Zad 1

liczba wyborów 8 elem. z 20:

$$\bar{\Omega} = C_{20}^{8} = \frac{20!}{8!\,12!} = 125970$$

liczba wyborów 3 z 6:

$$\bar{A} = C_{6}^{3} = \frac{6!}{3!\,3!} = 20$$

$$P(A) = \frac{\bar{A}}{\bar{\Omega}} = \frac{20}{125970} \approx 0.0159\%$$

Zad 2

zakład	A	B	C
ilość elem.	12	20	18
p. że 1. jakości	0.9	0.6	0.9

a – el. jest 1. jakości

b – jest z zakładu A

$$P(b|a) = \frac{P(b)P(a|b)}{P(a)}$$

$$P(a) = \frac{12\cdot 0.9 + 20\cdot 0.6 + 18\cdot 0.9}{50} = \frac{39}{50}$$

$$P(a|b) = 0.9$$

$$P(b) = \frac{12}{50}$$

$$P(b|a) = \frac{12}{50}\cdot\frac{9}{10}\cdot\frac{50}{39} = \frac{3}{10} = 0.3$$

Zad 4

$c = ?$

$$\int_0^2 c(2-x)dx = 1$$

$$c\int_0^2 (2-x)dx = 1$$

$$\int_0^2 (2-x)dx = \int_0^2 2dx - \int_0^2 x\,dx = 2[x]_0^2 - \left[\frac{x^2}{2}\right]_0^2 = 4-2=2$$

$$c \cdot 2 = 1$$

$$c = \frac{1}{2}$$

a) $F(x) = \int_0^x \frac{1}{2}(2-t)dt = P\{\xi < x\}$

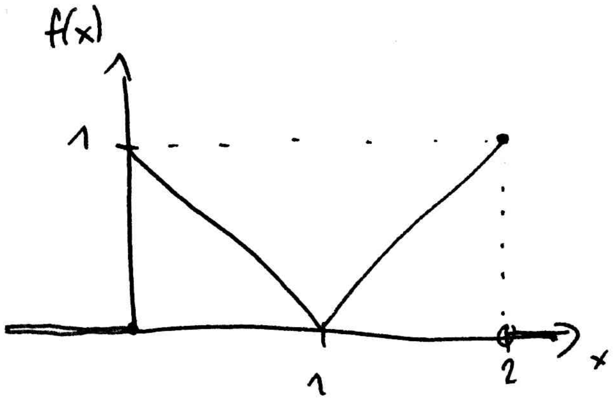
b) $E\xi = \int_0^2 \left(x \cdot \frac{1}{2}(2-x)\right)dx = \int_0^2 \left(x - \frac{x^2}{2}\right)dx = \int_0^2 x\,dx - \frac{1}{2}\int_0^2 x^2 dx =$

$$= \left[\frac{x^2}{2}\right]_0^2 - \frac{1}{2}\left[\frac{x^3}{3}\right]_0^2 = \frac{4}{2} - \frac{1}{2}\cdot\frac{8}{3} = 2 - \frac{4}{3} = \frac{2}{3}$$

$$E\xi = \frac{2}{3}$$

Zad 5

$$f(x) = \begin{cases} |x-1| & \text{dla } x \in \langle 0,2 \rangle \\ 0 & \text{dla } x \notin \langle 0,2 \rangle \end{cases}$$



$$E\xi = \int_0^2 x f(x)\,dx$$

$$E\xi = \int_0^1 x(1-x)\,dx + \int_1^2 x(x-1)\,dx = \int_0^1 (x-x^2)\,dx + \int_1^2 (x^2-x)\,dx =$$

$$= \int_0^1 x\,dx - \int_0^1 x^2\,dx + \int_1^2 x^2\,dx - \int_1^2 x\,dx = \left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1 + \left[\frac{x^3}{3}\right]_1^2 - \left[\frac{x^2}{2}\right]_1^2 =$$

$$= \frac{1}{2} - \frac{1}{3} + \frac{8}{3} - \frac{1}{3} - \frac{4}{2} + \frac{1}{2} = -1 + 2 = 1 \qquad \underline{E\xi = 1}$$

$$D^2\xi = \int_0^2 x^2 f(x)\,dx - (E\xi)^2$$

$$D^2\xi = \int_0^1 x^2(1-x)\,dx + \int_1^2 x^2(x-1)\,dx - 1 = \int_0^1 (x^2-x^3)\,dx + \int_1^2 (x^3-x^2)\,dx - 1 =$$

$$= \int_0^1 x^2\,dx - \int_0^1 x^3\,dx + \int_1^2 x^3\,dx - \int_1^2 x^2\,dx - 1 = \left[\frac{x^3}{3}\right]_0^1 - \left[\frac{x^4}{4}\right]_0^1 + \left[\frac{x^4}{4}\right]_1^2 - \left[\frac{x^3}{3}\right]_1^2 - 1 =$$

$$= \frac{1}{3} - \frac{1}{4} + \frac{16}{4} - \frac{1}{4} - \frac{8}{3} + \frac{1}{3} - 1 = -\frac{6}{3} - 1 + \frac{14}{4} = -3 + 3\frac{1}{2} = \frac{1}{2}$$

$$\underline{D^2\xi = \frac{1}{2}}$$

Zad 7

$$f(x)=\begin{cases} -\frac{3}{4}x^2+\frac{9}{2}x-6 & \text{dla } x\in\langle 2,4\rangle \\ 0 & \text{dla } x\notin\langle 2,4\rangle \end{cases}$$

- jest nieujemna
- jest ciągła
- $\int_2^4 f(x)dx=1$

spr: $\int_2^4\left(-\frac{3}{4}x^2+\frac{9}{2}x-6\right)dx=-\frac{3}{4}\int_2^4 x^2+\frac{9}{2}\int_2^4 x-6\int_2^4 dx=$

$$=-\frac{3}{4}\left[\frac{x^3}{3}\right]_2^4+\frac{9}{2}\left[\frac{x^2}{2}\right]_2^4-6\left[x\right]_2^4=-\frac{3}{4}\left(\frac{64}{3}-\frac{8}{3}\right)+\frac{9}{2}\left(\frac{16}{2}-\frac{4}{2}\right)-6\cdot 2=$$

$$=-\frac{\cancel{3}^{1}}{\cancel{4}_{1}}\cdot\frac{\cancel{56}^{14}}{\cancel{3}_{1}}+\frac{9}{\cancel{2}_{1}}\cdot\frac{\cancel{12}^{6}3}{\cancel{2}_{1}}-12=-14+27-12=1$$

Zad 8

$p=0.1 \quad D^2\xi=0.45 \quad n=?$

$D^2\xi=npq$

$q=1-p=0.9$

$0.45=0.1\cdot 0.9\cdot n$

$n=\frac{0.45}{0.09}=\frac{45}{9}=5$

Zad 9

$P(\xi > a) = \frac{1}{3}$

$P(\xi \leq a) = 1 - \frac{1}{3} = \frac{2}{3}$

$\sin 2x = \frac{2}{3}$

$2x = \arcsin\left(\frac{2}{3}\right)$

$x = \frac{\arcsin\left(\frac{2}{3}\right)}{2} \approx 0.36486$

Zad 10

$P(s=100) = \binom{150}{100} 0.7^{100} \cdot 0.3^{50} = \frac{150!}{100!\,50!} \cdot 0.7^{100} \cdot 0.3^{50} \approx 0.04674$