On the Auslander-Reiten conjecture for normal rings

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Auslander and Reiten [3] proposed the generalized Nakayama conjecture, which is rooted in the Nakayama conjecture [8]. In addition, they proposed another conjecture about projectivity of module by vanishing of Ext modules, which is called the Auslander-Reiten conjecture, and proved that this conjecture is true if and only if the generalized Nakayama conjecture is true. This long-standing conjecture is known to hold true for several classes of algebras.

The Auslander-Reiten conjecture remains meaningful for arbitrary commutative noetherian rings for formalization by Auslander, Ding, and Solberg [2]. The conjecture is known as follows: for a commutative noetherian ring R, every finitely generated R-module M such that $\mathrm{Ext}^i_R(M, M \oplus R) = 0$ for all $i \geq 1$ is projective. This conjecture is known to hold if R is a complete intersection [2], or if R is a locally excellent Cohen-Macaulay normal ring containing the field of rational numbers $\mathbb{Q}$ [5], or if R is a Gorenstein normal ring [1]. Recently, Kimura, Otake, and Takahashi [7] proved the conjecture for every Cohen-Macaulay normal ring. Even if R is not Cohen-Macaulay, it is known that R satisfies the conjecture if it is a quotient of a regular local ring and is a normal ring containing $\mathbb{Q}$ [4]. In this talk, we consider the above conjecture over normal rings.

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