

Morita equivalences and relative stable equivalences of Morita type for the principal blocks of finite groups

Kyoichi Suzuki* and Naoko Kunugi

Tokyo University of Science
E-mail: 1119703@ed.tus.ac.jp

Let k be a field of characteristic $p > 0$, and G a finite group. Then the group algebra kG has a unique decomposition into a direct sum of indecomposable algebras. Each summand is called a block of kG . In particular, kG has a unique block that is called the principal block of kG .

Morita equivalences between principal blocks have been constructed by lifting stable equivalences of Morita type. The notion of a stable equivalence of Morita type was introduced by Broué [1]. He also developed a method of constructing one between principal blocks. Linckelmann [3] gave an equivalent condition for stable equivalences of Morita type between indecomposable selfinjective algebras to be in fact Morita equivalences. By combining these results, Morita equivalences have been constructed in some cases (see for example [4] and [2]). However if finite groups have a common nontrivial central p -subgroup, then we cannot use Broué's method for the principal blocks. In order to have a method of constructing Morita equivalences for principal blocks in the situation, we consider the notion of relative stable equivalences of Morita type.

The notion of relative stable equivalences of Morita type for blocks was introduced by Wang and Zhang [5]. This is a generalization of stable equivalences of Morita type. In this talk, we consider finite groups having a common central p -subgroup Z , and generalize the results due to Broué and Linckelmann to stable equivalences of Morita type relative to Z between the principal blocks.

References

- [1] M. Broué, Equivalences of blocks of group algebras, in: Finite-Dimensional Algebras and Related Topics (Ottawa, 1992) (Eds. V. Dlab and L. L. Scott), Kluwer Acad. Publ., Dordrecht (1994), 1–26.
- [2] S. Koshitani and C. Lassueur, Splendid Morita equivalences for principal 2-blocks with dihedral defect groups, *Math. Z.* 294 (2020), no. 1-2, 639–666.
- [3] M. Linckelmann, Stable equivalences of Morita type for self-injective algebras and p -groups *Math. Z.* 223 (1996), no. 1, 87–100.

2020 Mathematics Subject Classification: 20C20

Keywords: Morita equivalence, Relative stable equivalence of Morita type

- [4] T. Okuyama, Some examples of derived equivalent blocks of finite groups, preprint.
- [5] L. Wang and J. Zhang, Relatively stable equivalences of Morita type for blocks, J. Pure Appl. Algebra 222 (2018), no. 9, 2703–2717.