Math Exam

1. Elementary Algebra

1.1 Simplify

$$\frac{x^{n+2}}{x^{n-2}}$$

$$= x^{n+2-(n-2)}$$

$$= x^{\cancel{n}+2-\cancel{n}+2}$$

$$= x^4$$

1.2 Solve for x

$$x^{-1} \times 8 = 2$$

$$\frac{8}{x} = 2$$

$$8 = 2x$$

$$x = \frac{8}{2}$$

$$x = 4$$

1.3 Calculate missing value

$a = 5$ $b = 10$

$(a^b)^0 = \ldots.$

$$(5^{10})^0 = 5^{10\cdot 0} = 5^0 = 1$$

1.4 Calculate $\frac{\sqrt{4x}}{\sqrt{x}}$

$= \frac{(4x)^{1/2}}{x^{1/2}}$

$= \frac{4^{1/2} \cancel{x^{1/2}}}{\cancel{x^{1/2}}}$

$= 2$.

1.5 Solve for x

$x^2 + (x+1)^2 = (x+2)^2$

$x^2 + x^2 + 2x + 1 = x^2 + 4x + 4$

$2x^2 + 2x + 1 = x^2 + 4x + 4$

$2x^2 - x^2 + 2x - 4x + 1 - 4 = 0$

$x^2 - 2x - 3 = 0$

$x^2 - 3x + x - 3 = 0$

$x(x-3) + 1(x-3) = 0$

$x = 3 \quad x = -1$

1.6 $\cancel{2x}$ $2^x > 1024$

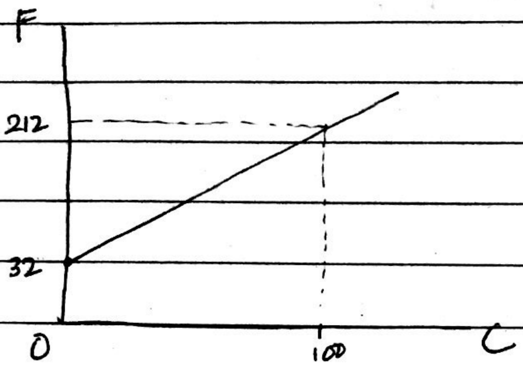
$2^x > 2^{10}$

$x > 10$

2. Functions of One Variable

2.1 $C_1 = 0^\circ c$ $\quad F_1 = 32^\circ$

$C_2 = 100^\circ c$ $\quad F_2 = 212^\circ$



$F = 32 + bC$

$$\text{slope} = \frac{F_2 - F_1}{C_2 - C_1}$$

$$= \frac{212 - 32}{100 - 0}$$

$$= \frac{180}{100}$$

$$= 1.8$$

$F = 32 + 1.8C$

$F = C$

$F = 32 + 1.8F$

$F - 1.8F = 32$

$-0.8F = 32$

$F = C = -40^\circ$

2.2 $f(x) = 5x + 4$ $\quad f(3) = y$ $\quad$ Find y

$f(3) = 5(3) + 4 = y$

$15 + 4 = y$

$y = 19$

2.3 Find all values of x that satisfy

$x^2 - 4x + 3 = 0$

$x^2 - 3x - x + 3 = 0$

$x(x-3) - 1(x-3) = 0$

$(x-3)(x-1) = 0$

$x = 3 \quad x = 1$

2.4 10 HUF for 90 years $i = 2\%$.

$$= 10 \times (1.02)^{90}$$
$$= 59.43 \text{ HUF}$$

2.5 $e^{\ln 5}$
$$= 5$$

3 Calculus

3.1 $$\sum_{i=1}^{\infty} \frac{12}{6i} = \frac{12}{6^1} + \frac{12}{6^2} + \frac{12}{6^3} \ldots$$
$$= 2 + \frac{2}{6} + \frac{2}{6^2} + \ldots$$
$$= 2\left(1 + \frac{1}{6} + \frac{1}{6^2} + \ldots\right)$$

sum of infinite series $= \frac{a}{1-b}$ $a = 1$ $b = 1/6$

$$= 2\left(\frac{1}{1-1/6}\right) = 2\left(\frac{1}{5/6}\right) = \frac{12}{5}$$

3.2 → Solve after 3.4

3.3 $f(x) = x^5 - 8$ at $x = -3$

$\frac{df(x)}{dx} = 5x^4$ slope at $x = -3$
$$= 5(-3)^4$$
$$= 405$$

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3.4 $$\frac{d}{dx} \frac{x^3+2x-1}{x-2}$$

$$\frac{df(x)}{dx} = \frac{(3x^2+2)(x-2) - (x^3+2x-1)(1)}{(x-2)^2}$$

$$= \frac{3x^3 - 6x^2 + 2x - 4 - (x^3+2x-1)}{(x-2)^2}$$

$$= \frac{3x^3 - x^3 - 6x^2 + \cancel{2x} - \cancel{2x} - 4 + 1}{(x-2)^2}$$

$$= \frac{2x^3 - 6x^2 - 3}{(x-2)^2}$$

3.2 Find following limit

$$\lim_{x \to 1} = \frac{6^{1-x}}{x} = \frac{6^{1-1}}{1} = \frac{6^0}{1} = 1$$

3.5 Find second derivative

$$\frac{d^2}{dx^2} 4x^4 + 4x^2$$

$$\frac{df(x)}{dx} = 16x^3 + 8x$$

$$\frac{d^2 f(x)}{dx^2} = 48x^2 + 8$$

3.6 Find derivative

$$\frac{d}{dx} \frac{\ln x}{e^x}$$

$$= \frac{1/x \cdot e^x - \ln x \cdot e^x}{(e^x)^2}$$

$$= \frac{\frac{e^x}{x} - \ln x \cdot e^x}{(e^x)^2}$$

$$= \frac{\cancel{e^x}(1/x - \ln x)}{(e^x)^{\cancel{2}}}$$

$$= \frac{\frac{1}{x} - \ln x}{e^x}$$

3.7

$$f(x) = 3x^2 - 5x + 2$$

$$f'(x) = 6x - 5$$

$$6x - 5 = 0$$

$$6x = 5$$

$$x = 5/6$$

$$f''(x) = 6 > 0$$ minimum → convex

$$f(x) = 3x^2 - 5x + 2 = 0$$

$$3x^2 - 3x - 2x + 2 = 0$$

$$3x(x-1) - 2(x-1) = 0$$

$$(3x-2)(x-1) = 0$$

$$3x = 2 \qquad x = 1$$

$$x = 2/3$$

X	0	2/3	5/6	1	6
f(x)		0		0	
f'(x)	-5	-1	0	+	+
shape	↓	↓	local minimum	↑	↑
f''(x)	+	+	+	+	+
shape	∪	∪	∪	∪	∪

3.8 $f(x,y) = x^2+y^3$ Calculate $f(2,3)$

$= 2^2+3^3$

$= 4+27$

$= 31$

3.9 $f(x,y) = \ln(x-y)$

$(x-y)>0$, the function is defined for all $x>y$

3.10 $\frac{d}{dx}\, x^5+xy^3$

$\frac{df(x,y)}{dx} = 5x^4+y^3$

$\frac{df(x,y)}{dy} = 3xy^2$

3.11 $f(x,y) = x^2y^2+10$

$\frac{df(x,y)}{dx} = 2xy^2 = 0$ — (1)

$\frac{df(x,y)}{dy} = 2x^2y = 0$ — (2)

$\cancel{2}\cancel{x}y^{\cancel{2}} = \cancel{2}x^{\cancel{2}}y$

$y = x \rightarrow$ substituting in (2)

$2y^2y = 0$ $2y^3 = 0$ $y = x = 0$

local min ~~for~~ at $(0,0)$

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3.12 max x^2y^2 s.t $x+y=10$

$$\mathcal{L} = x^2y^2 - \lambda(x+y-10)$$

$$\frac{d\mathcal{L}}{dx} = 2xy^2 - \lambda = 0$$

$$\frac{d\mathcal{L}}{dy} = 2x^2y - \lambda = 0$$

$$\frac{d\mathcal{L}}{d\lambda} = x+y-10 = 0$$

$$2xy^2 = \lambda \quad — \quad 1$$

$$2xy^2 = \lambda \quad — \quad 2$$

$$\cancel{2}\cancel{x}y^{\cancel{2}} = \cancel{2}x^{\cancel{2}}\cancel{y}$$

$$y = x$$

$$x + x - 10 = 0$$

$$2x = 10$$

$$x = 5 \qquad x = y = 5$$

4. Linear Algebra

4.1 $A = \begin{bmatrix} 2 & 6 \\ 5 & 1 \\ 1 & 9 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 1 & 7 \\ 2 & 8 & 2 \end{bmatrix}$

3×2 2×3

$A \cdot B = 3 \times 3$ matrix

$$= \begin{bmatrix} 2\cdot1+6\cdot2 & 2\cdot1+6\cdot8 & 2\cdot7+6\cdot2 \\ 5\cdot1+1\cdot2 & 5\cdot1+1\cdot8 & 5\cdot7+1\cdot2 \\ 1\cdot1+9\cdot2 & 1\cdot1+9\cdot8 & 1\cdot7+9\cdot2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+12 & 2+48 & 14+12 \\ 5+2 & 5+8 & 35+2 \\ 1+18 & 1+72 & 7+18 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 50 & 26 \\ 7 & 13 & 37 \\ 19 & 73 & 25 \end{bmatrix}$$

4.2

$$A = \begin{bmatrix} 2 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 9 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

B.A

$$\underset{2 \times 3}{\begin{bmatrix} 1 & 9 & 1 \\ 2 & 1 & 2 \end{bmatrix}} \qquad \underset{3 \times 2}{\begin{bmatrix} 2 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix}}$$

$$= \begin{bmatrix} 1\cdot2+9\cdot4+1\cdot1 & 1\cdot2+9\cdot6+1\cdot3 \\ 2\cdot2+1\cdot4+2\cdot1 & 2\cdot2+1\cdot6+2\cdot3 \end{bmatrix} \quad 2 \times 2$$

$$= \begin{bmatrix} 39 & 59 \\ 10 & 16 \end{bmatrix}$$

4.3 Transpose

$$A = \begin{bmatrix} 7.1 & 9.1 & 4.7 \\ 2 & 7.8 & 1.1 \\ 4 & 4.44 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 7.1 & 2 & 4 \\ 9.1 & 7.8 & 4.44 \\ 4.7 & 1.1 & 0 \end{bmatrix}$$

4.4

$$\begin{bmatrix} 1 & 9 \\ 2 & 8 \end{bmatrix}$$

calculate determinant

$$\det = 8 \cdot 1 - 9 \cdot 2$$
$$= 8 - 18$$
$$= -10$$

5 Probability Theory

5.1 Replacement with order

$$\# \Omega = n^k = 6^2 = 36$$

d_1 \ d_2	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

S.2 Drug user = D $\quad P(D) = 0.01$

Non-drug user = D' $\quad P(D') = 0.99$

Positive result = Pt $\quad P(Pt|D) = 0.99$

Negative result = Nt $\quad P(Nt|D') = 0.995$

$$P(Pt) = P(D)P(Pt/D) + P(D')P(Pt/D')$$

$$= (0.01)(0.99) + (0.99)(0.005)$$

$$= 0.0099 + 0.00495$$

$$= 0.01485$$

S.3

$$P(D|Pt) = \frac{P(Pt|D)\ P(D)}{P(Pt)}$$

$$= \frac{0.99 \times 0.01}{0.01485}$$

$$= 2/3 \text{ or } 0.667$$