Comparative Study of TSP Problem with GA and PSO Algorithm

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Introduction to Mathematical Modelling

1. Representation of real world problem in mathematical form with some simplified assumptions which helps to understand in fundamental and quantitative way.
2. It is complement to theory and experiments and often to integrate them.
3. Having widespread applications in all branches of Science and Engineering and Technology, Biology, Medicine and several other interdisciplinary areas.

Necessity of Mathematical Model

1. To perform experiments and to solve real world problems which may be risky and expensive or time consuming..
2. Emerged as a powerful, indispensable tool for studying a variety of problems in scientific research, product and process development and manufacturing..
3. Improves the quality of work and reduced changes, errors and rework.

Travelling Sales Problem

Given a list of cities and the distances between each pair of cities, what is the shortest possible route that visits each city and returns to the origin city?

Figure: 1000 Points

Figure: Optimized Route

Why TSP?

Find the shortest possible route that visits each city exactly once and returns to the origin city $\Rightarrow Hamiltoniancycle$.

The importance of the TSP does not arise from an overwhelming demand of salespeople to minimize their travel length. It can be used in:

- Transportation: school bus routes, service calls, delivering meals
- Manufacturing: an industrial robot that drills holes in printed circuit boards
- VLSI (microchip) layout
- Communication: planning new telecommunication network

TSP Description

1. Problem Statement: Given a complete weighted undirected graph, find the shortest Hamiltonian cycle(n nodes).
2. The size of the solution space in (n-1)!/2
3. Dynamic Programming gives us a solution in time $O(n^2 2^n)$
4. TSP is NP Complete

Particle Swarm Optimization

Particle swarm optimization (PSO) is one of the bio-inspired algorithms and it is a simple one to search for an optimal solution in the solution space. It is different from other optimization algorithms in such a way that only the objective function is needed and it is not dependent on the gradient or any differential form of the objective. It also has very few hyperparameters..

Particle Swarm Optimization was proposed by Kennedy and Eberhart in 1995. As mentioned in the original paper, sociobiologists believe a school of fish or a flock of birds that moves in a group "can profit from the experience of all other members". In other words, while a bird flying and searching randomly for food, for instance, all birds in the flock can share their discovery and help the entire flock get the best hunt.

Particle Swarm Optimization

Original velocity update equation:

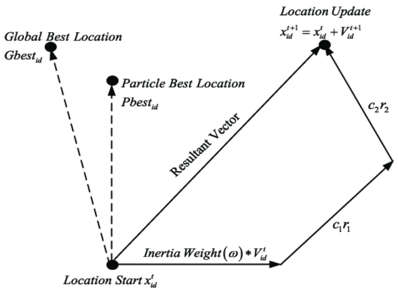
$$V_i(k+1) = Inertia + Cognitive + Social$$

$$V_i(k+1) = \omega * v_i(k) + c_1 * random_1() * (PBest_i - x_i(k)) + c_2 * random_2() * (GBest_i - x_i(k))$$

ω, c_1, c_2 : $constant$

$$X_i = X_{i-1} + V_i$$
$$X_i : Position of the i^{th} Particle$$
$$;V_i : Velocity of the i^{th} Particle$$

PSO Vector Diagram



Genetic Algorithm

Genetic Algorithm (GA) is a search-based optimization technique based on the principles of Genetics and Natural Selection. It is frequently used to find optimal or near-optimal solutions to difficult problems which otherwise would take a lifetime to solve.

Algorithm 1 PSO Algorithm

```
Initialize the population position vector xi^d and velocity vector vi^d randomly at j = 1, N is the population size
while (iteration limit is reached) do
    j = j + 1
    for loop over all d^th dimensional N particles do
        Update vi^d using Equation (1)
        Update xi^d using Equation (2)
        Evaluate the objective function
        Determine the current best for each particle pi^d
    end for
    Determine the current global best g^d
end while
Output the final gi^d and pi^d
```

Algorithm 2 GA Algorithm

```
Set Parameters
Choose encode method
Generate the initial population
while i < MaxIteration and Bestfitness < MaxFitness do
    Fitness Calculation
    Selection
    Crossover
    Mutation
end while
Decode the individual with maximum fitness
return the best solution
```

Inspiration - Evolution

1. Natural Selection:
    - "Survival of the Fittest"
    - favourable traits become common and unfavourable traits become uncommon in successive generations
2. Sexual Reproduction
    - Chromosomal crossover and genetic recombination.
    - population is genetically variable
    - adaptive evolution is facilitated
    - unfavourable mutations are eliminated

Selection

1. Fitness Function:
   - $f(x), x$ is a chromosome in the solution space
   - $f(x)$ may be:
     - an well-defined objective function to be optimised e.g. TSP and knapsack
2. Probability distribution for selection

$$P(X = x_i) = \frac{f(x_i)}{\sum_{j=1}^{M} f(x_i)} \tag{1}$$

3. Fitness proportional selection.

Operators-Crossover and Mutation

1. Crossover:
   - Applied with high probability
   - Position for crossover on the two parent chromosomes randomly selected
   - Offspring share characteristics of well-performing parents
   - Combinations of well-performing characteristics generated
2. Mutation:
   - Applied with low probability
   - Bit for mutation randomly selected
   - New characteristics introduced into the population
   - Prevents algorithm from getting trapped into a local optimum

Crossover and mutation operations in GA

Individuals in a standard genetic algorithm are represented by strings of binary digits or by symbols. In the current study, each individual is represented by a binary string. The genetic operators will then operate on the bits of the strings to identify the best permutation and combination of the bits of a string that determines the particular solution's characteristic. The operators, as in genetic programming, include reproduction, crossover, and mutation Fig. 3!

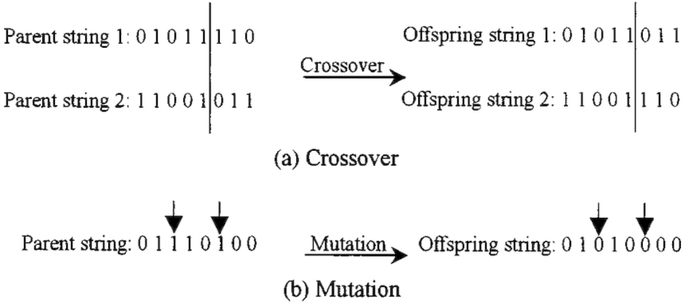


Figure: Crossover and mutation operations in genetic algorithm

Chromosomes with Regard to TSP

1. Crossover:
   - If there are 7 cities, we generate a double array of size 6 like below.
   - First Field is random number and second field is value 2-7
   - Then you sort the array based on first column
   - The sequence in second column gives you a chromosome

Table: Step 1

0.23	2
0.65	3
0.49	4
0.58	5
0.75	6
0.34	7

Table: Step 2

0.23	2
0.34	3
0.49	4
0.58	5
0.65	6
0.75	7

Crossover with Regard to TSP

1. Crossover:
   - Crossover part can change its pattern
   - The chrmosome is valid as none of the cities have repeated

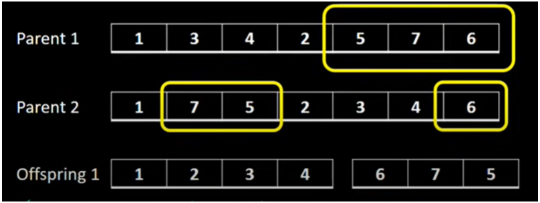


Figure: Crossover in genetic algorithm

Crossover with Regard to TSP

1. Crossover:
   - The purpose of mutation in GAs is to introduce diversity into the sampled population.
   - Mutation operators are used in an attempt to avoid local minima by preventing the population of chromosomes from becoming too similar to each other, thus slowing or even stopping convergence to the global optimum.

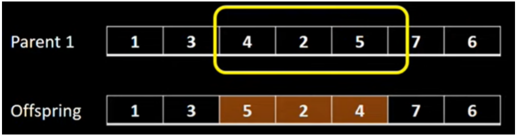
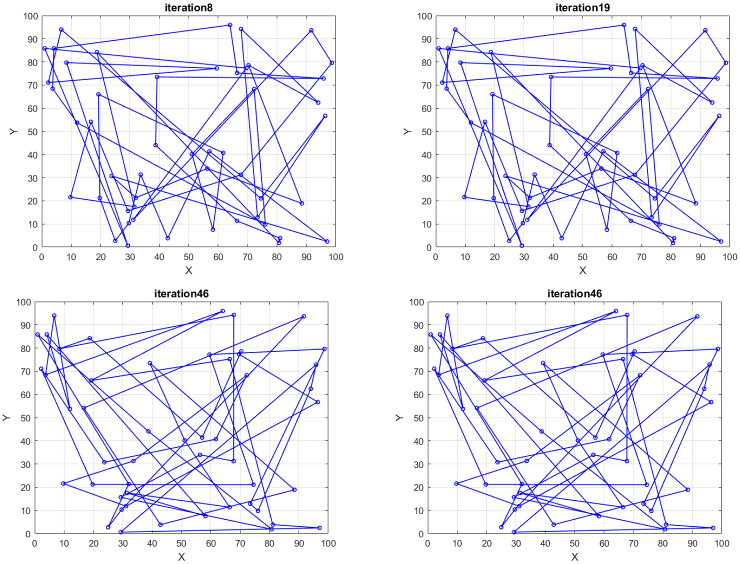
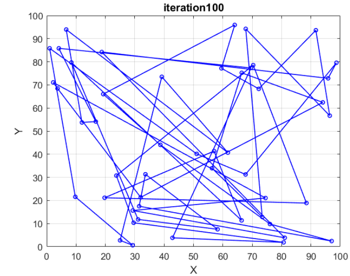


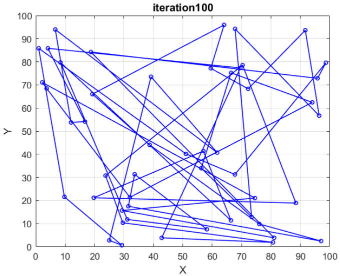
Figure: Crossover in genetic algorithm

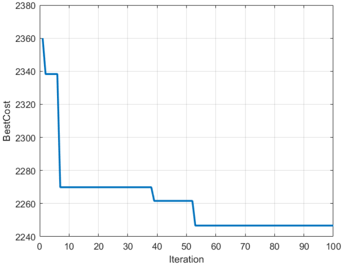
Results

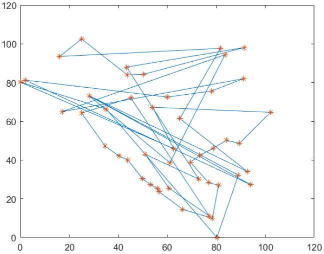


Results









Conclusion

1. The performance of particle swarm optimization and Genetic Algorithms are evaluated using Matlab Simulator.
2. In Optimization techniques cost and time are essential parameters. This project has presented the analysis of existing optimization techniques and their critical study. We have selected certain parameters associated with cost and time.
3. Optimization techniques such as Particle swarm optimization (PSO) and Genetic Algorithm (GA) according to those parameters. In all scenarios we have observed that PSO gives better result than GA with respect to cost and time.
4. In addition the increase in number of iterations and cities increases the execution time of both algorithms.