Quantum projective planes finite over their centers and Beilinson algebras

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In noncommutative algebraic geometry, a quantum polynomial algebra defined by Artin and Schelter [1] is a basic and important research object, which is a noncommutative analogue of a commutative polynomial algebra. Also, Artin and Schelter [1] gave the classifications of low dimensional quantum polynomial algebras. Moreover, Artin, Tate and Van den Bergh [2] found a nice correspondence between 3-dimensional quantum polynomial algebras and geometric pair (E, σ), where E is the projective plane or a cubic divisor in the projective plane, and σ is the automorphism of E. So, this result allows us to write a 3-dimensional quantum polynomial algebra A as the form $A = A(E, \sigma)$. For a 3-dimensional quantum polynomial algebra $A = A(E, \sigma)$, Artin, Tate and Van den Bergh [3] showed that A is finite over its center if and only if the order $|\sigma|$ of σ is finite. For a 3-dimensional quantum polynomial algebra $A = A(E, \sigma)$ with the Nakayama automorphism ν of A, the author and Mori [7] proved that the order $|\nu^*\sigma^3|$ of $\nu^*\sigma^3$ is finite if and only if the norm $||\sigma||$ of σ introduced by Mori [9] is finite if and only if the noncommutative projective plane in the sense Artin and Zhang [4] is finite over its center. In [6], for a 3-dimensional quantum polynomial algebra A of Type S', the author prove that the following conditions are equivalent; (1) the noncommutative projective plane is finite over its center. (2) The Beilinson algebra of A is 2-representation tame in the sense of Herschend-Iyama-Oppermann [5]. (3) The isomorphism classes of simple 2-regular modules over the Beilinson algebra of A are parametrized by the projective plane. Note that, this result holds for a Type S by Mori [9]. The Beilinson algebra of A is introduced by Minamoto and Mori [8], which is a typical example of 2-representation infinite algebra.

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