# $K_0$ of weak Waldhausen extriangulated categories

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A Waldhausen category is a generalization of an exact category. It is used to define the $K$-theory on a wide class of categories which generalizes Quillen's $K$-theory. In this talk, focusing on the Grothendieck groups, we modify the axiom of the Waldhausen structure so that it matches better with extriangulated categories. It enables us to define an abelian group $K_0(\mathsf{C})$ of a weak Waldhausen category $\mathsf{C}$ which generalizes the usual Grothendieck group of an extriangulated category. As one might expect, it behaves nicely in the context of Quillen's localization and resolution theorems. We obtain two applications: the first one generalizes exact sequences of the Grothendieck groups associated with the Serre/Verdier localization to some types of "one-sided exact" localizations; the second one reveals close relations between Quillen's theorems and Palu's index.

## References

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