The Eisenbud-Goto regularity conjecture for homogeneous prime ideals

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The Eisenbud-Goto regularity conjecture for homogeneous prime ideals in polynomial rings is one of the long-standing conjectures in Algebraic Geometry and Commutative Algebra. There is one to one correspondence between homogeneous prime ideals and projective irreducible varieties. Various methods have been developed according to categories of objects.

It is well known that the Eisenbud-Goto regularity conjecture is true for projective curves, smooth surfaces, smooth threefolds in $\mathbb{P}^5$, and toric varieties of codimension two. Since Jason McCullough and Irena Peeva constructed counterexamples in 2018 for the first time, there is a mysterious dichotomy between smooth varieties and singular varieties. In this talk, we'd like to present a survey on recent developments on regularity of homogeneous prime ideals in polynomial rings.

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2020 Mathematics Subject Classification: 14N05, 13D02

Keywords: homogeneous prime ideal, projective variety, minimal free resolution, Castelnuovo-Mumford regularity, Eisenbud-Goto conjecture

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