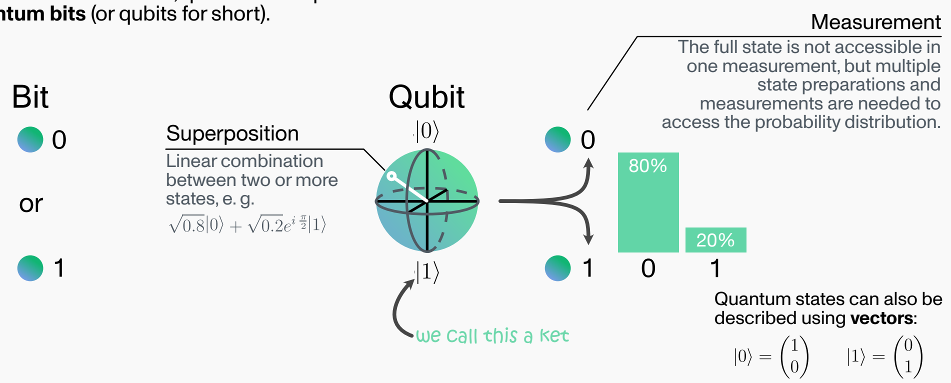
# Quantum Computing CHEAT SHEET *for circuit magicians*

## Bits and Qubits

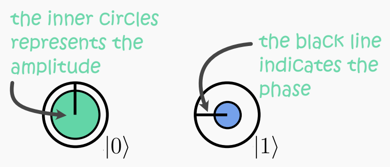
Instead of classical bits, quantum computers use **quantum bits** (or qubits for short).




Quantum states can also be described using **vectors**:

$|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ $|1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$

One way to picture quantum states is the circle notation:



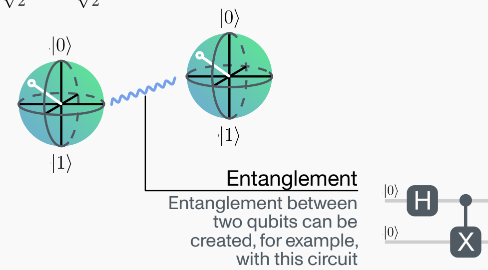

$\sqrt{0.8}|0\rangle + \sqrt{0.2}e^{i\frac{\pi}{2}}|1\rangle$

Multiple qubits form a **register**. The number of computational states doubles with each new qubit. A state with multiple qubits involved is often denoted like $|00\rangle = |0\rangle\otimes|0\rangle$ (where $\otimes$ is the tensor product)

| # qubits | # basis states | example |
|---|---|---|
| 1 | 2 | $\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$ |
| 2 | 4 | $\frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle + \frac{1}{2}|11\rangle$ possible linear combination of two-qubit states |
| 3 | 8 | $\frac{1}{2\sqrt{2}}|000\rangle - \frac{1}{2\sqrt{2}}|001\rangle - \frac{1}{2\sqrt{2}}|010\rangle + \frac{1}{2\sqrt{2}}|011\rangle - \frac{1}{2\sqrt{2}}|100\rangle + \frac{1}{2\sqrt{2}}|101\rangle - \frac{1}{2\sqrt{2}}|110\rangle + \frac{1}{2\sqrt{2}}|111\rangle$ |

Two or more qubits can be **entangled**, meaning that the state cannot be factorized as a product of states:

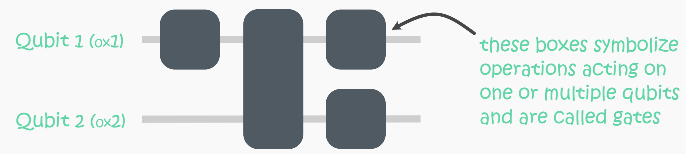
$\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle \neq (a|0\rangle + b|1\rangle)(c|0\rangle + d|1\rangle)$




## One-Qubit Gates

| Gate | Matrix | Ket and circle notation |
|---|---|---|
| **X** — **Pauli-X** is a 180° rotation around the x-axis; also known as the quantum NOT gate | $\begin{pmatrix}0&1\\1&0\end{pmatrix}$ | $a|0\rangle + b|1\rangle$ → X → $b|0\rangle + a|1\rangle$ |
| **Y** — **Pauli-Y** is a 180° rotation around the y-axis | $\begin{pmatrix}0&-i\\i&0\end{pmatrix}$ | $a|0\rangle + b|1\rangle$ → Y → $-ib|0\rangle + ia|1\rangle$ |
| **Z** — **Pauli-Z** is a 180° rotation around the z-axis | $\begin{pmatrix}1&0\\0&-1\end{pmatrix}$ | $a|0\rangle + b|1\rangle$ → Z → $a|0\rangle - b|1\rangle$ |
| **H** — **Hadamard** maps $|0\rangle$ to $|+\rangle$ and $|1\rangle$ to $|-\rangle$; used to create an equal superposition | $\frac{1}{\sqrt{2}}\begin{pmatrix}1&1\\1&-1\end{pmatrix}$ | $|0\rangle$ → H → $|+\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$; $|1\rangle$ → H → $|-\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$ |
| **S** — **S** is a 90° rotation around the z-axis; S S = Z ; The inverse $S^\dagger$ rotates in the opposite direction | $\begin{pmatrix}1&0\\0&i\end{pmatrix}$ | $a|0\rangle + b|1\rangle$ → S → $a|0\rangle + be^{i\frac{\pi}{2}}|1\rangle$ ($e^{i\frac{\pi}{2}}=i$) |
| **T** — **T** is a 45° rotation around the z-axis; T T = S ; The inverse $T^\dagger$ rotates in the opposite direction | $\begin{pmatrix}1&0\\0&e^{i\frac{\pi}{4}}\end{pmatrix}$ | $a|0\rangle + b|1\rangle$ → T → $a|0\rangle + be^{i\frac{\pi}{4}}|1\rangle$ |

**Quantum circuits** are a model to visualize operations on qubits.




**Binary and decimal**: You will find both the use of the binary representation of qubit states as well as the decimal representation.

| Decimal | Binary | Decimal | Binary |
|---|---|---|---|
| $|0\rangle$ | $|000\rangle$ | $|4\rangle$ | $|100\rangle$ |
| $|1\rangle$ | $|001\rangle$ | $|5\rangle$ | $|101\rangle$ |
| $|2\rangle$ | $|010\rangle$ | $|6\rangle$ | $|110\rangle$ |
| $|3\rangle$ | $|011\rangle$ | $|7\rangle$ | $|111\rangle$ |

means that the first and second qubit are $|1\rangle$ and the third qubit is $|0\rangle$

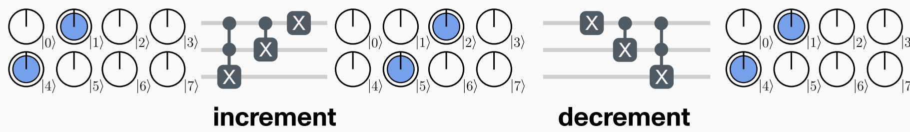
## Multi-Qubit Gates

| Gate | Matrix | Ket and circle notation |
|---|---|---|
| control qubit / target qubit — **CNOT** applies a Pauli-X gate to the target qubit if the state of the control qubit state is $|1\rangle$ | $\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}$ | $a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$ → $a|00\rangle + b|01\rangle + d|10\rangle + c|11\rangle$ |
| **CZ** applies a Pauli-Z gate to the target qubit if the state of the control qubit is $|1\rangle$ | $\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-1\end{pmatrix}$ | $a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$ → $a|00\rangle + b|01\rangle + c|10\rangle - d|11\rangle$ |
| **SWAP** swaps the state of 2 qubits; can be implemented using 3 alternating CNOTs | $\begin{pmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{pmatrix}$ | $a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$ → $a|00\rangle + c|01\rangle + b|10\rangle + d|11\rangle$ |
| **Toffoli** applies a Pauli-X gate to the target qubit if both control qubits are in state $|1\rangle$; can be used to construct a reversible version of the classical AND-gate | $\begin{pmatrix}1&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0\\0&0&1&0&0&0&0&0\\0&0&0&1&0&0&0&0\\0&0&0&0&1&0&0&0\\0&0&0&0&0&1&0&0\\0&0&0&0&0&0&0&1\\0&0&0&0&0&0&1&0\end{pmatrix}$ | $a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle + f|101\rangle + g|110\rangle + h|111\rangle$ → $a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle + f|101\rangle + h|110\rangle + g|111\rangle$ |

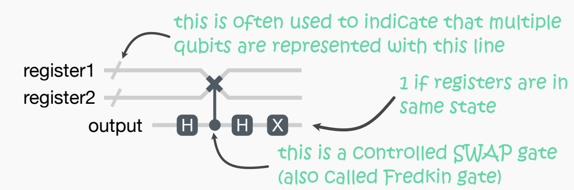
## Building Blocks for Quantum Algorithms

There are many clever ways to arrange quantum circuits. A couple of them are depicted below.

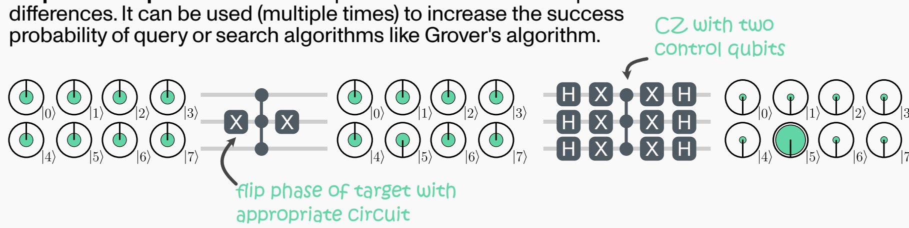
**Increment & decrement** are used to add or subtract one from a register and are an example of how to do arithmetic with quantum gates.



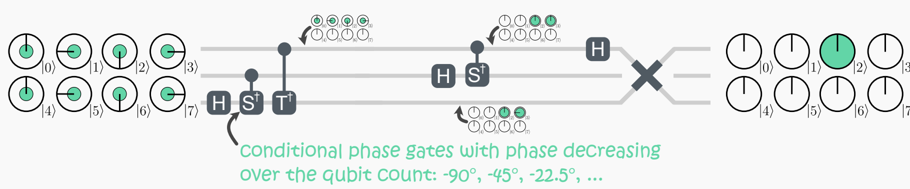

**Swap test** allows for checking how similar the states in two registers are.




**Amplitude Amplification** converts phase differences into amplitude differences. It can be used (multiple times) to increase the success probability of query or search algorithms like Grover's algorithm.




**Quantum Fourier Transform** can reveal the signal frequency in a register. Among other algorithms, it is used in Shor's algorithm for factoring numbers and computing the discrete logarithm.



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