Birkhoff-Pierce Problem and Riemann Hypothesis

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In this talk, we introduce the brief history of the Birkhoff-Pierce Problem, so-called "No.1 problem of ordered algebraic structure", and give the details of the concept "doubly convex set" introduced by Zhipeng Xu, a 2nd year undergraduate student of SJTU. The unified classification of directed partial orders over F(i) (where F is any non-archimedean o-field and $i^2 = -1$) is obtained using this concept. We also explain a connection of Birkhoff-Pierce Problem and Riemann Hypothesis.

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