Structure of right APIP rings

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We study the following two conditions in rings: (i) the right annihilator of some power of any element is an ideal, and (ii) the right annihilator of any nonzero element a contains an ideal generated by some power of any right zero-divisor of the element a. We investigate the structure of rings in relation to these conditions; especially, a ring with the condition (ii) is called right APIP. These conditions are shown to be not right-left symmetric. For a prime two-sided APIP ring R we prove that every element of R is either nilpotent or regular, and that if R is of bounded index of nilpotency then R is a domain. We also provide several interesting examples which delimit the classes of rings related to these properties.

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