ELEC 705
DIGITAL SIGNAL PROCESSING

Structure of DTS

6.0-6.5

Introduction

- A LTI system with rational system functions can be characterized by a LCCD equations.
- When such systems are implemented with discrete-time analog or digital hardware, the difference equation or the system function must be converted to an algorithm or structure that can be realized.
- System described by LCCD can be represented by structures consisting of an interconnection of the basic operations of addition, multiplication by a constant, and delay.

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- Block Diagram representations of LCCD equations
- Signal flow graph representations of LCCD equations
- Structures for IIR systems
- Transposed Forms
- Basic network structures for FIR Systems

Basic block diagram

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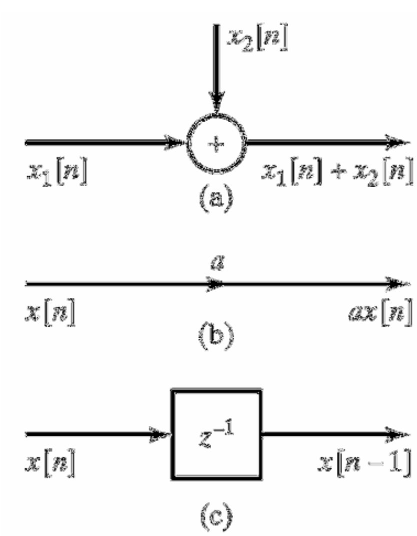
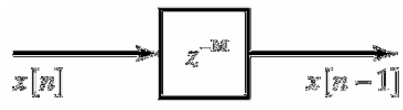


The number of input of the adders are limited to two.

In digital implementations, the delay can be implemented by providing a storage register.
In analog implementations, the delay are implemented by charge storage devices.

M-samples delay

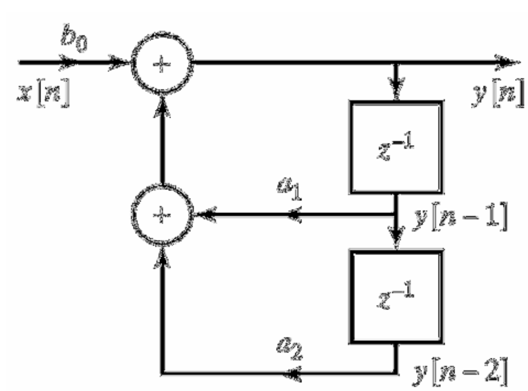


- The actual implementation of M samples of delay would generally be done by cascading M unit delays
  - In an integrated-circuit implementation, these unit delays might form a shift register that is clocked at the sampling rate of the input signal.
  - In a software implementation, M cascaded unit delays would be implemented as M consecutive memory registers.

Example 6.1

- Consider the second order difference equation:

$$y[n]= a_1y[n-1]+a_2y[n-2]+b_0x[n]$$



Block diagram

- Network structures serve as the basis for a program that implements the system.
- The block diagram is also the basis for determining a hardware system with VLSI technology.
- The block diagram conveniently depicts the complexity of the associated computational algorithm, the steps of the algorithm, and the amount of hardware required to realize the system.

Generalized N-th order difference equation

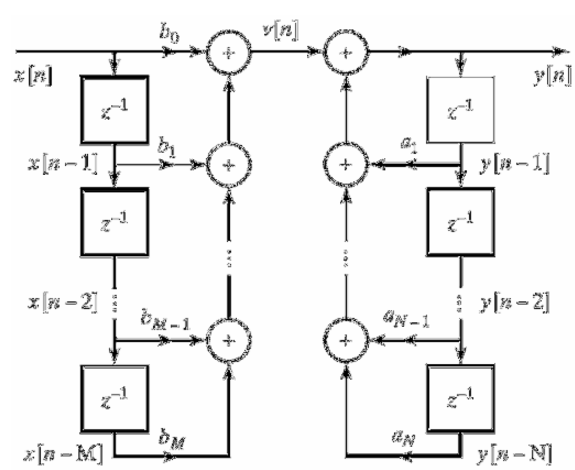
$$y[n]-\sum_{k=1}^{N}a_k y[n-k]=\sum_{k=0}^{M}b_k x[n-k]$$

$$y[n]=\sum_{k=1}^{N}a_k y[n-k]+\sum_{k=0}^{M}b_k x[n-k]$$

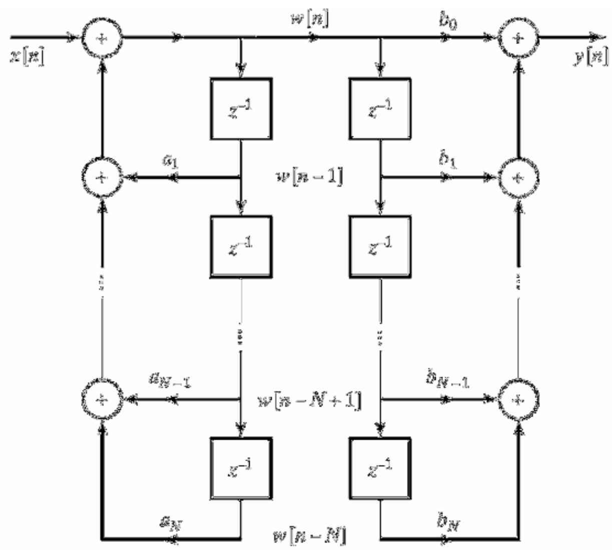
$$v[n]=\sum_{k=0}^{M}b_k x[n-k]$$

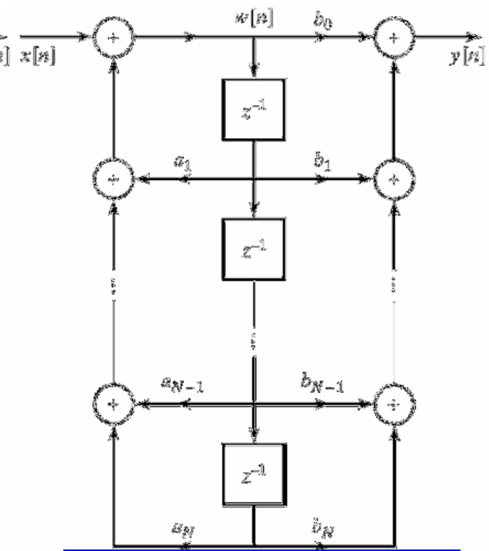
$$y[n]=\sum_{k=1}^{N}a_k y[n-k]+v[n]$$

Direct form I implementation



Modified block diagram

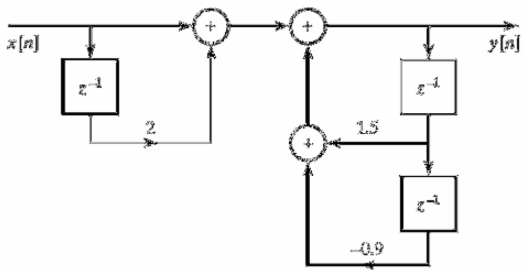


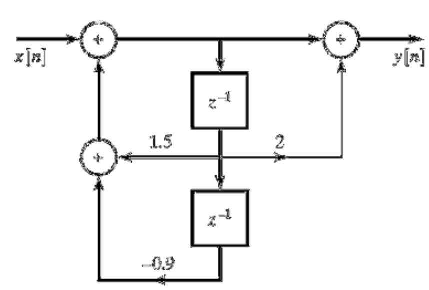


Direct form II implementation
Canonic form implementation

Example 6.2

$$H(z)=\frac{1+2z^{-1}}{1-1.5z^{-1}+0.9z^{-2}}$$



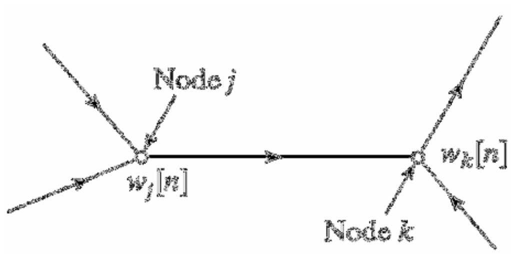


The feedback coefficients $\{a_k\}$ always have the opposite sign in the difference equation from their sign in the system function.

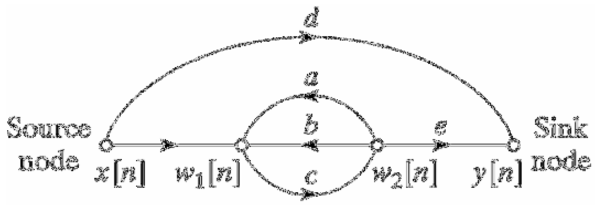
Contents

Signal Flow Graphs

- A signal flow graph is a network of directed branches that connect at nodes. Associated with each node is a variable or node value.

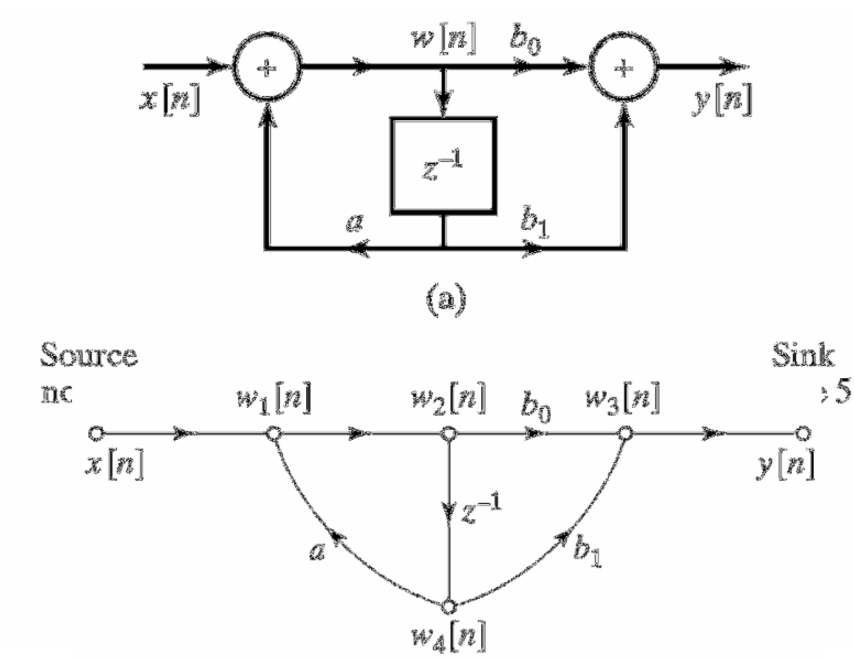


- Branch (j,k) denotes a branch originating at node j and terminating at node k, with the direction from j to k being indicated by an arrowhead on the branch.



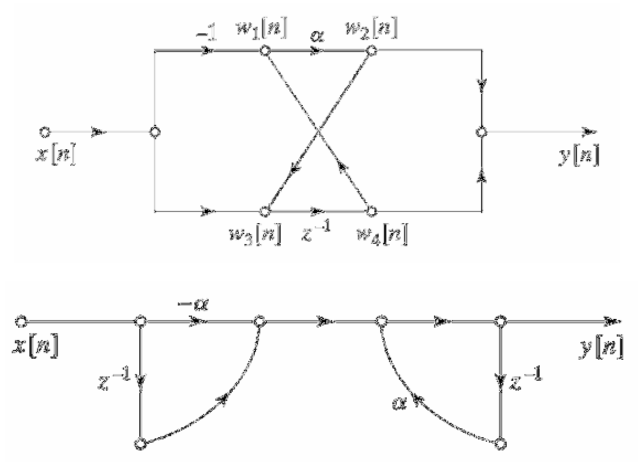
- Source nodes are nodes that have no entering branches.
  - Represent the injecting of external inputs or signal sources
- Sink nodes are nodes that have only entering branches.
  - Extract outputs from a graph

Delay branch



Example 6.3

Determination of the system function from a flow graph



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- Transposed Forms
- Basic network structures for FIR Systems

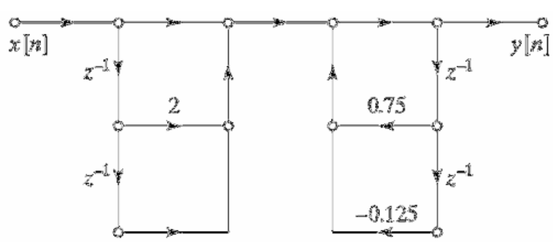
Basic Structures for IIR systems

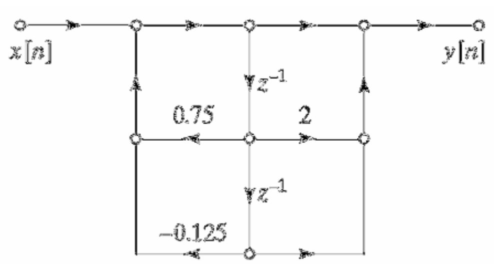
- For any given rational system function, a wide variety of equivalent sets of difference equations or network structures exists.
  - One consideration in the choice is computational complexity.
    - Multiplication is time-consuming and costly operation
    - Delay elements correspond to the memory requirement
  - Other considerations in practical applications
- IIR systems
    - Direct forms
    - Cascade form
    - Parallel form
    - Feedback in IIR systems

Direct Forms

- Example 6.4

$$H(z) = \frac{1+2z^{-1}+z^{-2}}{1-0.75z^{-1}+0.125z^{-2}}$$

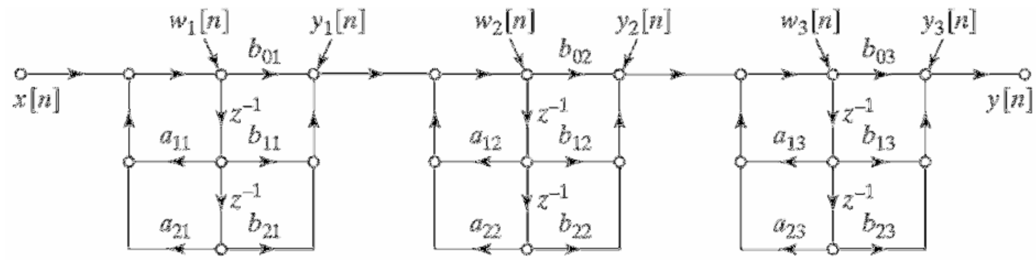




Cascade Form

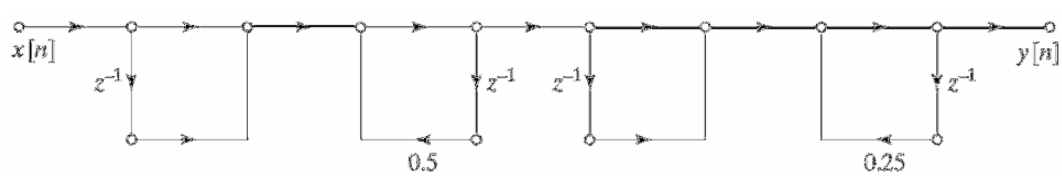
$$H(z) = A\frac{\prod_{k=1}^{M_1}(1-f_k z^{-1})\prod_{k=1}^{M_2}(1-g_k z^{-1})(1-g_k^* z^{-1})}{\prod_{k=1}^{N_1}(1-c_k z^{-1})\prod_{k=1}^{N_2}(1-d_k z^{-1})(1-d_k^* z^{-1})}$$

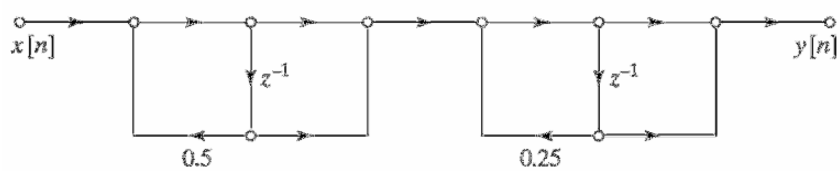
$$H(z) = \prod_{k=1}^{N_s}\frac{b_{0k}+b_{1k}z^{-1}+b_{2k}z^{-2}}{1-a_{1k}z^{-1}-a_{2k}z^{-2}}$$



Example 6.5

$$H(z) = \frac{1+2z^{-1}+z^{-2}}{1-0.75z^{-1}+0.125z^{-2}} = \frac{(1+z^{-1})(1+z^{-1})}{(1-0.5z^{-1})(1-0.25z^{-1})}$$

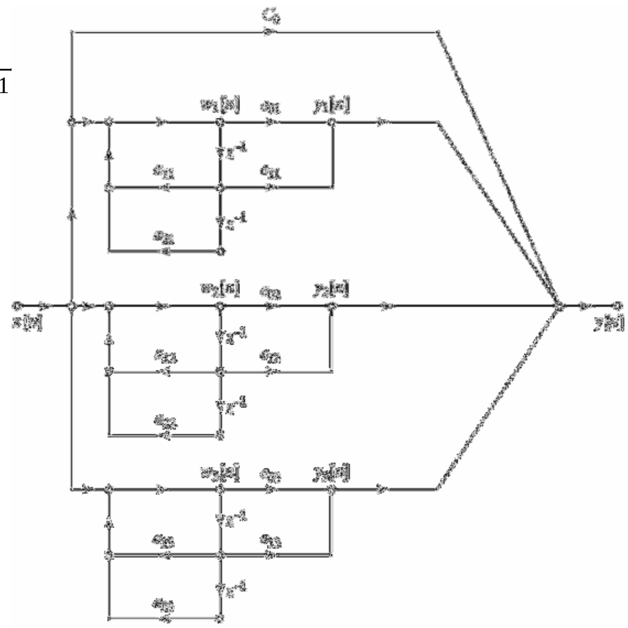




Parallel Form

$$H(z) = \sum_{k=0}^{N_p} c_k z^{-1} + \sum_{k=1}^{N_1} \frac{A_k}{1 - c_k z^{-1}} + \sum_{k=1}^{N_2} \frac{B_k(1 - e_k z^{-1})}{(1 - d_k z^{-1})(1 - d_k^* z^{-1})}$$

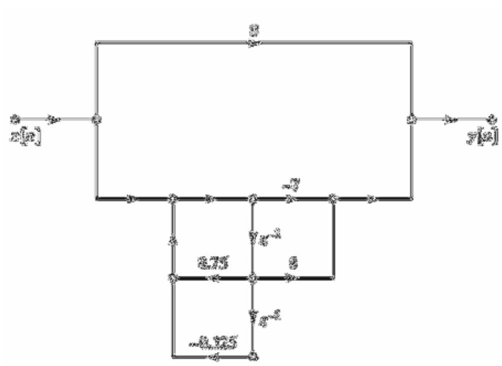
$$H(z) = \sum_{k=0}^{N_p} c_k z^{-1} + \sum_{k=1}^{N_s} \frac{e_{0k} + e_{1k} z^{-1}}{1 - a_{1k} z^{-1} - a_{2k} z^{-2}}$$

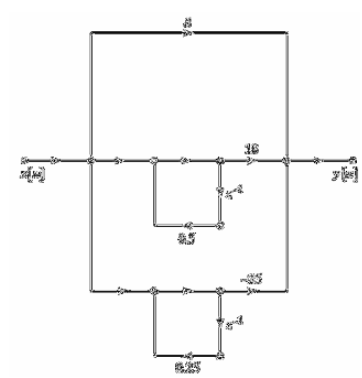


Example 6.6

$$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{1 - 0.75z^{-1} + 0.125z^{-2}} = 8 + \frac{-7 + 8z^{-1}}{1 - 0.75z^{-1} + 0.125z^{-2}}$$

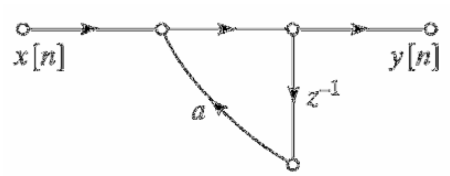
$$= 8 + \frac{18}{1 - 0.5z^{-1}} - \frac{25}{1 - 0.25z^{-1}}$$





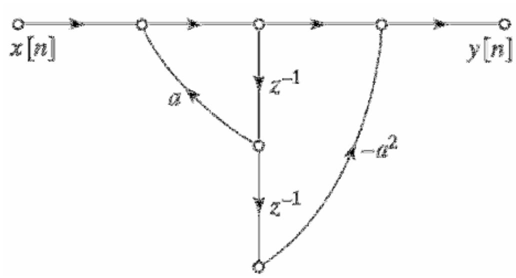
Feedback in IIR systems

- Feedback loop: closed paths that begin at a node and return to that node by traversing branches only in the direction of their arrowheads.
  - Implies that a node variable in a loop depends directly or indirectly on itself.
  - Example: y[n]=ay[n-1]+x[n]



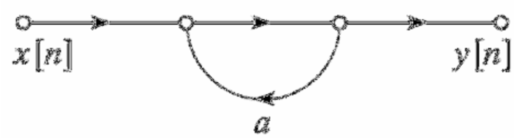
- If a network has no loops, the system function has only zeros
- And the network for an IIR system must include loops

- If a system has poles, feedback loops exist.
- Neither poles in the system function or loops in the network are sufficient for the impulse response to be infinitely long.
  - Example: Frequency sampling systems



Noncomputable network

- Flow graph does not represent a set of difference equations that can be solved successively for the node variable.
- The key to the computability of a flow graph is that all loops must contain at least one unit delay element.
- In manipulating flow graphs representing implementations of LTI systems, be careful not to create delay-free loops.



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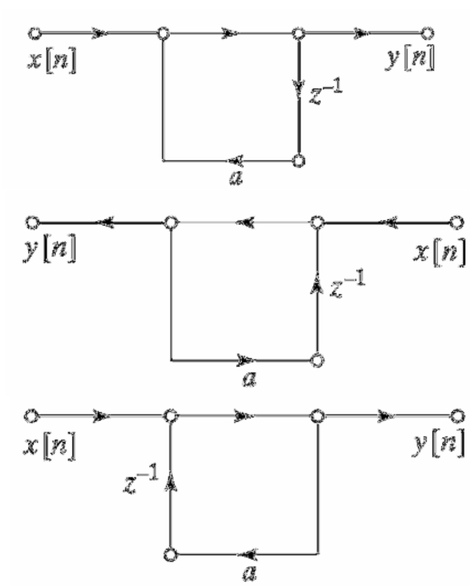
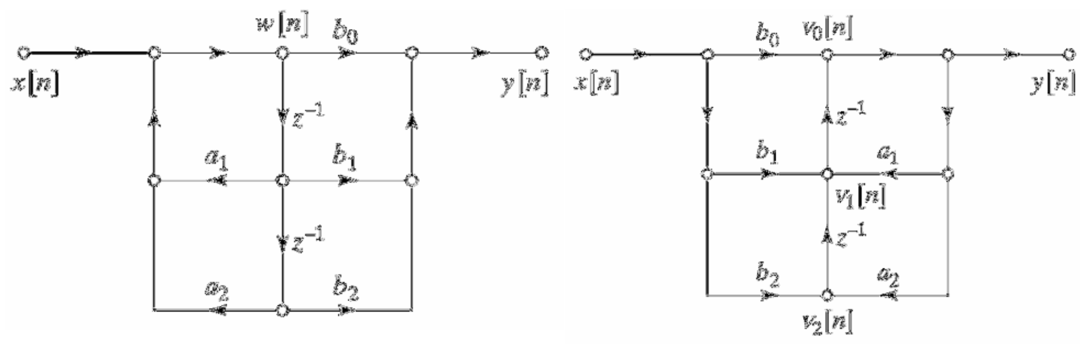
Transposition

- The theory of linear signal graphs provides a variety of procedures for transforming such graphs into different forms while leaving the overall system function between input and output unchanged.
- Transposition of a flow graph is accomplished by
  - Reversing the directions of all branches in the network while keeping the branch transmittances as they were.
  - Reversing the roles of the input and output so that source nodes become sink nodes and vice versa
- For single-input, single-output systems, the resulting flow graph has the same system function as the original graph.

Example 6.7



Example 6.8



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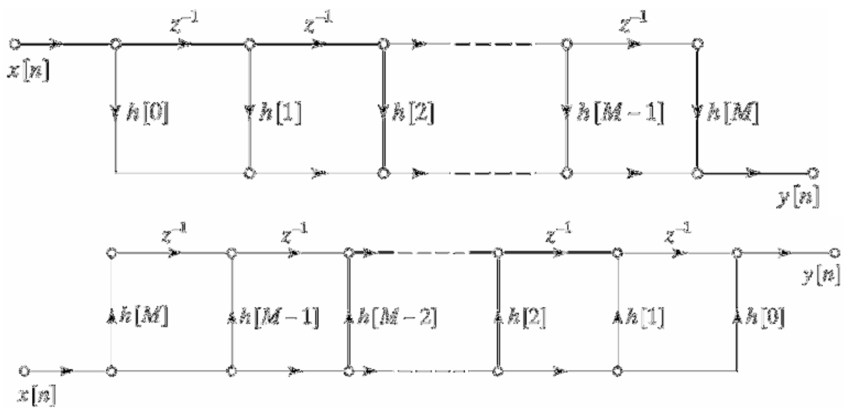
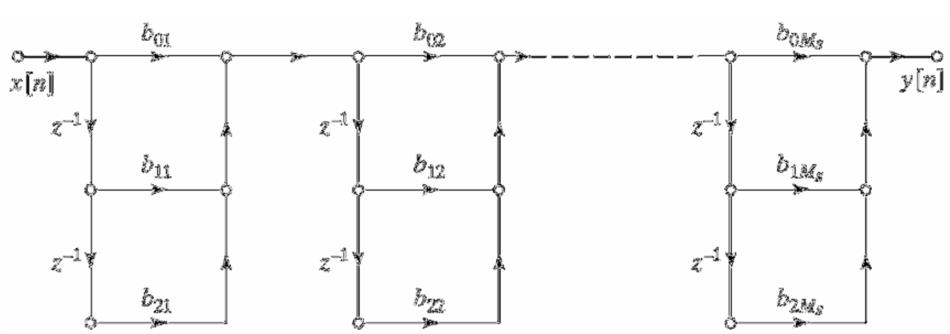
Direct Form

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$



Cascade Form

$$H(z) = \sum_{n=0}^{M} h[n] z^{-n} = \prod_{k=1}^{M_s} (b_0 k + b_{1k} z^{-1} + b_{2k} z^{-2})$$



Linear Phase FIR Systems

- A FIR system has linear phase if the impulse response satisfies either the even symmetric condition

  $h[n] = h[M-n] \quad n = 0,1,\ldots M$

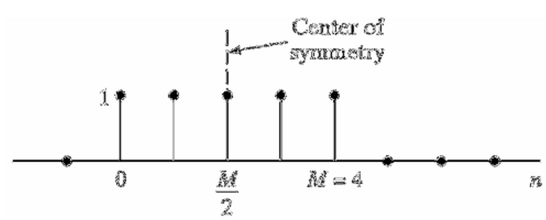
  Or the odd symmetric condition

  $h[n] = -h[M-n] \quad n = 0,1,\ldots M$

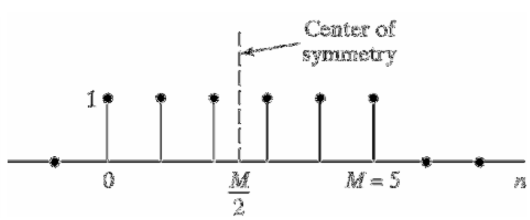
- The system has different characteristics depending on whether *N* is even or odd. Furthermore.
- Thus there are exactly four types of linear phase systems.

Types of Linear-Phase Sytems

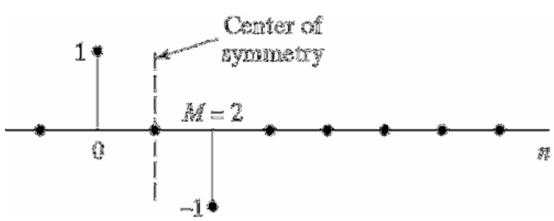
- M is an even integer
- M is an odd integer



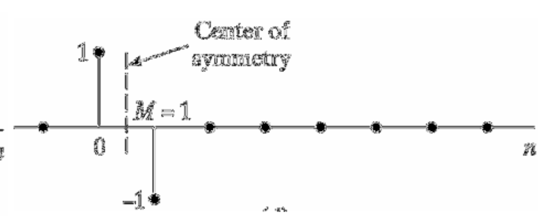
Type 1 $h[n] = h[M-n]$



Type 2 $h[n] = h[M-n]$



Type 3 $h[n] = -h[M-n]$



Type 4 $h[n] = -h[M-n]$

Structures for Linear-Phase FIR system

- The number of coefficient multipliers can be halved in linear-phase FIR system.

$$y[n]=\sum_{k=0}^{M}h[k]x[n-k]$$

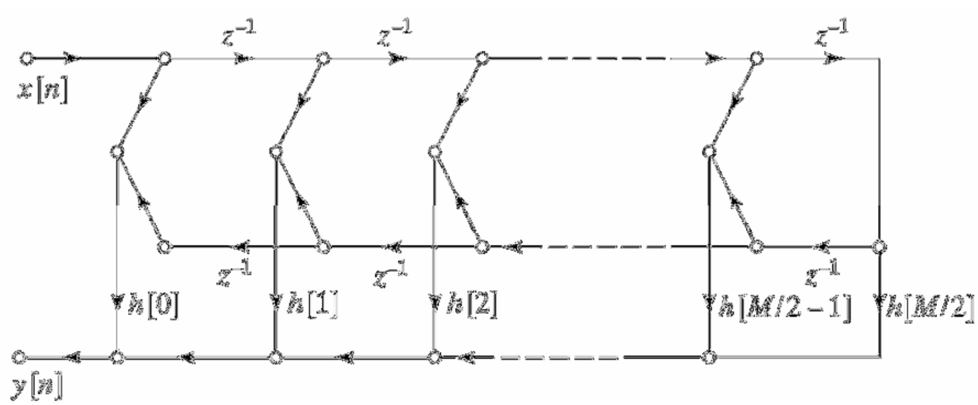
$$y[n]=\sum_{k=0}^{M/2-1}h[k](x[n-k]+x[n-M+k])+h[M/2]x[n-M/2] \qquad \text{Type I}$$

$$y[n]=\sum_{k=0}^{M/2-1}h[k](x[n-k]-x[n-M+k]) \qquad \text{Type III}$$

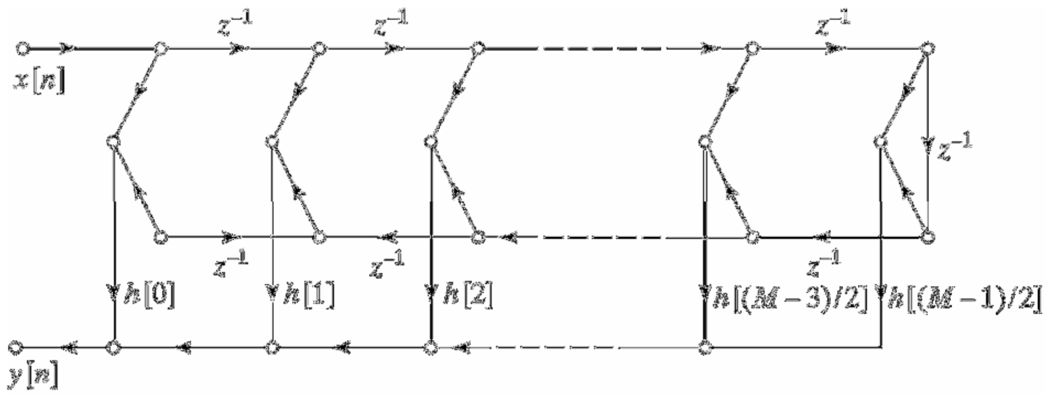
$$y[n]=\sum_{k=0}^{(M-1)/2}h[k](x[n-k]+x[n-M+k]) \qquad \text{Type II}$$

$$y[n]=\sum_{k=0}^{(M-1)/2}h[k](x[n-k]-x[n-M+k]) \qquad \text{Type IV}$$

FIR linear-phase system with even M



FIR linear-phase system with odd M



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