Rings, Modules, and the Transfer Krull Property

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Krull monoids can be studied with ideal theoretic as well as with divisor theoretic tools. Indeed, a commutative and cancellative monoid H is Krull if one of the following equivalent conditions hold.

(a) H has a divisor theory.

(b) There is a divisor homomorphism from H to a free abelian monoid.

(c) H is completely integrally closed and satisfies the ascending chain condition on divisorial ideals.

A commutative integral domain is a Krull domain if its monoid of nonzero elements is a Krull monoid. The concept of Krull monoids and domains goes back to the first half of the previous century. It was then successfully generalized to non-commutative rings as well as to commutative rings with zero-divisors.

A monoid homomorphism $\theta\colon H \to B$ is called a transfer homomorphism if it is surjective up to units and allows to lift factorizations. Arithmetic invariants (which measure the deviation of a monoid from being factorial) can be studied in the (oftentimes simpler) monoid B and then they can be pulled back by the homomorphism θ to the monoid H.

It can easily be seen that a Krull monoid allows a transfer homomorphism to a monoid of zero-sum sequences over its class group, whose arithmetic can be studied with methods from additive combinatorics. In the last decade it turned out that various classes of monoids and domains (which need neither be commutative nor cancellative nor completely integrally closed) allow a transfer homomorphism to a commutative Krull monoid and hence to a monoid of zero-sum sequences. Such monoids are called transfer Krull.

In this survey talk we provide examples of Krull monoids and of transfer Krull monoids, that are not Krull, which arise in ring and module theory, and we discuss some of their main algebraic and arithmetic properties.

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