On subalgebras of matrix algebra satisfying some polynomial identity

Michal Ziembowski

Warsaw University of Technology
E-mail: michal.ziembowski@pw.edu.pl

A subalgebra of the full matrix algebra $\mathrm{M}_n(K)$, K a field, satisfying the identity

$$[x_1, y_1]\,[x_2, y_2] \cdots [x_q, y_q] = 0$$

is called a D_q subalgebra of $\mathrm{M}_n(K)$.

We describe explicitly, up to conjugation, the structure of maximal D_q subalgebras of $\mathrm{M}_n(K)$ as block triangular subalgebras of $\mathrm{M}_n(K)$ with maximal commutative diagonal blocks. The sizes of the diagonal blocks are shown to play critical in deciding when two maximal D_q subalgebras of $M_n(K)$ are isomorphic. In case K is algebraically closed, we invoke Jacobson's characterization of maximal commutative subalgebras of $\mathrm{M}_n(K)$ with maximum (K-)dimension to obtain a complete classification of maximal D_q subalgebras of $\mathrm{M}_n(K)$ which are conjugates of block triangular subalgebras of $\mathrm{M}_n(K)$ with commutative diagonal blocks of maximum dimension.