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Assignment - 9

Consider a dataset with 2 points in 1D: $(x_1 = 0, y_1 = -1)$ and $(x_2 = \sqrt{2}, y_2 = 1)$. Map each point to 3D using the feature vector $\phi(x) = [1, \sqrt{2}x, x^2]^T$. This is equivalent to using a second order polynomial kernel. The SVM classifier has the form

$$\min \|\mathbf{w}\|^2 \ s.t.$$
$$y_1(\mathbf{w}^T\phi(x_1) + w_0) \geq 1$$
$$y_2(\mathbf{w}^T\phi(x_2) + w_0) \geq 1$$

(a). Find the corresponding points in 3D. That is, $\phi(x_1)$ and $\phi(x_2)$.

(b). What is the value of the margin? Notice since there are only 2 points in the dataset, those points are the support vectors. Hence, the margin is the distance between each of them in 3D to the decision boundary, which lies in the middle.

(c). The margin obtained from part (b) is in fact equal to $1/\|\mathbf{w}\|$. Determine the vector $\mathbf{w}$. Recall this vector is the line through $\phi(x_1)$ and $\phi(x_2)$, which is perpendicular to the decision boundary.

(d). Solve for w_0 using your value for $\mathbf{w}$ and the above equations. Since the points are on the decision boundary, the inequalities will become equalities.

(e). Write the function $f(x) = w_0 + \mathbf{w}^T\phi(x)$ as an explicit function of x.

a) $\phi(x_1) = (1,0,0)$ $\quad \phi(x_2) = (1,2,2)$

b) The decision boundary lies halfway between the two support vectors $\phi(x_1)$ and $\phi(x_2)$. The midpoint is $m = \frac{(1,0,0) + (1,2,2)}{2} = (1,1,1)$

Margin is the distance of each of the support vectors to this midpoint $\sqrt{(1-1)^2 + (0-1)^2 + (0-1)^2} = \sqrt{(1-1)^2 + (2-1)^2 + (2-1)^2}$
$= \sqrt{2}$

c) vector $(1,2,2)$ is longer than vector $(1,0,0)$

$\therefore \quad (1,2,2) - (1,0,0) = (0,2,2)$

To find vector w, normalize this vector by some constant so that the margin $= 1/\|w\|$ is satisfied where $\|w\| = 1/\sqrt{2}$ $\therefore w = (0, 1/2, 1/2)$

d) Solving for w_0

$$y_1 (w^T \phi(x_1) + w_0) = 1$$

$$-1 ((0, 1/2, 1/2) \cdot (1,0,0) + w_0) = 1$$

$$-w_0 = 1 \quad \text{or} \quad w_0 = -1$$

$$y_2 (w^T \phi(x_2) + w_0) = 1$$

$$+1 ((0, 1/2, 1/2) \cdot (1,2,2) + w_0) = 1$$

$$2 + w_0 = 1 \quad \text{or} \quad w_0 = -1$$

e) $f(x) = w_0 + w^T \phi(x)$

$$= -1 + (0, 1/2, 1/2) \cdot (1, \sqrt{2}x, x^2)$$

$$= -1 + \frac{\sqrt{2}x}{2} + \frac{1}{2}x^2$$