then we have, $\omega_k^{-\frac{n}{n-s}} = \frac{\sum_{j=1}^{k} X_j^n}{X_k^n}$. Then $\omega_j^{0\ \frac{n}{n-s}} = \frac{X_j^n}{\sum_{j=1}^{k} X_j^n}$ is a point of a maximum of function $\mu(\omega)$, $\omega \in M$ as $(\mu''_{\omega\omega} < 0)$. Let's calculate the value of function $\mu(\omega^0)$. It is easy to see, that

$$Z^s = \sum_{j=1}^{k} \omega_j X_j^s \le \sum_{j=1}^{k} \omega_j^0 X_j^s = \sum_{j=1}^{k} \left(\frac{X_j^n}{Z^n} \right)^{\frac{n-s}{n}} \cdot X_j^s = \sum_{j=1}^{k} \left(\frac{X_j^n X_j^{\frac{sn}{n-s}}}{Z^n} \right)^{\frac{n-s}{n}} =$$

$$= \sum_{j=1}^{k} \left(\frac{X_j^{1+\frac{s}{n-s}}}{Z} \right)^{n-s} = \sum_{j=1}^{k} \frac{X_j^n}{Z^{n-s}} \quad \text{i.e.} \quad Z^s \cdot Z^{n-s} = \sum_{j=1}^{k} X_j^n .$$

And hence $Z^n = \sum_{j=1}^{k} X_j^n$;$\forall \omega \in M$. It is easy to see that

$$Z = \mu(\omega_0) = \left(\sum_{j=1}^{k} \left(\frac{X_j^n}{\sum_{j=1}^{k} X_j^n} \right)^{\frac{n-s}{n}} \cdot X_j^s \right)^{1/s} = \left(\sum_{j=1}^{k} \left(\frac{X_j^n \cdot X_j^{\frac{sn}{n-s}}}{\sum_{j=1}^{k} X_j^n} \right)^{\frac{n-s}{n}} \right)^{1/s} = \left(\sum_{j=1}^{k} \frac{X_j^n}{Z^{n-s}} \right)^{1/s},$$

and $Z^s = \mu^s(\omega_0) = \frac{1}{Z^{n-s}} \sum_{j=1}^{k} X_j^n$, from here $Z = \mu(\omega_0) = \left(\sum_{j=1}^{k} X_j^n \right)^{1/n}$, i.e.

t takes place (6). Thus $Z=\mu(\omega_0)=\left(\sum\limits_{j=1}^{k}\left(\frac{X_j^n}{\sum\limits_{j=1}^{k}X_j^n}\right)^{\frac{n-s}{n}}\cdot X^s\right)^{1/s}=\left(\sum\limits_{j=1}^{k}X_j^n\right)^{1/n}$.

The theorem is proved.

Thus, the equation (1.4'), (1.5') is optimum in sense (4), and the tree of numbers appropriate by this equation represented on fig. 1 also is an optimum tree.

<u>The Theorem 3</u>. The tree of numbers to the appropriate equations (2.1) and (2.5) let is given. Then there is transformation K which translates the solutions (2.5) at $k=m-1$ on the solution (2.5) at $k=m$, i.e.

$$Y=KX \tag{2.9}$$

где

$$K=\begin{pmatrix} x & 0 & \cdots & 0 & 0 & 0\\ 0 & x & \cdots & 0 & 0 & 0\\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots\\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots\\ 0 & 0 & \cdots & x & 0 & 0\\ 0 & 0 & \cdots & 0 & y & 0\\ 0 & 0 & \cdots & 0 & 0 & z \end{pmatrix},\quad x^n+y^n=z^n\ ,$$

$$X=(a_{1,m-1},\dots,a_{m-1,m},N_{m-1},N_{m-1}),\ N_{m-1}=\left(\sum_{j=1}^{m-1}a_{im-1}^n\right)^{\frac{1}{n}}\ ,$$

$$Y=(a_{1m},a_{2m},\dots,a_{1m},N_m),\ N_m=\left(\sum_{j=1}^{m}a_{im}^n\right)^{\frac{1}{n}}.$$

<u>The proof</u>. Let (x,y,z) is the solution $x^n+y^n=z^n$ $n\geq 2$. Transformation (2.6) we shall copy as

$$a_{im}=xa_{im-1}\,,\quad a_{mm}=yN_{m-1}\,,\quad N_m=zN_{m-1},\quad i=1,2,\dots,m-1;\quad k=2,3\dots$$

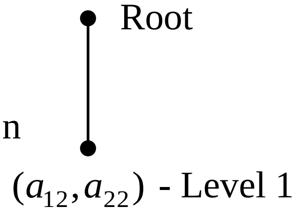
Let $(a_{im-1},\dots,a_{m-1m-1},N_{m-1})$ is the solution (2.5) at $k=m-1$. As $\sum\limits_{j=1}^{m-i}a_{jm-1}^n=N_{m-1}^n$ Multiplying on x^n we shall receive:

$x^n \sum_{j=1}^{m-1} a_{jm-1}^n = x^n N_{m-1}^n$ .From here $\sum_{j=1}^{m-1} (xa_{jm-1})^n = (z^n - y^n) N_{m-1}^n$,

And therefore $\sum_{j=1}^{m} a_{jm}^n = N_m^n$ That it was required to prove.

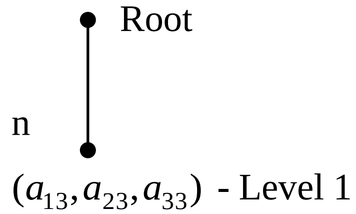
From the theorem 2 follows, that if we have a tree to the appropriate equation

$$N_2^n = a_{12}^n + a_{22}^n \quad ,$$



Then there is transformation K which to translate the given tree on

$$N_3^n = a_{13}^n + a_{22}^n + a_{33}^n \quad ,$$



Etc. and the number of elements at top to the appropriate level 1 is increased by unit and therefore the received trees are "growing", that corresponds to value $n = 2$.

At $n \geq 3$, most likely it cannot be approved, as the equation (1.5) at $k = 2$ in integers numbers not solving. But here, we can instead of N_k^n take any natural number $\tilde{N}_k$ and on the basis of Varring's theorem to receive representation [2] $\tilde{N}_k = a_{1k}^n + a_{2k}^n + ... + a_{kk}^n$,and numbers such as « *a root - level 1, a level 2, a level 3, ..., the level m* » in this case can not exist. But there is a decision of a problem. For trees such as « a root, a level 1 » in a case $n \geq 3$ we can make pasting with trees of numbers at $n = 2$ (see last tree).

We shall consider the equation $N_2^p = a_{12}^n + a_{22}^n$,...

and $N_m^p = a_{1m}^n + a_{2m}^n + ... + a_{mm}^n$, $\qquad m \geq 2,\ n \geq 2,\ 1 \leq p \leq n$. (2.8')

<u>**The Theorem 4**</u>. Transformation $a_{im} = xa_{im-1}, \quad i = \overline{1, m-1},\ a_{mm} = y\sqrt[n]{N_{m-1}^p}$,

$N_m = zN_{m-1}, \quad where \ \ x^n + y^n = z^p$ (2.9)

translates the solution of the equation

$$\sum_{i=1}^{m-1} a_{im-1}^{n} = N_{m-1}^{p} \tag{2.10}$$

on the solution of the equation

$$\sum_{i=1}^{m} a_{im}^{n} = N_{m}^{p} \tag{2.11}$$

Really, we shall increase both parts of the equation (1.10) on x^n with the account $x^n + y^n = z^p$ we have $\sum_{i=1}^{m-1}(a_{im-1}, x)^n = (z^p - y^n)N^{\ p}_{\ m-1}$.

From here, valid (1.9) we shall receive (1.11) and more over we have

$$N_m = z^{m-1},\ a_{1m} = x^{m-1},\ \ a_{2m} = yx^{m-2},\quad a_{im} = yz^{\frac{ip(-2)}{n}}x^{m-i},\ \ i = 2,...,m.,$$

Such any representation of type (1.8) with help of transformation (2.9) is transferred to the algebraic form

$$N_m^p = \left(x^{m-1}\right)^n + \sum_{i=2}^{m}\left(yx^{m-2}z^{\frac{p(i-2)}{n}}\right)^n,$$ or $$N_m^p = X^{\,m-1} + \sum_{i=2}^{m} YX^{\,m-i}Z^{\,i-2}$$ and

$N_m^p = X^{\,m-1} + \sum_{i=2}^{m} A_i X^{\,m-i}$. It should be out that transformations (1.7), (1.9) are described the process of numbers tree grows.

Now we shall consider now examples of Numbers Tree for different numbers.

1). Let $N = 25,\ \ n = 2$, then we have

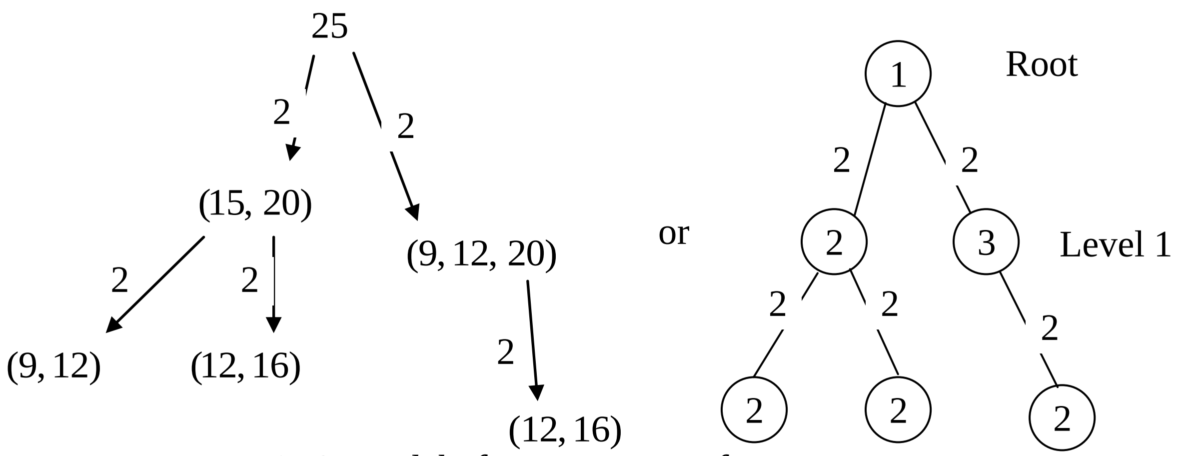


Fig.2. Model of Numbers Tree for $N = 25,\ \ n = 2$

k - means quantity(amount) of an element of the given top of a tree, and number on edges paw decomposition. From here follows, that representation (1.4) takes the following kind

$$25^2 = 9^2 + 2 \cdot 12^2 + 16^2 .$$

2). Now we shall consider number $N = 50$.

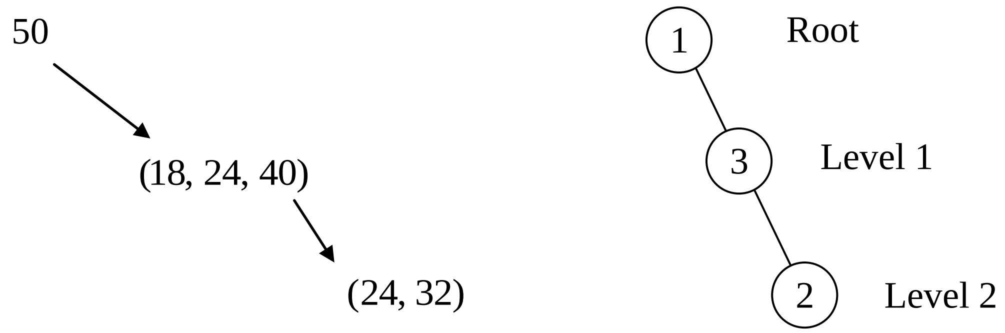


Fig.3. Model of Numbers Tree for $N = 50$

Hence $50^2 = 18^2 + 2 \cdot 24^2 + 32^2$

3). $N = 75$.

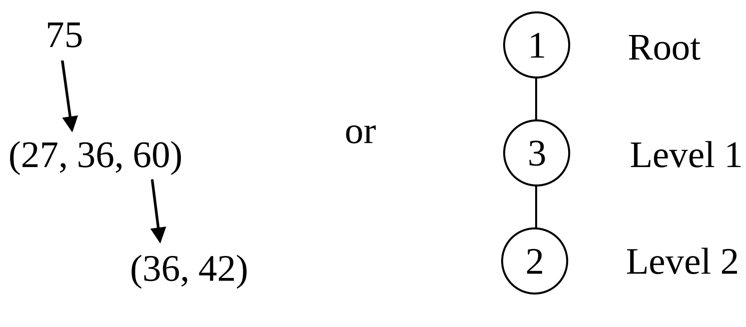


Fig.4. Model of Numbers Tree for $N = 75$

т.е. $75^2 = 27^2 + 2 \cdot 36^2 + 42^2$

4). $N = 100$.

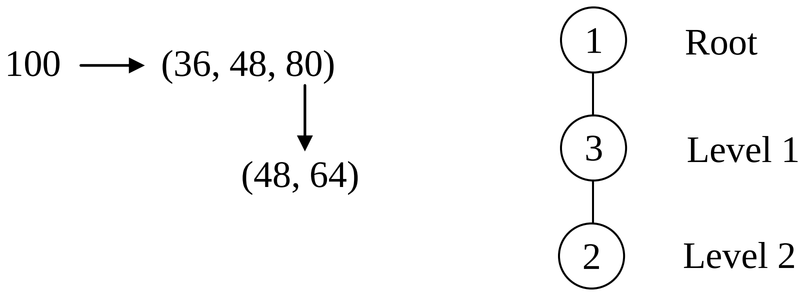


Fig.5. Model of Numbers Tree for $N = 100$

and therefore $100^2 = 36^2 + 2 \cdot 48^2 + 64^2$.

5). $N = 125$.

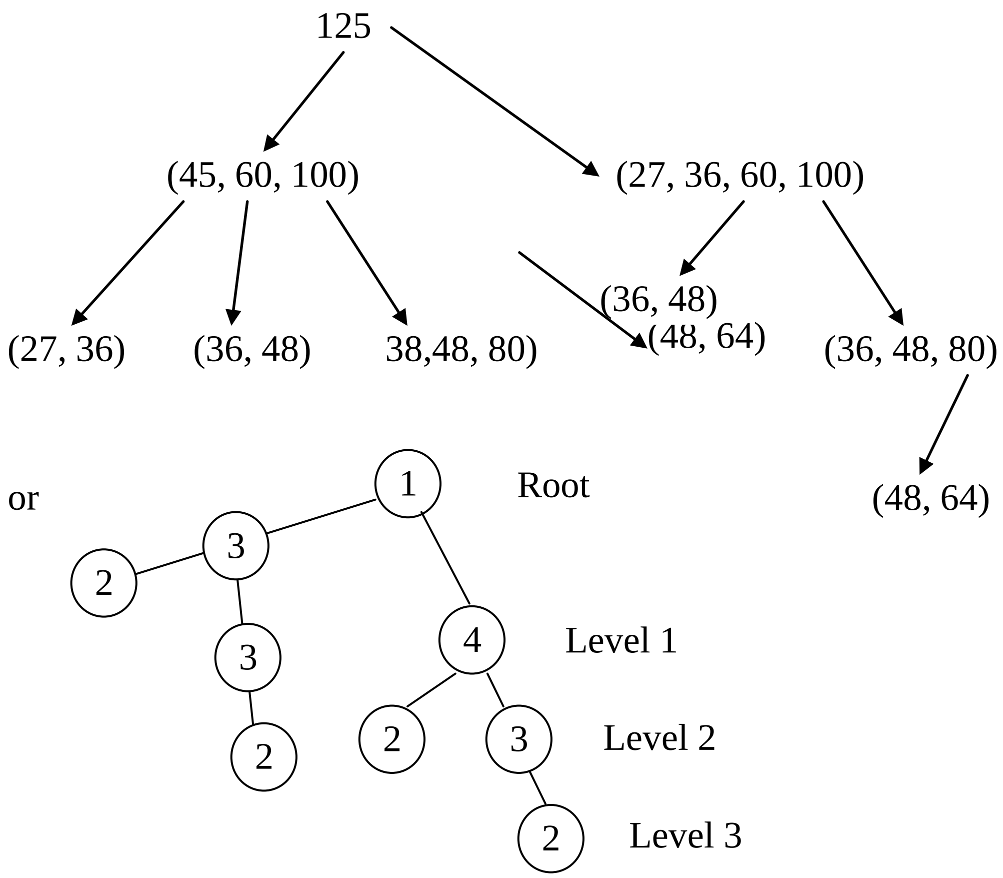


Fig.6. Model of Numbers Tree for $N = 125$

Then $125^2 = 27^2 + 3 \cdot 36^2 + 3 \cdot 48^2 + 64^2$

6). $N = 625,\ n = 2.$

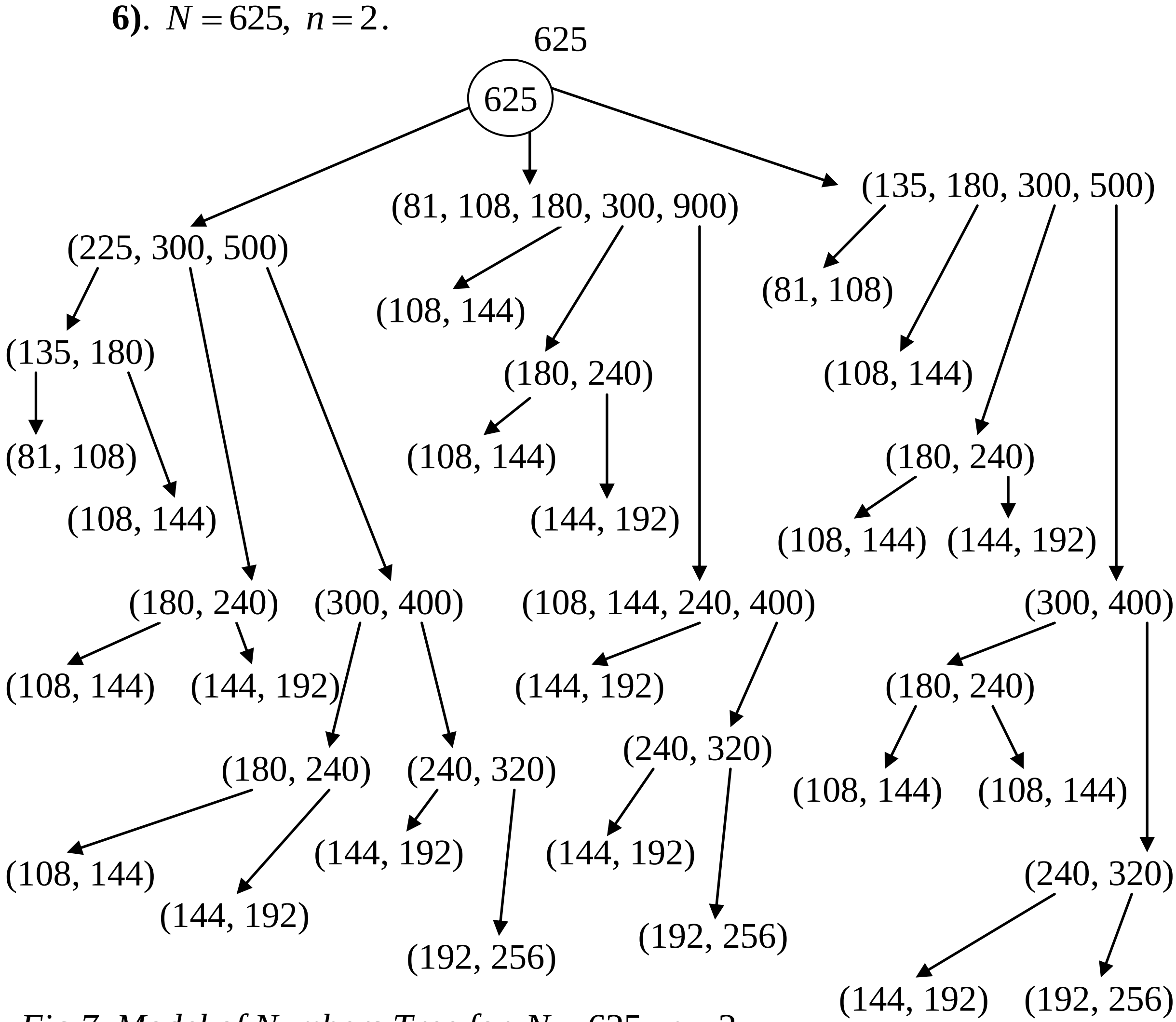
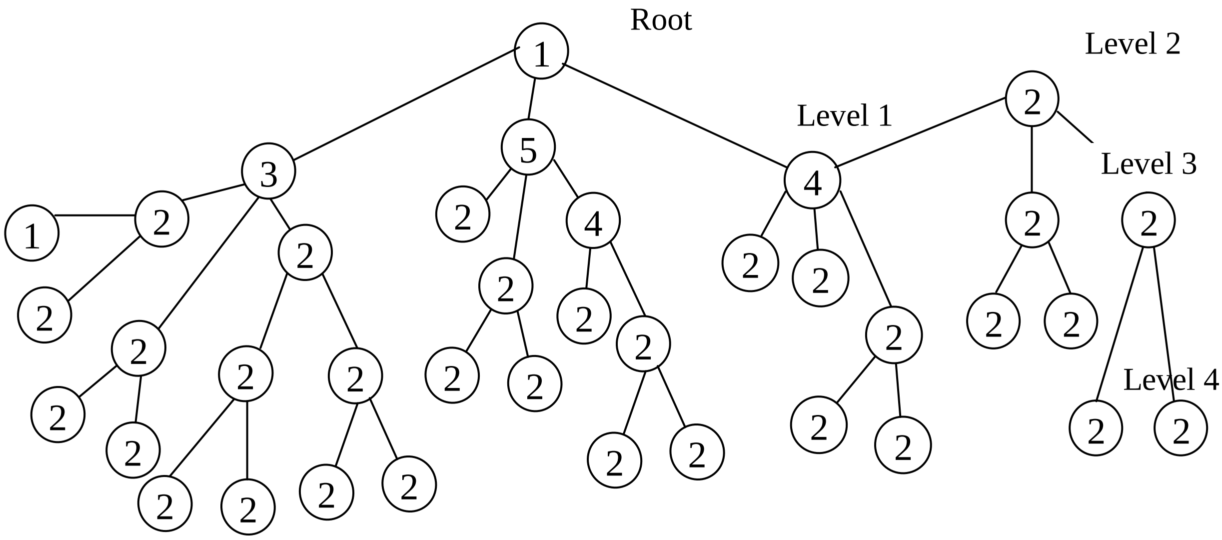


Fig.7. Model of Numbers Tree for $N = 625,\ n = 2$

or



Hence $625^2 = 81^2 + 4 \cdot 108^2 + 6 \cdot 144^2 + 4 \cdot 192^2 + 256^2$.

7). Similarly, for $N = 3125$ we shall receive representation

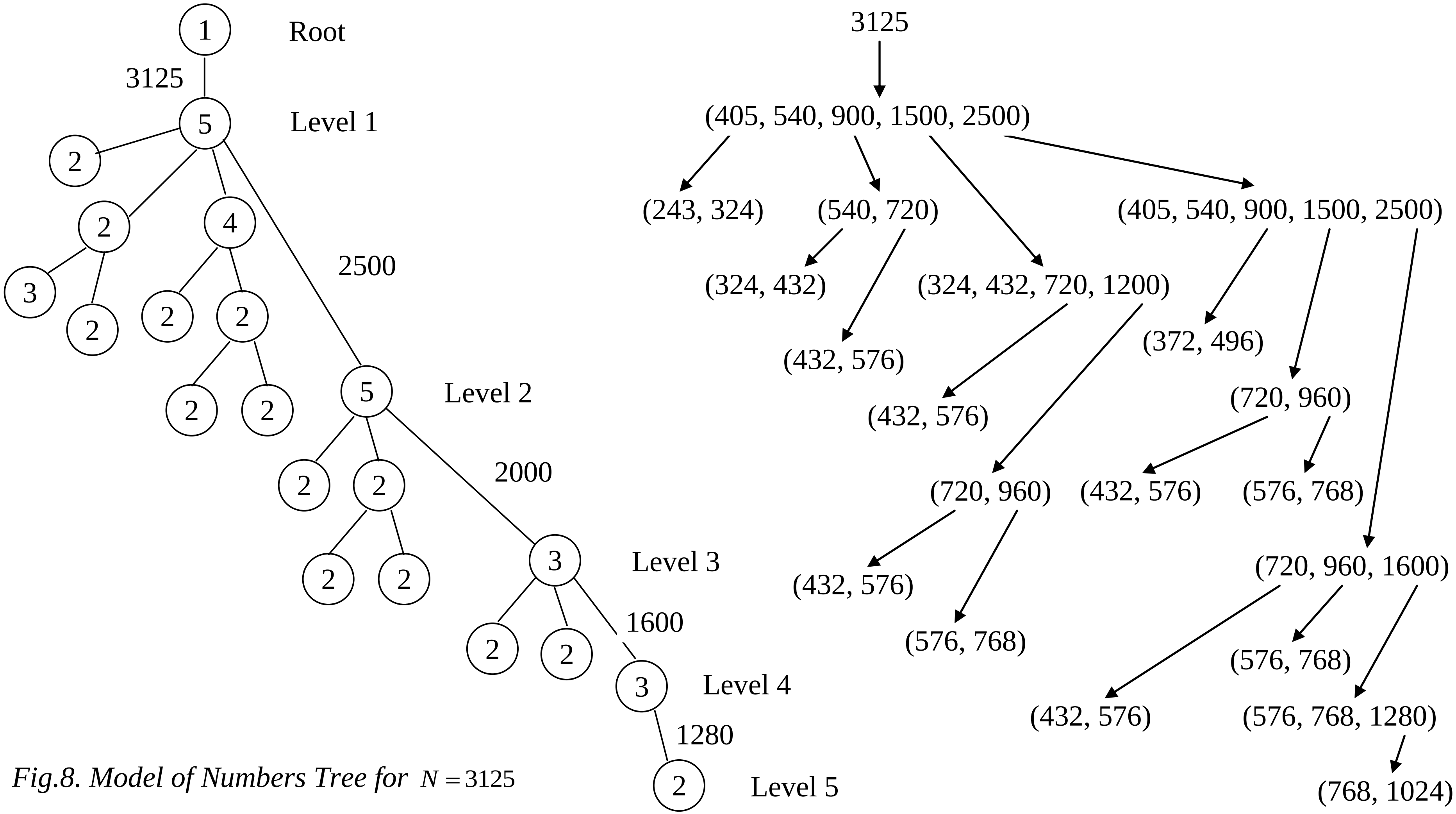


Fig.8. Model of Numbers Tree for $N = 3125$

and $3125^2 = 243^2 + 4 \cdot 324^2 + 372^2 + 8 \cdot 432^2 + 496^2 + 9 \cdot 576^2 + 5 \cdot 768^2 + 1024^2$.

For cases when $n > 2$ not always it is possible to construct graceful examples. We shall result one example. Let $N = 35, \quad n = 3$, then

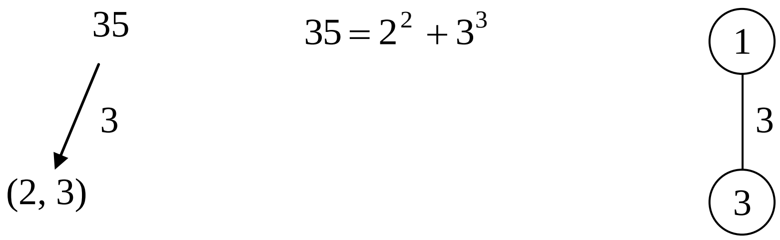


Fig.9

From the given example follows, that we have not received a tree, but only it(him) ветку. We shall name such "trees" not growing trees. For growth of such tree it is necessary to make "cuttings" from another (for example) $n = 2$ a tree and to insert them into not growing trees. For example

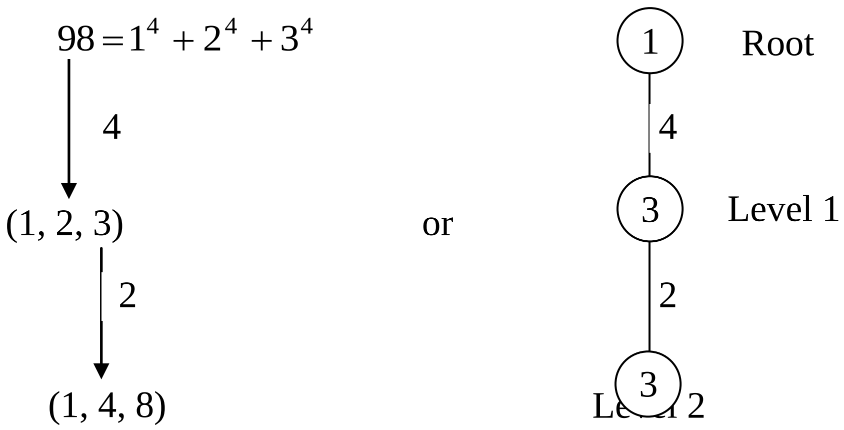


Fig.10.

$$\text{i.e.. } 98 = 1^4 + 2^4 + 1^2 + 4^2 + 8^2.$$

§3. Structure of solutions a number tree model

The tree of decisions represents one of ways of splitting of set of the data on classes or categories. The root of a tree implicitly contains all classified data, and leaves - the certain classes after performance of classification. Intermediate units of a tree represent items(points) of decision making on a choice or performance of testing procedures with attributes of elements of the data which serve for the further division of the data in this unit. Usually] the tree of decisions is determined as structure which consists from

Units - leaves, each of which represents the certain class;

Units of acceptance of decisions, специфицирующих the certain test procedures which should be executed in relation to one of values of attributes; the unit of acceptance of decisions is left with branches which quantity corresponds to quantity(amount) of possible(probable) outcomes of testing procedure. It is possible to consider a tree of decisions and from other point of view: intermediate units of a tree correspond to attributes of classified objects, and arches - to possible alternative values of these attributes. The example of a tree is submitted on figure. On this tree intermediate units represent attributes supervision, humidity, is windy. Leaves of a tree are marked by one of two classes P or H. It is possible to count, that P corresponds to a class of positive copies consultation, and H - to a class negative. For example, P can represent a class " to leave on walk ", and H - the class " to sit at home ". Though it is obvious, that the tree of decisions is the way of representation which is distinct from inducing rules, the tree can compare the certain rule of classification which gives for each object having the appropriate set of attributes (it(he) is submitted by set of intermediate units of a tree), the decision to what from classes to attribute(relate) this object (a set of classes is submitted by set of values of leaves of a tree). In the given example the rule will carry objects to class P or H. Directly it is possible to broadcast a tree in a rule shown below:

If supervision = is cloudy
v
Supervision = sun &
Humidity = it is normal
v
Supervision = rain &
It is windy = there is no
that P

The reason on which the preference is sometimes given trees of decisions, instead of to inducing rules, is, that there are rather simple algorithms of construction of a tree of decisions during processing training sample, and the constructed trees can be used further for correct classification of the objects which have been not submitted in training sample. Now we consider algorithms of extraction of precedents with use of various metrics and the account of factors of importance of parameters of object. On the basis of the modified method of definition of the nearest neighbour described above (the nearest neighbours) the appropriate algorithms of extraction of the precedents, using various metrics for definition of a degree of similarity of precedents with the current problem situation and taking into account factors of importance of parameters of object were developed. Let's consider algorithm of extraction of precedents from SP with use general Euclidian metrics. The entrance data: current situation T (i.e.

the values of the parameters describing a usual situation), CL - nonempty set of precedents (SP), w_1 ..., w_n - weights (factors of importance) parameters, m - quantity of considered(examined) precedents from SP and threshold value of a degree of similarity K. The target data: Set of precedents SC (Set of Cases) which have a degree of similarity (affinity) more or equal threshold value K. The intermediate data: Auxiliary variables i, j (loop variables).

1. SC=Æ, j=1 also we pass to the following step.
2. If j≤m we choose precedent Cj from set CL ($C_{j\hat{I}CL}$) and we pass to a step 3, differently all precedents from SP are considered and we pass to a step 6.
3. We consider distance in Euclidean to the metrics between chosen precedent Cj and current situation T (dCjT) in view of factors of importance of parameters:

$$Y_k = \max_{\omega \in M}\left(\sum_{i=1}^{k} \omega_i Y_{ik}{}^{s}\right)^{1/s}, \text{ where } M = \left\{\omega : 0 \le \omega_j \le 1, \sum_{j=1}^{k} \omega_j{}^{\frac{n}{n-s}} = 1, n > s, s > 0, k > 1\right\},$$

d_{CjT} =argY_k , Y_{ik} =(x_{iC} –x_{iT}). In case of absence of value of parameter x_{iCj} in the
description of precedent Cj we spend calculation of distance d $_{CjT}$, taking into account, that
xi_{Cj} = x_{iT}, and for a case when there is no value of parameter x_{iT} in the description of current situation T calculation of distance d_{CjT} is carried out, believing xiT = max {(x_{iCj} - $x_{iнач}$), (xiкон - x_{iCj})}. Further we pass to the following step.

4. On this step we calculate a degree of similarity S $_{(Cj, T)}$ =1-d_{CjT}/d_{MAX} or in percentage S $_{(Cj, T)}$ = (1-d_{CjT}/d_{MAX}) *100 % if threshold value K is given in percentage, (at calculation dMAX weights of parameters are taken into account) and we pass to a step 5.
5. If S (Cj, T) ≥K given precedent Cj is added in resulting set SC $_{(Cj\hat{I}SC)}$, i.e. the given precedent from БП is taken. After check j=j+1 also we pass to a step 2.
6. If SC=Æ precedents for the current problem situation are not found and we pass to a step 7 with distribution of the message for user about necessity of reduction of threshold value K, differently precedents for the current situation are successfully taken and we pass to the following step.
7. The end (end of algorithm).

In the result, the found precedents can be ordered on decrease of values of their degree of similarity to the current situation and are given to user.

Difference of other metric algorithms from considered consists that on the third step the distance is calculated with the help of other metrics. Realization of the generalized algorithm of extraction of precedents is in the long term possible (in the long term probable) on the basis of various metrics. Use of various metric algorithms of extraction of precedents in systems of expert diagnosing a technical condition of complex objects and, in particular, subsystems of the power unit provides more flexible work of mechanisms of search of the decision on the basis of precedents. At user there is an opportunity to consider various metrics for extraction of precedents from SP systems that provides a choice of more adequate metrics, capable to take into account specificity of a concrete decided(solved) problem) of expert diagnosing. It is necessary to note, that in algorithms of extraction of precedents for the account of factors of importance of parameters of object it can be carried out preliminary a stage (the Step 0) updating of values of borders of ranges of parameters and parameters that excludes necessity for the subsequent account of factors of importance at extraction of some precedents.

§4. Application of the extreme representations method to solution of differential equations . Construction of the modeling equations

Let function $u = u(x,t), \quad x \in E_m, t \geq 0$ $x = x(x_1, x_2, ..., x_m), \ x \in G$, $G \subseteq E^m$ -characterized a condition of some object (either process, or some substance) in a point x at the moment of time t and $\frac{\partial}{\partial t}, \frac{\partial}{\partial x_j}$, $j = \overline{1,m}$ some operators who are carrying out change of a condition of this object (or process) in general(common), and on a direction x_i. Then $a_j \frac{\partial u}{\partial x_j}$ means change of a condition of object on a direction x_j, $\frac{\partial u}{\partial x_j}$ is change of a condition of object as a whole. It is natural to assume, that $\frac{\partial u}{\partial t}$ is formed from the sum $\alpha_j a_j \frac{\partial u}{\partial x_j}$, where $0 < \alpha_j < 1$ and $\sum_{j=1}^{m} \alpha_j^{\frac{n}{n-s}} = 1$ $n > s$ $s \geq 1$.Let's make the following assumption having practical interpretation. The considered system (object) functions so that its condition as a whole was extreme (i.e. the best in any sense). In this connection we receive:

$$\frac{\partial u}{\partial t} = \max_{\alpha \in A} \left(\sum_{j=1}^{m} \alpha_j (a_j(x,t,u) \frac{\partial u}{\partial x_j})^s \right)^{1/s}, \quad s \geq 1, , \tag{4.1}$$

where

$$A = \left\{ \alpha = (\alpha_1, ..., \alpha_m) : \ 0 < \alpha_j < 1, \ \sum_{j=1}^{m} \alpha_j^{\frac{n}{n-s}} = 1 \right\},$$

n, s - are given numbers. $a_j = a_j(.)$ - the given functions of the arguments

$j=1,\ldots,m$; $x=(x_1,x_2,\ldots,x_m), x\in G$ $G\subseteq E^m$. The equation (4.1) describes process of the best condition of considered(examined) object.

<u>The theorem 1.1.</u> That function $u=u\ (x,t)$ satisfied to the equation (4.1), it is necessary and enough that the parity(ratio) took place

$$\left(\frac{\partial u}{\partial t}\right)^n=\sum_{j=1}^{m}\left(a_j(x,t,u)\frac{\partial u}{\partial x_j}\right)^n,\ n>s,\quad s\ge 1, \tag{4.2}$$

i.e. function $u=u(x,t)$ is the decision of the equation (4.2).

Let's consider a special case of the equation with extreme property such as the equation of carry (4.1):

$$\frac{\partial u}{\partial t}=\max_{\alpha\in A}\left(\sum_{j=1}^{m}\alpha_j\left(a_j\frac{\partial u}{\partial x_j}\right)^s\right)^{1/s}, \tag{4.3}$$

where $a_j=a_j(x_j)\in C^1(G),\ j=\overline{1,m}$ - the given functions of the arguments, G-the given set. We shall define(determine) for the equation (4.3) class of decisions. The equation (4.3) represents the two-parametrical nonlinear equation of carry with extreme property and at $s=1$ it turns to the usual equation of carry. Clearly, that the presence of his decision represents difficult, a problem(task), and the obvious decision it is impossible. We shall write for him(it) the appropriate equation such as (4.3) then we have:

$$\sum_{j=1}^{m}\left(a_j\frac{\partial u}{\partial x_j}\right)^n=\left(\frac{\partial u}{\partial t}\right)^n, \tag{4.4}$$

Following for finding of the decision of the equation (4.4) we shall set ***a class of possible decisions such as*** :

1. $\frac{\partial u}{\partial t}=C,\quad a_j\frac{\partial u}{\partial x_j}=C_j\ ,\ j=\overline{1,m}\ ,\ \sum_{j=1}^{m}C_j^m=C^n,$

2. $\frac{\partial u}{\partial t}=Cu\ ,\ a_j\frac{\partial u}{\partial x_j}=C_ju\ ,\ j=\overline{1,m}\ ,$ (4.5)

3. $\frac{\partial u}{\partial t}=Cu\ ,\ \frac{a_j}{(\delta-\varepsilon u)}\cdot\frac{\partial u}{\partial x_j}=C_ju$ 4. $\frac{\partial u}{\partial t}=CF(u,x,t),\ \frac{\partial u}{\partial x_j}=C_jF(u,x,t)$,

5. $\frac{\partial u}{\partial t}=f(u,x,t),\ \frac{\partial u}{\partial x_j}=f_j\ (u,x,t),$ where

$\sum_{j=1}^{m} C_j^n = C^n$ is algebraic representation of numbers tree model, and $\delta, \varepsilon, a_j(.) -$ are the given constants and functions $\delta, \varepsilon, a_j = a_j(x) > 0$, $\sum_{j=1}^{m} f_j^n = f^n$.

Definition. The first type a class of possible decisions we shall name a class of simple solutions, 2 is exponentiations class , 3-й-logistical, and 4, 5 are functional.

The Statement 1. If function $u = u(x,t)$ any decision of the equation (4.2) function $\varphi = \varphi(u) \in C^1$ and all functions of type $\varphi = \varphi(u)$, also are the solution of the equation (4.3).

Really, let function $u = u(x,t)$ any solution of the equation (4.3),as $\frac{\partial \varphi}{\partial t} = \frac{d\varphi}{du}\frac{\partial u}{\partial t}$ and $\frac{\partial \varphi}{\partial x_j} = \frac{d\varphi}{du}\frac{\partial u}{\partial x_j}$ substituting these values in the equation (4.3), we shall receive validity of the given statement.

From the given statement follows, that function $\varphi = \varphi(u)$ is the common decision of the equation (4.2). For definition of private(individual) decisions we shall consider(examine) classes of solutions of the equation (4.3)

The statement 2. For any common solution $\varphi = \varphi(u)$ where $u = u(x,t)$ any decision of the equation (4.3), takes place $\frac{d\varphi}{du} = 1$ on a class of simple solutions (a class 1) and $\frac{d\varphi}{du} = \varphi$ in a class exponential solutions (a class 2). Really, as

$$\frac{\partial \varphi}{\partial t} = \frac{d\varphi}{du}\frac{\partial u}{\partial t} = C,\ \frac{d\varphi}{du}C = C \quad \text{and} \quad \frac{\partial \varphi}{\partial t} = \frac{d\varphi}{du}\frac{\partial u}{\partial t} = Cu,\ \frac{d\varphi}{du}C = Cu,$$

That $\frac{d\varphi}{du} = 1$ and $\frac{d\varphi}{du} = \varphi$. Let's consider the equation (4.4) in a case, when $L_j = a_j \frac{\partial}{\partial x_j}$ $j = \overline{1,m}$, $L = \frac{\partial}{\partial t}$ then we have:

$$\frac{\partial u}{\partial t} = \alpha^{\mathrm{o}}{}_j^{-\frac{1}{n-s}} a_j \frac{\partial u}{\partial x_j}, \quad j=1, \ldots, m \qquad (4.5)$$

And also the initial equation connected to this system (4.4):

$$\left(\frac{\partial u}{\partial t}\right)^n = \sum_{j=1}^{m}\left(a_j\left(x_j\right)\frac{\partial u}{\partial x_j}\right)^n \qquad (4.6)$$

As characteristics of the equation look like

$$\int_0^t \alpha^0{}_j^{-\frac{1}{n-s}}(\tau)d\tau = \int_0^{x_j}\frac{d\varsigma}{a_j(\varsigma)} + const,$$

Those his(its) decisions are represented as follows

$$\phi_j = \phi_j(\zeta), \text{ j=1,2, …, m} \qquad (4.7)$$

where $\zeta = \frac{1}{k}\int_0^t \frac{1}{\alpha^0{}_j^{\frac{1}{n-s}}(\tau)}d\tau + \int_0^{x_j}\frac{d\varsigma}{a_j(\varsigma)}$.

The Theorem 5. At anyone $\phi_j(.)\in C^1$ functions $u(x,t) = \phi_j(\zeta|_{k=1})$, j=1,2, …, m from (4.7) are the decision (4.7), and function

$$u(x,t) = \sum_{j=1}^{m} c_j\,\phi_j(\zeta|_{k=m})$$

is the common decision the equation (4.7), where c_j, j=1,2, …, m any positive numbers. Validity of the theorem is established by direct image. Really, as

$a_j\frac{\partial u}{\partial x_j} = c_j\,\dot{\phi}_j(\zeta)$, $\alpha^0_j = \left(\frac{(c_j\,\dot{\phi}_j(\varsigma))^n}{\sum_{j=1}^{m}(c_j\,\dot{\phi}_j(\varsigma))^n}\right)^{\frac{n-s}{n}}$ and

$\frac{\partial u}{\partial t} = \sum_{j=1}^{m}\frac{1}{m}c_j\,\dot{\phi}_j(\zeta)\frac{1}{\alpha^0{}_j^{\frac{1}{n-s}}}$, $\frac{\partial u}{\partial t} = \left(\sum_{j=1}^{m}(c_j\,\dot{\phi}_j(\zeta))^n\right)^{1/n}$, and

Hence, $\sum_{j=1}^{m}\left(a_j\frac{\partial u}{\partial x_j}\right)^n = \left(\frac{\partial u}{\partial t}\right)^n$ that was required to prove.

We shall consider the equation of type

$$\sum_{j=1}^{m}\left(\frac{\partial u}{\partial x_j}\right)^n=\left(\frac{\partial u}{\partial t}\right)^n, \qquad (4.8)$$

where m,n natural numbers the big units, u required unknown function, $x=(x_1,....x_m)\in E^m, t\geq 0$ we shall Notice, that if $n=1$ we shall receive the usual equation of carry. As we saw for both equations, their decisions depend on some constants C_j and C which are the decision of the equation $\sum_{j=1}^{m} C_j^n = C^n$.

For their definition we use a method. We shall result these decisions. For example, in a case $n=2$ at any $m=2,3,4.....$, knowing the decision, the equations $C_1^2+C_2^2=C^2$ on the basis of transformation gradually we shall define decisions of the equations $C_1^2+C_2^2+....+C_m^2=C^2$. At $m=3,4,5...$, appropriate $C_1=3, C_2=4, C=5$ decisions $C_1, C_2,..., C_m$ with are given in the following tables

$m=3$	$m=4$	$m=5$
$C_1=9$	$C_1=81$	$C_1=81$
$C_2=12$	$C_2=108$	$C_2=108$
$C_3=20$	$C_3=180$	$C_3=180$
$C=25$	$C_4=300$	$C_4=300$
	$C_5=500$	$C_5=500$
	$C=625$	$C=625$

$m=9$	$m=10$	$m=11$
$C_1=6561$	$C_1=19683$	$C_1=59049$
$C_2=8748$	$C_2=26244$	$C_2=78732$
$C_3=14580$	$C_3=43740$	$C_3=131220$
$C_4=24300$	$C_4=72900$	$C_4=218700$
$C_5=40500$	$C_5=121500$	$C_5=364500$
$C_6=67500$	$C_6=202500$	$C_6=607500$
$C_7=112500$	$C_7=337500$	$C_7=1012500$
$C_8=187500$	$C_8=562500$	$C_8=1687500$
$C_9=312500$	$C_9=937500$	$C_9=2812500$
$C=390625$	$C_{10}=1562500$	$C_{10}=4687500$
	$C=1953125$	$C_{11}=7812500$
		$C=9765625$

§ 5. Differential equations with extreme properties 2-th order

Many processes (distributions processes of heats and waves, diffusion processes) belong to so-called model equations with extreme properties. In the general case such equations may be represented in the form of:

$Lu = \max\limits_{\alpha \in M} \left\{ \sum\limits_{j=1}^{m} \alpha_j \left(L_j u \right)^s \right\}^{1/s}$, or $(Lu)^n = \sum\limits_{j=1}^{m} \left(L_j u \right)^n$, $n > s > o,$ here L, L_j are given operators, which characterize the considered physical processes. Let it is given area $G \subseteq E^m$ with border Γ: $\Gamma = \cup \Gamma_j$, and then it is known, that the temperature mode inside area G and wave process satisfy to next equations[2]:

$$\frac{\partial u}{\partial t} = \sum_{j=1}^{m} \frac{\partial}{\partial x_j} \left(k_j \frac{\partial u}{\partial x_j} \right) , \quad \begin{matrix} x \in G \\ o <\le t \le t_k \end{matrix} , \quad \frac{\partial^2 u}{\partial t^2} = \sum_{j=1}^{m} \frac{\partial}{\partial x_j} \left(k_j \frac{\partial u}{\partial x_j} \right), \; x \in G,\; o < t \le t_k . \qquad (5.1)$$

Here $k_j = k_j(\cdot),\ j = \overline{1,m}$ are coefficients of heat conductivity (or wave velocity) characterizing a version of environment. We shall make the following assumption. Let the environment of heat conductivity is those, that

$\frac{\partial}{\partial x_j} \left(k_j \frac{\partial u}{\partial x_j} \right) = \alpha_j \frac{\partial}{\partial x_j} \left(k \frac{\partial u}{\partial x_j} \right),\ j = \overline{1,m},$ where $\alpha_j(\cdot) \ge o,\ \sum\limits_{j=1}^{m} \alpha_j^{\frac{n}{n-s}} = 1,\ n > s > o,\ m > 1,$ $\alpha = (\alpha_1 ... \alpha_m) \in M.$ The equation of heat conductivity (5.1) corresponds (meets) a case when s=1, i.e. heat exchange occurs under the usual law of Newton. Therefore at a choice of a set of parameters $\alpha = (\alpha_1 ... \alpha_m)$ from M at $s=1$. Then we have the equations with extreme properties

$$\frac{\partial u}{\partial t} = \max_{\alpha \in M|_{s=1}} \sum_{j=1}^{m} \alpha_j \frac{\partial}{\partial x_j} \left(K \cdot \frac{\partial u}{\partial x_j} \right), \text{ and } \frac{\partial^2 u}{\partial t^2} = \max_{\alpha \in M|_{s=1}} \sum_{j=1}^{m} \alpha_j \frac{\partial}{\partial x_j} \left(K \frac{\partial u}{\partial x_j} \right), \; x \in G,\; o < t \le t_k \qquad (5.2)$$

And therefore, on the basis of the basic theorem we shall receive the nonlinear equations of type

$$\left(\frac{\partial u}{\partial t} \right)^n = \sum_{j=1}^{m} \left[\frac{\partial}{\partial x_j} \left(K \frac{\partial u}{\partial x_j} \right) \right]^n , \quad \begin{matrix} x \in G \\ o < t \le t_k \end{matrix} \quad \left(\frac{\partial^2 u}{\partial t^2} \right)^n = \sum_{j=1}^{m} \left[\frac{\partial}{\partial x_j} \left(K \frac{\partial u}{\partial x_j} \right) \right]^n , \; x \in G,\; o \le t \le t_k \qquad (5.3)$$

The right parts of last equations corresponds to a maximum quantity of heat and a wave in area G, formed as a result of maximization of the previous equations (5.2) on a set $\alpha \in M|_{s=1}$. For the solutions of last equations (for example, the first equation (5.3)) is necessary to set a class of possible solutions or simple type,

$\frac{\partial u}{\partial t} = c$, $\frac{\partial}{\partial x_j} \left(K \frac{\partial u}{\partial x_j} \right) = c_j$, $j = \overline{1,m}$, or exponent type $\frac{\partial u}{\partial t} = cu$, $\frac{\partial}{\partial x_j} \left(K \frac{\partial u}{\partial x_j} \right) = c_j u,\ j = \overline{1,m},$

where $c_j,\ c$ are solutions of the coordination equation:

$\sum_{j=1}^{m} c_j^n = c^n$. Let area G is a rectangular with the sides: l_1, l_2 , And in the first equation (3) is given (similarly for the second), and also are given initial and boundary conditions: $u\big|_{t=0} = u_0(x_1, x_2),\ (x_1, x_2) \in \overline{G},\ u\big|_{x_j=0} = 0 = u\big|_{x_j=l_j},\ j = \overline{1,2}$. Then the solution is represented in the following kind:

$u(x_1, x_2, t) = \frac{2}{\sqrt{l_1 l_2}} \sum_{n_1, n_2 = 1} D_{n_1 n_1} e^{c_{n_1 n_2} t} \sin \frac{\pi n_1}{l_1} x_1 \sin \frac{\pi n_2}{l_2} x_2,$ where $D_{n_1 n_2}$ are coefficients Fourier of function $u_0(x_1, x_2)$, and parameters $c_{n_1 n_2} = c$ are the solutions of the coordination equation $c_1^n + c_2^n = c^n$, $c_{n_1 n_2} = n\sqrt{\left(\frac{\pi n_1}{l_1}\right)^n + \left(\frac{\pi n_2}{l_2}\right)^n},\ n_j = 1,2,3,...,\ j = \overline{1,2}$.

. Let $G \subset E^2$ is a rectangular with the sides l_1, l_2 and next initial and boundary conditions are given

$u\big|_{t=0} = u_0(x_1, x_2),\ u_t\big|_{t=0} = u_2(x_1, x_2),\ u\big|_{x_i=0} = 0,$ Then the solution of the second equation (3) is represented as

$$u(x_1, x_2, t) = \sum_{n_1 n_2 = 1} \left(A_{n_1 n_2} \cos \sqrt{c_{n_1 n_2}}\, t + B_{n_1 n_2} \sin \sqrt{c_{n_1 n_2}}\, t \right) \sin \frac{(2n_1 + 1)\pi x_1}{2l_1} \sin \frac{(2n_2 + 1)\pi x_2}{2l_2},$$

where $c_{n_1 n_2} = n\sqrt{\left(\frac{(2n_1 + 1)\pi x_1}{2l_1}\right)^n + \left(\frac{(2n_2 + 1)\pi x_2}{2l_2}\right)^n}$, $A_{n_1 n_2}$, $B_{n_1 n_2}$ are defined from initial conditions.

The given equation refers to as equation *Navier-Stokes* and is the basic at calculation of movement viscous incompressible liquids consisting of three equations which are written down as one vector equation. However generally it is not solved methods of modern mathematics and in practice it is necessary to be limited to the decision of only private problems. Solutions of these equations are unknown, and thus even it is not known, how them to solve. However we have found classes of possible general and smooth solutions that equation on the basis of entered earlier to us a principle of extreme conditions that takes place in processes and objects of the real world. We shall be considering that the disorder of energy and pressure will be maximal. Let function $u = u(x, t),\quad t \geq 0$, $x = (x_1, x_2, ..., x_m),\ x \in G,\ G \subseteq E^m$ is a state of some object (or process) in a point x at the moment of time t and $L,\ L_j\ \ j=\overline{1,m}$ are some operators who are carrying out change of a condition of this object (or process) in general and on a direction x_i. Then $L_j u$ means change of a condition of object on a direction x_j; $L_j u$ is change of a condition of object as a whole. It is natural to assume, that Lu is formed from the sum $\alpha_j L_j u$, where and $0 \le \alpha_j \le 1$ и $\sum\limits_{j=1}^{m} \alpha_j^{\frac{n}{n-s}} = 1$, $n > s$, $s > 0$, i.e. $\alpha = (\alpha_1 ... \alpha_m) \in M$. *Let us formulate the following principle having the important practical interpretation:* ***any system (or object) functioning so that its state as a whole was extreme (i.e. best in any sense) in the future.*** On the base of this principle, we have:

$$Lu = \max_{\alpha \in M} \left(\sum_{j=1}^{m} \alpha_j (L_j u)^s \right)^{1/s}, \quad s > 0, \tag{5.4}$$

where $M = \{\alpha = (\alpha_1, \ldots, \alpha_m):\ 0 \le \alpha_j \le 1,\ \sum\limits_{j=1}^{m} \alpha_j^{\frac{n}{n-s}} = 1\}$, $n > s$, s >0 are given numbers. The equation (5.4) describes process of the best functioning of considered system (or object, or processes).

<u>The Theorem</u>. That the function $u = u\ (x, t)$ satisfied the equation (5.4) it is necessary and enough, that

$$(Lu)^n = \sum_{j=1}^{m} \left(L_j u \right)^n, \tag{5.5}$$

and moreover, it is fair

$$Lu = \alpha^0{}_j^{-\frac{1}{n-s}} L_j u, \quad \text{where} \quad \alpha_j^0 = \left(\frac{(L_j u)^n}{\sum\limits_{j=1}^{m} (L_j u)^n} \right)^{\frac{n-s}{n}}.$$

We consider well known equation

$$\frac{\partial v_i}{\partial t} + \sum_{j=0} v_j \frac{\partial v_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + d \sum_{j=0} \frac{\partial^2 v_i}{\partial x_j{}^2} \quad -\infty \prec x_i \prec \infty,\ \ t \ge 0$$

Let $v = \sum\limits_i \alpha_i \vartheta_i$, $\alpha_i \in \mathrm{M}$. Then we have

$\frac{\partial v}{\partial t} + \sum\limits_i v_i \frac{\partial v}{\partial x_i} = -\frac{1}{\rho} \sum\limits_i \frac{\partial p}{\partial x_i} + d \sum \frac{\partial^2 v}{\partial x_i{}^2} \quad x_j = x_j / \alpha_j$. We also suppose that $v_i = \delta_i v$, where $\delta_i \in \mathrm{M}^{n,s}, s=1$ *then* $\sum_j \delta_j \frac{\partial v^2}{\partial x_j} = \left[-\frac{1}{\rho}\frac{\partial p}{\partial x} + d\sum_j \frac{\partial^2 v}{\partial x_j{}^2} - \frac{\partial v}{\partial t} \right].$ We shall maximize

left part of last equation on parameters $\delta = (\delta, ... \delta_m) \in \mathrm{M}$ and have

$$\sum_j \left(\frac{\partial v^2}{\partial x_j} \right)^n = \left[-\frac{1}{\rho}\frac{\partial p}{\partial x} + v\sum_j \frac{\partial^2 v}{\partial x_j{}^2} - \frac{\partial u}{\partial t} \right]^n \tag{5.6}$$

For solution (3) we consider next class possibility solution: 1). $\frac{dv^2}{dx_j} = c_j$ the simple class and 2). $\frac{dv^2}{dx_j} = c_j v^2$ the exponential class, where $\sum c_j{}^n = c^n$ is coordination equations. In the beginning we stall consider the exponential class

$$\frac{\partial v^2}{\partial x_j{}^2} = c_j v^2 \quad and \quad \frac{\partial u}{\partial t} = \sum_j \frac{\partial^2 u}{\partial x_j{}^2} + cv + \frac{1}{\rho}\frac{\partial p}{\partial x} \tag{5.7}$$

Such we can take $\frac{1}{\rho}\sum_i \alpha_i \frac{\partial p}{\partial x_j} = \rho \sum \frac{\partial^2 u}{\partial x^2{}_j} - \frac{\partial u}{\partial t} + cv$ and we have

$$\sum \left(\frac{\partial p}{\partial x_j} \right)^n = \left[\rho \sum \frac{\partial^2 u}{\partial x_j} - cv \right].$$

It to allow proving, that the decision exists and is enough smooth function and it will allow essentially changing ways of realization hydro-aerodynamic calculations. We shall write some classes of possible solutions (them much - by virtue of a generality of the description of real processes by this the equation) for the initial equation

$$\frac{\partial V^2}{\partial x_i} = \begin{cases} C_i \text{, the simple class} \\ C_i V\text{, the exponential class} \end{cases}, \quad \frac{\partial P}{\partial x_i} = \begin{cases} d_i \text{, the simple class} \\ d_i V, \\ d_i P\text{, the exponential class} \end{cases},$$

$$\left(v\sum_{j=1}^{3} \frac{\partial^2 V}{\partial x_j^2} - \frac{\partial V}{\partial t} - \begin{cases} C\text{, the simple class} \\ CV \\ CP\text{, the exponential class} \end{cases} \right. = \frac{1}{\rho} \begin{cases} d_i \text{, the simple class} \\ d_i V, \\ d_i P\text{, the exponential class} \end{cases},$$

where C_j, C, d_j, d be show solutions the so-called equation of the coordination of capacity (the coordination equation): $\sum_{j=1}^{m} C_j^n = C^n$ and $\sum_{j=1}^{m} d_j^n = d^n$, and for a simple class we shall write the appropriate general and smooth solution*:*

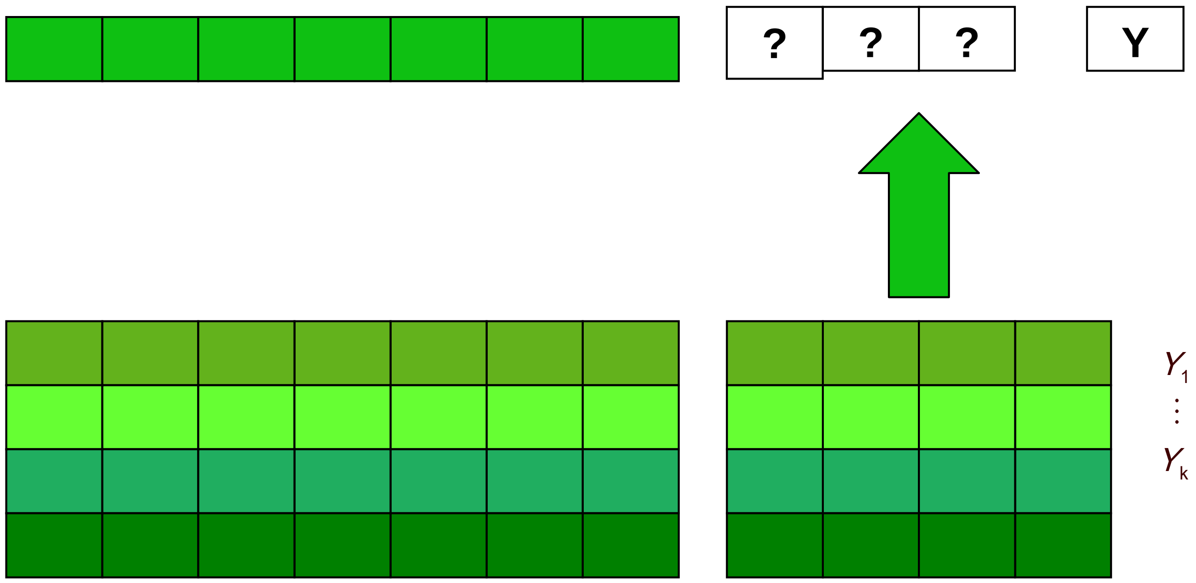
$$V(x,t)=\sqrt{\left(\int_{-\infty}^{\infty}G(x,\xi)V(\xi,0)d\xi+\int_{0}^{t}dt\int_{-\infty}^{\infty}G(x,\xi)\,(C+\frac{d}{\rho})\,d\xi\right)^{2}+\sum_{j=1}^{3}C_{j}x_{j}}\quad,\ P(x)=P(0)+\sum_{j=1}^{3}d_{j}x_{j}\ ,$$

$$G(x,\xi)=\left(\frac{1}{2\sqrt{\pi\,\nu\,t}}\right)^{3}e^{\frac{\sum_{j=1}^{3}\left(x_{j}-\xi_{j}\right)^{2}}{4\nu t}}\ ,\ x=(x_{1},x_{2},x_{3}),\quad \xi=(\xi_{1},\xi_{2},\xi_{3}),\ V=\sum_{i=1}^{2}\alpha_{i}V_{i}$$

§6. Models of Development of Losses in the Worst Condition by Kinds with Long Settlement - a modification method of the nearest neighbour

Now we consider questions of construction and investigation a new method of calculation a size of losses by kinds with long settlement: a modification method of the nearest neighbor. The proposition method is to define a size of losses in the worst condition of system and is based on using of so-called model of numbers tree. By using this method for model, data showed and carried out some computational experiments. It is noticed that the propose method basis on numbers tree model, is a simple and universal method for definition of value of Losses in the Worst Condition by Kinds with Long Settlement and it is easy programmed on all computer languages. Receiving formulas are controlled our calculations. As the methodology actuaries calculations uses the probability theory, given and the long-term statistical data, financial calculations that on faculty are in full read to a demography rates connected with System mathematical and the statistical regularities establishing mutual relation between the insurer and the insurant. They reflect as mathematical formulas the mechanism of formation (education) and an expenditure of insurance fund in long-term insurance operations. To them also carry calculations of tariffs on any kind of insurance: life’s pensions, from accidents, property, work capacity. The methodology actuary’s calculations use the probability theory, given to demography and the long-term statistical data, financial calculations. By means of the last in tariffs the income which is received by the insurer from use as credit resources of the accumulated payments of insurants is taken into account. Except for that rates on actuaries are read to calculations which is connected to one of the widespread problems(tasks) of such -statistical calculations connected to definition of norms and conditions of insurance, is, that the sum of insurance payments minus relying payments guaranteed reception by insurance firm (or the state organization) expected results. Making "tree" of decisions, it is necessary to draw "trunk" and the "branches" displaying structure of a problem. "Trees" from left to right settle down."Branches" designate possible alternative decisions which can be accepted, and the possible outcomes arising as a result of these decisions. On the circuit we use two kinds of "branches": The first - the dashed lines connecting squares possible(probable) decision, the second - the continuous lines connecting circles of possible(probable) outcomes. Square "units" designate places where it is made a decision, round "units" - occurrence of outcomes. As accepting the decision can not influence occurrence of outcomes, it needs to calculate probability of their occurrence only. When all decisions and their outcomes are specified on "tree", each of variants is counted, and in the end its monetary income is put down. All charges caused by the decision, are put down appropriate "branch".

As is known, the forecast of the future losses, using observably average development of the nearest neighbours, usually define under the following circuit [1,3]:



On the basis of given to the circuit the model of the nearest neighbour assumes the following

$$Y = \sum_{i=1}^{k} \omega_i Y_i, \text{ where } \omega_i \ge 0, i = 1,...,k; \quad \sum_{i=1}^{k} \omega_i = 1. \qquad (6.1)$$

We shall consider more general model, than model (6.1):

$$Y_k = \left(\sum_{i=1}^{k} \omega_i Y_{ik}^{\ s} \right)^{1/s}, \qquad (6.2)$$

where $\sum_{i=1}^{k} \omega_i^{\frac{n}{n-s}} = 1$, $n > s > 0$, $\omega_i \ge 0$, $i = 1,...,k$; $k=2,3,$.

Having entered a designation $X_{ik} = \omega_i^{1/s} Y_{ik}$ model (6.2) we shall copy as

$$Y_k = \left(\sum_{i=1}^{k} X^s{}_{ik} \right)^{1/s} \text{ or}$$

$$Y_k^{\ s} = \sum_{i=1}^{k} X^s{}_{ik} \text{ and } \sum_{i=1}^{k} \left(\frac{X_{ik}}{Y_{ik}} \right)^{\frac{ns}{n-s}} = 1, \quad n>s>0,\ k=2,3,4\ldots\ldots \qquad (6.3)$$

The first equation (6.3) is the equation of a degree *s* with *k+1* unknown and has infinite number of decisions. For allocation of the necessary decisions it is necessary that we use echo the equation (6.2).

***Model of the worst development of losses*:** For everything $\sum_{i=1}^{k}\omega_i^{\frac{n}{n-s}}=1$, *n > s > 0* $\omega_i \geq 0,\ i=1,\dots,k$ right part of the equation (6.2) has the maximal value that there corresponds the worst condition of system, i.e.

$$Y_k = \max_{\omega\in M}\left(\sum_{i=1}^{k} \omega_i Y_{i\,k}{}^{s}\right)^{1/s} \tag{6.4}$$

where $M=\left\{\omega: 0\leq\omega_j\leq 1,\ \sum_{j=1}^{k}\omega_j^{\frac{n}{n-s}}=1,\ n>s,\ s>0,\ k>1\right\}$.

The equation (6.4) we shall call "*Model of the worst development of losses".*

Generalizations

1. The basic result of the given work consists in the description of any complex object of type

$$Lu=\max_{\alpha\in M}\left(\sum_{i=1}^{m}\alpha_i\ (L_i\ \ u)^s\right)^{\frac{n}{p\,s}},$$

$$(Lu)^n=\sum_{j=1}^{m}\left(L_j u\right)^n \quad ,\ n>s>o,$$

$$M=\left\{(\alpha_1,\dots,\alpha_m)=\alpha;\ \sum_{j=1}^{m}\alpha_j^{\frac{n}{n-s}}=1,\ \ n>s>0,\ \ 0<\alpha_j<1\right\}$$

By means of a vector ***(a₁ … … aₘ)*** ∈ ***Eᵐ*** which is defined from representation (6.1)

$$N_m^p = \sum_{i=1}^{m} a_{i\,m}^{n}$$

and it is reduced to its description by means of final number of elementary objects of type $(x,y)\in \mathrm{E}^2$ as a polynomial be relative x_n:

$$N_m^p = x^{n(m-1)}+\sum_{i=2}^{m} y^n z^{p(i-2)} x^{n(m-i)}$$, or

$$N_m^p = X^{m-1}+\sum_{i=2}^{m} YZ^{i-2}X^{m-i}$$ and more over

$$N_m^p = X^{m-1}+\sum_{i=2}^{m} A_i X^{m-i}$$. i.e. curve high degrees (it is possible also elliptic) used at protection of the information or a kind

$N_m^p - Y\sum_{i=3}^{m} X^{m-i}Z^{i-2} = X^{(m-1)} + YX^{m-2}$, where $Z=X+Y$ and all decisions of the equation $x^n + y^n = z^P$ at $p=n$ are represented as

$$x = zt^{1/n}, \quad y = z(1-t)^{1/n}, \quad p = n, \ t \in (0,1). \tag{6.5}$$

From here follows, that

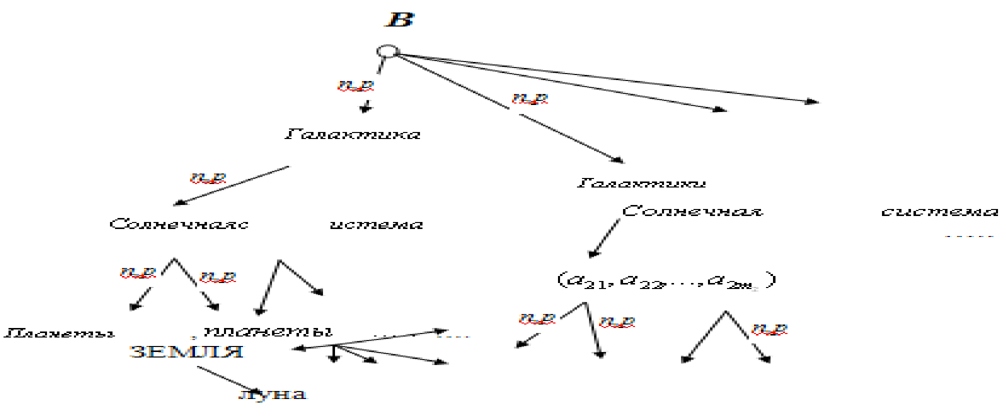
1). Any complex object such as (6.1), т.е a vector $(a,....a_m)$ from E^m described by means of elementary objects $(x,y)\in E^2$ laying on elliptic curves.

2). The result (6.5) gives the simple decision of a problem the Ferma. Really, предполагав ненатуральности decisions from (6.5) we shall receive $X^s = \int_0^1 x^s dt = \frac{n}{n+s} z^s, \ \ Y^s = \int_0^1 y^s dt = \frac{n}{n+s} z^s$ And at $n = s$ we have $X^{-n} = \frac{1}{2} z^n u \ \ X = \frac{1}{\sqrt[n]{2}} z$ With other party $X^2 = \frac{n}{n+2} z^2$, from here $\left(1 + \frac{2}{n} - \sqrt[n]{4}\right) z^2 = 0 \ \Rightarrow z = 0$, $x = 0, \ y = 0$ at $n \succ 2$. It is necessary to note, that in $n=2$ *decisions* (11) can be integers and any integers. For example, all decisions such as (11) in $t = \left(\frac{2j}{j^2+1}\right)^2$ - numbers (number) of an integer, and they are submitted as: $x = j \ \ , y = \frac{j^2-1}{2} \ \ , z = \frac{j^2+1}{2}$ at odd j and $x = 2j \ \ y = \left(j^2 - 1\right), \ \ z = \left(j^2 + 1\right)$ at even j.

2. Analytical number tree model and information security. $P_m^p(x,y) = N_m^p$ is a polynomial of a degree *(m-1)* on x_n and it is a class curve higher degrees which are very well used at protection of the information. For example, we shall write a class of such curves in special cases with пohelp of displays

$N_m = z^{m-1}$, $a_{1m} = x^{m-1}$, $a_{2m} = yx^{m-2}$, $\quad a_{im} = yz^{\frac{i-2}{n}p} x^{m-i}$, $\quad i = 1, ..., m$; I.e. curves $P^p{}_2(x,y) = X + Y$, $P_3^p(x,y) = X^2 + YX + YZ$ and $P_4^p(x,y) = X^3 + YX^2 + YZX + YZ^2$, …., and also type $N_4^2 = X^3 + YX^2 + YZX + Z^2Y$, $Z=X+Y$, which belongs to a class elliptic curves. Pairs (x, y) is usual to name "point" which can "be put" about other similar point of an elliptic curve. « The sum » two points, in turn, too "lays" on an elliptic curve. Except for the points laying on an elliptic curve, « the zero point » is considered also. The sum of two points A with coordinates (XA is considered, that, YA) and B with coordinates (XB, YB) is equal O, if XA = XB, YA = - YB (mod p). The zero point does not lay on an elliptic curve, but, nevertheless, participates in calculations; it can be considered as indefinitely removed from a curve. Set of points of an elliptic curve together with a zero point and with the entered operation of addition we shall name "group". For each elliptic curve the number of points in group certainly, but is great enough. The important role in algorithms of the signature with use of elliptic curves is played with "multiple" points. Point Q refers to as a point of frequency rate k if for some point P k time it is executed equality**: P = Q + Q + Q +. + Q = kQ**. If for some point P there is such number k, that kP = 0, this number is named the order of point P. Multiple points of an elliptic curve are analogue of degrees of numbers in a simple field. The problem of calculation of frequency rate of a point is equivalent to a problem of calculation of the discrete logarithm. Follows noticed that reliability of the digital signature is based on complexity of calculation of "frequency rate" of a point of an elliptic curve and. Though equivalence of a problem discrete logarithm and problems of calculation of frequency rate also is proved, the second has the big complexity. For this reason at construction of algorithms of the signature in group of points of an elliptic curve appeared possible to do without shorter keys in comparison with a simple field at maintenance of the greater stability. A confidential key, consider some random number. The open key considers coordinates of some point on elliptic curve P which is defined as P = xQ where Q - special image the chosen point of an elliptic curve named « a base point. Coordinates of point Q together with factors of the equation specifying a curve, are parameters of the circuit of the signature and they should be known to all participants of an exchange messages. From here follows, that anyone "modern" cryptosystem can "be shifted" on elliptic curves.

3. Model (1-3), and also the formula (6.8) are a basis of model "Universe" with "Galaxies", « Solar systems » and planets (with satellites and without satellites). Really, in this case we have the following conceptual model a tree:



The appropriate equations takes the following kind. For satellites of planets: $N_{CPl}{}^{p} = x^{2n} + 2x^n y^n + y^n = (x^n + y^n)^2$, and planets $N_{Pl}{}^{p_1} = k_{kcPl}(x^n + y^n)^{2p}$.

Similarly, for solar systems and galaxies we have the following equations:

$N_{Pl}{}^{p_2} = N_{1i}^{p_1} + \ldots + N_{9i}^{p_1} = k_{kcPl}(x^n + y^n)^{2pp_1}$,

$N_G{}^{p_3} = N_{1i}^{p_2} + ... + N_{9i}^{p_2} = k_{kcG}(x^n + y^n)^{2pp_1p_2}$. And at last
$N_U{}^{p_4} = N_{1i}^{p_3} + ... + N_{9i}^{p_3} = k_{kcU}(x^n + y^n)^{2pp_1p_2p_3}$. Here k_{kcPl} $k_{kcC\acute{n}}$ k_{kcG} k_{kcG}, k_{kcU} are accordingly mean quantity of planets, solar systems and galaxies.

Addition

The model spaces with weights and its applications

We shall consider the set from our works:

$$M_n^s = \left\{ \alpha = (\alpha_1(t),...,\alpha_m(t)) : \sum_{i=1}^{m} \alpha_j^{\frac{n}{n-s}}(t) = 1, \quad 0 \le \alpha_j(t) \le 1,\ t \in T \quad, \quad j = 1,..,m; n > s, s > 0 \right\},$$

where m, n, s are natural, $n > s, s > 0, m \ge 2$, T is a arbitrary set from $[0,\infty]$. Let $\alpha \in M_n^s$ and $x \in L_m^n(T)$ with norm $\|x\|_{L_m^n(T)} = \left(\int_T \sum_{j=1}^{m} \left|x_j\right|^n dt \right)^{1/n} < \infty$. Let the set of functional

$$\mu(\alpha) = \left(\int_T \left(\sum_{j=1}^{m} \alpha_j \left|x_j\right|^s \right)^{\frac{n}{s}} dt \right)^{1/n}, \tag{1}$$

for all $\alpha \in M_n^s$ and $x \in L_m^n(T)$ is given. The set of functional (1) with norm $\|\mu\| = \sup_{\alpha \in M_n^s} \mu(\alpha)$ is the normed space. Denote it by M, $\alpha \in M_n^s$. Introducing the vector $y = K|x|$, where k is the diagonal matrix with elements $\alpha_j^{1/s}$ we have that space with norm $\mu(\alpha) = \|y\|_{L_m^{n,s}(T)} \left(\int_T \left(\sum_{j=1}^{m} \left|y_j\right|^s \right)^{\frac{n}{s}} dt \right)^{1/n}$ is also the normed space. It is denoted by M (α).

<u>Theorem 1.</u> At $\alpha = \alpha^0 \in M_n^s$, where $\alpha_j^0 = \left(\frac{\left|x_j\right|^n}{\sum_{j=1}^{m} \left|x_j\right|^n} \right)^{\frac{n-s}{n}}$, $j = \overline{1,m}$ the functional (1) has a maximal value $Z=\|\mu\|=\left(\int\limits_T \sum\limits_{j=1}^{m}\left|x_j\right|^n dt\right)^{1/n}=\|x\|_{L_m^n(T)}$,

$\|Y\|_{L_m^{s,s}(T)}\le Z,\ \|Y\|_{L_m^{n,s}}\le Z$ and what is more all maximal values of functional (1) with different $x\in L_m^n(T)$ are solutions of the equation $Z^n=\sum\limits_{j=1}^{m} X_j^{\ n}$, where $X_j=\left(\int\limits_T \left|x_j\right|^n dt\right)^{1/n}$, j=1,2,…m. Besides $\mathrm{M}\subseteq \mathrm{M}\,(\alpha)$ for all $\alpha\in M_n^s$.

The proof. As

$$\mu(\alpha)=\left(\int\limits_T\left(\sum_{j=1}^{m}\alpha_j\left|x_j\right|^s\right)^{\frac{n}{s}}dt\right)^{1/n}=\left(\int\limits_T\left[\sum_{j=1}^{m-1}\alpha_j\left|x_j\right|^s+\left(1-\sum_{j=1}^{m-1}\alpha_j^{\frac{n}{n-s}}\left|x_m\right|^s\right)^{\frac{n-s}{n}}\right]^{\frac{n}{s}}dt\right)^{1/n}.$$

and

$$\mu(\alpha+\Delta\alpha)=\left(\int\limits_T\left[\sum_{j=1}^{m-1}(\alpha_j+\Delta\alpha_j)\left|x_j\right|^s+\left(1-\sum_{j=1}^{m-1}(\alpha_j+\Delta\alpha_j)^{\frac{n}{n-s}}\cdot\left|x_m\right|^s\right)^{\frac{n-s}{n}}\right]^{\frac{n}{s}}dt\right)^{1/n}=$$

$$=\left(\int\limits_T\sum_{j=1}^{m-1}\left(\alpha_j\left|x_j\right|^s+\Delta\alpha_j\left|x_j\right|^s\right)+\left(1-\sum_{j=1}^{m-1}\alpha_j^{\frac{n}{n-s}}\left|x_m\right|^s-\frac{n}{n-s}\sum_{j=1}^{m-1}\alpha_j^{\frac{n}{n-s}-1}\cdot\Delta\alpha_j\cdot\left|x_m\right|^s-\right.\right.$$

$$\cdot\left(\frac{n}{n-s}\sum_{j=1}^{m-1}\alpha_j^{\frac{n}{n-s}-1}\cdot\Delta\alpha_j\left|x_m\right|^s-\sum_{j=1}^{m-1}\alpha_j^{\frac{n}{n-s}-2}\cdot\Delta\alpha_j^2\cdot\left|x_m\right|^s\right)+\frac{1}{2}\cdot\frac{n}{n-s}\cdot\frac{(n-s)}{n}\cdot$$

$$\left.\left.-\frac{1}{2}\frac{n}{n-s}\left(\frac{n}{n-s}-1\right)\sum_{j=1}^{m-1}\alpha_j^{\frac{n}{n-s}-2}\cdot\Delta\alpha_j^{\ 2}\cdot\left|x_m\right|^s+\cdots\right)^{\frac{n-s}{n}}\right]^{\frac{n}{s}}dt\Bigg)=\left(\int\limits_T\left[\sum_{j=1}^{m-1}\alpha_j\left|x_j\right|^s+\right.\right.$$

$$\cdot\left(\frac{n-s}{n}-1\right)\cdot\left(1-\sum_{j=1}^{m-1}\alpha_j^{\frac{n-s}{}}\cdot\left|x_m\right|^s\right)^{\frac{n-s}{n}-2}\cdot\left(\sum_{j=1}^{m-1}\alpha_j^{\frac{n}{n-s}}\cdot\left|x_m\right|^s\cdot\Delta\alpha_j\right)^2+m\Bigg]^{n/s}dt\Bigg)^{1/n}.$$

Continuing this expansion from condition $grad\ \mu(\alpha)\ |_{\alpha=0}=0$ we have the equation

$\left|x_j\right|^s-\left(1-\sum_{j=1}^{m-1}a_j^{\frac{n}{n-s}}\right)^{-\frac{s}{n}}\cdot a_j^{\frac{s}{n-s}}\cdot\left|x_m\right|^s=0$ for definition maximal points of the functional (1). As $\left|x_j\right|^s-a_m^{-\frac{s}{n-s}}\cdot a_j^{\frac{s}{n-s}}\cdot\left|x_m\right|^s=0$ and

$\alpha_m^{-\frac{s}{n-s}}\cdot\alpha_j^{\frac{s}{n-s}}=\frac{\left|x_j\right|^s}{\left|x_m\right|^s}$ of $\frac{\alpha_j^{\frac{n}{n-s}}}{\alpha_m^{\frac{n}{n-s}}}=\frac{\left|x_j\right|^n}{\left|x_m\right|^n}$ after summation with regard to condition $\sum_{j=1}^{m}\alpha_j^{\frac{n}{n-s}}(t)=1$ we have $\alpha_m^{-\frac{n}{n-s}}=\frac{\sum_{j=1}^{m}\left|x_j\right|^n}{\left|x_m\right|^n}$. Hence

$\alpha_j^{0\frac{n}{n-s}}(t)=\frac{\left|x_j(t)\right|^n}{\sum_{j=1}^{m}\left|x_j(t)\right|^n}$. Such under $\alpha=\alpha^0$ the functional $\mu(\alpha)$ has a maximal (using condition $d^2\mu\ |_{\alpha^0}<0$) value. Besides, at this point we have

$$\mu(\alpha^0)=\left(\int_T\left(\sum_{j=1}^{m}\alpha_j^0\left|x_j\right|^s\right)^{n/s}dt\right)^{1/n}=\left(\int_T\left(\sum_{j=1}^{m}\left(\frac{\left|x_j\right|^n}{\sum_{j=1}^{m}\left|x_j\right|^n}\right)^{\frac{n-s}{n}}\cdot\left|x_j\right|^s\right)^{n/s}dt\right)^{1/n}=$$

$$\left(\int_T\sum_{j=1}^{m}\left|x_j\right|^n dt\right)^{1/n}=\|x\|_{L_m^n(T)}=\|\mu\|.$$

Introducing

$X_j = \left(\int_T \left| x_j \right|^n dt \right)^{1/n}, \ j = \overline{1,m} \quad Z = \|\mu\|$ we have the equation [5-8]:

$$\sum_{j=1}^{m} X_j^n = Z^n. \tag{2}$$

It is to easy that M $\in$M (α) for all $a \in M_n^s$. The set of such functional with norm $\|\mu\| = \sup_{\alpha \in M_n^s} \mu(\alpha)$ formed the normed space of type m. Besides, if introduce notations $Y_1(t) = \alpha(t)^{1/s} x_1(t),\ Y_2(t) = \alpha(t)^{1/s} x_2(t)$ we have the normed space of type M (α). Now we consider the model space of functions m with norm

$$\hat{\mu}(\alpha) = \left(\int_T \sum_{j=1}^{m} \left| x_j \right|^n d\beta_j(t) \right)^{1/n}, \tag{3}$$

where $x \in L_m^n(T),\ \beta(t) = \int_0^t \alpha_j[t]dt,\ \alpha \in M_s^n$. Introducing notations $\hat{Z} = \hat{\mu}(\alpha)$, $\hat{x}_j = \left(\int_o^T \left| x_j(t) \right|^n d\beta_j \right)^{1/n}$, we have equations

$$\sum_{j=1}^{m} \hat{x}_j^{\,n} = \hat{Z}^n, \tag{2'}$$

The set of points $\left(\hat{x}_1 ... \hat{x}_m\right)$ with norm $\hat{Z}$ is formed the Euclidean model space $\tilde{M}^m(\alpha),\ \alpha \in M_s^n$ and for functional of type (3) from $\tilde{M}$: $\tilde{M} \subseteq \widehat{M}(\alpha.)$ take place.

Theorem 2. For any natural n>1 between to set of solutions (2) (or (2')), i.e. under m=k-1 and m=k) it take place next presentations:

$$\tilde{x}_{jk} = \tilde{x}_{12} \tilde{x}_{jk-1},\quad \tilde{x}_{kk} = \tilde{x}_{22} \tilde{z}_{k-1},\ \tilde{z}_k = \tilde{z}_2 \tilde{z}_{k-1},\ j = 1,2,..k-1 \tag{4}$$

where k=3, 4…,$(\tilde{x}_{12}, \tilde{x}_{22})$ is some point of the special Plane with distance $\tilde{z}_2 = \sqrt[n]{\tilde{x}_{12}^n + \tilde{x}_{22}^n}$. The transformation of (4) may be written in the form of $\tilde{X}_m = \mu_t \tilde{X}_m$, where the group transformations

$$\tilde{\mu}_t = \begin{pmatrix} \tilde{x}^t & 0 \ldots\ldots & 0 & 0 & 0 \\ 0 & \tilde{x}^t \ldots\ldots & 0 & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & 0 \ \ldots\ldots & \tilde{x}^t & 0 & 0 \\ 0 & 0 \ \ldots\ldots & 0 & \tilde{x}^t & 0 \\ 0 & 0 \ \ldots\ldots & 0 & 0 & \tilde{z}^t \end{pmatrix}$$

, where $\tilde{z} = \tilde{x} + \tilde{y},\ t = 1/n$, transfer any points $\left(\tilde{x}_{1m-1}\cdots\tilde{x}_{m-1m-1}\right)\in\tilde{M}^{m-1}$ with metric $\tilde{z}_{m-1}$ into some corresponding point properties of $\left\{\tilde{\mu}_t\right\}$ are given in [6]. The Plane of $\tilde{M}^2$ is special plane all points of the model spaces $\tilde{M}^n(\alpha)$, $\alpha\in M_n^s$ are depending from points of this Plane. The set $\tilde{M}^2$ is called prescribed set [8]. Hence and $N_m^p=a_{1m}^n+a_{2m}^n+...+a_{mm}^n$, $m\geq 2,\ n\geq 2,\ 1\leq p\leq n$.

<u>The Theorem 3</u>. Transformation (4) in the form of

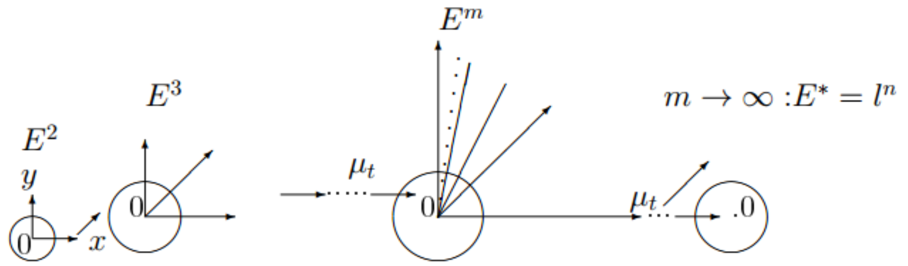
$$\begin{cases} a_{im}=xa_{im-1}, \quad i=\overline{1,m-1} \\ a_{mm}=y\sqrt[n]{N_{m-1}^p} \\ N_m=zN_{m-1}, \quad where\ \ x^n+y^n=z^p \end{cases} \tag{4'}$$

translates the solution of the equation $\sum_{i=1}^{m-1}a_{im-1}^n=N_{m-1}^p$

on the solution of the equation

$$\sum_{i=1}^{m}a_{im}^n=N_m^p \tag{5}$$

and more over



Really, we shall increase both parts of the equation (9) on x^n with the account $x^n+y^n=z^p$ we have $\sum_{i=1}^{m-1}(a_{im-1},x)^n=(z^p-y^n)N^{\ p}_{\ m-1}$. From here, we shall receive (5) and more over we have $N_m=z^{m-1}$, $a_{1m}=x^{m-1}$, $a_{2m}=yx^{m-2}$, $a_{im}=yz^{\frac{ip(-2)}{n}}x^{m-i}$, $i=2,...,m$. such any representation of type (5) with help of transformation (4') is transferred to the form

$$N_m^p=\left(x^{m-1}\right)^n+\sum_{i=2}^{m}\left(yx^{m-2}z^{\frac{p(i-2)}{n}}\right)^n \tag{6}$$

It should be out that transformations (4’) with (6) are described the process of numbers tree grows.

Some Generalizations and conclusions:

a). The basic result of the given work consists in the description of any complex object of type

$$Lu = \max_{\alpha \in M} \left(\sum_{i=1}^{m} \alpha_i \, (L_i \;\; u)^s\right)^{\frac{n}{p\,s}},$$

and corresponding *algebraic representation* of type

$(Lu)^p = \sum_{j=1}^{m} \left(L_j u\right)^n \quad , n > s > o, \;\; M = \left\{(\alpha_1,...,\alpha_m) = \alpha; \; \sum_{j=1}^{m} \alpha_j^{\frac{n}{n-s}} = 1, \; n > s > 0, \; 0 < \alpha_j < 1\right\}$ which described by means of a vector $(a_1 \ldots \ldots a_m)$ from E^m . We consider that it is defined from representation (1) $N_m^p = \sum_{i=1}^{m} a_{i\,m}^n$ and it is reduced to its description by means of final number of elementary objects of type $(x, y) \in E^2$ as a polynomial formula be relative x^n:

$$N_m^p = x^{n(m-1)} + \sum_{i=2}^{m} y^n z^{p(i-2)} x^{n(m-i)} \,, \qquad (6')$$

i.e. curve high degrees (it is possible also elliptic) used at protection of the information or a kind

$$N_m^p = X^{m-1} + \sum_{i=2}^{m} YZ^{i-2} X^{m-i}, \qquad N_m^p = X^{m-1} + \sum_{i=2}^{m} A_i X^{m-i} \;\; \text{or}$$

$$N_m^p - Y \sum_{i=3}^{m} X^{m-i} Z^{i-2} = X^{(m-1)} + YX^{m-2},$$ where $X = x^n, Y = y^n, Z = z^{\mathrm{P}}$,

$A_i = YZ^{i-2}$, $Z=X+Y$ and all solutions of the equation $x^n + y^n = z^{\mathrm{P}}$ at $p=n$ are represented as

$$x = zt^{1/n}, \quad y = z(1-t)^{1/n}, \quad p = n, \;\; t \in (0,1). \qquad (7)$$

__**Theorem 4.**__ Let the function

$u = u(\tilde{x}_1 ... \tilde{x}_m, \tilde{z}), (\hat{x}_1 ... \hat{x}_m) \in \tilde{M}^m \; , \; \tilde{z} = \left(\sum_{j=1}^{m} \tilde{x}_j^n\right)^{1/n}$,is the density of some information flow (substance, moving object and so on) and $L_j, \; j = 1,2...m$, L are some operators which are realizing changes of these information’s flows then $(Lu)^n = \sum_{j=1}^{m} (L_j u)^n$ is its general equations. In the case of

$L_j = \dfrac{\partial^k}{\partial \tilde{x}_j^k}$, $L = \dfrac{\partial^k}{\partial \tilde{z}^k}$, $k \geq 1$ we have

$$u(\tilde{x}_1 \cdots \tilde{x}_m, \hat{z}) = P_{k-1}(\tilde{x}_1 \cdots \tilde{x}_m, \tilde{z}) = P_{k-1}(\hat{x}_1 \cdots \tilde{x}_m, \tilde{z}) + \sum_{j=1}^{m} c_j \frac{\hat{z}_j^k}{k!} + c\frac{\hat{z}^k}{k!}, \quad \sum_{j=1}^{m} c_j{}^n = c^n,$$

are

$$u(\tilde{x}_1,...,\tilde{x}_m,\tilde{t}) = P_{k-1}(\tilde{x}_1,...,\tilde{x}_m,t) + \sum_{j=1}^{m} \tilde{x}_j^{\,k+\frac{2}{n}} + c_0^n t^{\,k+\frac{2}{n}}, \ (\tilde{x}_1,...,\tilde{x}_m,t) \in \left\{ \sum \tilde{x}_j^2 = c_0^n t^2, \ c_0 > 0 \right\}$$

,

$P_{k-1}(\cdot)$ is a polynomial of the (k-1)-th order. From here follows, that - Any complex object such as (1), т.е a vector $(a,....a_m)$ from E^m described by means of elementary objects $(x, y) \in \mathrm{E}^2$ laying on elliptic curves,

- The result (7) gives the simple solution of a all known problem
- And processes with all distributions functions $u = u(\hat{x}_1 ... \hat{x}_m, \hat{z})$ are taking place in different model spaces $\tilde{M}^m(\alpha)$, $m \geq 2$ are controlled by prescribed points of the Plane $\tilde{M}^2$. Prescribed points of the Plane $\tilde{M}^2$ are independent points and were installed by Creator.

<u>Corollary.</u> Newton type Law $F = \gamma \dfrac{m_1^{\alpha_1} m_2^{\alpha_2} ... m_k^{\alpha_k}}{R^k}$ i.e $\dfrac{R^k F}{\gamma} = (m_1)^{\alpha_1} ... (m_k)^{\alpha_k}$ and from condition $\sum_{i=1}^{k} \alpha_i^{\frac{n}{n-1}} = 1$ we have $f = \alpha_1 \ln m_1 + \cdots + \alpha_k \ln m_k$. Hence $f^n = (\ln m_1)^n + \cdots + (\ln m_k)^n$, where $f = \ln \dfrac{R^k F}{\gamma}$. More over Newton type Law and Einstein Equation of type $E^n = (mc^2)^n + (Pc)^n$ with help our representation we have algebraic presentation in the text way $Z^n = \sum_{i=1}^{k} X_{i=1}^n$. And from here and (5)-(5”) we received polynomial formula of type (5”) with data for Newton law $X_1 = \ln m_1, ..., X_k = \ln m_k, Z = f$ and for Einstein Equation parameters $x = mc^2, y = Pc, z = E, p = 2, n = 2$. It should be noted that the mass of space objects would be determined as follows $N(x, 0, t) = \int_0^\infty B(N) da$,

$$\frac{\partial N}{\partial t} + \frac{\partial N}{\partial a} + \sum_{j=1}^{m} V_i \frac{\partial N}{\partial x_i} = F(N) + \sum_{j=1}^{m} \frac{\partial}{\partial x_i}\left(D_j \frac{\partial N}{\partial x_i}\right).$$

Example 1. Definitions of radiuses of optimum spheres containing model solar systems. It is easy to see, that radius of optimum spheres containing model solar systems with the centre on the Earth and on the Sun are accordingly defined under formulas

$R_z= (92^n+42^n+78^n+628^n+1276^n+2719^n+4346^n+5750^n+150^n)^{1/n}$,

$R_c= (58^n+108^n +150^n+228^n+778^n+1426^n+2869^n+4496^n+5900^n)^{1/n}$

computer calculations of value which depending on parameter of curvature of space n are given as following tables 1 and accordingly figures 1,2,3. In the table 1 computer calculations of formulas (6) are given and their difference, and in figures 1,2,3 their diagrams appropriate to these values are given.

Table 1

N	**Rc**	**Rz**	**Rc-Rz**
2	**8123,02**	**7836,105**	**286,9142**
3	**6863,079**	**6654,122**	**208,9574**
4	**6414,142**	**6230,927**	**183,2149**
5	**6203,457**	**6032,365**	**171,0926**
6	**6089,816**	**5925,543**	**164,2727**
7	**6023,233**	**5863,205**	**160,0277**
8	**5982,066**	**5824,852**	**157,2144**
9	**5955,655**	**5800,384**	**155,2712**
10	**5938,258**	**5784,368**	**153,8908**
11	**5926,575**	**5773,683**	**152,8913**
12	**5918,612**	**5766,454**	**152,1582**
13	**5913,123**	**5761,507**	**151,6159**
14	**5909,305**	**5758,092**	**151,2125**
15	**5906,629**	**5755,718**	**150,9111**
16	**5904,743**	**5754,057**	**150,6854**
17	**5903,405**	**5752,889**	**150,5161**
18	**5902,453**	**5752,064**	**150,3889**
19	**5901,772**	**5751,479**	**150,2932**
20	**5901,284**	**5751,063**	**150,2212**

<u>Example</u> 2. The following example is the law of "universal gravitation" between planets of model solar system in the form of

$$F = \gamma \frac{m_1^{\alpha_1} m_2^{\alpha_2} \dots m_k^{\alpha_k}}{R^n},$$

where F is gravitation force of solar system , m_j accordingly weights of j-th planets. α_j a share influence of j-th planets weight in solar system, R is radius of spheres containing our model solar systems. Using of our method we have optimal relations:

$$\alpha_j^{\frac{n}{n-1}} = k \frac{h_j^n}{\sum_{j=1}^{m} h_j^n}, \quad j = \overline{1,m},$$

$$(\bar{F})^n = \sum_{j=1}^{n} h_j^n \,, \qquad (8)$$

where $h_j = \ln(m_j)$, $j = \overline{1,k}$, $\bar{F} = \ln(\frac{F R^n}{\gamma})$,

$$\bar{F} = \gamma \frac{e^{\sqrt[n]{\sum_{j=1}^{k} h_j^n}}}{R^n}, \quad \sum_{j=1}^{k} \alpha_j^{\frac{n}{n-1}} = 1.$$

The second relation of (8) is well-known equation of type (2). Now we are using our method for calculation of model parameters of model solar system.

Table 2

0,06	**3,583519**	**36**	**2**	**(Fg/Rm)**	**911,0487501**	**(Fg/Rm**	**30,18358412**
0,87	**6,257668**	**522**	**3**		**12200,81131**		**23,02128547**
1	**6,39693**	**600**	**4**		**185501,7494**		**20,75329227**
0,11	**4,189655**	**66**	**5**				
318	**12,15898**	**190800**	**6**				
95,1	**10,95186**	**57060**	**7**				
14,5	**9,071078**	**8700**	**8**				
17,3	**9,247636**	**10380**	**9**				
0,002	**0,182322**	**1,2**	**10**				
330000	**19,10378**	**1,98E+08**	**11**				
	g=	**m0=**	**Rn=**	**n=**	**F=**		**m solsys=**
	1,00E-06	**1,00E+22**	**8123,02**	**2**	**3,72E+13**		**1,98E+30**
			6863,079	**3**	**3,35E+13**		
			6414,142	**4**	**3,24E+13**		

Remark. Functional (1) also characterizes the model producing functional introduced in our works [1-10]. Indeed, consider the case m = 2. It is known that the product model in this case is presented as follows:

$$f[K,L] = f_0 A \left(\int_T \left[\alpha \left(\frac{K(t)}{K_0} \right)^{-\rho} + \left(1 - \alpha^{\frac{n}{n-s}} \right)^{\frac{n}{n-s}} \left(\frac{L(t)}{L_0} \right)^{-\rho} \right] dt \right)^{-1/\rho},$$

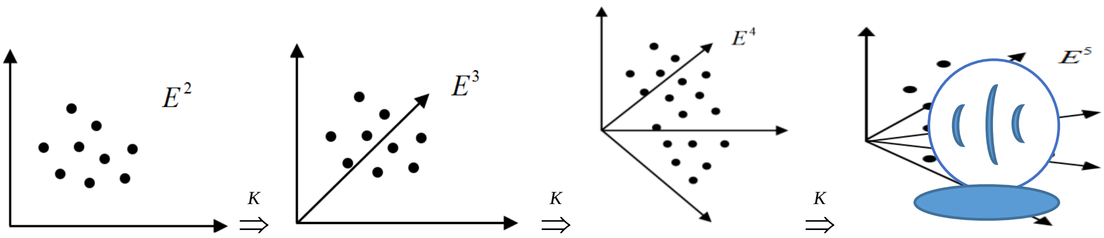
and

$$\mu(\alpha)=\left[\frac{f(K,L)}{f_0\,A}\right]^{-\rho_0},\ x_1(t)=\left(\frac{K(t)}{K_0}\right)^{-\rho_0},\quad x_2=\left(\frac{L(t)}{L_0}\right)^{-\rho_o}.$$

$$\mu(\alpha)=\left(\int_T\left[\alpha(t)x_1^{\,s}(t)+\left(1-\alpha^{\frac{n}{n-s}}(t)\right)^{\frac{n-s}{n}}x_2^{\,s}(t)\right]dt\right)^{1/s}\quad \text{и}\ \ x^n+y^n=z^p,\ x=\|x_1\|,\ y=\|x_2\|.$$

Remark 2. The big bang and the very ordering of the generating objects of the corresponding spaces. We introduce a generating functional $\mu(\alpha)=\left(\left(\int_T\sum_j\alpha_j|x_j|^s\right)^{\frac{n}{s}}dt\right)^{\frac{1}{n}}$ on the set of nonlinear decomposition of units of type $M_n^s=\left\{\alpha=(\alpha_1,\dots,\alpha_m):\sum_j\alpha_j^{\frac{n}{n-s}}(t)=1,\ \ t\geq 0, 1<\alpha_j(t)\leq 1\right\},\quad x\in W_n^p\cdot$

We call the explosion the case when, with a certain factor, the generating functional reaches its maximum i.e. and are ordered: $\exists\ \alpha^0$ for which $\mu(\alpha^0)=\max\limits_{\alpha\in M}\mu(\alpha)$ and a transformation group that order the generating objects to the corresponding spaces i.e.



Here E^2- is a plane, E^3-three-dimensional space, E^4- this is E^3- plus the age of the generated object, E^5- this is E^4-plus time, ...

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Брошюра посвящена разработке и исследованию модельных функционалов и соответствующих им модельных пространств, которые имеют многочисленные применения в различных отраслях науки и техники. Для этой цели в качестве области определения рассматриваемых функционалов берется нелинейное множество- разбиение единиц аналога разбиение единиц линейного случая из функционального анализа. Показано, что введение функционалы с нормой типа max (sup) образуют нормированное пространства. Кроме того, сами функционалы являются нормами некоторых функции из функциональных пространств. Для этих модельных пространств построены, получены алгебраические представления соответствующих элементов и объектов и процессов. И они представлены в виде некоторых полиномов, которые зависят от точек некоторой специально построенной гиперплоскости. Найдены групп преобразования переводящие точек пространств из двумерного в соответствующих точек трехмерного, а затем в четырехмерного и.т.д. и наоборот. Результаты применены для различных объектов и процессов. В двумерном случае показаны эти функционалы являются производственными функциями и на максимуме которых получены довольно известные уравнения.

Показано, что известные некоторые физические законы качественно совпадают. Приведены примеры для различрых областей науки техники.