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Assignment - 7

Consider N i.i.d. samples drawn from a Poisson distribution. The PMF is defined as follows:

$$Poisson(x|\lambda) = e^{-\lambda}\frac{\lambda^x}{x!} \text{ for } x \in \{0,1,2,\dots\} \text{ where } \lambda > 0 \text{ is the rate parameter.}$$

Find λ_{MLE}.

Show your work.

N i.i.d samples ~ Poisson $(x|\lambda) \rightarrow p(x|\lambda)$

$$= \prod_{i=1}^{N} e^{-\lambda} \frac{\lambda^{x^{(i)}}}{x^{(i)}!}$$

log likelihood $\ell(\lambda) = \log \prod_{i=1}^{N} e^{-\lambda} \frac{\lambda^{x^{(i)}}}{x^{(i)}!}$

$$= \sum_{i=1}^{N} \log \left(e^{-\lambda} \frac{\lambda^{x^{(i)}}}{x^{(i)}!} \right)$$

$$= \sum_{i=1}^{N} \left(\log e^{-\lambda} + \log \lambda^{x^{(i)}} - \log x^{(i)}! \right)$$

$$= \sum_{i=1}^{N} \left(-\lambda + x^{(i)} \log \lambda - \log x^{(i)}! \right)$$

$$= -N\lambda + \sum_{i=1}^{N} x^{(i)} \log \lambda - \sum_{i=1}^{N} \log x^{(i)}!$$

$$\frac{d\ell(\lambda)}{d\lambda} = -N + \frac{1}{\lambda}\sum_{i=0}^{N} x^{(i)} = 0$$

$$\Rightarrow \quad \lambda_{MLE} = \frac{1}{N}\sum_{i=1}^{N} x^{(i)}$$