Factorization in the (composite) Hurwitz rings

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Let R be a commutative ring with identity and $H(R)$ be the set of formal power series over R. Addition on $H(R)$ is defined termwise. A multiplication, called $*$-product, on $H(R)$ is defined as follows: For $f = \sum_{n=0}^{\infty} a_n x^n, g = \sum_{n=0}^{\infty} b_n x^n \in H(R)$,

$$f * g = \sum_{n=0}^{\infty} c_n x^n, \ c_n = \sum_{k=0}^{n} \binom{n}{k} a_k b_{n-k}, \text{ where } \binom{n}{k} = \frac{n!}{(n-k)!k!}.$$

Then $H(R)$ is a commutative ring with identity under these two operations [1] and then call $H(R)$ the ring of Hurwitz series over R [2]. The ring $h(R)$ of Hurwitz polynomials over R is the subring of $H(R)$ consisting of polynomials $\sum_{k=0}^{n} a_k x^k$. We simply call $H(R)$ and $h(R)$ Hurwitz rings.

For an extension $A \subseteq B$ of commutative rings with identity, let $H(A, B) = \{f \in H(B) \mid$ the constant term of f belongs to $A\}$ and $h(A, B) = \{f \in h(B) \mid$ the constant term of f belongs to $A\}$. Then these are commutative rings with identity. We call $H(A, B)$ and $h(A, B)$ composite Hurwitz rings.

In this talk, we introduce several factorization properties (UFD, BFD, and FFD) on (composite) Hurwitz rings.

References

[1] W.F. Keigher, Adjunctions and comonads in differential algebra, Pacific J. Math. 59 (1975) 99-112.

[2] W.F. Keigher, On the ring of Hurwitz series, Comm. Algebra 25 (1997) 1845-1859.

2020 Mathematics Subject Classification: 13B25, 13F15

Keywords: Composite Hurwitz series ring, composite Hurwitz polynomial ring, UFD, bounded factorization domain, finite factorization domain