Para nodos equiespaciados

$\Rightarrow \quad x_j = x_0 + jh \qquad j = 0, \dots, n \qquad h - \text{step/paso}$

Derivada progresiva o derecha

$$f(x+h) = f(x) + h f'(x) + \frac{h^2}{2} f''(x) + \dots \qquad h \ll 1$$

$$\Rightarrow \quad f'(x) = \frac{f(x+h) - f(x)}{h} - \underbrace{\frac{h}{2} f''(x)}_{O(h)}$$

Discretizando: $\quad x = x_j$

$$f'(x_j) = \frac{f(x_{j+1}) - f(x_j)}{h} \qquad (1)$$

Derivada regresiva o izquierda

$$f(x-h) = f(x) - hf'(x) + h^2/_2 f''(x)$$

$$f'(x) = \frac{f(x) - f(x-h)}{h} + \underbrace{\frac{hf''(x)}{2}}_{O(h)}$$

Para $\quad x = x_j$

$$f'(x_j) = \frac{f(x_j) - f(x_j - h)}{h} \qquad (2)$$

Notar que el error es del orden de $O(h)$

Derivada Central

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2} f''(x) + \frac{h^3}{6} f'''(x) + \dots$$

$$f(x-h) = f(x) - hf'(x) + \frac{h^2}{2} f''(x) - \frac{h^3}{6} f'''(x) + \dots$$

$$\Rightarrow f'(x) = \frac{f(x+h) - f(x-h)}{2h} - \underbrace{\frac{h^2}{6} f'''(x)}_{O(h^2)}$$

$$f'(x_j) \simeq \frac{f(x_{j+1}) - f(x_{j-1})}{2h} \qquad (3)$$

El error en la aproximación se mide a través de la distancia con el valor real.

$$\delta(Df(x_i)) = f'(x_i) - dfa(x_i).$$

También se puede calcular un error global de estimación

$$\delta_G(Df(x_i)) = \sqrt{\frac{\sum_{i=1}^{n} \left(f'(x_i) - dfa(x_i)\right)^2}{\sum_{i=1}^{n} \left(f'(x_i)\right)^2}}$$

Para la formula de 2 derivada

$$f(x_0+dx) = f(x_0) + \left.\frac{\partial f}{\partial x}\right|_{x_0} + \frac{1}{2}\left.\frac{\partial^2 f}{\partial x^2}\right|_{x_0} dx^2 + \dots$$

$$f(x_0-dx) = f(x_0) - \left.\frac{\partial f}{\partial x}\right|_{x_0} + \frac{1}{2}\left.\frac{\partial^2 f}{\partial x^2}\right|_{x_0} dx^2 + \dots$$

Sumando ecuaciones

$$f(x_0+dx) + f(x_0-dx) = 2f(x_0) + \left.\frac{\partial^2 f}{\partial x^2}\right|_{x_0} dx^2 + O(dx^4)$$

$$\Rightarrow \quad \left.\frac{\partial^2 f}{\partial x^2}\right|_{x_0} = \frac{f(x_0+dx) + f(x_0-dx) - 2f(x_0)}{dx^2} + O(dx^2)$$

Notar que la Precision $\sim dx^2$

$$F''(x_j) \simeq \frac{f(x+h) + f(x_j-h) - 2f(x_j)}{h^2}$$

$$\simeq \frac{f(x_{j+1}) + f(x_{j-1}) - 2f(x_j)}{h^2}$$