Hochschild homology dimension and a class of Hochschild extension algebras of truncated quiver algebras

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In this talk, we show that higher Hochschild homology groups do not vanish for a class of Hochschild extension algebras of truncated quiver algebras by the standard duality module. Let $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$ and A a finite dimensional algebra over a field K. The Hochschild homology dimension $\mathrm{HH}\dim A$ of A is defined by $\mathrm{HH}\dim A = \inf\{n \in \mathbb{N}_0 \mid \mathrm{HH}_n(A) = 0\}$. Han conjectured that if $\mathrm{HH}\dim A$ is finite, then the global dimension of A is finite, in [2]. This is the homology version of Happel's question. After that, many classes of algebras which have infinite Hochschild homology dimension were discovered. In [1], Bergh and Madsen showed that the Hochschild homology dimension of trivial extension algebras by the standard duality module are infinite for selfinjective algebras, local algebras and graded algebras. We focus on truncated quiver algebras, and we consider Hochschild extension algebras by the standard duality module and their Hochschild homology dimension. Hochschild extension algebras are given by 2-cocycles $A \times A \to DA = \mathrm{Hom}_K(A, K)$. In particular, the trivial extension algebra is the Hochschild extension algebra by the zero map. It is well known that there is a one-to-one correspondence between the set of the equivalent classes of Hochschild extension and the second Hochschild cohomology group $H^2(A, DA)$ of A with coefficients in DA which is isomorphic to the second Hochschild homology group $\mathrm{HH}_2(A)$ of A as K-modules. Let $m \geq 2$, Q a finite quiver, R_Q the arrow ideal of KQ and $B = KQ/R_Q^m$. For each $n \geq 0$, the n-th Hochschild homology group $\mathrm{HH}_n(B)$ is $\mathbb{N}_0$-graded, and its l part $\mathrm{HH}_{n,l}(B)$ was computed in [3]. In particular, $\mathrm{HH}_2(B) = \oplus_{l=m}^{2m-1} \mathrm{HH}_{2,l}(B)$. In this talk, we show that for any 2-cocycle $\alpha : B \times B \to DB$ corresponding to an element in $\oplus_{l=m}^{2m-2} \mathrm{HH}_{2,l}(B)$ or $\mathrm{HH}_{2,2m-1}(B)$, the Hochschild homology dimension of the Hochschild extension algebra by α is infinite.

References

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2020 Mathematics Subject Classification: 16E40
Keywords: Hochschild extension algebras, Hochschild homology, truncated quiver algebras