

# ELCE 705 Digital Signal Processing

## Simulation Project 2

**Due date: Oct. 10, 7:00 pm.**

**Report should be submitted via UMMoodle.**

MATLAB hints:

- Use function *stem* to plot discrete-time signals and use function *plot* to plot continuous-time signals. Other useful plotting functions may include *plot*, *subplot*, *hold on*, *axis*, *xlabel*, *ylabel*, *title*.
- When more than one signal is shown in the figure, please use different colors and symbols to distinguish them.

### Problem 1: Linear Constant Coefficient Difference Equations

A causal system is given by:

$$y[n] - 0.4y[n-1] + 0.75y[n-2] + 0.2y[n-3] = 2.2403x[n] + 2.4908x[n-1] + 2.2403x[n-2]$$

The impulse response of the causal discrete-time system can be calculated by matlab function *impz*, as following:

```
num=[2.2403 2.4908 2.2403];  
den=[1 -0.4 0.75];  
h=impz(num, den, N);    % compute N samples of the impulse response
```

The following program can be used to calculate the output  $y[n]$ :

```
num=[2.2403 2.4908 2.2403];  
den=[1 -0.4 0.75];  
ic=[0 0 0]; %set zero initial conditions, number of zero is determined by the order of the  
difference equation  
y=filter(num, den, x, ic);    % x is the input sequence
```

- (a) Calculate the output of the discrete-time system by using matlab function *filter*, when input  $x_1[n] = \cos(2\pi \cdot 0.1 \cdot n)$  for  $n=0 \sim 30$ . Show the input and output sequences in your report.
- (b) Calculate the impulse response  $h[n]$  with the length of 10, 20 and 30 samples respectively, by matlab function *impz*. Show them in the report and compare the results.
- (c) Using matlab function *conv* to calculate the convolution sum of  $x_1[n]$  and the  $h[n]$  to get the output of the given system. Use the three  $h[n]$  obtained in part (b) respectively. Show the results in your report and compare them. Comparison with the result obtained in part (a) is also necessary. Comments on your result.

## Problem 2: FIR system

A causal system is given by:

$$y[n] = 0.9x[n] - 0.45x[n-1] + 0.35x[n-2] + 0.002x[n-3]$$

- (a) Calculate the output of the discrete-time system by using matlab function **filter**, when input  $x_1[n] = \cos(2\pi \cdot 0.1 \cdot n)$  for  $n=0 \sim 30$ . Show the input and output sequences in your report.
- (b) Generate the impulse response  $h[n]$  of the given causal LTI system and provide the results in your report. In this case, the parameter  $N$  is not necessary for using the function **impz**. Could you explain why? Is the given system a stable system and why? Please provide your answer in the report.
- (c) Using matlab function **conv** to calculate the convolution sum of  $x_1[n]$  and  $h[n]$ , compare the results with that obtained in part (a).

## Problem 3: DTFT Computation

The DTFT of a sequence  $x[n]$  of the form of Eq. (1) can be computed easily by using the Matlab function **freqz**.

$$X(e^{j\omega}) = \frac{b_0 + b_1 e^{-j\omega} + \dots + b_M e^{-j\omega M}}{a_0 + a_1 e^{-j\omega} + \dots + a_N e^{-j\omega N}} \quad (1)$$

There are several methods to use the function **freqz**, one example is given as follows:

`w = -pi:0.01:pi; % Assign the frequency spectrum range and step size`

`h=freqz(num,den,w);`

- (a) Using Matlab function **freqz** to obtain the discrete-time Fourier transform of Eq. (1). Show its real part, imaginary part, magnitude response and phase response in the range  $-2\pi \leq \omega \leq 2\pi$  in the report.

$$X(e^{j\omega}) = \frac{0.0181 + 0.0543e^{-j\omega} + 0.0543e^{-j2\omega} + 0.0181e^{-j3\omega}}{1 - 1.76e^{-j\omega} + 1.1829e^{-j2\omega} - 0.2781e^{-j3\omega}} \quad (1)$$

Is the DTFT a periodic function of  $\omega$ ? If it is, what is the period? Explain the type of symmetries exhibited by the four figures.

- (b) Modify your program to evaluate the DTFT of the following finite-length sequence:

$$g[n] = [0.1170 \ 0.4132 \ 0.7500 \ 0.9698 \ 0.9698 \ 0.7500 \ 0.4132 \ 0.1170];$$

Draw its real part, imaginary part, magnitude response and phase response in the range  $-\pi \leq \omega \leq \pi$ . Is there any difference in the phase response of (b) and (a)?

Hints: You can use `h=freqz(num, den, w)` to obtain the DTFT, then use matlab function **real**, **imag**, **abs**, **angle** to draw the required figures. Other method to get the

DTFT of the given sequence is also acceptable.

#### Problem 4: z-transform

(a) Draw the real part, imaginary part, magnitude and phase response of the following z-transform when its value is evaluated on the unit circle.

$$H(z) = \frac{2 + 5z^{-1} + 9z^{-2} + 5z^{-3} + 3z^{-4}}{5 + 45z^{-1} + 2z^{-2} + z^{-3} + z^{-4}} \quad (1)$$

(b) Computer the poles and zeros of the z-transform in (1). Express (1) in factored form with the obtained poles and zeros, and generate the pole-zero plot of (1).

*The pole-zero plot of a rational z-transform  $H(z)$  can be readily obtained using the function **zplane**. There are two versions of this function. If the z-transform is given in the form of a rational function as in (1), the command to use is **zplane(num, den)** where num and den are row vectors containing the coefficients of the numerator and denominator polynomials of  $H(z)$  in ascending powers of  $z^{-1}$ . On the other hand, if the zeros and poles of  $H(z)$  are given, the command to use is **zplane(zeros, poles)** where zeros and poles are column vectors.*

*The function **tf2zp** can be used to determine the zeros and poles of a rational z-transform  $H(z)$ . The program statement to use is  $[z, p, k] = \text{tf2zp}(\text{num}, \text{den})$  where num and den are row vectors containing the coefficients of the numerator and denominator polynomials of  $H(z)$  in ascending powers of  $z^{-1}$  and the output file contains the gain constant  $k$  and the computed zeros and poles given as column vectors  $z$  and  $p$ , respectively.*

(c) From the pole-zero plot generated in part(b), determine the number of regions of convergence (ROC) of  $H(z)$ . Show explicitly all possible ROCs. Can you tell from the pole-zero plot whether or not the DTFT exists?

(d) Determine the partial fraction expansion using **residuez**.

(e) Determine the rational form of a z-transform whose zeros are at  $z_1=0.3$ ,  $z_2=2.5$ ,  $z_3=-0.2+0.4j$ ,  $z_4=-0.2-0.4j$ ; the poles are at  $p_1=0.5$ ,  $p_2=-0.75$ ,  $p_3=0.6+0.7j$ ,  $p_4=0.6-0.7j$ ; and the gain constant  $k$  is 3.9. Generate the corresponding pole-zero plot.

*The reverse process of converting a z-transform given in the form of zeros, poles, and the gain constant to a rational form can be implemented using the function **zp2tf**. The program statement to use is  $[\text{num}, \text{den}] = \text{zp2tf}(z, p, k)$ .*

~~~ The End ~~~