Some characterizations of w-Noetherian rings and SM rings

De Chuan Zhou*, Hwankoo Kim, Xiao Lei Zhang and Jin Xie

Southwest University of Science and Technology
Hoseo University
Shandong University of Technology
Mianyang Teachers' College
E-mail: zdechuan11119@163.com

In this paper, we characterize w-Noetherian rings and SM rings. More precisely, in terms of the u-operation on a commutative ring R, we prove that R is w-Noetherian if and only if the direct limit of rGV-torsion-free injective R-modules is injective and that R is SM, which can be regarded as a regular w-Noetherian ring, if and only if the direct limit of GV-torsion-free (or rGV-torsion-free) reg-injective R-modules is reg-injective. As a by-product of the proof of the second statement, we also obtain that the direct and inverse limits of u-modules are both u-modules and that SM rings are regular w-coherent.

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