ELCE 702 HW03

M-BJ-5515-2 TANG 幻 唐祺

1. $y' + 2.5y = 1.6x$

first order linear ODE:

$$\frac{dy}{dx} + P(x)y = k(x)$$

$$\Rightarrow y = Ce^{-\int P(x)dx} + e^{-\int P(x)dx}\int k(x)\cdot e^{\int P(x)dx}dx$$

$$= C\cdot e^{-\int 2.5dx} + e^{-\int 2.5dx}\int 1.6x\cdot e^{\int 2.5dx}dx$$

$$= C\cdot e^{-2.5x} + e^{-2.5x}\times 1.6\int x\cdot e^{2.5x}dx$$

$$\int uv'dx = u\cdot v - \int u'vdx \Rightarrow \int x\cdot e^{2.5x}dx = \frac{x}{2.5}e^{2.5x} - \frac{1}{2.5}e^{2.5x}$$

$$\therefore y = C\cdot e^{-2.5x} + 1.6e^{-2.5x}\cdot\left(\frac{x}{2.5}e^{2.5x} - \frac{1}{2.5}e^{2.5x}\right)$$

$$= C\cdot e^{-2.5x} + \frac{16}{25}x - \frac{16}{25}$$

2. $25yy' - 4x = 0$

$$25y\frac{dy}{dx} = 4x$$

$$\int 25y\,dy = \int 4x\,dx$$

$$\frac{25}{2}y^2 = 2x^2 + C$$

$$y^2 = \frac{4}{25}x^2 + C$$

$$y = \pm\frac{2}{5}x + C$$

3. $(3xe^y + 2y)dx + (x^2e^y + x)dy = 0$

$$\int(3xe^y + 2y)dx = -\int(x^2e^y + x)dy$$

$$\frac{3}{2}x^2e^y + 2xy = -x^2e^y - xy$$

$$3xy = -\frac{5}{2}x^2e^y$$

$$3y = -\frac{5}{2}e^y$$

4. $y' = \sqrt{1-y^2}$, $y(0) = \frac{1}{\sqrt{2}}$

$$\frac{dy}{dx} = \sqrt{1-y^2}$$

$$\int\frac{1}{\sqrt{1-y^2}}dy = \int dx$$

$$\arcsin y = x + C$$

apply $y(0) = \frac{1}{\sqrt{2}}$

$$\arcsin\frac{1}{\sqrt{2}} = C$$

$$C = \frac{\pi}{4}$$

$$\therefore \arcsin y = x + \frac{\pi}{4}$$

5. $3\sec y\,dx + \frac{1}{3}\sec x\,dy = 0$, $y(0) = 0$

$$3\sec y\,dx = -\frac{1}{3}\sec x\,dy$$

$$-9\int\frac{1}{\sec x}dx = \int\frac{1}{\sec y}dy$$

$$-9\int\cos x\,dx = \int\cos y\,dy$$

$$-9\sin x = \sin y + C$$

apply $y(0) = 0$

$$0 = C$$

$$\therefore \sin y = -9\sin x$$