

# Drazin inverses and pseudo core inverses of a sum of morphisms

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Let  $\mathcal{C}$  be an additive category. Suppose that  $\varphi : X \rightarrow X$  is a morphism of  $\mathcal{C}$  with Drazin inverse  $\varphi^D$ , and  $\eta : X \rightarrow X$  is a morphism of  $\mathcal{C}$  such that  $1 + \varphi^D \eta$  is invertible and  $\eta(\varphi^D \varphi - 1)\varphi = \varphi(\varphi^D \varphi - 1)\eta = 0$ . Let  $\alpha = (1 + \varphi^D \eta)^{-1}$ ,  $\beta = (1 + \eta \varphi^D)^{-1}$ ,  $\varepsilon = (1 - \varphi \varphi^D)\eta\alpha(1 - \varphi^D \varphi)$ ,  $\gamma = \alpha(1 - \varphi^D \varphi)\eta\varphi^D\beta$ ,  $\sigma = \alpha\varphi^D\varphi\alpha^{-1}(1 - \varphi\varphi^D)\beta$ . It is proved that  $f = \varphi + \eta - \varepsilon$  is Drazin invertible with  $f - f^2 f^D = \varphi - \varphi^2 \varphi^D$  if and only if  $1 - \gamma$  is invertible if and only if  $1 - \sigma$  is invertible. This result extends the case of group inverses and reduces the two invertible morphisms used by Chen et al. to one. The relevant result for pseudo core inverses of a sum of morphisms is also given.

The talk reports a joint work with Jianlong Chen.

## References

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