ELCE 705
DIGITAL SIGNAL PROCESSING

FIR Filter Design Techniques

7.5-7.6

Contents

- Introduction
- Design of FIR Filters

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Basic filter types:

- lowpass filters (to pass low frequencies from zero to a certain cut-off frequency ω_C and to block higher frequencies)
- highpass filters (to pass high frequencies from a certain cut-off frequency ω_C to π and to block lower frequencies)
- bandpass filters (to pass a certain frequency range $[\omega_{\min}, \omega_{\max}]$, which does not include zero, and to block other frequencies)
- bandstop filters (to block a certain frequency range $[\omega_{\min}, \omega_{\max}]$, which does not include zero, and to pass other frequencies)

Basic filters

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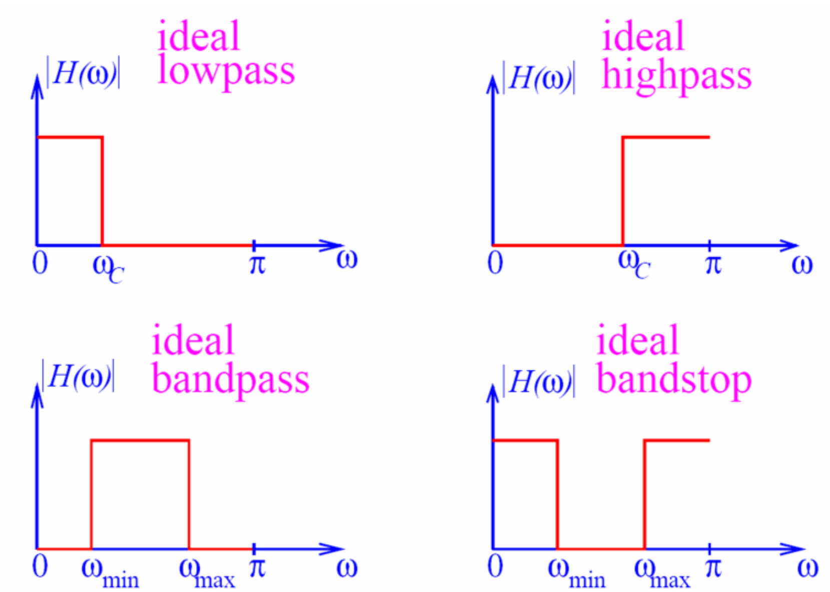
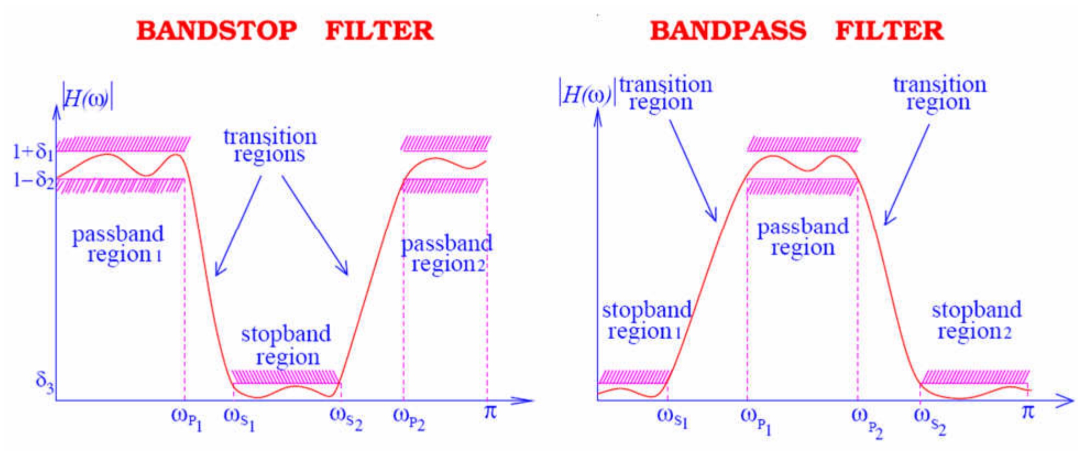
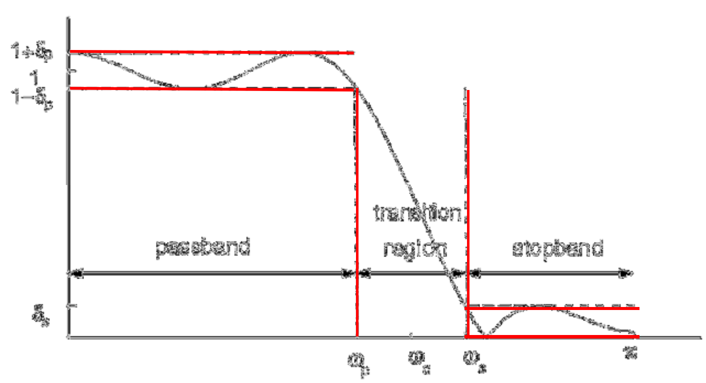




Design discrete-time filters

- Determine the system function
  - frequency response falls within the prescribed tolerances.
- This is a problem in functional approximation.
  - Designing IIR filters implies approximation by a rational function of z
  - Designing FIR filters implies polynomial approximation.



Contents

- Introduction
- Design of FIR Filters by Windowing

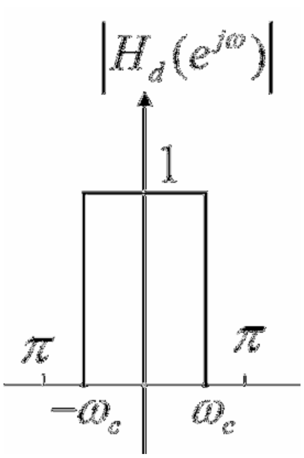
Finite impulse response (FIR) filter design

- A FIR filter is characterized by the equations

$$y[n] = \sum_{k=0}^{N-1} h[k]x[n-k] \qquad H(z) = \sum_{k=0}^{N-1} h[k]z^{-k}$$

- The following are useful properties of FIR filters:
  - Stable — the system function contains no poles.
  - Linear phase response. The result is no frequency dispersion, which is good for pulse and data transmission.
  - Finite length register effects are simpler to analyze and of less consequence than for IIR filters.
  - Simple to implement, and all DSP processors have architectures that are suited to FIR filtering.
  - For large N (many filter taps), the FFT can be used to improve performance.

Review example (Chapter 2)

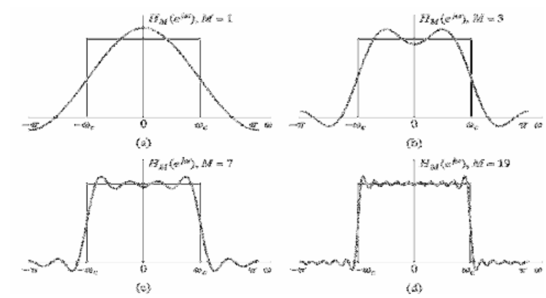


$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega})e^{j\omega n}\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega n}\,d\omega = \frac{\sin(\omega_c n)}{\pi n}$$

Noncausal
infinite long

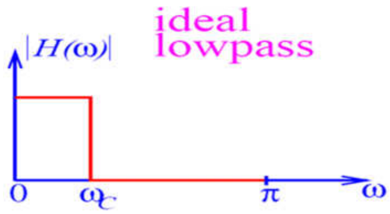
$$H_M(e^{j\omega}) = \sum_{n=-M}^{M} \frac{\sin \omega_c n}{\pi n} e^{-j\omega n}$$



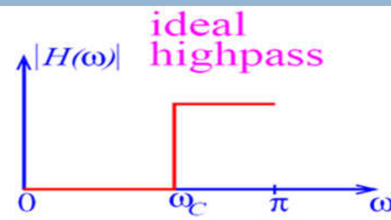
Window method for FIR filter design

- Step1: Using the inverse Fourier transform to determine $h_d\,[n]$ from the desired filter response $H_d\,(e^{j\omega})$
  - $h_d\,[n]$ in general is an infinite duration sequence
  - The corresponding filter is not realizable.
- Step2: window method
  - A FIR filter is obtained by multiplying a window w[n] with $h_d\,[n]$ to obtain a finite duration $h[n]$ of length N.
  - If $h_d[n]$ is even or odd symmetric and w[n] is even symmetric, then $h_d[n]$w[n] is a linear phase filter.
- Two important design criteria are the length and shape of the window w[n].

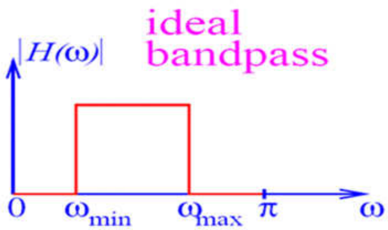
Basic filters



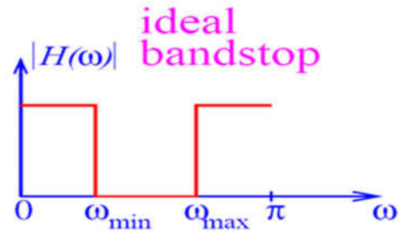
$$h_{lp}[n] = \frac{\sin[\omega_c (n - M/2)]}{\pi (n - M/2)}$$



$$h_{lp}[n] = \frac{\sin[\pi (n - M/2)]}{\pi (n - M/2)} - \frac{\sin[\omega_c (n - M/2)]}{\pi (n - M/2)}$$



$$h_{lp}[n] = \frac{\sin[\omega_{\max} (n - M/2)]}{\pi (n - M/2)} - \frac{\sin[\omega_{\min} (n - M/2)]}{\pi (n - M/2)}$$



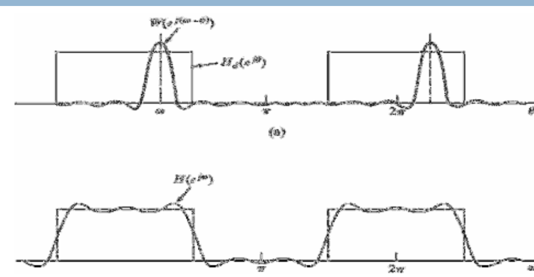
$$h_{lp}[n] = \frac{\sin[\pi (n - M/2)]}{\pi (n - M/2)} - \frac{\sin[\omega_{\max} (n - M/2)]}{\pi (n - M/2)} + \frac{\sin[\omega_{\min} (n - M/2)]}{\pi (n - M/2)}$$

Convolution operation

$$h[n] = h_d[n] w[n]$$

where $w[n] = \begin{cases} 1 & 0 \le n \le M \\ 0 & else \end{cases}$

$$H(e^{j\omega}) = H_d(e^{j\omega}) * W(e^{j\omega})$$



- The *mainlobe* width of $W(e^{j\omega})$ affects the transition width of $H(e^{j\omega})$
  - Increasing the length N of $h[n]$ reduces the mainlobe width and the transition width of the overall response.
- The *sidelobes amplitude* of $W(e^{j\omega})$ affect the passband and stopband tolerance of $H(e^{j\omega})$.
  - Be controlled by changing the shape of the window.
  - Change M does not affect the sidelobe behaviour.

FIR Filter Design

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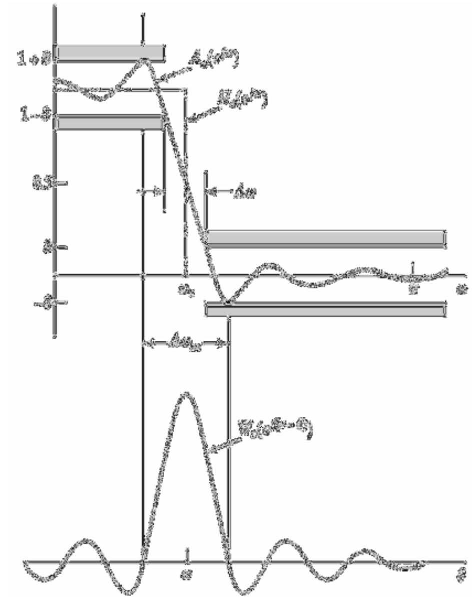
- The width of the transition band
  - Determined by the width of the main lobe of the Fourier transform of the window.
- The passband and stopband ripples
  - Determined by the side lobes of the Fourier transform of the window.
  - Ripples of the two bands are approximately the same.
- Change the shape and duration of the window can control the result FIR filter.



Rectangular Window

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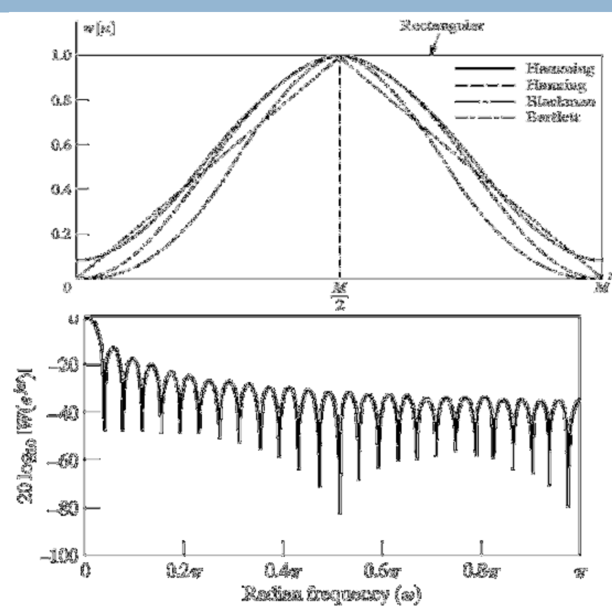
- Simplest window possible

$$w[n] = \begin{cases} 1 & 0 \le n \le M \\ 0 & \text{else} \end{cases}$$

- Narrowest main lob
  - $4\pi/(M+1)$
  - Sharpest transitions at discontinuities in frequency
- Large side lobs
  - -13 dB
  - Large oscillation around discontinuities
- M increase, side lobe remains constant for rectangular window



Commonly used windows

- Bartlett

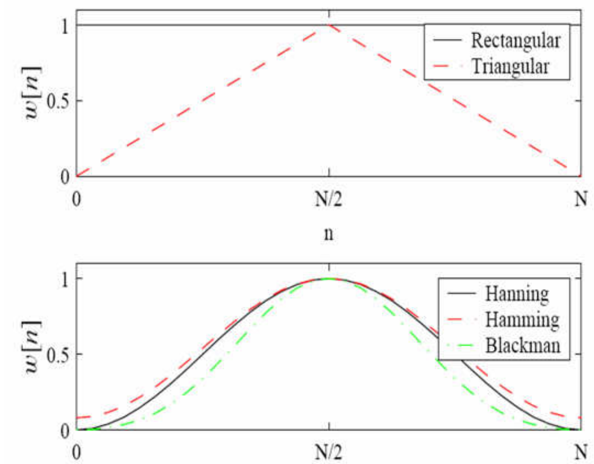
$$w[n] = \begin{cases} 2n/M & 0 \le n \le M/2 \\ 2 - 2n/M & M/2 \le n \le M \\ 0 & \text{else} \end{cases}$$

- Hann

$$w[n] = \begin{cases} \frac{1}{2}\left[1 - \cos\left(\frac{2\pi n}{M}\right)\right] & 0 \le n \le M \\ 0 & \text{else} \end{cases}$$

- Hamming

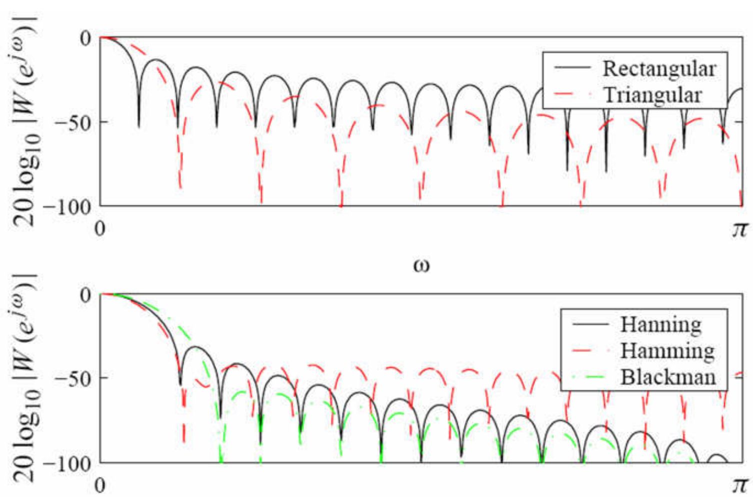
$$w[n] = \begin{cases} 0.54 - 0.46\cos\left(\frac{2\pi n}{M}\right) & 0 \le n \le M \\ 0 & \text{else} \end{cases}$$



Taper the window smoothly to zero at each end. The height of the side lobes are reduced at the cost of a wider main lobe

FT of the windows

All windows trade off a reduction in sidelobe level against an increase in mainlobe width.



Comparisons

TABLE 7.2 COMPARISON OF COMMONLY USED WINDOWS

Type of Window	Peak Side-Lobe Amplitude (Relative)	Approximate Width of Main Lobe	Peak Approximation Error, $20\log_{10}\delta$ (dB)	Equivalent Kaiser Window, β	Transition Width of Equivalent Kaiser Window
Rectangular	−13	$4\pi/(M+1)$	−21	0	$1.81\pi/M$
Bartlett	−25	$8\pi/M$	−25	1.33	$2.37\pi/M$
Hann	−31	$8\pi/M$	−44	3.86	$5.01\pi/M$
Hamming	−41	$8\pi/M$	−53	4.86	$6.27\pi/M$
Blackman	−57	$12\pi/M$	−74	7.04	$9.19\pi/M$

- The rectangular window
  - The narrowest main lobe, so it yield the sharpest transitions of $H(e^{j\omega})$ at a discontinuity of $H_d(e^{j\omega})$
  - The highest side lobe, which result in oscillations of $H(e^{j\omega})$ of considerable size

Example

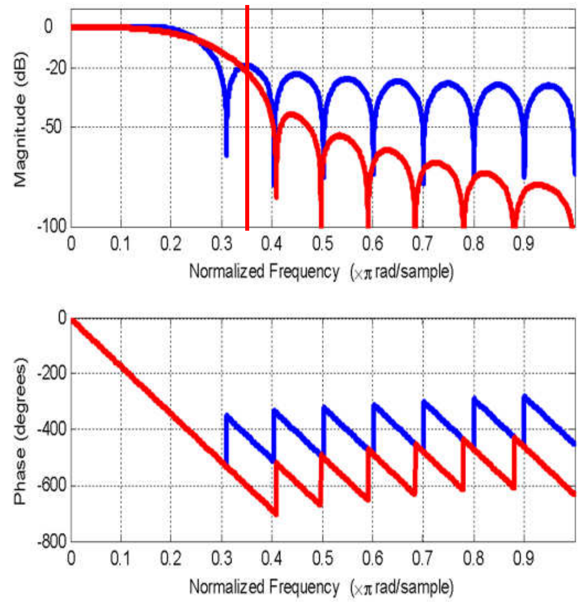
- The specifications for the filter are

  - Passband cutoff frequency: $\omega_p = 0.15\pi$
  - Stopband cutoff frequency: $\omega_s = 0.35\pi$
  - Passband ripple[1]: $-3 \text{ dB} \le \left|H(e^{j\omega})\right| \le 0 \text{ dB}, \ |\omega| \le \omega_p$
  - Stopband attenuation: $\left|H(e^{j\omega})\right| \le -20 \text{ dB}, \ \omega_s \le |\omega| \le \pi$

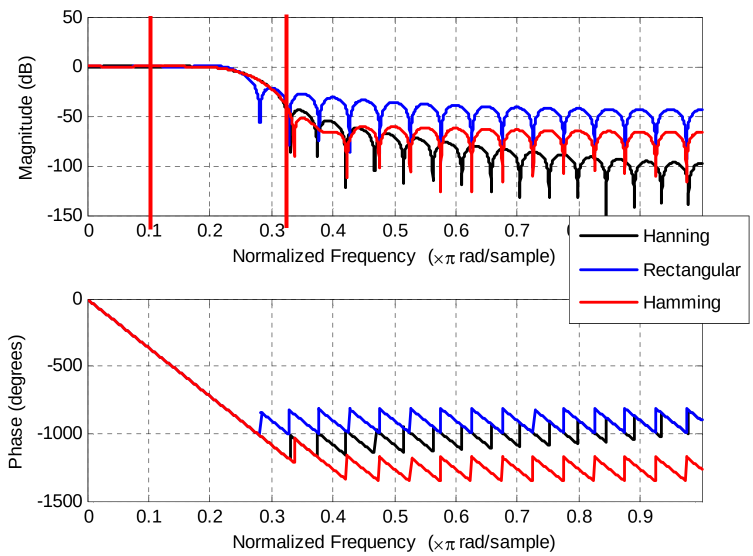
FIR filter M=19, Rectangular Window

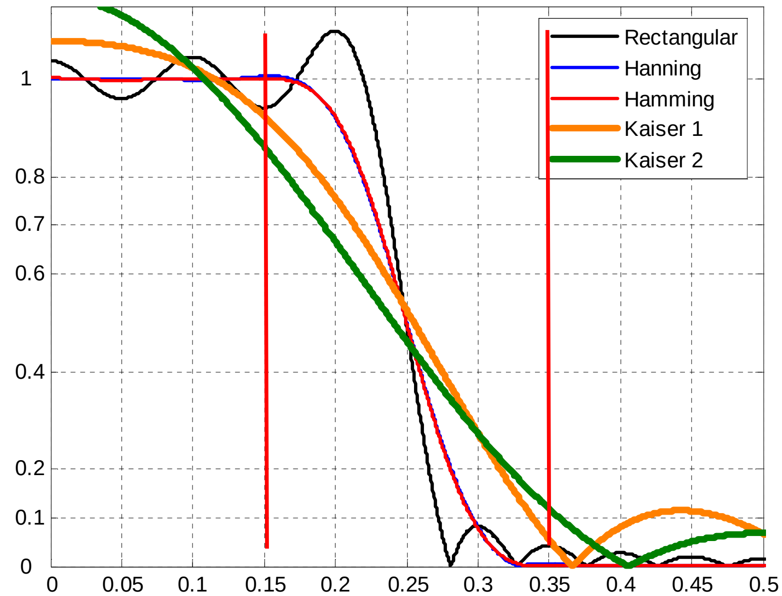

```
n=0:1:19;
hn=0.25*sinc(0.25*(n-19/2));
w=0:0.001:pi;
freqz(hn,1,w)

hnn=hn.*hanning(20)';
hold on;
freqz(hnn,1,w)
```



40-order FIR filter



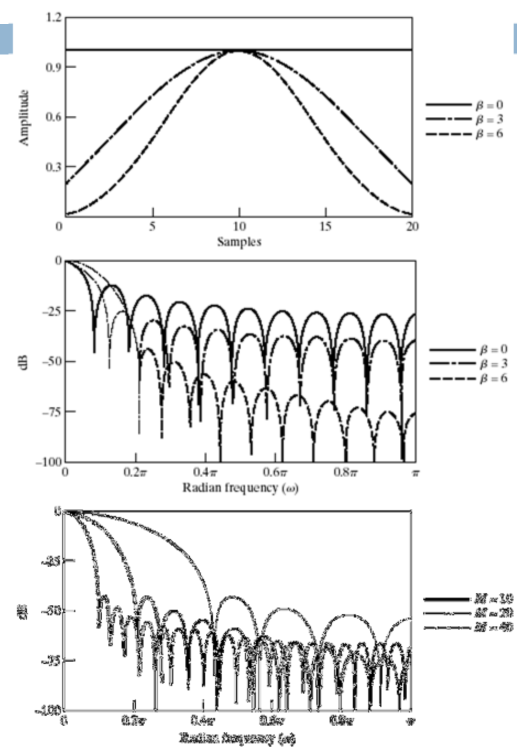


Kaiser Window Filter Design Method

- Parameterized equation forming a set of windows
  - Parameter to change main-lob width and side-lob area trade-off

$$w[n] = \begin{cases} \dfrac{I_0\left[\beta\sqrt{1-\left(\dfrac{n-M/2}{M/2}\right)^2}\right]}{I_0(\beta)} & 0 \le n \le M \\ 0 & \text{else} \end{cases}$$

  - $I_0(.)$ represents zeroth-order modified Bessel function of 1st kind



Kaiser Window

- In practice, the window shape is chosen first based on passband and stopband tolerance requirements.

$$A = -20\log_{10}\delta$$

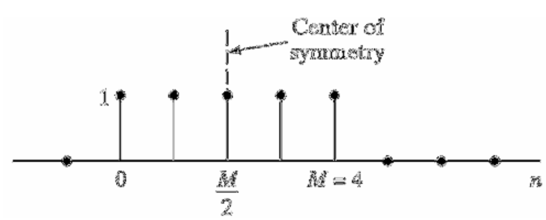
$$\beta = \begin{cases} 0.1102(A-8.7) & , \quad A > 50 \\ 0.5842(A-21)^{0.4} + 0.07886(A-21), & 21 \le A \le 50 \\ 0.0 & , \quad A < 21 \end{cases}$$

- The window size is then determined based on transition width requirements.

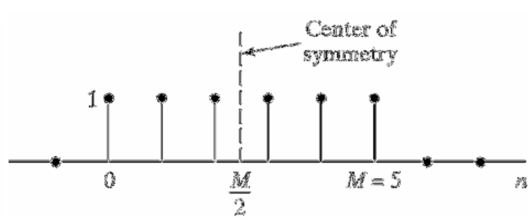
$$M = \frac{A-8}{2.285\Delta\omega} \qquad \Delta\omega = \omega_s - \omega_p$$

Types of Linear-Phase Sytems

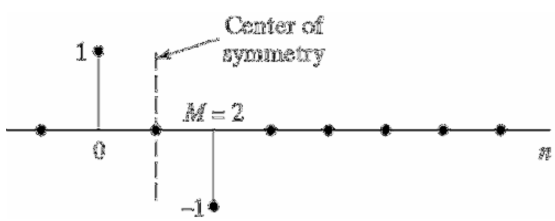
- M is an even integer
- M is an odd integer



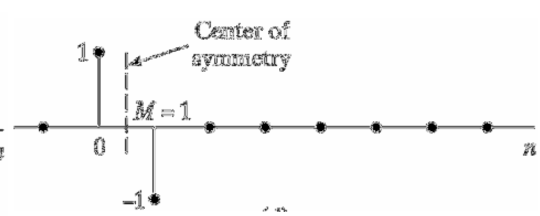
Type 1 $h[n] = h[M-n]$



Type 2 $h[n] = h[M-n]$



Type 3 $h[n] = -h[M-n]$



Type 4 $h[n] = -h[M-n]$

- A type II filter has the property that it is *always* zero for $\omega=\pi$, and is not appropriate for a highpass filter.
- Filters of type III and IV introduce a 90 phase shift, and have a frequency response that is always zero at $\omega= 0$ which makes them unsuitable for as lowpass filters.
- Additionally, the type III response is always zero at $\omega=\pi$, making it unsuitable as a highpass filter.
- The type I filter is the most versatile of the four.

Example- highpass filter

The ideal highpass filter with generalized linear phase has the frequency response

$$H_{\text{hp}}(e^{j\omega}) = \begin{cases} 0, & |\omega| < \omega_c, \\ e^{-j\omega M/2}, & \omega_c < |\omega| \le \pi. \end{cases} \tag{7.78}$$

$$h_{\text{hp}}[n] = \frac{\sin \pi(n - M/2)}{\pi(n - M/2)} - \frac{\sin \omega_c(n - M/2)}{\pi(n - M/2)}, \qquad -\infty < n < \infty.$$

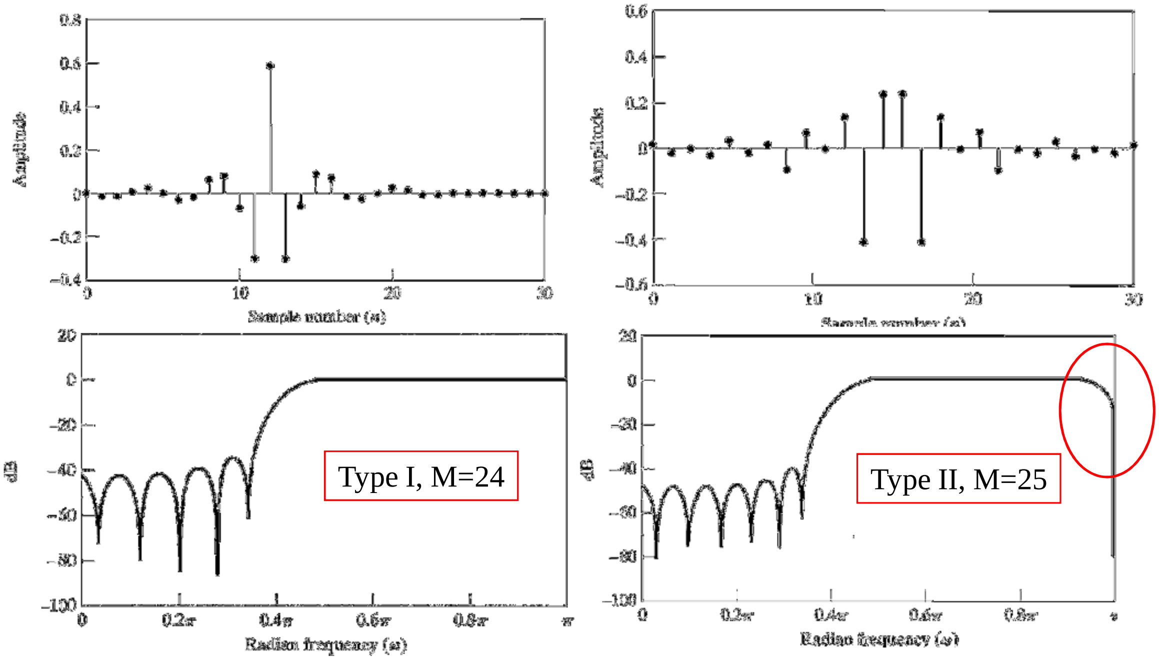
Suppose that we wish to design a filter to meet the highpass specifications

$$|H(e^{j\omega})| \le \delta_2, \qquad |\omega| \le \omega_s$$

$$1 - \delta_1 \le |H(e^{j\omega})| \le 1 + \delta_1, \qquad \omega_p \le |\omega| \le \pi$$

where $\omega_s = 0.35\pi$, $\omega_p = 0.5\pi$, and $\delta_1 = \delta_2 = \delta = 0.02$. Since the ideal response also has a discontinuity, we can apply Kaiser's formulas in Eqs. (7.75) and (7.76) with $A = 33.98$ and $\Delta\omega = 0.15\pi$ to estimate the required values of $\beta = 2.65$ and $M = 24$.

Obtained high-pass filter



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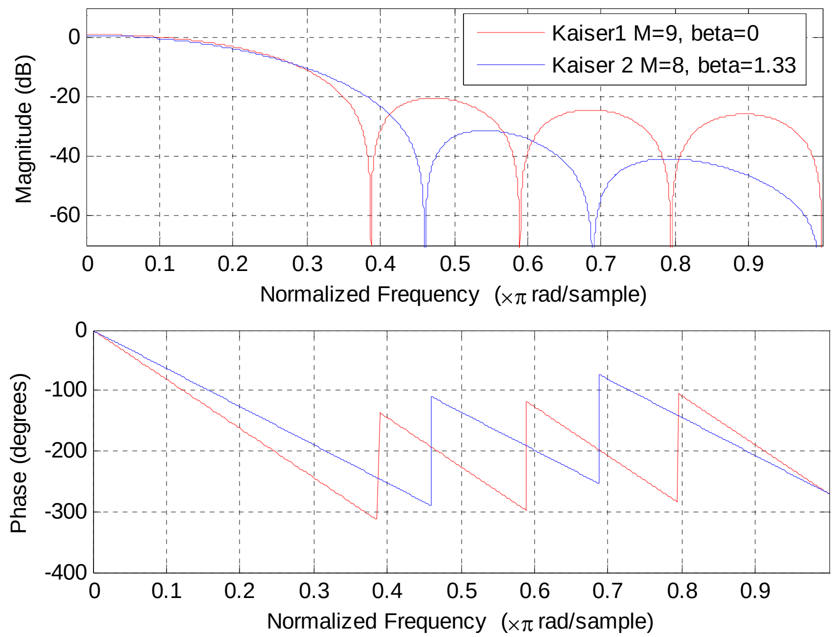
Kaiser Window



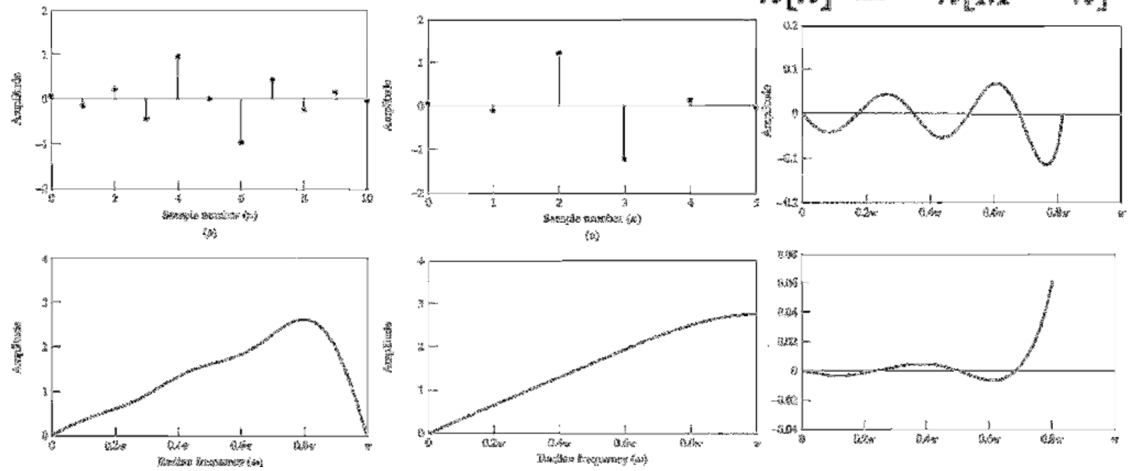
Kaiser window design of a differentiator

- Samples of the derivative of the continuous-time signal

$$H_{\text{diff}}(e^{j\omega}) = (j\omega)e^{-j\omega M/2}, \qquad -\pi < \omega < \pi$$

$$h_{\text{diff}}[n] = \frac{\cos \pi(n - M/2)}{(n - M/2)} - \frac{\sin \pi(n - M/2)}{\pi(n - M/2)^2}, \qquad -\infty < n < \infty.$$

$$h[n] = -h[M - n]$$



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