Characterizations of standard derived equivalences of diagrams of dg categories and their gluings

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A diagram consisting of differential graded (dg for short) categories and dg functors is formulated as a colax functor X from a small category I to the 2-category $kdgCat$ of small dg categories, dg functors and dg natural transformations for a fixed commutative ring k. If I is a group regarded as a category with only one object $*$, then X is nothing but a colax action of the group I on the dg category $X(*)$. In this sense, this X can be regarded as a generalization of a dg category with a colax action of a group. We define a notion of standard derived equivalence between such colax functors by generalizing the corresponding notion between dg categories with a group action. Our first main result gives some characterizations of this notion, one of which is given in terms of generalized versions of a tilting object and a quasi-equivalence. On the other hand, for such a colax functor X, the dg categories $X(i)$ with i objects of I can be glued together to have a single dg category $Gr(X)$, called the Grothendieck construction of X. Our second main result insists that for such colax functors X and X', the Grothendieck construction $Gr(X')$ is derived equivalent to $Gr(X)$ if there exists a standard derived equivalence from X' to X. These are new even for dg categories with a group action. In particular, the second result gives a new tool to show the derived equivalence between the orbit categories of dg categories with a group action, which will be illustrated in some examples. This is joint work with Asashiba Hideto.

References

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