Classifying Serre subcategories of the category of Maximal Cohen-Macaulay modules

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Classifying several subcategories is an active subject in the representation theory of algebras. One of the most classical results is Gabriel's classification of Serre subcategories. He classified Serre subcategories of the module category of a commutative noetherian ring in terms of its prime spectrum. So far, the classification of subcategories has focused on the case of abelian categories (or triangulated categories) such as the above example.

In this talk, we will talk about a classification of Serre subcategories of a certain exact category which is not abelian. Here, an exact category is a generalization of an abelian category, and this framework enables us to study homological properties of, for instance, the category of maximal Cohen-Macaulay modules, which is not an abelian category. We will give the classification result of Serre subcategories of a torsionfree class of the module category of a commutative noetherian ring. Here, a torsionfree class is a subcategory of an abelian category closed under subobjects and extensions, and it has a natural structure of an exact category. This result extends Gabriel's classification to torsionfree classes. As an immediate consequence, we can classify the Serre subcategories of the category of maximal Cohen-Macaulay modules over a one-dimensional Cohen-Macaulay ring.

References

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