The structure of rings which are symmetric on the set of all idempotents

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Alghazzawi and Leroy studied the structure of subsets satisfying the properties of symmetric and commutatively closed, that is, $abc \in S$ for $a, b, c \in R$ implies $acb \in S$, and $ab \in S$ for $a, b \in R$ implies $ba \in S$, respectively, where S is a subset of a ring R. In this article we discuss the structure of rings which are symmetric on zero (resp., idempotents). Such rings are also called *symmetric* (resp., *I-symmetric*). We first prove that if a polynomial $\sum_{i=0}^{m} a_i x^i$ over a symmetric ring is a unit then a_0 is a unit and a_i is nilpotent for all $i \geq 1$; based on this result, we obtain that for a reduced ring R, the group of all units of the polynomial ring over R coincides with one of R, and that polynomial rings over I-symmetric rings are identity-symmetric. It is proved that for an abelian semiperfect ring R, R is I-symmetric if and only if the units in R form an Abelian group if and only if R is commutative. It is also proved that for an I-symmetric ring R, R is π-regular if and only if $R/J(R)$ is a commutative regular ring and $J(R)$ is nil, where $J(R)$ is the Jacobson radical of R.

References

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