A new class of generalized inverses in rings with involution

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Let R be a $*$-ring and let a, w, $v \in R$. This paper contributes to define two new classes of generalized inverses, called the w-core inverse and the dual v-core inverse in R. An element $a \in R$ is w-core invertible if there exists some $x \in R$ such that $awx^2 = x$, $xawa = a$ and $(awx)^* = awx$. Such an x is called a w-core inverse of a, which encompasses several known generalized inverses such as the core inverse, the e-core inverse and the Moore-Penrose inverse. Several characterizations and representations for elements to be w-core invertible are given, based on $\{1, 3\}$-inverses, inverses along an element, (b, c)-inverses, projections, ideals and units. Also, the connections between the w-core inverse and other generalized inverses are given.

References

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