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Assignment - 6

Consider the function $\ell(\mathbf{w}) = y\log\big(\sigma(\mathbf{w}^T\mathbf{x})\big) + (1-y)\log\big(1-\sigma(\mathbf{w}^T\mathbf{x})\big)$

Where $\mathbf{w}$ and $\mathbf{x}$ are k^{th} dimensional vectors. Assume 1 training example.

Find $\nabla_{w_j}\ell(\mathbf{w})$, that is, the partial derivative of $\ell(\mathbf{w})$ with respect to the j^{th} element of vector $\mathbf{w}$.

- Recall if $\sigma(x) = \frac{1}{1+e^{-x}}$ then $\sigma'(x) = \sigma(x)\big(1-\sigma(x)\big)$
- Show your work!

$$\nabla_{w_j} l(w) = \nabla_{w_j}\left(y\log(\sigma(w^Tx)) + (1-y)\log(1-\sigma(w^Tx))\right)$$

$$= \frac{y}{\sigma(w^Tx)}\nabla_{w_j}\sigma(w^Tx) + \frac{(1-y)}{-(1-\sigma(w^Tx))}\nabla_{w_j}\sigma(w^Tx)$$

where,

$$\nabla_{w_j}\sigma(w^Tx) = (\sigma(w^Tx))(1-\sigma(w^Tx))\cdot\nabla_{w_j}(w^Tx)$$

$$= (\sigma(w^Tx))(1-\sigma(w^Tx))\cdot x_j$$

Hence,

$$\nabla_{w_j} l(w) = \frac{y}{\sigma(w^Tx)}(\sigma(w^Tx))(1-\sigma(w^Tx))\cdot x_j$$

$$- \frac{(1-y)}{(1-\sigma(w^Tx))}(\sigma(w^Tx))(1-\sigma(w^Tx))\cdot x_j$$

$$\nabla_{w_j} l(w) = y\left(1 - \sigma(w^T x)\right) \cdot x_j - (1-y)\left(\sigma(w^T x)\right) \cdot x_j$$

$$\nabla_{w_j} l(w) = \left(y - \sigma(w^T x)\right) \cdot x_j$$