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Abstract:	Competitive balance indicates the degree of control participating teams have over a sports competition. We have a wide range of indices to measure competitive balance, but we will use the Distance to Competitive Balance, a standardized index that complies cardinality property. We focus our attention on the UEFA Champions League, before and after competition, and we measure the competitive balance of the UEFA Champions League qualifying stage groups between the 1999/2000 and 2017/2018 seasons. We assume that a highly competitive group will be more "difficult" in terms of qualifying than a highly concentrated one. Supporters say that their team was unlucky to be in a "difficult" group… But, will the supporters have sufficient reason?

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1. Introduction

The competitive balance in a sports competition shows us the degree to which the strongest teams tend to accumulate victories. Really, competitive balance is the situation where, in any head-to-head matchup, each team has an equal likelihood of winning. Then, a league exhibits competitive balance if, for all team pairs *i* and *j*, the probability that team *i* will win a match-up is 50%. If a league exhibits perfect competitive balance, every game would be competitive, and every team a potential champion. At the end of the competition, the focus is on the distribution of results achieved. So, competitive balance indicates the degree of control participating teams have over a sports competition (Szymanski 2003). The relationship with the concept of concentration is inverse: greater control of results by few teams, means less competitive balance; and, on the contrary, if all teams participating in a competition obtain the same result, competitive balance peaks.

In this sense, what is the fan interest? This is another theme. Obviously, a fan wants his team to win every competition. So, when a team wins a championship, there is no doubt that supporters of rival teams will disagree. It will be understandable that such supporters will look for and "find" excuses to justify the defeat of their team and the triumph of their rivals. In particular, if the competition has required a preliminary qualifying group stage, as is the case for the UEFA Champions League (UCL), they will argue that it was unfair and that, if anything, the winning team was in an "easy group" from the start. Obviously, they will say that their team was unlucky to be in a "difficult group", which made it difficult for them to win in the end. Of course, there are

also the referees... It is therefore of interest to determine what is an "easy group" and what is a "difficult group". This is directly related to the concept of competitive balance. And, how can we measure competitive balance?

We have a wide range of indices to measure competitive balance. When these indices are applied to the score vector give us a number representing the level of competitive balance. These indices, related to those which measure levels of inequality and concentration (although the relationship of both these concepts with competitive balance is clearly inverted) must be standardised to incorporate specific concepts related to sports economics (possible changes in the number of teams between seasons and the bilateralism of the matches, which prevent any one team from ending up with all the points at stake). This standardisation requires knowing the minimum and maximum values that an index can theoretically reach. However, determining these values has not been a contentious issue until recently, despite the fact that the scoring system affects the maximum value of the concentration index. Whatever the case, within football, this problem has been solved because the *perfectly unbalanced* distribution of points, which generates the maximum value of this index, has been determined (Avila-Cano and Triguero-Ruiz 2018).

In this article we assume that a highly competitive group will be more "difficult" in terms of qualifying than a highly concentrated one and we analyse if the UCL winners started the competition in groups that could be considered easier than average.

Therefore, we will measure the competitive balance of the UCL qualifying stage groups between the 1999/2000 and 2017/2018 seasons. Since that date the format has been 32 teams grouped into 8 groups. We will measure the competitive balance:

i. Ex ante, based on the coefficients assigned to the teams by UEFA itself (UEFA 2019). These coefficients are used by UEFA itself to form the groups. The groups are formed

according to the degree of strength that the teams have displayed over the previous season, historically, and in terms of their corresponding national leagues. The coefficient summarises these attributes.

ii. Ex post, i.e. after the competition, based on the results obtained by each team in their matches against their rivals.

We will then consider in each season whether the UCL winning team started the competition in an "easy" or a "difficult" group, both at the beginning of the competition and after the competition has run its course. Will the most sceptical supporters of the opposing teams have sufficient reason?

The article is structured as follows: In Section 2 we define the competitive balance measure that we will use. Section 3 presents the results of the competitive balance indices applied, compares the results and analyses if the UCL winners started the competition in groups that could be considered easier than average. Finally, our conclusions are presented.

2. Measuring competitive balance

The competitive balance of any competition is measured with a variety of indices associated with the concepts of inequality and concentration. In the literature it is currently accepted that these indices must be standardised, which enables the measurements to be compared inasmuch as:

i. They incorporate the possibility that the number of teams in the competition can vary, in such a way that the standardisation can correct the possible variation in the minimum value of the index, which is reached when all teams have the same number of points.

ii. They reflect the impossibility of reaching a monopoly configuration, as the maximum value of concentration of results, given the bilateral nature of the games.

In this way the standardised index is constructed, relativising its value in terms of both the minimum that it can fall to, and the maximum distance (difference between the index's maximum and minimum values). Notable among these indices are the standardised Herfindahl-Hirschman (HHI_{Norm}) proposed by Owen, Ryan and Weatherston (2007) and the Distance to Competitive Balance (*DCB*) proposed by Triguero-Ruiz and Avila-Cano (2019). In this article, we use this standardized index to measure the competitive balance in the major European football leagues because *DCB* complies the ideal cardinality property, which, taking the unit interval as the range, is represented by a mathematical distance, thereby ensuring that the ratios are maintained. *DCB* can be expressed as a function of the Herfindahl-Hirschman index.

We consider a league with $n \in \mathbb{N}$ teams. Let us denote by p_i, $i \in \{1, 2, ..., n\}$, the number of points obtained at the end of the championship by team i, whereas the vector $\mathbf{p} = (p_1, p_2, ..., p_n)$ represents the final score of the championship. The subscript i indicates the position of each team in the final ranking. Given vector $\mathbf{p}$, we define the vector of shares $\mathbf{s} = (s_1, s_2, ..., s_n)$, where $s_i = \frac{p_i}{\sum_{i=1}^{n} p_i}$. An index that measures the competitive balance will be a function defined on the vector of shares, which assigns a real number belonging to the unit interval.

Let $\mathbf{s} = (s_1, ..., s_n)$ denote the vector of the teams' points in leagues with a maximum of n teams, where $s_i \in [0,1]$, and $\sum_{i=1}^{n} s_i = 1$. For each $\mathbf{s}$, the *DCB* is constructed as the ratio between the Euclidean distance at the minimum concentration (equal points) and the maximum distance that can be reached. The latter is represented by a configuration, s_i^{max}, which can be obtained from

n and the current scoring system in the championship (Avila-Cano and Triguero-Ruiz 2018). In this way, we have:

$$DCB(\boldsymbol{s}) = \sqrt{\frac{n \cdot \sum_{i=1}^{n} s_i^2 - 1}{n \cdot \sum_{i=1}^{n} (s_i^{max})^2 - 1}} \quad (1)$$

None of the most currently used indices comply with the cardinality property. This is the relevance of *DCB*, particularly useful in comparative studies, because: (i) The values obtained with the measurement have a meaning (the concentration percentage with respect to the maximum achievable); and (ii) the differences (and proportions) between such values also have a meaning. Given that HHI is a quadratic function, it does not comply with triangular inequality, so it is not a distance and does not comply with the cardinality property.

Moreover, *DCB* index is interpretable in terms of percentage of concentration, which means that its differences are significant in terms of percentage points and can be rewritten accordingly:

$$DCB = \sqrt{HHI_{Norm}} = \sqrt{\frac{HHI - HHI_{min}}{HHI_{max} - HHI_{min}}} \quad (2)$$

where HHI is the index without standarisation, defined as the sum of the square of the points quotas of the teams in the league; $HHI_{min} = {}^{1}/_{N}$ is the minimum value in a league of *N* teams; and HHI_{max} is the maximum value, which is less than the unit as it is unable to reach the monopoly configuration, and which must be calculated according to the number of teams and the scoring system of the championship.

2.1. The perfectly unbalanced distribution

Calculating the *DCB* implies having to know the value of HHI_{max}. This is generated by the distribution which we call *perfectly unbalanced*. This distribution of the final points of a tournament is known as the *complete cascade (CCD)*. In this distribution each team has beaten all teams below them in the final ranking and lost to all those above; the last in the ranking has lost to all: Fort and Quirk (1995); Gayant and Le Pape (2012; 2015); Horowitz (1997); Larsen, Fenn and Spenner (2006); Owen, Ryan, and Weatherston (2007); o Utt and Fort (2002).

Avila-Cano and Triguero-Ruiz (2018) have demonstrated that, for a {3,1,0} pattern the *perfectly unbalanced distribution* is a *truncated cascade (TCD)*. In this distribution there are a group of teams who have a cascade of wins, and the rest draw all games that they have not lost against the former. It can be demonstrated that the *truncated cascade distribution* generalises the *complete cascade distribution*.

Most major European football leagues have 20 teams (Bundesliga, 18). For *n=20*, if it meets the established stability condition $HHI_{max} = 0.0684211$ and the distribution that it generates is a complete cascade and if the condition is not met, $HHI_{max} = 0.0754015$ and is generated by the *truncated cascade distribution* with seven winning teams.

Since in each UCL competition group, the four teams play a double round-robin league with the score system of points for a win (p_w), draw (p_t) and loss (p_l): $\{p_w, p_t, p_l\} = \{3,1,0\}$, the stability condition is not met. Thus, the truncated cascade distribution is as follows: the first team will have won all its matches (earning 18 points) whereas, each of the other three teams will have lost the two games, and drawn with the other teams; so, each of the three will have a total of 4 points. In total 30 points will have been shared out, and the quota distribution will be (0.60, 2/15, 2/15, 2/15). This distribution generates the maximum concentration of results, so that the

corresponding value of the *HHI* index in this distribution is: $HHI_{max} = 0.413$. The Distance to Competitive Balance index generates a unit value: *DCB*=1.

3. Is the UCL champion's group easier?

We have calculated the *DCB* indices for each of the eight groups from the 1999/2000 seasons to 2017/2018. This exercise has been done from both ex ante and ex post perspectives. Having done so, we count on 152 observations of ex ante competitive balance corresponding to each of the eight groups for each of the nineteen seasons, and another 152 observations for the ex post competitive equilibrium. Table 1 shows this information.

Additionally, the Table 1 have highlighted the data of the champion's group every season. We have constructed confidence intervals for a 99% probability from the value of the t-Student statistic. Therefore, for each season, we can identify whether the value of the *DCB* index falls inside or outside the confidence interval. If it lies outside, below the lower limit, we infer that the empirical value of the *DCB* is significantly small, within the season as a whole, and that the group was very competitive and therefore "difficult". If the empirical value of the *DCB* is above the upper limit of the interval, the group will be uncompetitive and therefore "easy".

Table 1. *DCB* index in the UCL group stage (1999-2018): EX ANTE → EX POST. Highlighted the champion's group.

Group / Season	A	B	C	D	E	F	G	H	Confidence interval ($t_{99\%}$ = 2.3646) Ex ante analysis		Confidence interval ($t_{99\%}$ = 2.3646) Ex post analysis	
									Lower	Upper	Lower	Upper
1999/00	0.7165→0.5682	*0.6519→0.7171*	*0.4879→0.3780*	0.6714→0.4660	0.6929→0.5689	0.5004→0.4509	0.4016→0.6993	0.3173→0.2893	0.430	0.680	0.392	0.642
2000/01	*0.5848→0.6386*	0.6799→0.6193	0.5759→0.4513	0.3976→0.1634	*0.5235→0.2736*	0.6351→0.3576	0.5056→0.4167	0.5091→0.3943	0.478	0.625	0.280	0.548
2001/02	0.6323→0.5576	0.3321→0.4374	0.3089→0.2916	0.3138→0.3001	0.4281→0.3919	0.5898→0.7715	0.5769→0.3636	*0.5245→0.7336*	0.351	0.575	0.324	0.638
2002/03	*0.5002→0.2677*	0.6943→0.8508	0.6963→0.3749	0.3601→0.3971	0.3466→0.4305	0.6856→0.5689	0.3363→0.5957	*0.6066→0.8137*	0.392	0.664	0.362	0.713
2003/04	0.3610→0.1948	*0.5002→0.1634*	0.3908→0.5446	0.4544→0.4902	*0.5497→0.6882*	0.7789→0.7171	0.3857→0.4420	*0.4184→0.2218*	0.366	0.594	0.250	0.616
2004/05	0.4592→0.5590	0.5332→0.6307	0.6046→0.7241	0.6427→0.6630	0.3753→0.5339	*0.5814→0.4660*	*0.5580→0.8137*	0.4292→0.4420	0.445	0.601	0.497	0.711
2005/06	*0.6852→0.8267*	*0.6827→0.8375*	0.6385→0.6550	0.3456→0.2736	*0.5869→0.4020*	0.5882→0.7241	0.4982→0.5342	0.7430→0.4969	0.491	0.701	0.425	0.763
2006/07	*0.4853→0.7314*	0.5557→0.4767	0.5928→0.4882	0.4459→0.5889	0.4600→0.7336	*0.5715→0.2893*	0.4858→0.4902	0.7307→0.3636	0.463	0.619	0.387	0.653
2007/08	*0.4673→0.3001*	0.5137→0.3943	0.4034→0.4158	0.4666→0.3791	0.3851→0.5867	*0.4473→0.8518*	0.3256→0.7528	0.5263→0.8330	0.386	0.498	0.378	0.750
2008/09	0.7826→0.4660	*0.6355→0.2361*	0.5153→0.6660	*0.5284→0.7221*	*0.6538→0.3401*	0.4110→0.7655	0.4257→0.5844	0.6258→0.6282	0.467	0.677	0.394	0.708
2009/10	*0.6195→0.8330*	0.6797→0.4882	0.5417→0.4902	*0.6970→0.7809*	0.7354→0.8267	0.7484→0.3807	0.6729→0.6088	0.6188→0.5682	0.607	0.721	0.478	0.766
2010/11	0.3613→0.3943	0.3881→0.4660	0.8215→0.7655	0.7363→0.6916	0.7220→0.5946	0.7754→0.7715	0.5667→0.6866	*0.6224→0.7715*	0.479	0.770	0.521	0.764
2011/12	0.5339→0.7314	*0.6802→0.2361*	0.4618→0.7336	*0.6067→0.9291*	0.7125→0.4767	0.5173→0.3919	0.4058→0.2617	0.9617→0.8110	0.463	0.757	0.351	0.792
2012/13	*0.5109→0.8330*	0.6288→0.5648	0.4312→0.3943	0.4564→0.7171	0.7072→0.6395	0.6522→0.6193	0.7231→0.5298	0.6601→0.4834	0.501	0.691	0.483	0.712
2013/14	0.6069→0.7064	0.6596→0.6903	0.4445→0.6550	*0.6275→0.8248*	*0.5159→0.5016*	0.2950→0.7143	0.5646→0.7171	*0.6336→0.5342*	0.441	0.646	0.580	0.756
2014/15	0.6886→0.5135	0.8413→0.8137	0.4756→0.3576	0.4567→0.7600	0.6267→0.6126	*0.6487→0.8330*	0.6955→0.6088	0.4927→0.5940	0.505	0.727	0.502	0.771
2015/16	*0.7190→0.8267*	0.2569→0.4305	0.7638→0.5682	0.3700→0.4427	0.7617→0.5797	0.6795→0.5832	*0.6611→0.7314*	0.5143→0.5946	0.431	0.750	0.483	0.706
2016/17	*0.5046→0.8475*	0.3865→0.3456	0.6121→0.7087	0.7769→0.7598	0.3485→0.4969	0.8358→0.7221	0.6101→0.7451	0.6189→0.7819	0.444	0.729	0.536	0.816
2017/18	0.2624→0.7715	0.6540→0.8248	*0.6923→0.5906*	0.5814→0.7299	0.9212→0.5533	0.5172→0.6521	0.5599→0.6569	*0.7037→0.9507*	0.454	0.769	0.607	0.826

The following exercise is a straightforward application of the results above. Their rivals tend to stress the sporting weakness of the UCL champions group. Has this traditionally been true? This question is related to the competitive balance of this sporting championship and can be understood from a twofold perspective:

i. Ex ante, refers to the fact that, from the very beginning that the groups are formed, the team that will finally win the championship enjoys a relative advantage, given that the weakness of their group rivals facilitates their qualifying.

ii. Ex post, however, refers to the fact that, however tough a group may initially be, in the end, after the competition has been played, the group was *easier* than expected.

For this reason, we have to identify when a group is easier or more difficult. The answer is found in the ease or difficulty with which a team qualifies for the next round. We must remember that, out of the four teams competing in the league by double round-robin and with the score system {3,1,0} only the first two qualify, i.e. the two who accumulate the most points after playing every game.

When, therefore, can we consider a group to be *more difficult*? As we understand it, when the four teams have similar levels of strengths, competitive balance is greater. So, the toughest groups will be those where competitive balance is greater, i.e. the concentration of results is lower. We also believe that this should happen in a significantly different way to the rest of the groups in the round. Note that, the lowest concentration of results lies in the perfectly competitive distribution in which all the teams have a results quota of ¼ and $HHI = {}^{1}/_{4}$. Therefore, the *DCB* index value will be null.

As we understand it, a group is *easier* when the competitive balance is minimal, i.e. the concentration of results is maximum. Note that this extreme case of maximum concentration occurs with a distribution of final scores in a *Truncated cascade* of *q=1*, with *q* being the number of winning teams. The results distribution will be (18, 4, 4, 4) and $HHI = 0.413$. Therefore, the *DCB* index value will be the unit.

Other situations that are also found in the upper half of the concentration (and variance) of results, and, so, are nearer to the minimum competitive balance (maximum value of *DCB*) are:

i. The *Complete cascade*, habitually proposed (sometimes erroneously) to identify the maximum value of the concentration index, required to normalise the index. The score

distribution is (18, 12, 6, 0), with $HHI = 0.389$ and $DCB = 0.922$.

ii. The *Truncated cascade* in *q=2*: the first has beaten all others; the second has beaten the remaining two, who, in turn, draw with each other in their two games. The final score vector will be (18, 12, 2, 2), with $HHI = 0.412$ and $DCB = 0.995$.

iii. The *Truncated cascade* in q=2 changed, where the first two qualifying teams draw with each other in their games, and beat the other two teams, who, in turn, draw in their games. The score distribution will be (14, 14, 2, 2), with $HHI = 0.391$ and $DCB = 0.929$.

In these three cases, the value is lower than the distribution found by Avila-Cano & Triguero-Ruiz (2018), which is called the *Truncated cascade distribution* in *q=1*, where *q* is the number of teams who do not draw.

Consequently, once the ex ante and ex post *DCB* have been calculated for each group and season, and each case has been statistically analysed to classify it in relation to whether or not the distance from the group average for each season is statistically significant, we can identify each group thus: *Easy*, with *DCB* significantly higher than average, or *difficult*, with *DCB* significantly lower than average.

What conclusions can we draw? Firstly, in Table 2 we can see that, over the last two decades, only one season (2003/2004), and in the case of FC Porto, was the group apparently easier when it was formed, and after the competition it was clear this was indeed so. In another two cases, 2006/2007, with AC Milan and 2009/2010 with FC Internazionale, ex ante the groups appeared to be easy, but, ex post, turned out to be difficult.

Secondly, in 8 out the 19 seasons, the UCL winning team did start in an easier group. With the exception of the three, aforementioned cases, in the 5 remaining seasons it was found that as the competition progressed, these groups were in line with the average (2000/2001, FC Bayern;

1999/2000, 2001/2002, 2013/2014 y 2016/2017, Real Madrid CF).

Thirdly, with the exception of the 2002/2003 season, in which AC Milan was the only team to start in a significantly more competitive (more difficult) group, the 10 remaining UCL winners started the season in groups similar to the average. In four of these cases it was found that the progress of these groups over the competition was significantly easier (2007/2008, Manchester United FC; 2014/2015, FC Barcelona; 2015/2016 & 2017/2018, Real Madrid, CF). In another six cases, the groups were normal, both ex ante and ex post (2004/2005, Liverpool FC; 2005/2006, 2008/2009 and 2010/2011, FC Barcelona; 2011/2012, Chelsea FC; and 2012/2013, FC Bayern).

Thus, and in light of the data (Table 2) it does not seem that the original affirmation holds true i.e. the ease of the UCL starting groups of those teams who finally win. More than half of the time this is not true ex ante, without it being confirmed by the outcome of the competition. Moreover, in just one of the last 19 seasons, has this been proved to be true, with a reduced ex post competitive balance as well. Contrarily, in two seasons, the group of the team that finally won the UCL, in the end, proved to be difficult.

Table 2. UCL finalists (1999/2000-2017/18). *DCB* ex ante & *DCB* ex post of their groups in UCL group phase. EASY GROUP in **bold** (Out of the confidence interval, upper tail). HARD GROUP in underlined (Out of the confidence interval, lower tail)

	UCL winner	Group	*DCB* ex ante	*DCB* ex post	Runner-up	Group	*DCB* ex ante	*DCB* ex post
1999/00	Real Madrid CF	E	**0.693**	0.569	Valencia CF	F	0.500	0.451
2000/01	FC Bayern	F	**0.635**	0.358	Valencia CF	C	0.576	0.451
2001/02	Real Madrid CF	A	**0.632**	0.558	Bayern Leverkusen	F	**0.590**	**0.772**
2002/03	A.C. Milan	G	0.336	0.596	Juventus FC	E	0.347	0.431
2003/04	FC Porto	F	**0.779**	**0.717**	AS Monaco FC	C	0.391	0.545
2004/05	Liverpool FC	A	0.459	0.559	AC Milan	F	0.581	0.466
2005/06	FC Barcelona	C	0.638	0.655	Arsenal FC	B	0.683	0.838
2006/07	A.C. Milan	H	**0.731**	0.364	Liverpool FC	C	0.593	0.488
2007/08	Manchester United FC	F	0.447	**0.852**	Chelsea FC	B	**0.514**	0.394
2008/09	FC Barcelona	C	0.515	0.666	Manchester United FC	E	0.654	0.340
2009/10	FC Internazionale	F	**0.748**	0.381	FC Bayern	A	0.619	**0.833**
2010/11	FC Barcelona	D	0.736	0.692	Manchester United FC	C	**0.822**	**0.766**
2011/12	Chelsea FC	E	0.712	0.477	FC Bayern	A	0.534	0.731
2012/13	FC Bayern	F	0.652	0.619	BV Borussia	D	0.456	**0.717**
2013/14	Real Madrid CF	B	**0.660**	0.690	Atlético de Madrid	G	0.565	0.717
2014/15	FC Barcelona	F	0.649	**0.833**	Juventus FC	A	0.689	0.514
2015/16	Real Madrid CF	A	0.719	**0.827**	Atlético de Madrid	C	**0.764**	0.568
2016/17	Real Madrid CF	F	**0.836**	0.722	Juventus FC	H	0.619	0.782
2017/18	Real Madrid CF	H	0.704	**0.951**	Liverpoool FC	E	**0.921**	0.553

4. Conclusions

Competitive balance in a sports competition is usually obtained from the distribution of points (or wins) achieved by each team at the end of the championship. We can also measure the competitive balance before the competition starts. Then, we should use information on the overall strengths of the teams. For the case of the UCL, a good summary can be found in the UEFA coefficients. From these distributions, indices such as the HHI and *DCB* are calculated, and require standardisation in order to take into account the special characteristics of sports competitions. Standardisation requires knowing the maximum and minimum theoretical values of the index.

The distribution that has usually been considered to generate the minimum competitive balance, which we have called the *complete cascade distribution*, is not valid for any points award pattern. So, measuring competitive balance is affected by the scoring system used. The *perfectly unbalanced distribution* under the {3,1,0} pattern is the *truncated cascade*, which allows the maximum value of concentration to be obtained.

In this article we have calculate the DCB indices, ex ante and ex post, for the stage group UCL between 1999/2000 and 2017/2018. We can reject the idea that the UCL groups are homogenous in terms of ex ante and ex post competitive balance, but can we say that the team that wins the UCL every season was in an easy group? We have consider that a group is easier when the competitive balance is minimal, i.e. the concentration of results is maximum. On the contrary, if the concentration is minimal, the competitive balance is high, and the group will be difficult as there is more competition.

The last two decades, only one season, the winner (FC Porto) was in an easy group ex ante and ex post. In other 7 seasons, the UCL winning team did start in an easier group: in 5 seasons it was found that as the competition progressed, these groups were in line with the average, and in two cases ex ante the groups appeared to be easy, but, ex post, turned out to be difficult. Only one team to start in a significantly more competitive (more difficult) group, and the 10 remaining UCL winners started the season in groups similar to the average. In four of these cases it was found that the progress of these groups over the competition was significantly easier In another six cases, the groups were normal, both ex ante and ex post. Therefore, the question: will the more sceptical supporters of the opposing teams have sufficient motives? can be answered in the negative.

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