**Advanced**

**Risk Management**

**Assignment 4**

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**Part A: Cointegration Analysis (Long-Run Relationship)**

We use the following methods to conduct cointegration analysis

1. **Engle-Granger Two-Step Test (ADF/CADF)**

* Step 1: Regress one yield on another:
* Step 2: Test the residuals for stationarity using the ADF test:

Null: Residuals have a unit root → no cointegration

Reject if p-value < threshold (e.g., 0.05 or 0.1)

**2. Johansen Test (Multivariate Cointegration)**

Applies to a system of k non-stationary variables. Tests the rank of cointegration matrix in VAR:

where captures long-run relationships.

Trace Statistic used:

**Statistical Interpretation**

1. **ADF and CADF Tests (Engle-Granger Two-Step)**

The p-value heatmaps and the output significantly cointegrated pairs using the two tests are shown in Appendix (Figure 3, 4, 6)

• The ADF test found 3 significant cointegrated pairs (p < 0.05), mainly among short-term maturities:

(SOFR, DGS1MO), (SOFR, DGS3MO), (DGS1MO, DGS3MO)

• The CADF test detected broader cointegration, especially between SOFR and multiple points on the yield curve — from 1-month to 30-year.

This suggests that SOFR has a long-run equilibrium relationship with almost the entire Treasury curve.

1. **Johansen Test (Multivariate Approach)**

The p-value heatmaps and the output significantly cointegrated pairs using the Johansen test are shown in Appendix (Figure 5, 7)

The Johansen test found extensive evidence of cointegration:

• All SOFR–Treasury pairs are cointegrated at p ≈ 0.01.

• Adjacent maturity pairs (e.g., DGS1MO–DGS3MO, DGS3MO–DGS6MO) also exhibit strong cointegration.

This is consistent with the **expectation hypothesis**, where yields across maturities are linked through expectations of future short rates.

**Economic Interpretation**

**1. Yield Curve Structure & Monetary Policy Transmission**

The strong cointegration between SOFR and short- to long-term yields supports the notion that SOFR is a central driver of the entire Treasury yield curve.

Since SOFR is a risk-free overnight rate influenced by the Fed, its long-run relationship with 1–30Y Treasury yields suggests that:

Monetary policy shocks at the short end propagate through expectations into longer maturities.

**2. Term Structure Theories**

The results reflect no-arbitrage and expectations hypotheses:

Cointegrated yields imply that their spreads are mean-reverting, aligning with models where long-term yields are averages of expected future short rates.

For example, the tight relationship between DGS1MO, DGS3MO, and DGS6MO indicates predictability in the slope and curvature of the short end of the yield curve.

**3. Implications for Risk Management and Modeling**

These findings justify modeling the yield curve as a cointegrated system (e.g., Vector Error Correction Models).

Risk managers can exploit cointegration in hedging strategies:

e.g., hedging long-duration positions using shorter bonds based on stable cointegration vectors.

**4. SOFR as a Policy Anchor**

The evidence that SOFR is cointegrated with all Treasury maturities implies:

SOFR is an effective anchor for the yield curve, validating its use in Quantitative Easing/Tightening operations and benchmark reform.

**Part B: SOFR-Based Econometric Analysis of the U.S. Treasury Yield Curve**

We use following method for SOFR-based econometric analysis**.**

**1. ADF Unit Root Test**

Test each series for stationarity:

Null: = 0 → non-stationary

Reject if p-value < 0.05

**2. Granger Causality Test**

Test if past values of SOFR help predict other yields:

Null:

If rejected → SOFR Granger-causes the yield

**3. VAR Model**

Estimate vector autoregression (VAR) for multiple interest rates:

Each rate is regressed on lags of itself and others for understanding dynamic interactions and forecasting

**Statistical Interpretation**

**1. ADF Unit Root Tests**

All six rates, including SOFR, fail to reject the null of a unit root (p > 0.87), indicating that they are non-stationary in levels.

This supports modeling the rates in a VAR in differences or using VECM.

**2. Granger Causality: SOFR → Other Rates**

All SOFR → [DGS3MO, DGS6MO, DGS2, DGS10, DGS30] show significant Granger causality (min p-value = 0.0).

Interpretation:

Changes in SOFR help forecast movements in short- and long-term Treasury yields.

Indicates unidirectional causality from the Fed's overnight policy rate to the entire yield curve.

**3. VAR Estimation**

VAR(5) model chosen with strong AIC/HQIC values.

In the SOFR equation, lags of SOFR itself are highly significant (L1–L5, all p < 0.01), showing strong autoregression.

Most lagged Treasury rates are insignificant in predicting SOFR → supports SOFR as the exogenous driver.

In Treasury yield equations, SOFR lags often enter significantly — especially in DGS3MO, DGS6MO, DGS2.

Some lags of SOFR significantly affect DGS10 and DGS30 (e.g., L5.SOFR in DGS10 and DGS30 equations with p = 0.006 and 0.020).

**Economic Interpretation**

**1. SOFR as a Monetary Policy Tool**

Granger causality and VAR results confirm that SOFR transmits monetary policy to both short- and long-term interest rates.

**2. Short-Run Dynamics**

The dynamic coefficients of SOFR on other yields (especially DGS3MO, DGS6MO, DGS2) are significant and mostly **positive**, suggesting an increase in SOFR leads to rising yields across the curve, consistent with tightening monetary policy.

**3. Policy Transmission Lag**

The effect of SOFR is strongest at **lag 2–5**, which may reflect:

• The time it takes for repo market changes to filter into longer-dated Treasuries.

• Market expectations incorporating forward guidance and inflation outlooks.

**4. Yield Curve Slope Control**

• SOFR affects not just short rates but also long-term yields (DGS10, DGS30).

• Implies that the Fed, through SOFR-based operations (e.g., overnight repos, balance sheet guidance), can influence the entire yield curve, validating SOFR’s role in QE/QT regimes.

**Part C: Policy Interpretation — SOFR as a Policy Instrument**

Yes, our results strongly support the use of SOFR as an effective monetary policy tool to influence the US Treasury yield curve. The statistical evidence confirms SOFR’s pivotal role in driving yields across all maturities, validating its use in central bank operations such as Quantitative Easing (QE) and Quantitative Tightening (QT).

**Supporting Evidence**

**1. Long-Run Cointegration (Part A)**

• Johansen and Engle-Granger tests reveal widespread cointegration between SOFR and short-, medium-, and long-term Treasury yields.

• This implies that long-run equilibrium relationships exist between SOFR and the entire yield curve, consistent with term structure theories (e.g., expectations hypothesis).

• Economically, this means shocks to SOFR are transmitted persistently, influencing yield curve levels and shape.

**2. Short-Run Dynamics and Causality (Part B)**

• Granger causality tests confirm unidirectional causality from SOFR to all yields (3M to 30Y), reinforcing SOFR’s role as a policy driver.

• The VAR model shows significant SOFR lag coefficients in yield equations, especially for short- and medium-term maturities.

• Residual analysis further shows weak SOFR dependence on Treasury yields, suggesting that SOFR innovations are exogenous, which is ideal for policy instruments.

**Central Bank Perspective**

**1. Transmission Effectiveness**

The cointegration and causality results demonstrate that the Fed’s manipulation of SOFR can **shape the yield curve over both the short and long term**.

This is crucial for:

* **Interest rate targeting**
* **Controlling inflation expectations**
* **Managing term premia through QE/QT**

**2. Robustness Across Maturities**

* SOFR affects yields up to 30 years, with statistically significant influence in both short and long horizons.
* This robustness supports SOFR’s use in forward guidance and longer-horizon economic signaling.

**3. Operational Feasibility**

* The clean Granger causality (no feedback loop) suggests that central banks can rely on SOFR as a controllable and effective lever, without being significantly influenced by market noise from longer-term securities.

**Risks or Limitations**

While the evidence is strongly supportive, some considerations remain:

* The lag structure suggests that SOFR takes several days or weeks to fully transmit through the curve.
* Market expectations, risk premia, and external shocks (e.g., geopolitical or inflationary) may temporarily reduce SOFR's influence on longer maturities.

In conclusion, the statistical results from both long-run and short-run analyses indicate that SOFR is a powerful and reliable instrument for the Federal Reserve to implement monetary policy through the Treasury yield curve. Its influence is broad, statistically significant, and economically interpretable, making it well-suited for steering macroeconomic conditions via QE, QT, and rate guidance strategies.

**Appendix**

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**Figure 1:** Plot of the yield data from 2018/04/03-2025/05/22

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**Figure 2:** Coefficient of correlation heatmap of yield data

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**Figure 3:** p-value heatmap of ADF test of each pair of yield data

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**Figure 4:** p-value heatmap of CADF test of each pair of yield data

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**Figure 5:** p-value heatmap of Johansen test of each pair of yield data

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**Figure 6:** Significant cointegrated pair in ADF test and CADF test

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**Figure 7:** Significant cointegrated pair in Johansen test

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**Figure 8:** ADF test and Granger Causality test result for part B

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**Figure 9:** VAR model summary

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**Figure 10:** Correlation matrix of residuals in VAR model estimation

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**Figure 11:** VAR Estimation – SOFR Equation Coefficients

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**Figure 12:** VAR Estimation – DGS3MO Equation Coefficients

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**Figure 13:** VAR Estimation – DGS6MO Equation Coefficients

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**Figure 14:** VAR Estimation – DGS10 Equation Coefficients

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**Figure 15:** VAR Estimation – DGS30 Equation Coefficients

**The full code is also available in**