On polyform modules

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The rational extension of a ring is developed by R.E. Johnson in 1951 and Y. Utumi in 1956 in order to generalize the quotients ring theory. In 1958, G.D. Findlay and J. Lambek defined a relationship between three modules over a ring R, $A_R \leq B_R(C_R)$, which means that A_R is a relatively dense submodule of B_R to C_R. Recently, in [1], as an application of the relatively dense property, the equivalent condition for the rational hull of the finite direct sum of modules to be the direct sum of the rational hulls of those modules is shown. Polyform modules, known as non-M-singular modules, are studied by R. Wisbauer (1981) and J. M. Zelmanowitz (1986). Recall that a right R-module is said to be *polyform* if every essential submodule of M is a dense submodule. It is known that the endomorphism ring of the quasi-injective hull of a polyform module is von Neumann regular. See [2, Section 11] for more properties of polyform modules.

In this talk, we provide polyform modules and introduce the notion of relatively polyform modules over a ring R. Some new characterizations and properties of polyform modules are obtained. For a retractable polyform module M_R, it is proved that the maximal right ring of quotients of $\mathrm{End}_R(M)$ is $\mathrm{End}_R(\widetilde{E}(M))$, where $\widetilde{E}(M)$ is the rational hull of M. Moreover, it is proved that the class of all polyform right R-modules is closed under submodules and rational extensions.

References

[1] G. Lee, The rational hull of modules, Submitted

[2] R. Wisbauer, *Modules and Algebras: bimodule structure and group actions on algebras*, Pitman Monographs and Surveys in Pure and Applied Mathematics, 81, Longman, Harlow (1996)

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