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Assignment - 4

Given the cost function $J(w_0, w_1) = \frac{1}{2m}\sum_{i=1}^{m}\left(w_0 + w_1\mathbf{x}^{(i)} - y_i\right)^2$, determine the definiteness of its Hessian matrix and the convexity of the function. Assume 1-D dataset and $m = 1$. Show your work.

Cost function when $m = 1$,

$$J(w_0, w_1) = \frac{1}{2}(w_0 + w_1 x - y)^2$$

$$\frac{\partial J(w)}{\partial w_0} = w_0 + w_1 x - y$$

$$\frac{\partial J(w)}{\partial w_0^2} = 1$$

$$\frac{\partial J(w)}{\partial w_0 \partial w_1} = x$$

$$\frac{\partial J(w)}{\partial w_1} = (w_0 + w_1 x - y)x$$

$$\frac{\partial^2 J(w)}{\partial w_1^2} = x^2$$

$$\frac{\partial J(w)}{\partial w_1 \partial w_0} = x$$

$$H(w) = \begin{bmatrix} \dfrac{\partial^2 J(w)}{\partial w_0^2} & \dfrac{\partial J(w)}{\partial w_0 \partial w_1} \\[2ex] \dfrac{\partial J(w)}{\partial w_1 \partial w_0} & \dfrac{\partial^2 J(w)}{\partial w_1^2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & x \\ x & x^2 \end{bmatrix}$$

$$\begin{vmatrix} 1-\lambda & x \\ x & x^2-\lambda \end{vmatrix} = 0$$

$$1-\lambda\,(x^2-\lambda) - x^2 = 0$$

$$\cancel{x^2} - x^2\lambda - \lambda + \lambda^2 - \cancel{x^2} = 0$$

$$\lambda^2 - \lambda\,(1+x^2) = 0$$

$$\lambda = 0 \quad ; \qquad \lambda = 1+x^2 > 0$$

Thus, Hessian matrix is positive semi-definite and the cost function is convex