

Derivación por Definición

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Exempli gratia:

1)

$$e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = \lim_{h \rightarrow 0} (1 + h)^{\frac{1}{h}}$$

$$f : \mathbb{R} \rightarrow \mathbb{R}$$

$$f(x) = e^x \Rightarrow f'(x_0) = e^{x_0}$$

Demostración:

Sea $x \neq x_0$,

$$\frac{f(x) - f(x_0)}{x - x_0} = \frac{e^x - e^{x_0}}{x - x_0} = e^{x_0} \frac{e^{x-x_0} - 1}{x - x_0}$$

Sea $h = e^{x-x_0} - 1 \Rightarrow 1 + h = e^{x-x_0}$,

$$\ln(1 + h) = \ln e^{x-x_0}$$

$$\ln(1 + h) = (x - x_0) \ln e$$

$$\ln(1 + h) = x - x_0$$

Si $x \rightarrow x_0$, entonces $h \rightarrow 0$,

$$e^{x_0} \frac{e^{x-x_0} - 1}{x - x_0} = e^{x_0} \frac{h}{1 + h} = e^{x_0} \frac{1}{\frac{1}{h} \ln(1 + h)} = e^{x_0} \cdot \frac{1}{\ln(1 + h)^{\frac{1}{h}}}$$

Entonces,

$$\begin{aligned} f'(x_0) &= \lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{x \rightarrow x_0} e^{x_0} \cdot \frac{1}{\ln(1 + h)^{\frac{1}{h}}} = \lim_{x \rightarrow x_0} e^{x_0} \cdot \frac{\lim_{x \rightarrow x_0} 1}{\lim_{x \rightarrow x_0} \ln(1 + h)^{\frac{1}{h}}} \\ &= e^{x_0} \cdot \frac{1}{\lim_{x \rightarrow x_0} \ln(1 + h)^{\frac{1}{h}}} \end{aligned}$$

Luego, por la continuidad de \ln ,

$$f'(x_0) = e^{x_0} \frac{1}{\ln[\lim_{x \rightarrow x_0} (1+h)^{\frac{1}{h}}]} = e^{x_0} \frac{1}{\ln(e)} = e^{x_0}$$

2)

$$f(x) = x^3 - 12x$$

Sea $x \neq x_0$,

$$\begin{aligned} \frac{f(x) - f(x_0)}{x - x_0} &= \frac{(x^3 - 12x) - (x_0^3 - 12x_0)}{x - x_0} = \frac{(x^3 - x_0^3) - (12x - 12x_0)}{x - x_0} \\ &= \frac{(x - x_0)(x^2 + xx_0 + x_0^2) - 12(x - x_0)}{x - x_0} = \frac{(x - x_0)[x^2 + xx_0 + x_0^2 - 12]}{x - x_0} \end{aligned}$$

$$f'(x_0) = \lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{x \rightarrow x_0} (x^2 + xx_0 + x_0^2 - 12) = x_0^2 + x_0^2 + x_0^2 - 12 = 3x_0^2 - 12$$

Sea $h \neq 0$,

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{(x+h)^3 - 12(x+h) - x^3 + 12x}{h} = \frac{(x+h)[(x+h)^2 - 12] - x^3 + 12x}{h} \\ &= \frac{(x+h)[x^2 + 2xh + h^2 - 12] - x^3 + 12x}{h} \\ &= \frac{x^3 + 2x^2h + h^2 - 12x + x^2h + 2xh^2 + h^3 - 12x - x^3 + 12x}{h} \\ &= \frac{3x^2h + 3xh^2 + h^3 - 12h}{h} = 3x^2 + 3xh + h^2 - 12 \\ \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} &= \lim_{h \rightarrow 0} (3x^2 + 3xh + h^2 - 12) = 3x^2 - 12 \end{aligned}$$