Factorization into valuation ideals and valuation elements

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Let D be an integral domain, let I be an ideal of D and let x be a nonzero nonunit of D. We say that I is a valuation ideal of D if there exists a valuation overring V of D such that $IV \cap D = I$. Moreover, x is said to be a valuation element of D if xD is a valuation ideal. The domain D is called a (weak) VIFD if every nonzero proper (principal) ideal of D is a finite product of valuation ideals of D. D is said to be a VFD if every nonzero nonunit of D is a finite product of valuation elements of D. Finally, we say that D is an almost VFD if for each nonzero nonunit $y \in D$, there is a positive integer n such that y^n is a finite product of valuation elements of D.

In this talk, we discuss the connections between (weak) VIFDs and (almost) VFDs. Based on earlier work (1.) on VFDs, we establish various similar results. We provide a characterization of treed (weak) VIFDs and mention several necessary conditions for D to be an almost VFD. Besides that, we present descriptions of almost VFDs and weak VIFDs in terms of almost Schreier domains and quasi Schreier domains, respectively. As an application, we characterize when the polynomial ring over D is an almost VFD. To supplement our results, we touch upon the t- and w-analogues of (weak) VIFDs.

This is joint work with Gyu Whan Chang.

References

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2020 Mathematics Subject Classification: 13A15, 13F05, 13G05
Keywords: Almost Schreier domain, Prüfer domain, treed domain, valuation ideal