Demostraciones por Epsilon y Delta

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Exampli gratia:

1) Sea $f(x) = 3x - 1$. Dado $\epsilon = 0{,}01$, encuentre un $\delta > 0$, tal que si $0 < |x - 4| < \delta$, entonces $|f(x) - 11| < \epsilon$.

$$\lim_{x \to 4} (3x - 1) = 11$$

$$a = 4$$

$$L = 11$$

$$0 < |x - a| < \delta \Rightarrow |f(x) - L| < \epsilon$$

$$x \neq a$$

Probemos que $\lim_{x \to 4} (3x - 1) = 11$.

Demostración:

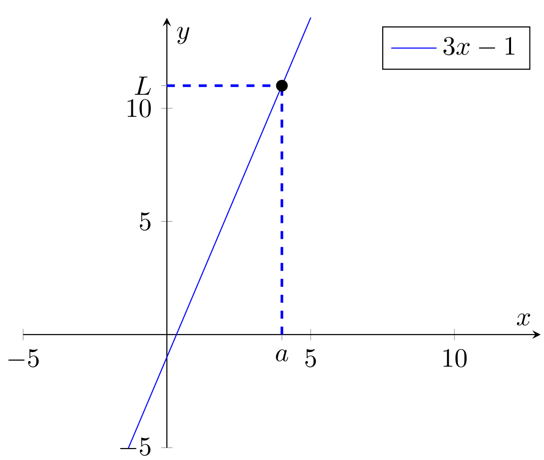
Sea $\epsilon > 0$ arbitrario.

Si elegimos $\delta = \frac{\epsilon}{3}$.

Suponemos que $0 < |x - 4| < \delta$.

Se verifica que,

$$|f(x) - 11| = |3x - 1 - 11| = |3x - 12| = 3|x - 4| < 3\delta = 3(\frac{\epsilon}{3}) = \epsilon$$



Si $\epsilon = 0{,}01 = \frac{1}{100}$, entonces $\delta = \frac{1}{3} \cdot \frac{1}{100} = \frac{1}{300}$.

2) Demuestre que $\lim\limits_{x \to -2} \sqrt{3x+22} = 4$.

Demostración:

Sea $\epsilon > 0$ arbitrario.

Si elegimos $\delta = 3\epsilon$

Suponemos que $0 < |x+2| < \delta$

Se verifica que

$$|\sqrt{3x+22} - 4|, \text{ donde } \sqrt{3x+22} = \frac{(\sqrt{3x+22})(\sqrt{3x+22})}{\sqrt{3x+22}}$$

$$, \text{ tenemos que } |\frac{3x+22-16}{\sqrt{3x+22}+4}| = |\frac{3x+6}{\sqrt{3x+22}+4}|$$

$$= 3|x+2| \cdot \frac{1}{\sqrt{3x+22}+4}, \text{ porque } \sqrt{3x+22}+4 > 0$$

$$< 3\delta \cdot \frac{1}{\sqrt{3x+22}+4} < 3\delta, \text{ porque } \sqrt{3x+22}+4 > 1$$

$$= 3(\frac{\epsilon}{3}) = \epsilon$$

3) Demuestre que $\lim\limits_{x \to 3} 6x^2 = 54$.

Demostración:

Sea $\epsilon > 0$ arbitrario,

Si elegimos $\delta = \{1, \frac{\epsilon}{42}\}$.

Supongamos que $0 < |x-3| < \delta \Rightarrow |x-3| < 1 \iff -1 < x-3 < 1 = 5 < x+3 < 7 = -7 < x+3 < 7 = |x+3| < 7$.

Se verifica que

$$|6x^2 - 54| = |6(x^2-9)| = 6|x-3| \cdot |x+3| < 6\delta \cdot 7 = 42\delta = 42(\frac{\epsilon}{42}) = \epsilon$$

4) Demuestre que $\lim\limits_{x \to 3} \frac{x+1}{x+2} = \frac{4}{5}$.

Demostración:

Sea $\epsilon > 0$. Para encontrar $\delta > 0$, suponemos $|x-3| < \delta$ y estimamos $|\frac{x+1}{x+2} - \frac{4}{5}| = |\frac{5x+5-4x-8}{5(x+2)}| = \frac{1}{5}|\frac{x-3}{x+2}|$.

Si $\delta = 1$, $|x-3| < 1 \iff -1 < x-3 < 1 = 4 < x+2 < 6 = \frac{1}{4} > \frac{1}{x+2} > \frac{1}{6}$.

$$\frac{1}{6} < \frac{1}{x+2} < \frac{1}{4} \iff |\frac{1}{x+2}| < \frac{1}{4} \iff \frac{1}{|x+2|} < \frac{1}{4}$$

Entonces,

$$|f(x) - L| = \frac{1}{5}|x-3| \cdot \frac{1}{|x+2|}$$

$$< \frac{1}{5}\delta \cdot \frac{1}{4} = \frac{1}{20}\delta = \frac{1}{20}(20\epsilon) = \epsilon$$

$$\therefore \delta = min\{1, 20\epsilon\}$$

5) Encuentre el límite.

$$\lim_{x \to 1} \frac{\frac{1}{\sqrt{x}} - 1}{x - 1} = \frac{0}{0}$$

$$\lim_{x \to 1} \frac{\frac{1-\sqrt{x}}{\sqrt{x}}}{\frac{x-1}{1}} = \lim_{x \to 1} \frac{1-\sqrt{x}}{\sqrt{x}(x-1)} = \lim_{x \to 1} \frac{-(\sqrt{x}-1)}{\sqrt{x}(\sqrt{x}-1)(\sqrt{x}+1)} = \lim_{x \to 1} \frac{-1}{\sqrt{x}(\sqrt{x}+1)} = -\frac{1}{2}$$

6) Demuestre que $\lim\limits_{x \to 3} \frac{x+1}{x+2} = \frac{4}{5}$.

Demostración:

Sea $\epsilon > 0$ arbitrario. Para obtener $\delta > 0$, suponemos que $0 < |x-3| < \delta$ y estimamos,

$$|\frac{x+1}{x+2} - \frac{4}{5}| = |\frac{5x+5-4x-8}{5(x+2)}| = \frac{1}{5}|\frac{x-3}{x+2}| = \frac{1}{5}|x-3| \cdot \frac{1}{|x+2|}$$

Si $\delta = 1$, $|x-3| < 1 \iff -1 < x-3 < 1 \iff 4 < x+2 < 6 \iff \frac{1}{4} > \frac{1}{x+2} > \frac{1}{6}$.