The n-torsionfreeness of syzygies of the residue field

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The notion of n-torsionfree modules was introduced by Auslander and Bridger [1] as a natural generalization of the notion of torsionfree modules over integral domains. The n-torsionfreeness of syzygies plays an important role in Auslander's approximation theory [1, 2]. As a dual theory of approximation theories, the existence of finite projective hulls has been well-studied [2, 4, 5, 6]. We will see that the existence of finite projective hulls is closely related to the $(n + 1)$-torsionfreeness of n-syzygies. As an application, we consider the n-torsionfreeness of syzygies of the residue field k over a local ring R. Let $t = \operatorname{depth} R$ and $\Omega^t k$ be the tth syzygy of the residue field k. Dey and Takahashi [3] proved that $\Omega^t k$ is $(t + 1)$-torsionfree, and it is a $(t + 2)$nd syzygy if and only if the local ring R has type one. Motivated by their results, we study higher torsionfreeness of the syzygy $\Omega^t k$.

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