Rings and radicals related to n-primariness

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An ideal I of a ring R is called *n-primary*(resp., *T-primary*) provided that $AB \subseteq I$ for ideals A, B of R implies that $(A+I)/I$ or $(B+I)/I$ is nil of index n(resp., $(A+I)/I$ or $(B+I)/I$ is nil) in R/I, where $n \geq 1$. It is proved that for a proper ideal I of a principal ideal domain R, I is T-primary if and only if I is of the form p^kR for some prime element p and $k \geq 1$ if and only if I is 2-primary, through which we study the structure of matrices over principal ideal domains. We prove that for a T-primary ideal I of a ring R, R/I is prime when the Wedderburn of R/I is zero. In addition we provide a method of constructing strictly descending chain of n-primary radicals from any domain, where the *n-primary radical* of a ring R means the intersection of all the n-primary ideals of R.

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