Elliptic algebras

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This talk is based on joint work with Alex Chirvasitu and S. Paul Smith.

In 1989, Feigin and Odesskii introduced a family of noncommutative graded algebras $Q_{n,k}(E,\tau)$ called elliptic algebras, which are parametrized by an elliptic curve E, a point $\tau \in E$, and coprime integers $n > k \geq 1$. The algebra $Q_{n,k}(E,\tau)$ is generated by n variables in degree 1 and its relations are defined in terms of Belavin's elliptic solution to the quantum Yang-Baxter equation with spectral parameter.

When $k = 1$, the algebras $Q_{n,1}(E,\tau)$ are called (higher dimensional) Sklyanin algebras in honor of Sklyanin's discovery of $Q_{4,1}(E,\tau)$ in 1982. Sklyanin algebras have been widely studied and recognized as important examples of Artin-Schelter regular algebras. Although Feigin and Odesskii proved and claimed a number of remarkable results on $Q_{n,k}(E,\tau)$ in their series of papers, there are still many things that are only known for $Q_{n,1}(E,\tau)$, but not known for general $Q_{n,k}(E,\tau)$.

For example, it has long been expected that the $Q_{n,k}(E,\tau)$ has the same Hilbert series as the polynomial ring in n variables. This was originally claimed by Feigin and Odesskii, and proved by Tate and Van den Bergh when $k = 1$. One of our main results is that $Q_{n,k}(E,\tau)$ has the same Hilbert series as the polynomial ring in n variables when τ is not a torsion point.

In this talk, starting with motivation and background in noncommutative algebraic geometry, I will explain some properties of Feigin-Odesskii's algebras and how they are obtained from the quantum Yang-Baxter equation.

References

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