

Periodic dimensions for modules and algebras

Satoshi Usui

Tokyo Metropolitan College of Industrial Technology

E-mail: `usui@metro-cit.ac.jp`

Let R be a Noetherian semiperfect ring. Eventually periodic R -modules are finitely generated R -modules having periodic higher syzygies. Until now, eventually periodic R -modules have been characterized in various ways in the literature such as [1, 2, 3, 4, 5, 6]. Simultaneously, the degrees and the periods of their first periodic syzygies have been also studied. See also [7] other than the above references.

The aim of this talk is to investigate the degree of the first periodic syzygy of an eventually periodic module. To do this, we will introduce the notion of the periodic dimension of a module, which indicates the desired degree when the module is eventually periodic. In this talk, after providing basic properties of periodic dimensions, we will give our main result that periodic dimensions can be computed from Gorenstein dimensions, and vice versa. This improves some known results on the degrees of the first periodic syzygies of eventually periodic modules over Gorenstien rings.

References

- [1] L. Avramov, Modules of finite virtual projective dimension, *Invent. Math.* 96 (1) (1989) 71–101.
- [2] P. A. Bergh, Complexity and periodicity, *Colloq. Math.* 104 (2) (2006) 169–191.
- [3] A. Croll, Periodic modules over Gorenstein local rings, *J. Algebra* 395 (2013) 47–62.
- [4] V. N. Gasharov, I. V. Peeva, Boundedness versus periodicity over commutative local rings, *Trans. Amer. Math. Soc.* 320 (2) (1990) 569–580.
- [5] S. Küpper, *Two-sided Projective Resolutions, Periodicity and Local Algebras*, Logos Verlag, Berlin, 2010.
- [6] S. Usui, Characterization of eventually periodic modules in the singularity categories, *J. Pure Appl. Algebra* 226 (12) (2022) Paper No.107145.
- [7] V. Dotsenko, V. Gélinas, P. Tamaroff, Finite generation for Hochschild cohomology of Gorenstein monomial algebras, *Selecta Math. (N.S.)* 29 (1) (2023) Paper No.14.

2020 Mathematics Subject Classification: 16E05, 16E10, 16G10, 16G50

Keywords: eventually periodic modules, Gorenstein dimensions, eventually periodic algebras, Gorenstein algebras