Amme 3500 : System Dynamics & Control

Design via Frequency Response

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Course Outline

Week	Date	Content	Assignment Notes
1	1 Mar	Introduction	
2	8 Mar	Frequency Domain Modelling	
3	15 Mar	Transient Performance and the s-plane	
4	22 Mar	Block Diagrams	**Assign 1 Due**
5	29 Mar	Feedback System Characteristics	
6	5 Apr	Root Locus	**Assign 2 Due**
7	12 Apr	Root Locus 2	
8	19 Apr	Bode Plots	**No Tutorials**
	26 Apr	BREAK	
9	3 May	Bode Plots 2	
10	10 May	State Space Modeling	**Assign 3 Due**
11	17 May	State Space Design Techniques	
12	24 May	Advanced Control Topics	
13	31 May	Review	**Assign 4 Due**
14		Spare	

Frequency Response

- In week 7 we looked at modifying the transient and steady state response of a system using root locus design techniques
  - Gain adjustment (speed, steady state error)
  - Lag (PI) compensation (steady-state error)
  - Lead (PD) compensation (speed, stability)
- We will now examine methods for designing for a particular specification by examining the *frequency response* of a system
- We still rely on approximating CL behaviour as 2nd Order

Time vs. Freq. Domain Analysis

- Control system performance generally judged by time domain response to certain test signals (step, etc.)
  - Simple for < 3 OL poles or ~2nd order CL systems.
  - No unified methods for higher-order systems.
- Freq response easy for higher order systems
  - Qualitatively related to time domain behaviour
  - More natural for studying sensitivity and noise susceptibility

Frequency Response Specifications

- Crossover frequency: $|G(j\omega_c)| = 1$
- Gain Margin:

$$K\,(\mathrm{dB})\ \text{s.t.}\ |KG(j\omega)| = 1\ \text{at}\ \omega\ \text{where}\ \angle G(j\omega) = 180^\circ$$

- Phase Margin:

$$PM = 180^\circ - \angle G(j\omega_c)$$

- Bandwidth (CL specification)

$$|G(j\omega_{BW})| = -3\ \mathrm{dB} \qquad \omega_c \le \omega_{BW} \le 2\omega_c$$

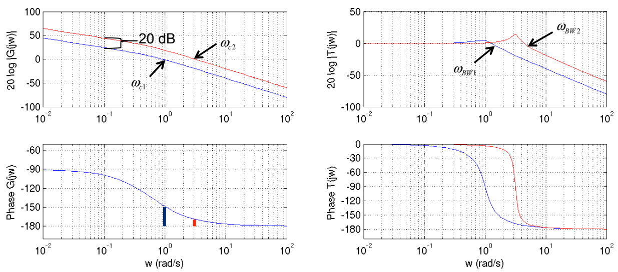
- Less intuitive than RL, but easier to draw for high order systems.

Transient Response via Gain

- The root locus demonstrated that we can often design controllers for a system via gain adjustment to meet a particular transient response
- We can effect a similar approach using the frequency response by examining the relationship between phase margin and damping

Gain Adjustment and the Frequency Response

$$G(s) = \frac{\omega_n^2}{s(s+2\zeta\omega_n)}$$



Design using Phase Margin

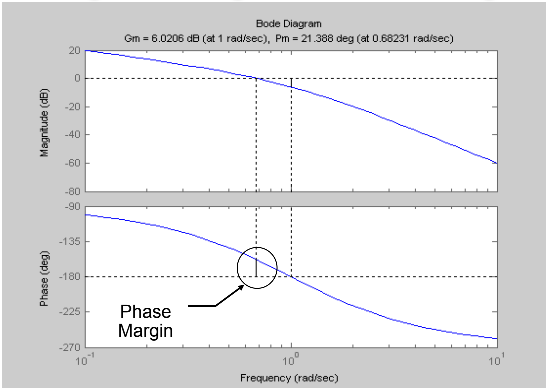
- Recall that the Phase Margin is closely related to the damping ratio of the system
- For a unity feedback system with open-loop function

$$G(s) = \frac{\omega_n^2}{s(s+2\zeta\omega_n)}$$

- We found that the relationship between PM and damping ratio is given by

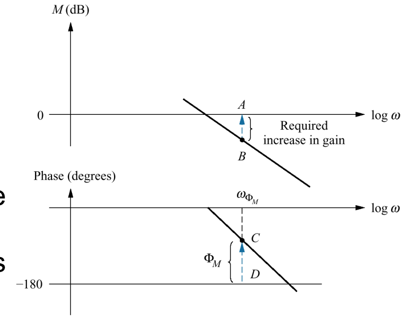
$$PM = \tan^{-1}\frac{2\zeta}{\sqrt{-2\zeta^2+\sqrt{1+4\zeta^4}}} \qquad \zeta \approx \frac{PM}{100},\ PM < 70^\circ$$

Design using Phase Margin



Design using Phase Margin

- Given a desired overshoot, we can convert this to a required damping ratio and hence PM
- Examining the Bode plot we can find the frequency that gives the desired PM

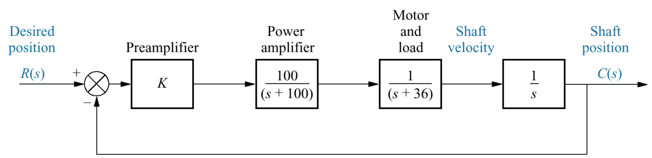


Design using Phase Margin

- The design procedure therefore consists of
  - Draw the Bode Magnitude and phase plots
  - Determine the required phase margin from the percent overshoot
  - Find the frequency on the Bode phase diagram that yields the desired phase margin
  - Change the gain to force the magnitude curve to go through 0dB

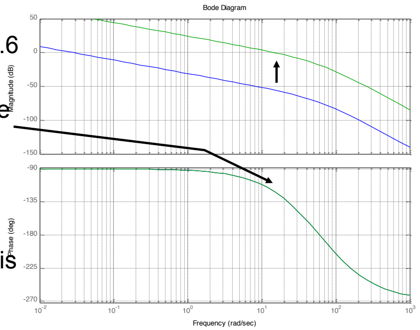
Phase Margin Example

- For the following position control system shown here, find the preamplifier gain K to yield a 9.5% overshoot in the transient response for a step input



Phase Margin Example

- Draw the Bode plot
- For 9.5% overshoot, z=0.6 and PM must be 59.2°
- Locate frequency with the required phase at 14.8 rad/s
- The magnitude must be raised by 55.3dB to yield the cross over point at this frequency
- This yields a K = 583.9



Designing Compensation

- As we saw previously, not all specifications can be met via simple gain adjustment
- We examined a number of compensators that can bring the root locus to a desired design point
- A parallel design process exists in the frequency domain

Designing Compensation

- In particular, we will look at the frequency characteristics for the
  - PD Controller
  - Lead Controller
  - PI Controller
  - Lag Controller
- Understanding the frequency characteristics of these controllers allows us to select the appropriate version for a given design

PD Controller

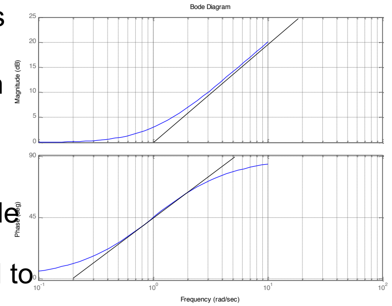
- The ideal derivative compensator adds a pure differentiator, or zero, to the forward path of the control system

$$U(s) = K(s + z_c)$$

- The root locus showed that this will tend to stabilize the system by drawing the roots towards the zero location
- We saw that the pole and zero locations give rise to the break points in the Bode plot

PD Controller

- The Bode plot for a PD controller looks like this
- The stabilizing effect is seen by the increase in phase at frequencies above the break frequency
- However, the magnitude grows with increasing frequency and will tend to amplify high frequency noise



Lead Compensation

- Introducing a higher order pole yields the lead compensator

$$U(s)=\frac{K(s+z_c)}{(s+p_c)}\quad z_c<p_c$$

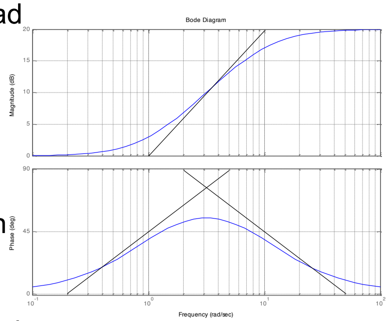
- This is often rewritten as

$$U(s)=\frac{Ts+1}{\alpha Ts+1}=\frac{1}{\alpha}\frac{s+1/T}{s+1/\alpha T},\ \alpha<1$$

where 1/α is the ratio between pole-zero break points
- The name Lead Compensation reflects the fact that this compensator imparts a phase lead

Lead Compensation

- The Bode plot for a Lead compensator looks like this
- The frequency of the phase increase can be designed to meet a particular phase margin requirement
- The high frequency magnitude is now limited



Lead Compensation

- The lead compensator can be used to change the damping ratio of the system by manipulating the Phase Margin
- The phase contribution consists of

$$\phi=\tan^{-1}T\omega-\tan^{-1}\alpha T\omega$$

- The peak occurs at

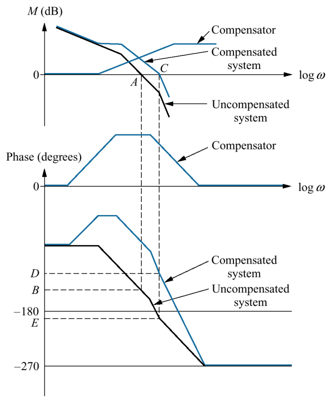
$$\omega_{\max}=\frac{1}{T\sqrt{\alpha}}$$

with a phase shift and magnitude of

$$\phi_{\max}=\sin^{-1}\frac{1-\alpha}{1+\alpha},\ \left|U(j\omega_{\max})\right|=\frac{1}{\sqrt{\alpha}}$$

Lead Compensation

- This compensator allows the designer to raise the phase of the system in the vicinity of the crossover frequency



Lead Compensation Design

- The design procedure consists of the following steps:
  1. Find the open-loop gain K to satisfy steady-state error or bandwidth requirements
  2. Evaluate phase margin of the uncompensated system using the value of gain chosen above
  3. Find the required phase lead to meet the damping requirements
  4. Determine the value of α to yield the required increase in phase

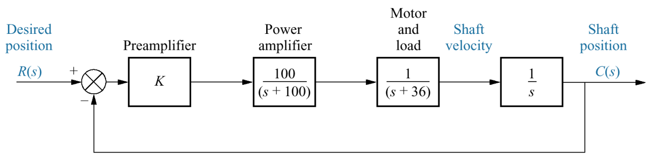
$$\phi_{\max} = \sin^{-1}\frac{1-\alpha}{1+\alpha}$$

Lead Compensation Design

5. Determine the new crossover frequency $\left|U(j\omega_{\max})\right| = \frac{1}{\sqrt{\alpha}}$
6. Determine the value of T such that ω_{max} lies at the new crossover frequency $\omega_{\max} = \frac{1}{T\sqrt{\alpha}}$
7. Draw the compensated frequency response and check the resulting phase margin.
8. Check that the bandwidth requirements have been met.
9. Simulate to be sure that the system meets the specifications (recall that the design criteria are based on a 2nd order system).

Lead Compensation Example

- Returning to the previous example, we will now design a lead compensator to yield a 20% overshoot and K_v=40, with a peak time of 0.1s



Lead Compensation Example

- From the specifications, we can determine the following requirements
  - For a 20% overshoot we find ζ=0.456 and hence a Phase Margin of 48.1°
  - For peak time of 0.1s with the given ζ, we can find the require closed loop bandwidth to be 46.6rad/s

$$\omega_{BW} = \frac{\pi}{T_p\sqrt{1-\zeta^2}}\sqrt{(1-2\zeta^2)+\sqrt{4\zeta^4-4\zeta^2+2}}$$

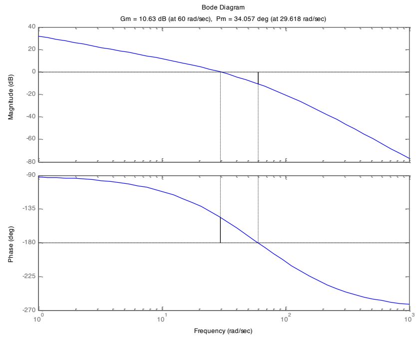
  - To meet the steady state error specification

$$K_v = 40 = \lim_{s\to 0} sG(s) = \frac{K100}{3600}$$

$$K = 1440$$

Lead Compensation Example

- From the Bode plot, we evaluate the PM to be 34° for a gain of 1440
- We can't simply increase the gain without violating the other design constraints
- We use a Lead Compensator to raise the PM



Lead Compensation Example

- We require a phase margin of 48.1°
- The lead compensator will also increase the phase margin frequency so we add a correction factor to compensate for the lower uncompensated system's phase angle
- The total phase contribution required is therefore

  48.1° – (34° – 10°) = 24.1°

Lead Compensation

- Based on the phase requirement we find

$$\phi_{\max} = \sin^{-1}\frac{1-\alpha}{1+\alpha}$$

$$\alpha = 0.42 \ for \ \phi_{\max} = 24.1^o$$

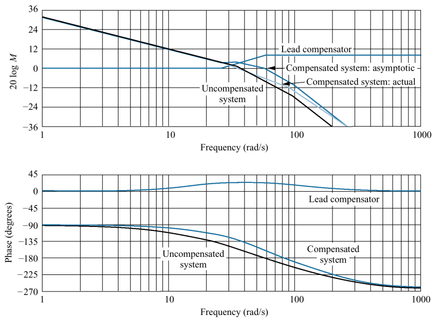
- The resulting magnitude is

$$|U(j\omega)| = \frac{1}{\sqrt{\alpha}} = 3.77dB$$

- Examining the Bode magnitude, we find that the frequency at which the magnitude is -3.77dB is ω_{max}=39rad/s
- The break frequencies can be found at 25.3 and 60.2

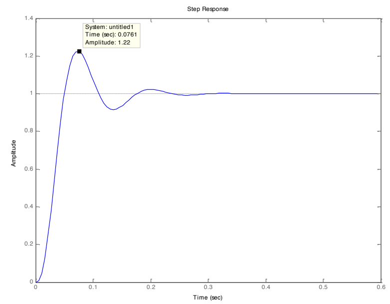
Lead Compensation Example

- The compensator is
$$U(s) = 2.38\frac{s+25.3}{s+60.2}$$
- The resulting system Bode plot shows the impact of the phase lead



Lead Compensation Example

- We need to verify the performance of the resulting design
- The simulation appears to validate our second order assumption

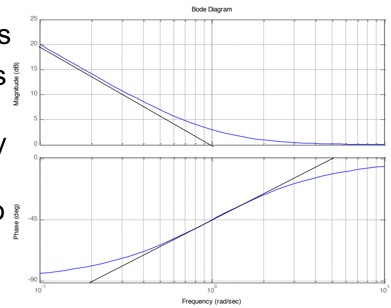


PI Controller

- We saw that the integral compensator takes on the form
$$U(s) = K\frac{s+z_c}{s}$$
- This results in infinite gain at low frequencies which reduces steady-state error
- A decrease in phase at frequencies lower than the break will also occur

PI Controller

- The Bode plot for a PI controller looks like this
- The break frequency is usually located at a frequency substantially lower than the crossover frequency to minimize the effect on the phase margin



Lag Compensation

- Lag compensation approximates PI control

$$U(s) = \frac{K(s+z_c)}{(s+p_c)} \quad z_c > p_c$$

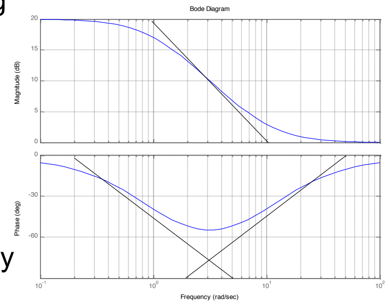
- This is often rewritten as

$$U(s) = \frac{Ts+1}{\alpha Ts+1}, \alpha > 1$$

  where α is the ratio between zero-pole break points
- The name Lag Compensation reflects the fact that this compensator imparts a phase lag

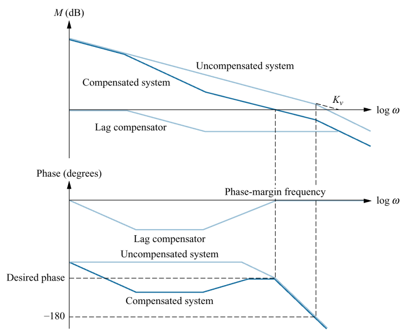
Lag Compensation

- The Bode plot for a Lag compensator looks like this
- This compensator effectively raises the magnitude for low frequencies
- The effect of the phase lag can be minimized by careful selection of the centre frequency



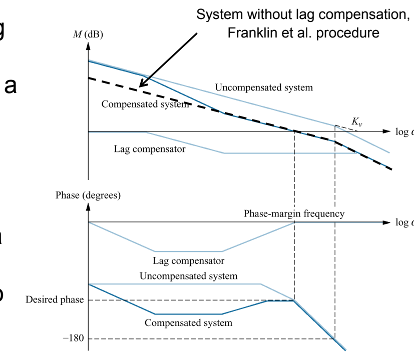
Lag Compensation

- In this case we are trying to raise the gain at low frequencies without affecting the stability of the system



Lag Compensation

- Nise suggests setting the gain K for s.s. error, then designing a lag network to attain desired PM.
- Franklin suggests setting the gain K for PM, then designing a lag network to raise the low freq. gain w/o affecting system stability.

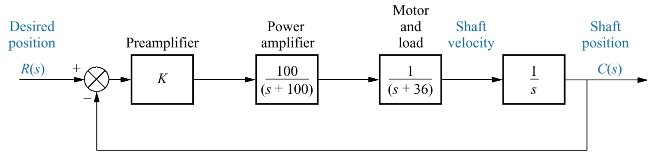


Lag Compensation Design

- The design procedure (Franklin et al.) consists of the following steps:
  1. Find the open-loop gain K to satisfy the phase margin specification without compensation
  2. Draw the Bode plot and evaluate low frequency gain
  3. Determine a to meet the low-frequency gain error requirement
  4. Choose the corner frequency ω=1/T to be one octave to one decade below the new crossover frequency
  5. Evaluate the second corner frequency ω=1/αT
  6. Simulate to evaluate the design and iterate as required

Lag Compensation Example

- Returning again to the previous example, we will now design a lag compensator to yield a ten fold improvement in steady-state error over the gain-compensated system while keeping the overshoot at 9.5%



Lag Compensation Example

- In the first example, we found the gain K=583.9 would yield our desired 9.5% overshoot with a PM of 59.2° at 14.8rad/s
- For this system we find that

$$K_v = \lim_{s\to 0} sG(s)$$

$$= 583.9\frac{100}{3600} = 16.22$$

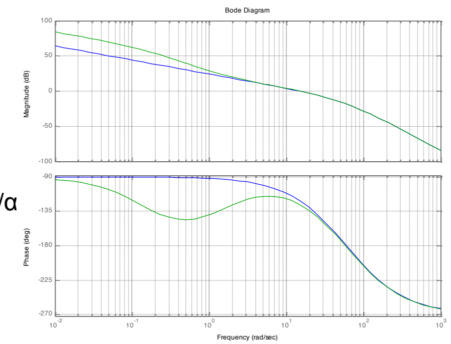
- We therefore require a K_v of 162.2 to meet our specification
- We need to raise the low frequency magnitude by a factor of 10 (or 20dB) without affecting the PM

Lag Compensator Example

- First we draw the Bode plot with K=583.9
- Set the zero at one decade, 1.48rad/s, lower than the PM frequency
- The pole will be at 1/α relative to this so

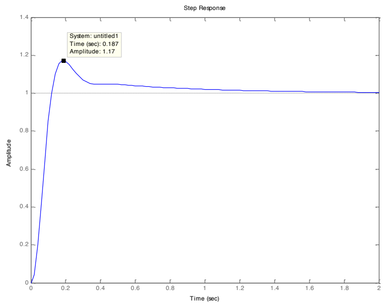
$$U(s) = \frac{s+1.483}{s+0.1483}$$

$$= 10\frac{0.674s+1}{6.74s+1}$$



Lag Compensator Example

- The resulting system has a low frequency gain K_v of 162.2 as per the requirement
- The overshoot is slightly higher than the desired
- Iteration of the zero and pole locations will yield a lower overshoot if required



Lag-Lead Compensation

- As with the Root Locus designs we considered previously, we often require both lead and lag components to effect a particular design
- This provides simultaneous improvement in transient and steady-state responses
- In this case we are trading off three primary design parameters
  - Crossover frequency ω_c which determines bandwidth, rise time and settling time
  - Phase margin which determines the damping coefficient and hence overshoot
  - Low frequency gain which determines steady state error characteristics

Conclusions

- We have looked at techniques for designing controllers using the frequency techniques
- There is once again a trade-off in the requirements of the system
- By selecting appropriate pole and zero locations we can influence the system properties to meet particular design requirements

Further Reading

- Nise
  - Sections 11.1-11.5
- Franklin & Powell
  - Section 6.7