DERIVATIVES & PARTIAL DERIVATIVES

DERIVATIVE RULES

- **Constant Rule:** $f(x) = c$ then $f'(x) = 0$
- **Constant Multiple Rule:** $g(x) = c \cdot f(x)$ then $g'(x) = c \cdot f'(x)$
- **Power Rule:** $f(x) = x^n$ then $f'(x) = nx^{n-1}$
- **Sum and Difference Rule:** $h(x) = f(x) \pm g(x)$ then $h'(x) = f'(x) \pm g'(x)$
- **Product Rule:** $h(x) = f(x)g(x)$ then $h'(x) = f'(x)g(x) + f(x)g'(x)$
- **Quotient Rule:** $h(x) = \dfrac{f(x)}{g(x)}$ then $h'(x) = \dfrac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$
- **Chain Rule:** $h(x) = f(g(x))$ then $h'(x) = f'(g(x))g'(x)$

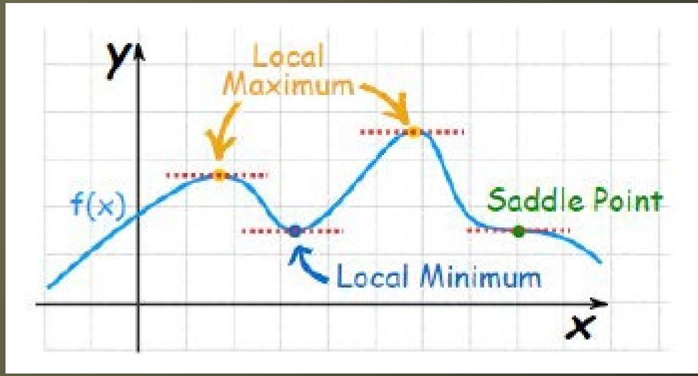
- **Exponential Derivatives**
  - $f(x) = a^x$ then $f'(x) = \ln(a)a^x$
  - $f(x) = e^x$ then $f'(x) = e^x$

- **Logarithm Derivatives**
  - $f(x) = \log_a(x)$ then $f'(x) = \dfrac{1}{\ln(a)x}$
  - $f(x) = \ln(x)$ then $f'(x) = \dfrac{1}{x}$

EXAMPLES

- $y = 3 \;\rightarrow y' = 0$
- $y = 10\,x^8 \rightarrow y' = 80\,x^7$
- $y = x^3 \ln(x) \rightarrow y' = 3\,x^2 . \ln(x) + x^2$
- $y = \ln\left(x^2 + 1\right) \rightarrow y' = \frac{2\,x}{x^2 + 1}$

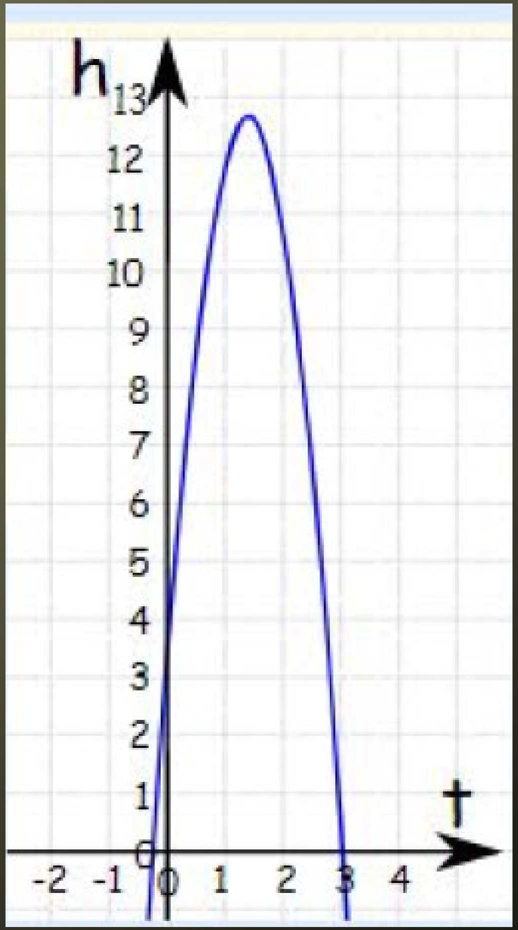
FINDING MAXIMA AND MINIMA



- Given a function $y = f(x)$
- x^* is a critical point if $y' = \frac{df}{dx} = 0$, solve for x
- x^* is maximum if $y'' = \frac{d^2f}{dx^2} < 0$ evaluated at x^*
- x^* is minimum if $y'' = \frac{d^2f}{dx^2} > 0$ evaluated at x^*
- x^* is neither maximum or minimum if $y'' = \frac{d^2f}{dx^2} = 0$ evaluated at x^*

EXAMPLE

- $h(t) = -5\,t^2 + 14\,t + 3$
- $h' = -10\,t + 14 = 0 \rightarrow t = 1.4$
- $h'' = -10 < 0 \rightarrow t = 1.4$ (maximum)
- $h(1.4) = 12.8$



PARTIAL DERIVATIVES

- Taking the partial derivative of a multi-variable function is accomplished by taking the derivative of the function with respect to one variable and treating all the other variables as constants.

EXAMPLES

1. If $z = f(x,y) = x^4y^3 + 8x^2y + y^4 + 5x$, then the partial derivatives are

$$\frac{\partial z}{\partial x} = 4x^3y^3 + 16xy + 5 \qquad \text{(Note: } y \text{ fixed, } x \text{ independent variable, } z \text{ dependent variable)}$$

$$\frac{\partial z}{\partial y} = 3x^4y^2 + 8x^2 + 4y^3 \qquad \text{(Note: } x \text{ fixed, } y \text{ independent variable, } z \text{ dependent variable)}$$

2. If $z = f(x,y) = (x^2 + y^3)^{10} + \ln(x)$, then the partial derivatives are

$$\frac{\partial z}{\partial x} = 20x(x^2 + y^3)^9 + \frac{1}{x} \qquad \text{(Note: We used the chain rule on the first term)}$$

$$\frac{\partial z}{\partial y} = 30y^2(x^2 + y^3)^9 \qquad \text{(Note: Chain rule again, and second term has no } y\text{)}$$

3. If $z = f(x,y) = xe^{xy}$, then the partial derivatives are

$$\frac{\partial z}{\partial x} = e^{xy} + xye^{xy} \qquad \text{(Note: Product rule (and chain rule in the second term)}$$

$$\frac{\partial z}{\partial y} = x^2e^{xy} \qquad \text{(Note: No product rule, but we did need the chain rule)}$$

ANOTHER EXAMPLE

$$J(\mathbf{w}) = \frac{1}{m}\sum_{i=1}^{m}\left(w_0 + w_1\mathbf{x}^{(i)} - y_i\right)$$

$$\frac{\partial J(\mathbf{w})}{\partial w_0} = \nabla_{w_0} J(\mathbf{w}) = \frac{1}{m}\sum_{i=1}^{m} 1$$

$$\frac{\partial J(\mathbf{w})}{\partial w_1} = \nabla_{w_1} J(\mathbf{w}) = \frac{1}{m}\sum_{i=1}^{m} \mathbf{x}^{(i)}$$