Tilting theory for finite dimensional 1-Iwanaga-Gorenstein algebras

Kota Yamaura*, Yuta Kimura and Hiroyuki Minamoto

University of Yamanashi
Osaka Metropolitan University
Osaka Metropolitan University
E-mail: kyamaura@yamanashi.ac.jp

A finite dimensional graded algebra $A = \bigoplus_{i=0}^{\ell} A_i$ is called graded d-Iwanaga-Gorenstein if the graded injective dimensions of A_A and $_A A$ are at most d. A finitely generated graded A-module M is called Cohen-Macaulay if it satisfies $\mathrm{Ext}_A^i(M, A) = 0$ for all $i > 0$. We denote by $\mathrm{CM}^{\mathbb{Z}} A$ the category of graded Cohen-Macaulay A-modules. Since this category is a Frobenius category, its stable category $\underline{\mathrm{CM}}^{\mathbb{Z}} A$ has a structure of a triangulated category.

In representation theory of graded Iwanaga-Gorenstein algebras, one of main concerns is to study the existence of tilting objects in $\underline{\mathrm{CM}}^{\mathbb{Z}} A$. By tilting theory for algebraic triangulated categories [2], if $\underline{\mathrm{CM}}^{\mathbb{Z}} A$ has a tilting object T, then there is an equivalence $\underline{\mathrm{CM}}^{\mathbb{Z}} A \cong \mathrm{K}^{\mathrm{b}}(\mathrm{proj}\,\Gamma)$ of triangulated categories where $\Gamma = \mathrm{End}_{\underline{\mathrm{CM}}^{\mathbb{Z}} A}(T)$ and $\mathrm{K}^{\mathrm{b}}(\mathrm{proj}\,\Gamma)$ is the homotopy category of bounded complexes of projective Γ-modules. This equivalence allows us to study Cohen-Macaulay A-modules from viewpoint of representations of Γ. Therefore, a fundamental problem is that when $\underline{\mathrm{CM}}^{\mathbb{Z}} A$ has a tilting object.

In the case $d = 0$, the problem was solved. It was shown that $\underline{\mathrm{CM}}^{\mathbb{Z}} A$ has a tilting object if and only if the 0-th subring A_0 of A is of finite global dimension. In the case $d \geq 1$, many authors studied the problem for various classes of (not necessarily finite dimensional) graded Iwanaga-Gorenstein algebras (refer to a survey article [1]).

In this talk, we develop tilting theory for finite dimensional 1-Iwanaga-Gorenstein algebras. We will introduce an invariant $g(A)$ for a finite dimensional graded algebra A. Then, we show that in the case where A is 1-Iwanaga-Gorenstein, an inequality for $g(A)$ gives a sufficient condition that a specific Cohen-Macaulay module V becomes a tilting object in $\underline{\mathrm{CM}}^{\mathbb{Z}} A$.

As an application, we study the problem for the case that A is the truncated preprojective algebra $\Pi(Q)_w$ of a quiver Q associated to $w \in W_Q$. We show that if the underling graph of Q is tree, then $\underline{\mathrm{CM}}^{\mathbb{Z}} \Pi(Q)_w$ has a tilting object.

References

[1] O. Iyama, Tilting Cohen-Macaulay representations, Proceedings of the International Congress of Mathematicians-Rio de Janeiro 2018. Vol. II. Invited lectures, 125-162, World Sci. Publ., Hackensack, NJ, 2018.

[2] B. Keller, Deriving DG categories, Ann. Sci. Ecole Norm. Sup. (4) 27 (1994), no. 1, 63-102.