CHAPTER 4

Making Choices

In this chapter, we will learn how to use the details in a structured problem to find a preferred alternative. "Using the details" typically means analysis: making calculations, creating graphs, and examining the results so as to gain insight into the decision. We will see that the kinds of calculations we make are essentially the same in solving decision trees and influence diagrams. We also introduce risk profiles and dominance considerations, ways to make decisions without doing all those calculations.

We begin by studying the analysis of decision models that involve only one objective or attribute. Although most of the examples we give use money as the attribute, it could be anything that can be measured as discussed in Chapter 3. After discussing calculation of expected values and the use of risk profiles for single-attribute decisions, we turn to decisions with multiple attributes and present some simple analytical approaches. The chapter concludes with a discussion of software for doing decision-analysis calculations on personal computers.

Our main example for this chapter is from the famous Texaco-Pennzoil court case.

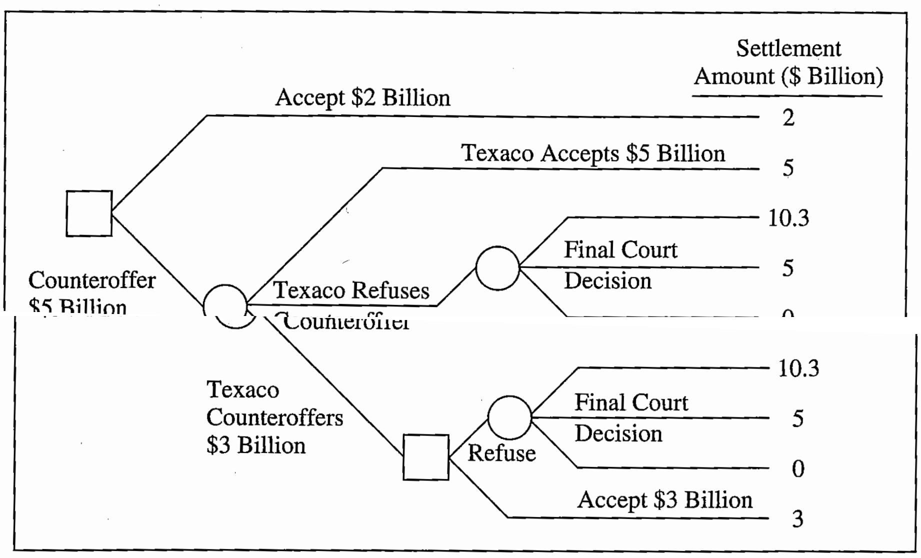
TEXACO VERSUS PENNZOIL

In early 1984, Pennzoil and Getty Oil agreed to the terms of a merger. But before any formal documents could be signed, Texaco offered Getty a substantially better price, and Gordon Getty, who controlled most of the Getty stock, reneged on the Pennzoil deal and sold to Texaco. Naturally, Pennzoil felt as if it had been dealt with unfairly and immediately filed a lawsuit against Texaco alleging that Texaco had interfered illegally in the Pennzoil-Getty negotiations. Pennzoil won the case; in late 1985, it was awarded $11.1 billion, the largest judgment ever in the United States at that time. A Texas appeals court reduced the judgment by $2 billion, but interest and penalties drove the total back up to $10.3 billion. James Kinnear, Texaco's chief executive officer, had said that Texaco would [illegible] tained court permission to secure the judgment by filing liens against Texaco's assets. Furthermore, Kinnear had promised to fight the case all the way to the U.S. Supreme Court if necessary, arguing in part that Pennzoil had not followed Security and Exchange Commission regulations in its negotiations with Getty. In April 1987, just before Pennzoil began to file the liens, Texaco offered to pay Pennzoil $2 billion to settle the entire case. Hugh Liedtke, chairman of Pennzoil, indicated that his advisors were telling him that a settlement between $3 and $5 billion would be fair.

What do you think Liedtke (pronounced "lid-key") should do? Should he accept the offer of $2 billion, or should he refuse and make a firm counteroffer? If he refuses the sure $2 billion, then he faces a risky situation. [illegible] billion, a reasonable amount in Liedtke's mind. If he counteroffered $5 billion as a settlement amount, perhaps Texaco would counter with $3 billion or simply pursue further appeals. Figure 4.1 is a decision tree that shows a simplified version of Liedtke's problem.

The decision tree in Figure 4.1 is simplified in a number of ways. First, we assume that Liedtke has only one fundamental objective: maximizing the amount of the settlement. No other objectives need be considered. Also, Liedtke has a more varied set of decision alternatives than those shown. He could counteroffer a variety of possible values in the initial decision, and in the second decision, he could counteroffer some amount between $3 and $5 billion. Likewise, Texaco's counteroffer, if it

Figure 4.1
Hugh Liedtke's decision in the Texaco-Pennzoil affair.



makes one, need not be exactly $3 billion. The outcome of the final court decision could be anything between zero and the current judgment of $10.3 billion. Finally, we have not included in our model of the decision anything regarding Texaco's option of filing for bankruptcy.

Why all of the simplifications? A straightforward answer (which just happens to have some validity) is that for our purposes in this chapter we need a relatively simple decision tree to work with. But this is just a pedagogical reason. If we were to try to analyze Liedtke's problem in all of its glory, how much detail should be included? As you now realize, all of the relevant information should be included, and the model should be constructed in a way that makes it easy to analyze. Does our representation accomplish this? Let us consider the following points.

1 *Liedtke's objective.* Certainly maximizing the amount of the settlement is a valid objective. The question is whether other objectives, such as minimizing attorney fees or improving Pennzoil's public image, might also be important. Although Liedtke may have other objectives, the fact that the settlement can range all the way from zero to $10.3 billion suggests that this objective will swamp any other concerns.

2 *Liedtke's initial counteroffer.* The counteroffer of $5 billion could be replaced by an offer for another amount, and then the decision tree reanalyzed. Different amounts may change the chance of Texaco accepting the counteroffer. At any rate, other possible counteroffers are easily dealt with.

3 *Liedtke's second counteroffer.* Other possible offers could be built into the tree, leading to a Texaco decision to accept, reject, or counter. The reason for leaving these out reflects an impression from the media accounts (especially *Fortune,* May 11, 1987, pp. 50–58) that Kinnear and Liedtke were extremely tough negotiators and that further negotiations were highly unlikely.

4 *Texaco's counteroffer.* The $3 billion counteroffer could be replaced by a fan representing a range of possible counteroffers. It would be necessary to find a "break-even" point, above which Liedtke would accept the offer and below which he would refuse. Another approach would be to replace the $3 billion value with other values, recomputing the tree each time. Thus, we have a variety of ways to deal with this issue.

5 *The final court decision.* We could include more branches, representing additional possible outcomes, or we could replace the three branches with a fan representing a range of possible outcomes. For a first-cut approximation, the possible outcomes we have chosen do a reasonably good job of capturing the uncertainty inherent in the court outcome.

6 *Texaco's bankruptcy option.* A detail left out of the case is that Texaco's net worth is much more than the $10.3 billion judgment. Thus, even if Texaco does file for bankruptcy, Pennzoil probably would still be able to collect. In reality, negotiations can continue even if Texaco has filed for bankruptcy; the purpose of filing is to protect the company from creditors seizing assets while the company proposes a financial reorganization plan. In fact, this is exactly what Texaco needs

to do in order to figure out a way to deal with Pennzoil's claims. In terms of Liedtke's options, however, whether Texaco files for bankruptcy appears to have no impact.

The purpose of this digression has been to explore the extent to which our structure for Liedtke's problem is requisite in the sense of Chapter 1. The points above suggest that the main issues in the problem have been represented in the problem. While it may be necessary to rework the analysis with slightly different numbers or structure later, the structure in Figure 4.1 should be adequate for a first analysis. The objective is to develop a representation of the problem that captures the essential features of the problem so that the ensuing analysis will provide the decision maker with insight and understanding.

One small detail remains before we can solve the decision tree. We need to specify the chances associated with Texaco's possible reactions to the $5 billion counteroffer, and we also need to assess the chances of the various court awards. The probabilities that we assign to the outcome branches in the tree should reflect Liedtke's beliefs about the uncertain events that he faces. For this reason, any numbers that we include to represent these beliefs should be based on what Liedtke has to say about the matter or on information from individuals whose judgments in this matter he would trust. For our purposes, imagine overhearing a conversation between Liedtke and his advisors. Here are some of the issues they might raise:

- Given the tough negotiating stance of the two executives, it could be an even chance (50%) that Texaco will refuse to negotiate further. If Texaco does not refuse, then what? What are the chances that Texaco would accept a $5 billion counteroffer? How likely is this outcome compared to the $3 billion counteroffer from Texaco? Liedtke and his advisors might figure that a counteroffer of $3 billion from Texaco is about twice as likely as Texaco accepting the $5 billion. Thus, because there is already a 50% chance of refusal, there must be a 33% chance of a Texaco counteroffer and a 17% chance of Texaco accepting $5 billion.

- What are the probabilities associated with the final court decision? In the *Fortune* article cited above, Liedtke is said to admit that Texaco could win its case, leaving Pennzoil with nothing but lawyer bills. Thus, there is a significant possibility that the outcome would be zero. Given the strength of Pennzoil's case so far, there is also a good chance that the court will uphold the judgment as it stands. Finally, the possibility exists that the judgment could be reduced somewhat (to $5 billion in our model). Let us assume that Liedtke and his advisors agree that there is a 20% chance that the court will award the entire $10.3 billion and a slightly larger, or 30%, chance that the award will be zero. Thus, there must be a 50% chance of an award of $5 billion.

Figure 4.2 shows the decision tree with these chances included. The chances have been written in terms of probabilities rather than percentages.

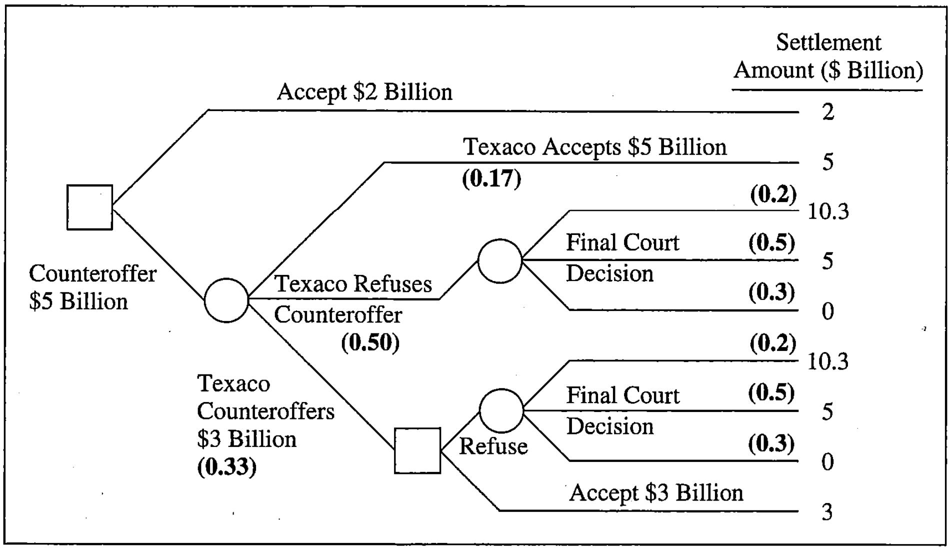


Figure 4.2
Hugh Liedtke's decision tree with chances (probabilities) included.

Decision Trees and Expected Monetary Value

One way to choose among risky alternatives is to pick the alternative with the highest *expected value* (EV). When the decision's consequences involve only money, we can calculate the *expected monetary value* (EMV). Finding EMVs when using decision trees is called "folding back the tree" for reasons that will become obvious. (The procedure is called "rolling back" in some texts.) We start at the endpoints of the branches on the far right-hand side and move to the left, (1) calculating expected values (to be defined momentarily) when we encounter a chance node, or (2) choosing the branch with the highest value or expected value when we encounter a decision node. These instructions sound rather cryptic. It is much easier to understand the procedure through a few examples. We will start with a simple example, the double-risk dilemma shown in Figure 4.3.

Recall that a double-risk dilemma is a matter of choosing between two risky alternatives. The situation is one in which you have a ticket that will let you participate in a game of chance (a lottery) that will pay off $10 with a 45% chance, and nothing with a 55% chance. Your friend has a ticket to a different lottery that has a 20% chance of paying $25 and an 80% chance of paying nothing. Your friend has offered to let you have his ticket if you will give him your ticket plus one dollar. Should you agree to the trade and play to win $25, or should you keep your ticket and have a better chance of winning $10?

Figure 4.3 displays your decision situation. In particular, notice that the dollar consequences at the ends of the branches are the net values as discussed in Chapter 3. Thus, if you trade tickets and win, you will have gained a net amount of $24, having paid one dollar to your friend.

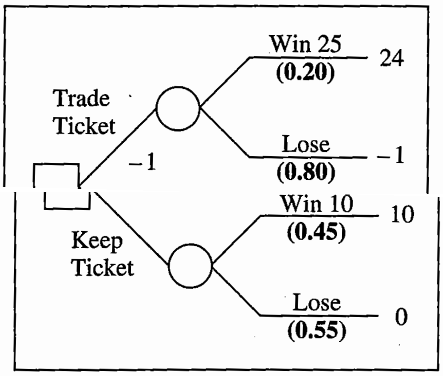


Figure 4.3
A double-risk dilemma.

To solve the decision tree using EMV, begin by calculating the expected value of keeping the ticket and playing for $10. This expected value is simply the weighted average of the possible outcomes of the lottery, the weights being the chances with which the outcomes occur. The calculations are

$$\begin{aligned}\text{EMV(Keep Ticket)} &= 0.45(10) + 0.55(0)\\ &= \$4.5\end{aligned}$$

One interpretation of this EMV is that playing this lottery many times would yield an average of approximately $4.50 per game. Calculating EMV for trading tickets gives

$$\begin{aligned}\text{EMV(Trade Ticket)} &= 0.20(24) + 0.80(-1)\\ &= \$4\end{aligned}$$

Now we can replace the chance nodes in the decision tree with their expected values, as shown in Figure 4.4. Finally, choosing between trading and keeping the ticket amounts to choosing the branch with the highest expected value. The double slash through the "Trade Ticket" branch indicates that this branch would not be chosen.

This simple example is only a warm-up exercise. Now let us see how the solution procedure works when we have a more complicated decision problem. Consider Hugh Liedtke's situation as diagrammed in Figure 4.2. Our strategy, as indicated, will be to work from the right-hand side of the tree. First, we will calculate the expected value of the final court decision. The second step will be to decide whether it is better for Liedtke to accept a $3 billion counteroffer from Texaco or to refuse and take a chance on the final court decision. We will do this by comparing the expected value of the judgment with the sure $3 billion. The third step will be to calculate the

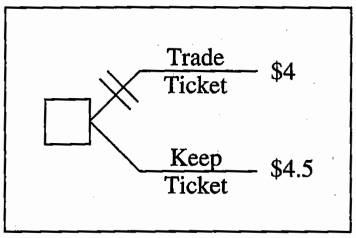


Figure 4.4
Replacing chance nodes with EMVs.

expected value of making the $5 billion counteroffer, and finally we will compare this expected value with the sure $2 billion that Texaco is offering now.

The expected value of the court decision is the weighted average of the possible outcomes:

$$\begin{aligned} \text{EMV(Court Decision)} &= [\text{P(Award} = 10.3) \times 10.3] + [\text{P(Award} = 5) \times 5] \\ &\quad + [\text{P(Award} = 0) \times 0] \\ &= [0.2 \times 10.3] + [0.5 \times 5] + [0.3 \times 0] \\ &= 4.56 \end{aligned}$$

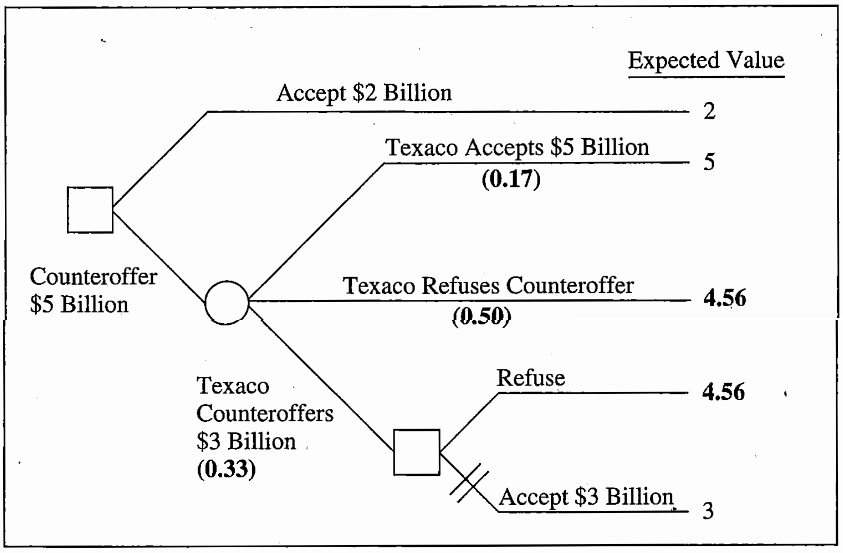
We replace both uncertainty nodes representing the court decision with this expected value, as in Figure 4.5. Now, comparing the two alternatives of accepting and refusing Texaco's $3 billion counteroffer, it is obvious that the expected value of $4.56 billion is greater than the certain value of $3 billion, and hence the slash through the "Accept $3 Billion" branch.

To continue folding back the decision tree, we replace the decision node with the preferred alternative. The decision tree as it stands after this replacement is shown in Figure 4.6. The third step is to calculate the expected value of the alternative "Counteroffer $5 Billion." This expected value is

$$\begin{aligned} \text{EMV(Counteroffer \$5 Billion)} &= [\text{P(Texaco Accepts)} \times 5] \\ &\quad + [\text{P(Texaco Refuses)} \times 4.56] \\ &\quad + [\text{P(Texaco Counteroffers)} \times 4.56] \\ &= [0.17 \times 5] + [0.50 \times 4.56] + [0.33 \times 4.56] \\ &= 4.63 \end{aligned}$$

Replacing the chance node with its expected value results in the decision tree shown in Figure 4.7. Comparing the values of the two branches, it is clear that the expected value of $4.63 billion is preferred to the $2 billion offer from Texaco. According to

Figure 4.5
Hugh Liedtke's decision tree after calculating expected value of court decision.



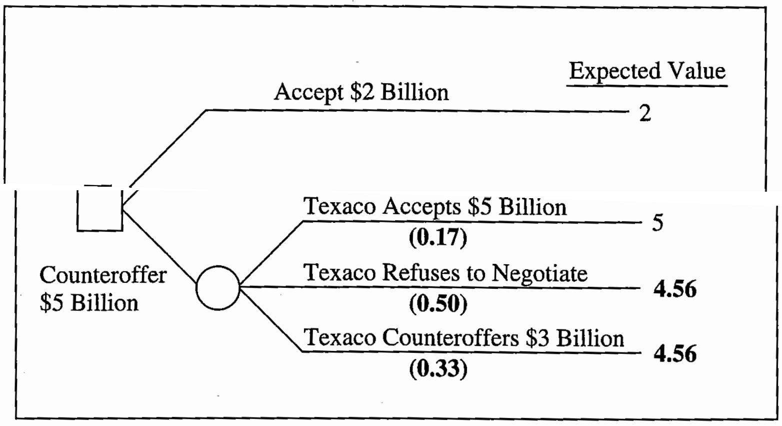


Figure 4.6
Hugh Liedtke's decision tree after decision node replaced with expected value of

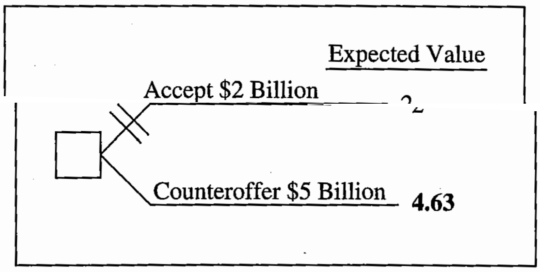


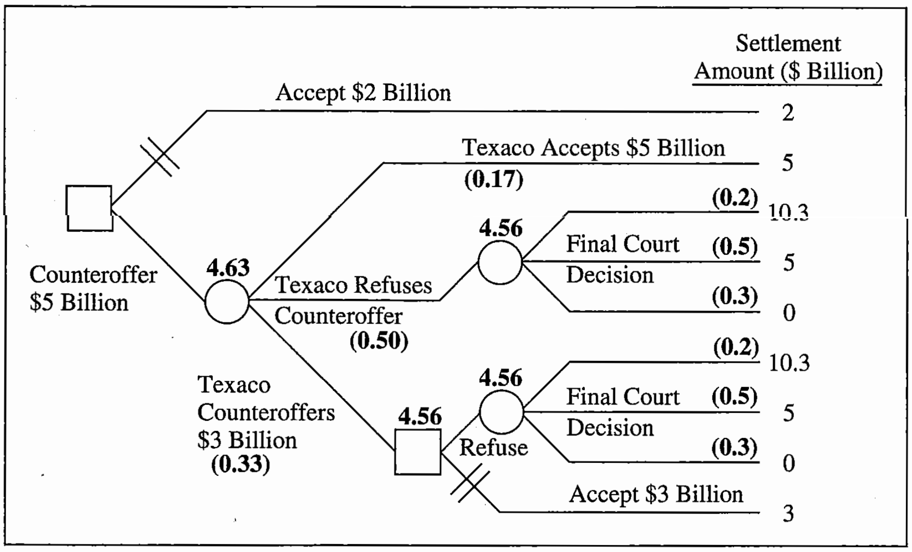
Figure 4.7
Hugh Liedtke's decision tree after original tree completely folded back.

this solution, which implies that decisions should be made by comparing expected values, Liedtke should turn down Texaco's offer but counteroffer a settlement of $5 billion. If Texaco turns down the $5 billion and makes another counteroffer of $3 billion, Liedtke should refuse the $3 billion and take his chances in court.

We went through this decision in gory detail so that you could see clearly the steps involved. In fact, in solving a decision tree, we usually do not redraw the tree at each step, but simply indicate on the original tree what the expected values are at each of the chance nodes and which alternative is preferred at each decision node. The solved decision tree for Liedtke would look like the tree shown in Figure 4.8, which shows all of the details of the solution. Expected values for the chance nodes are placed above the nodes. The 4.56 above the decision node indicates that if Liedtke gets to this decision point, he should refuse Texaco's offer and take his chances in court for an expected value of $4.56 billion. The decision tree also shows that his best current choice is to make the $5 billion counteroffer with an expected payoff of $4.63 billion.

The decision tree shows clearly what Liedtke should do if Texaco counteroffers $3 billion: He should refuse. This is the idea of a contingent strategy. If a particular course of events occurs (Texaco's counteroffer), then there is a specific course of action to take (refuse the counteroffer). Moreover, in deciding whether to accept Texaco's current $2 billion offer, Liedtke must know what he will do in the event that Texaco returns with a counteroffer of $3 billion. This is why the decision tree is solved backward. In order to make a good decision at the current time, we have to know what the appropriate contingent strategies are in the future.

Figure 4.8
Hugh Liedtke's solved decision tree.

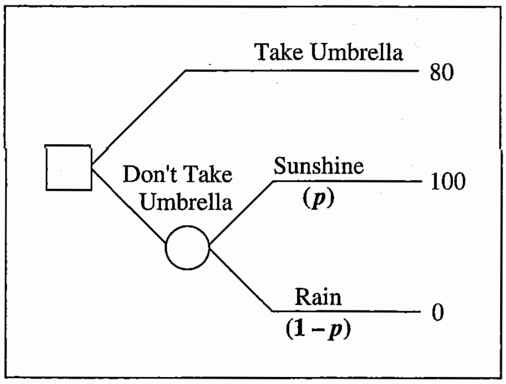


Solving Influence Diagrams: Overview

Solving decision trees is straightforward, and EMVs for small trees can be calculated by hand relatively easily. The procedure for solving an influence diagram, though, is somewhat more complicated. Fortunately, computer programs such as PrecisionTree are available to do the calculations. In this short section we give an overview of the issues involved in solving an influence diagram. For interested readers, the following optional section goes through a complete solution of the influence diagram of the Texaco-Pennzoil decision.

While influence diagrams appear on the surface to be rather simple, much of the complexity is hidden. Our first step is to take a close look at how an influence diagram translates information into an internal representation. An influence diagram "thinks" about a decision in terms of a symmetric expansion of the decision tree from one node to the next.

Figure 4.9
Umbrella problem.

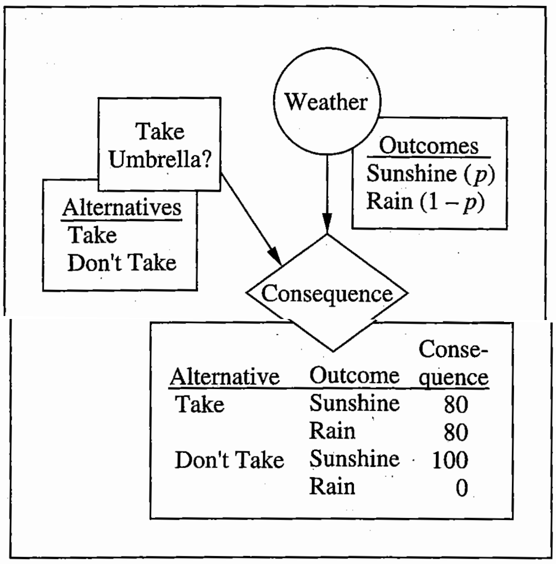


For example, suppose we have the basic decision tree shown in Figure 4.9, which represents the "umbrella problem" (see Exercise 3.9). The issue is whether or not to take your umbrella. If you do not take the umbrella, and it rains, your good clothes (and probably your day) are ruined, and the consequence is zero (units of satisfaction). However, if you do not take the umbrella and the sun shines, this is the best of all possible consequences with a value of 100. If you decide to take your umbrella, your clothes will not get spoiled. However, it is a bit of a nuisance to carry the umbrella around all day. Your consequence is 80, between the other two values.

If we were to represent this problem with an influence diagram, it would look like the diagram in Figure 4.10. Note that it does not matter whether the sun shines or not if you take the umbrella. If we were to reconstruct exactly how the influence diagram "thinks" about the umbrella problem in terms of a decision tree, the representation would be that shown in Figure 4.11. Note that the uncertainty node on the "Take Umbrella" branch is an unnecessary node. The payoff is the same regardless of the weather. In a decision-tree model, we can take advantage of this fact by not even drawing the unnecessary node. Influence diagrams, however, use the symmetric decision tree, even though this may require unnecessary nodes (and hence unnecessary calculations).

With an understanding of the influence diagram's internal representation, we can talk about how to solve an influence diagram. The procedure essentially solves the symmetric decision tree, although the terminology is somewhat different. Nodes are *reduced;* reduction amounts to calculating expected values for chance nodes and choosing the largest expected value at decision nodes, just as we did with the decision tree. Moreover, also parallel with the decision-tree procedure, as nodes are reduced, they are removed from the diagram. Thus, solving the influence diagram in Figure 4.10 would require first reducing the "Weather" node (calculating the expected values) and then reducing the "Take Umbrella?" node by choosing the largest expected value.

Figure 4.10
Influence diagram of the umbrella problem.



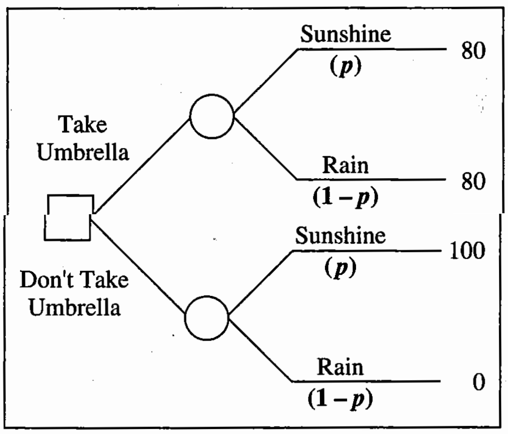


Figure 4.11
How the influence diagram "thinks" about the umbrella problem.

Solving Influence Diagrams: The Details (Optional)

Consider the Texaco-Pennzoil case in influence-diagram form, as shown in Figure 4.12. This diagram shows the tables of alternatives, outcomes (with probabilities), and consequences that are contained in the nodes. The consequence table in this case is too complicated to put into Figure 4.12. We will work with it later in great detail, but if you want to see it now, it is displayed in Table 4.1.

Figure 4.12 needs explanation. The initial decision is whether to accept Texaco's offer of $2 billion. Within this decision node a table shows that the available alternatives are to accept the offer or make a counteroffer. Likewise, under the "Pennzoil Reaction" node is a table that lists "Accept 3" and "Refuse" as alternatives. The chance node "Texaco Reaction" contains a table showing the probabilities of Texaco accepting a counteroffer of $5 billion, making an offer of $3 billion, or refusing to

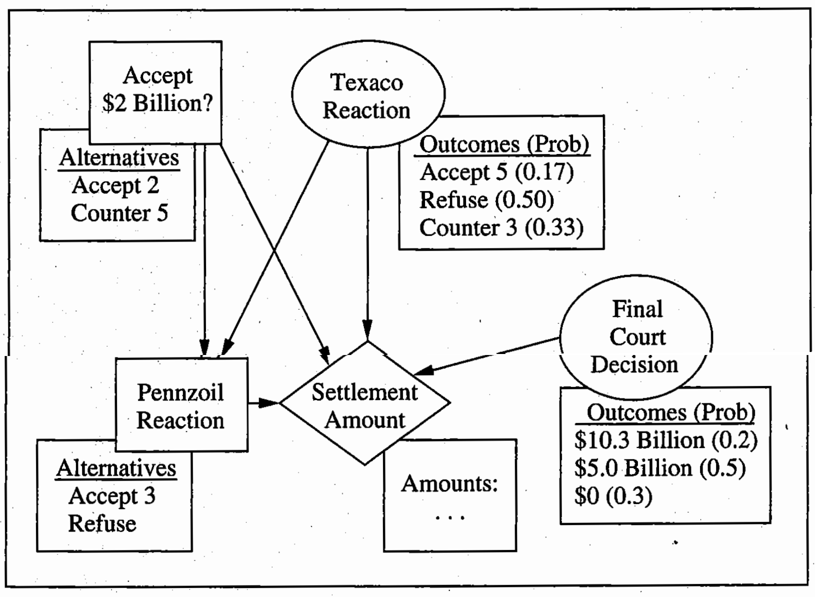


Figure 4.12
Influence diagram for Liedtke's decision.

Table 4.1
Consequence table for the influence diagram of Liedtke's decision.

Accept $2 Billion?	Texaco Reaction ($ Billion)	Pennzoil Reaction ($ Billion)	Final Court Decision ($ Billion)	Settlement Amount ($ Billion)
Accept 2	Accept 5	Accept 3	10.3	2.0
			5	2.0
			0	2.0
		Refuse	10.3	2.0
			5	2.0
			0	2.0
	Offer 3	Accept 3	10.3	2.0
			5	2.0
			0	2.0
		Refuse	10.3	2.0
			5	2.0
			0	2.0
	Refuse	Accept 3	10.3	2.0
			5	2.0
			0	2.0
		Refuse	10.3	2.0
			5	2.0
			0	2.0
Offer 5	Accept 5	Accept 3	10.3	5.0
			5	5.0
			0	5.0
		Refuse	10.3	5.0
			5	5.0
			0	5.0
	Offer 3	Accept 3	10.3	3.0
			5	3.0
			0	3.0
		Refuse	10.3	10.3
			5	5.0
			0	0.0
	Refuse	Accept 3	10.3	10.3
			5	5.0
			0	0.0
		Refuse	10.3	10.3
			5	5.0
			0	0.0

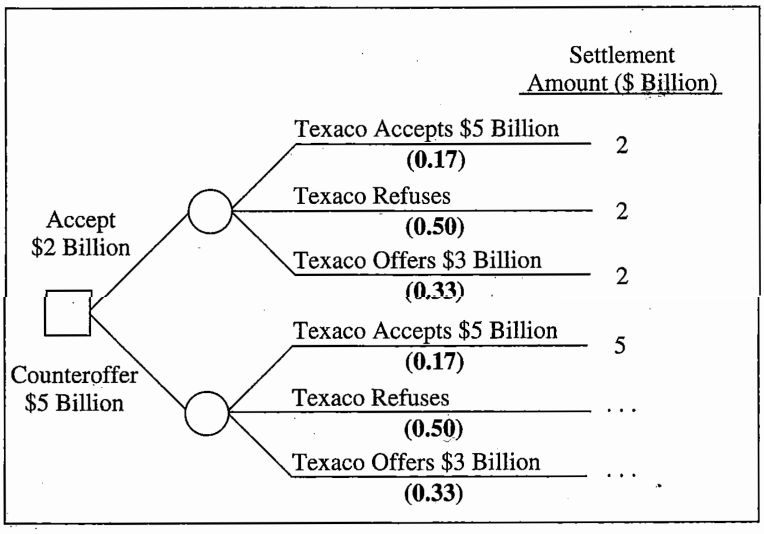
negotiate. Finally, the "Final Court Decision" node has a table with its outcomes and associated probabilities.

The thoughtful reader should have an immediate reaction to this. After all, whether Texaco reacts depends on whether Liedtke makes his $5 billion counteroffer in the first place! Shouldn't there be an arrow from the decision node "Accept $2 Billion" to the "Texaco Reaction" node? The answer is yes, there could be such an arrow, but it is unnecessary and would only complicate matters. The reason is that, as with the umbrella example above, the influence diagram "thinks" in terms of a symmetric expansion of the decision tree. Figure 4.13 shows a portion of the tree that deals with Liedtke's initial decision and Texaco's reaction. An arrow in Figure 4.12 from "Accept $2 Billion" to "Texaco Reaction" would indicate that the decision made (accepting or rejecting the $2 billion) would affect the chances associated with Texaco's reaction to a counteroffer. But the uncertainty about Texaco's response to a $5 billion counteroffer does not depend on whether Liedtke accepts the $2 billion. Essentially, the influence diagram is equivalent to a decision tree that is symmetric.

For similar reasons, there are no arrows between "Final Court Decision" and the other three nodes. If some combination of decisions comes to pass so that Pennzoil and Texaco agree to a settlement, it does not matter what the court decision would be. The influence diagram implicitly includes the "Final Court Decision" node with the agreed-upon settlement regardless of the "phantom" court outcome.

How is all of this finally resolved in the influence-diagram representation? Everything is handled in the consequence node. This node contains a table that gives Liedtke's settlement for every possible combination of decisions and outcomes. That table (Table 4.1) shows that the settlement is $2 billion if Liedtke accepts the current offer, regardless of the other outcomes. It also shows that if Liedtke counteroffers $5 billion and Texaco accepts, then the settlement is $5 billion regardless of the court decision or Pennzoil's reaction (neither of which have any impact if Texaco accepts the $5 billion). The table also shows the details of the

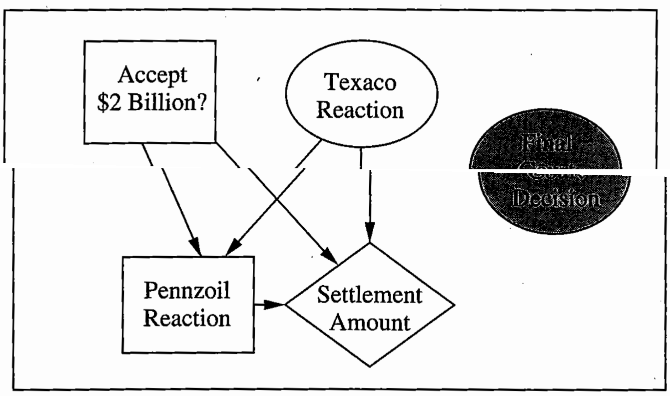
Figure 4.13
How the influence diagram "thinks" about the Texaco-Pennzoil case.



court outcomes if either Texaco refuses to negotiate after Liedtke's counteroffer or if Liedtke refuses a Texaco counteroffer. And so on. The table shows exactly what the payoff is to Pennzoil under all possible combinations. The column headings in Table 4.1 represent nodes that are predecessors of the value node. In this case, both decision nodes and both chance nodes are included because all are predecessors of the value node. We can now discuss how the algorithm for solving an influence diagram proceeds. Take the Texaco-Pennzoil diagram as drawn in Figure 4.12. As mentioned above, our strategy will be to *reduce nodes* one at a time. The order of reduction is reminiscent of our solution in the case of the decision tree. The first node reduced is "Final Court Decision," resulting in the diagram in Figure 4.14. In this first step, expected values are calculated using the "Final Court Decision" probabilities, which yields Table 4.2. All combinations of decisions and possible outcomes of Texaco's reaction are shown. For example, if Liedtke counteroffers $5 billion and Texaco refuses to negotiate, the expected value of $4.56 billion is listed regardless of the decision in the "Pennzoil Reaction" node (because that decision is meaningless if Texaco initially refuses to negotiate). If Liedtke accepts the $2 billion offer, the expected value is listed as $2 billion, regardless of other outcomes. (Of course, there is nothing uncertain about this outcome; the value that we know will happen is the expected value.) If Liedtke offers 5, Texaco offers 3, and finally Liedtke refuses to continue negotiating, then the expected value is given as 4.56. And so on.

The next step is to reduce the "Pennzoil Reaction" node. The resulting influence diagram is shown in Figure 4.15. Now the table in the consequence node (Table 4.3) reflects the decision that Liedtke should choose the alternative with the highest expected value (refuse to negotiate) if Texaco makes the counteroffer of $3 billion. Thus, the table now says that, if Liedtke offers $5 billion and Texaco either refuses to negotiate or counters with $3 billion, the expected value is $4.56 billion. If Texaco accepts the $5 billion counteroffer, the expected value is $5 billion, and if Liedtke accepts the current offer, the expected value is $2 billion. (Again, there is nothing uncertain about these values; the expected value in these cases is just the value that we know will occur.)

Figure 4.14
First step in solving the influence diagram.



The third step is to reduce the "Texaco Reaction" node, as shown in Figure 4.16. As with the first step, this involves taking the table of consequences (now expected values) within the "Settlement Amount" node and calculating expected values again. The resulting table has only two entries (Table 4.4). The expected value of Liedtke accepting $2 billion is just $2 billion, and the expected value of countering with $5 billion is $4.63 billion.

The fourth and final step is simply to figure out which decision is optimal in the "Accept $2 Billion?" node and to record the result. This final step is shown in Figure 4.17. The table associated with the decision node indicates that Liedtke's optimal choice is to counteroffer $5 billion. The payoff table now contains only one value, $4.63 billion, the expected value of the optimal decision.

Reviewing the procedure, you should be able to see that it followed basically the same steps that we followed in folding back the decision tree.

Table 4.2
Table for Liedtke's decision after reducing the "Final Court Decision" node.

Accept $2 Billion?	Texaco Reaction	Pennzoil Reaction	Expected Value ($ Billion)
Accept 2	Accept 5	Accept 3	2
		Refuse	2
	Offer 3	Accept 3	2
		Refuse	2
	Refuse	Accept 3	2
		Refuse	2
Offer 5	Accept 5	Accept 3	5
		Refuse	5
	Offer 3	Accept 3	3
		Refuse	4.56
	Refuse	Accept 3	4.56
		Refuse	4.56

Figure 4.15
Second step in solving the influence diagram.

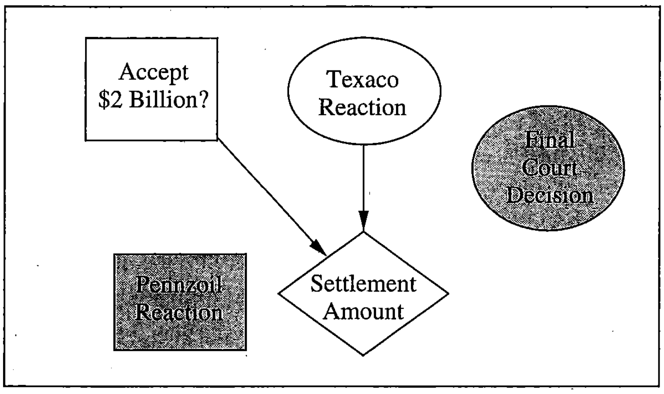


Table 4.3
Table for Liedtke's decision after reducing "Final Court Decision" and "Pennzoil Reaction" nodes.

Accept $2 Billion?	Texaco Reaction	Expected Value ($ Billion)
Accept 2	Accept 5	2.00
	Offer 3	2.00
	Refuse	2.00
Offer 5	Accept 5	5.00
	Offer 3	4.56
	Refuse	4.56

Table 4.4
Table for Liedtke's decision after reducing "Final Court Decision," "Pennzoil Reaction," and "Texaco Reaction" nodes.

Accept $2 Billion?	Expected Value ($ Billion)
Accept 2	2.00
Offer 5	4.63

Figure 4.16
Third step in solving the influence diagram.

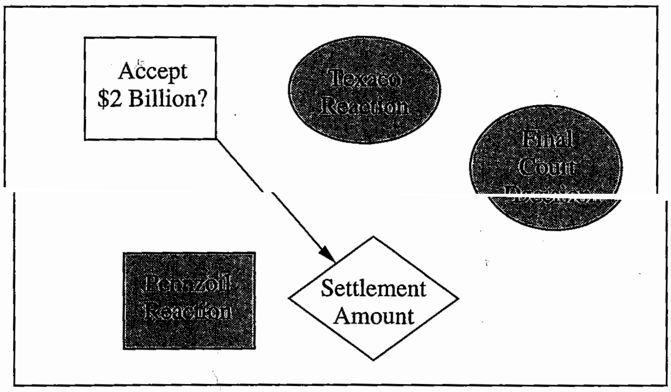
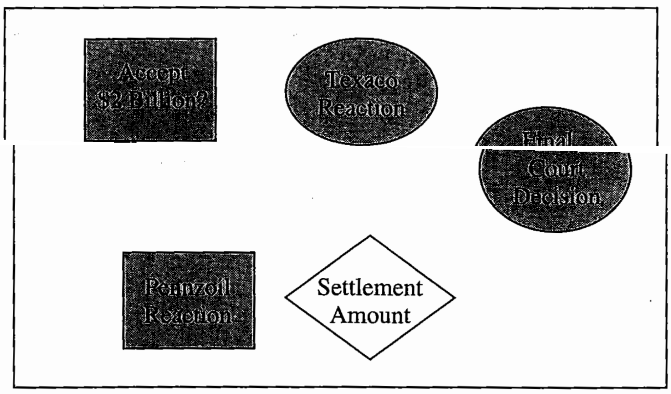


Figure 4.17
Final step in solving the influence diagram.



Solving Influence Diagrams: An Algorithm (Optional)

The example above should provide some insight into how influence diagrams are solved. Fortunately, you will not typically have to solve influence diagrams by hand; computer programs are available to accomplish this. It is worthwhile, however, to spend a few moments describing the procedure that is used to solve influence diagrams. A set procedure for solving a problem is called an *algorithm*. You have already learned the algorithm for solving a decision tree (the folding-back procedure). Now let us look at an algorithm for solving influence diagrams.

1 First, we simply clean up the influence diagram to make sure it is ready for solution. Check to make sure the influence diagram has only one consequence node (or a series of consequence nodes that feed into one "super" consequence node) and that there are no cycles. If your diagram does not pass this test, you must fix it before it can be solved. In addition, if any nodes other than the consequence node have arrows into them but not out of them, they can be eliminated. Such nodes are called *barren nodes* and have no effect on the decision that would be made. Replace any intermediate-calculation nodes with chance nodes. (This includes any consequence nodes that feed into a "super" consequence node representing a higher-level objective in the objectives hierarchy.) For each possible combination of the predecessor node outcomes, such a node has only one outcome that happens with probability 1.

2 Next, look for any chance nodes that (a) directly precede the consequence node and (b) do *not* directly precede any other node. Any such chance node found should be reduced by calculating expected values. The consequence node then inherits the predecessors of the reduced nodes. (That is, any arrows that went into the node you just reduced should be redrawn to go into the consequence node.)

This step is just like calculating expected values for chance nodes at the far right-hand side of a decision tree. You can see how this step was implemented in the Texaco-Pennzoil example. In the original diagram, Figure 4.12, the "Final Court Decision" node is the only chance node that directly precedes the consequence node *and* does not precede any decision node. Thus it is reduced by the expected-value procedure, resulting in Table 4.2. The consequence node does not inherit any new direct predecessors as a result of this step because "Final Court Decision" has no direct predecessors.

3 Next, look for a decision node that (a) directly precedes the consequence node and (b) has as predecessors all of the other direct predecessors of the consequence node. *If you do not find any such decision node, go directly to Step 5.* If you find such a decision node, you can reduce it by choosing the optimum value. When decision nodes are reduced, the consequence node does not inherit any new predecessors. This step may create some barren nodes, which can be eliminated from the diagram.

This step is like folding a decision tree back through a decision node at the far right-hand side of the tree. In the Texaco-Pennzoil problem, this step was implemented when we reduced "Pennzoil Reaction." In Figure 4.14, this node satisfies the criteria for reduction because it directly precedes the consequence node, and the other nodes that directly precede the consequence node also precede "Pennzoil Reaction." In reducing this node, we choose the option for "Pennzoil Reaction" that gives the highest expected value, and as a result we obtain Table 4.3. No barren nodes are created in this step.

4 Return to Step 2 and continue until the influence diagram is completely solved (all nodes reduced). This is just like working through a decision tree until all of the nodes have been processed from right to left.

5 You arrived at this step after reducing all possible chance nodes (if any) and then not finding any decision nodes to reduce. How could this happen? Consider the influence diagram of the hurricane problem in Figure 3.12. None of the chance nodes satisfy the criteria for reduction, and the decision node also cannot be reduced. In this case, one of the arrows between chance nodes must be reversed. This is a procedure that requires probability manipulations through the use of Bayes' theorem (Chapter 7). We will not go into the details of the calculations here because most of the simple influence diagrams that you might be tempted to solve by hand will not require arrow reversals.

Finding an arrow to reverse is a delicate process. First, find the correct chance node. The criteria are that (a) it directly precedes the consequence node and (b) it does not directly precede any decision node. Call the selected node A. Now look at the arrows out of node A. Find an arrow from A to chance node B (call it A → B) such that there is no other way to get from A to B by following arrows. The arrow A → B can be reversed using Bayes' theorem. Afterward, both nodes inherit each other's direct predecessors *and* keep their own direct predecessors.

After reversing an arrow, return to Step 2 and continue until the influence diagram is solved. (More arrows may need to be reversed before a node can be reduced, but that only means that you may come back to Step 5 one or more times in succession.)

This description of the influence-diagram solution algorithm is based on the complete (and highly technical) description given in Shachter (1986). The intent is not to present a "cookbook" for solving an influence diagram because, as indicated, virtually all but the simplest influence diagrams will be solved by computer. The description of the algorithm, however, is meant to show the parallels between the influence-diagram and decision-tree solution procedures.

Risk Profiles

The idea of expected value is appealing, and comparing two alternatives on the basis of their EMVs is straightforward. For example, Liedtke's expected values are $2 billion and $4.63 billion for his two immediate alternatives. But you might have noticed that these two numbers are not exactly perfect indicators of what might happen. In particular, suppose that Liedtke decides to counteroffer $5 billion: He might end up with $10.3 billion, $5 billion, or nothing, given our simplification of the situation. Moreover, the interpretation of EMV as the average amount that would be obtained by "playing the game" a large number of times is not appropriate here. The "game" in this case amounts to suing Texaco—not a game that Pennzoil will play many times!

That Liedtke could come away from his dealings with Texaco with nothing indicates that choosing to counteroffer is a somewhat risky alternative. In later chapters we will look at the idea of risk in more detail. For now, however, we can intuitively grasp the relative riskiness of alternatives by studying their *risk profiles*.

A risk profile is a graph that shows the chances associated with possible consequences. Each risk profile is associated with a *strategy*, a particular immediate alternative, as well as specific alternatives in future decisions. For example, the risk profile for the "Accept $2 Billion" alternative is shown in Figure 4.18. There is a 100% chance that Liedtke will end up with $2 billion. The risk profile for the strategy "Counteroffer $5 Billion; Refuse Texaco Counteroffer" is somewhat more complicated and is shown in Figure 4.19. There is a 58.5% chance that the eventual settlement is $5 billion, a 16.6% chance of $10.3 billion, and a 24.9% chance of nothing. These numbers are easily calculated. For example, take the $5 billion amount. This can happen in three different ways. There is a 17% chance that it happens because Texaco accepts. There is a 25% chance that it happens because Texaco refuses and the judge awards $5 billion. (That is, there is a 50% chance that Texaco refuses times a 50% chance that the award is $5 billion.) Finally, the chances are 16.5% that the settlement is $5 billion because Texaco counteroffers $3 billion, Liedtke refuses and goes to court, and the judge awards $5 billion. That is, 16.5% equals 33% times 50%. Adding up, we get the chance of $5 billion = 17% + 25% + 16.5% = 58.5%.

In constructing a risk profile, we collapse a decision tree by multiplying out the probabilities on sequential chance branches. At a decision node, only one branch is taken; in the case of "Counteroffer $5 Billion; Refuse Texaco Counteroffer," we use only the indicated alternative for the second decision, and so this decision node need not be included in the collapsing process. You can think about the process as one in which nodes are gradually removed from the tree in much the same sense as we did

Figure 4.18
Risk profile for the "Accept $2 Billion" alternative.

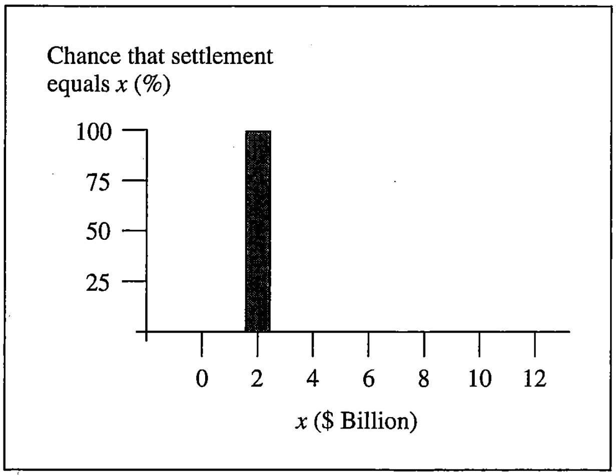
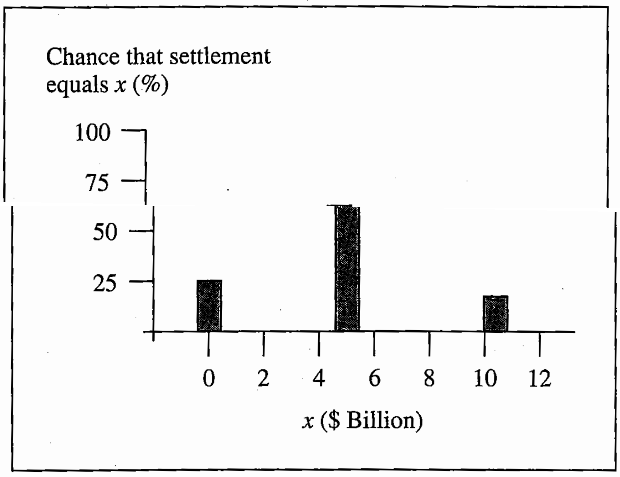


Figure 4.19
Risk profile for the "Counteroffer $5 Billion; Refuse Texaco Counteroffer" strategy.

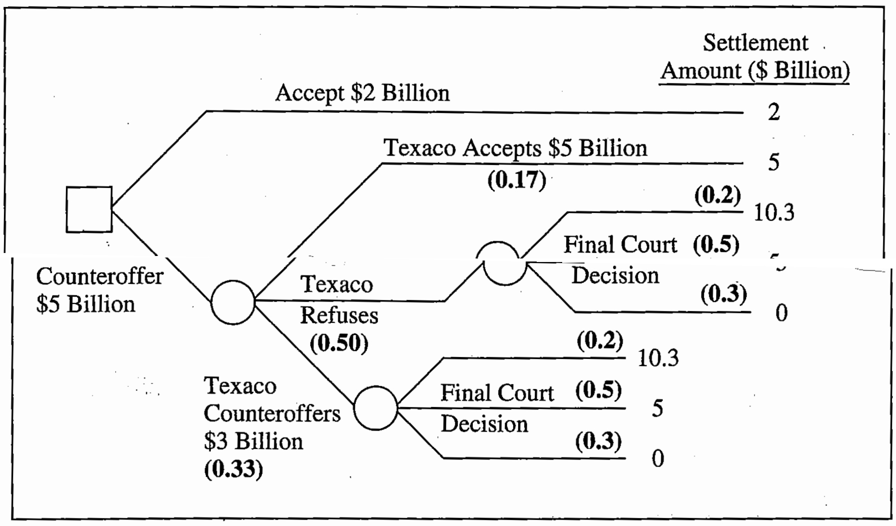


with the folding-back procedure, except that in this case we keep track of the possible outcomes and their probabilities. Figures 4.20, 4.21, and 4.22 show the progression of collapsing the decision tree in order to construct the risk profile for the "Counteroffer $5 Billion; Refuse Texaco Counteroffer" strategy.

By looking at the risk profiles, the decision maker can tell a lot about the riskiness of the alternatives. In some cases a decision maker can choose among alternatives on the basis of their risk profiles. Comparing Figures 4.18 and 4.19, it is clear that the worst possible consequence for "Counteroffer $5 Billion; Refuse Texaco Counteroffer" is less than the value for "Accept $2 billion." On the other hand, the largest amount ($10.3 billion) is much better than $2 billion. Hugh Liedtke has to decide whether the risk of perhaps coming away empty-handed is worth the possibility of getting more than $2 billion. This is clearly a case of a basic risky decision, as we can see from the collapsed decision tree in Figure 4.22.

Risk profiles can be calculated for strategies that might not have appeared as optimal in an expected-value analysis. For example, Figure 4.23 shows the risk profile for

Figure 4.20
First step in collapsing the decision tree to make a risk profile for "Counteroffer $5 Billion; Refuse Texaco Counteroffer" strategy. The decision node has been removed to leave only the outcomes associated with the "Refuse" branch.



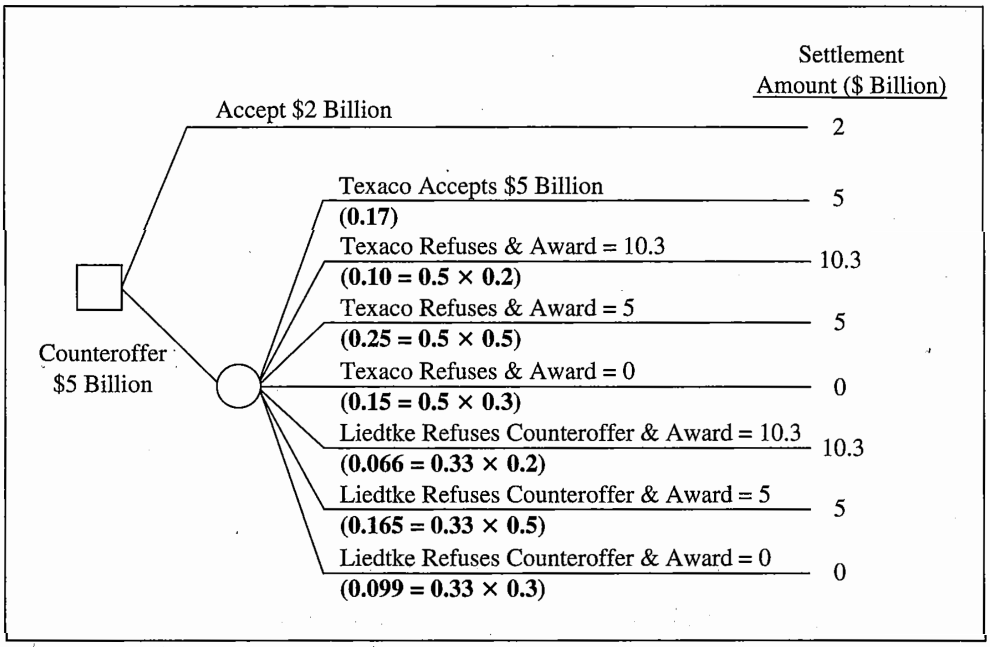


Figure 4.21 *Second step in collapsing the decision tree to make a risk profile.* The three chance nodes have been collapsed into one chance node. The probabilities on the branches are the product of the probabilities from sequential branches in Figure 4.20.

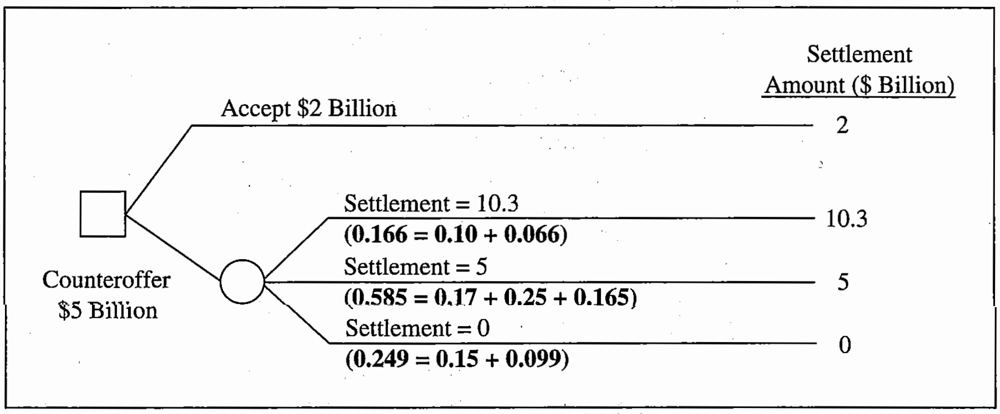


Figure 4.22 *Third step in collapsing the decision tree to make a risk profile.* The seven branches from the chance node in Figure 4.21 have been combined into three branches.

"Counteroffer $5 Billion; Accept $3 Billion," which we ruled out on the basis of EMV. Comparing Figures 4.23 and 4.19 indicates that this strategy yields a smaller chance of getting nothing, but also less chance of a $10.3 billion judgment. Compensating for this is the greater chance of getting something in the middle: $3 or $5 billion.

Although risk profiles can in principle be used as an alternative to EMV to check every possible strategy, for complex decisions it can be tedious to study many risk profiles. Thus, a compromise is to look at strategies only for the first one or two decisions, on the assumption that future decisions would be made using a decision rule such as maximizing expected value, which is itself a kind of strategy. (This is the approach used by many decision-analysis computer programs, PrecisionTree included.) Thus, in the Texaco-Pennzoil example, one might compare only the "Accept $2 Billion" and "Counteroffer $5 Billion; Refuse Texaco Counteroffer" strategies.

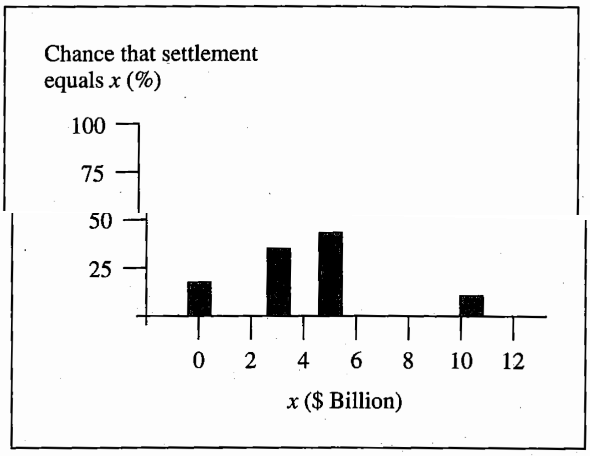


Figure 4.23
Risk profile for the "Counteroffer $5 Billion; Accept $3 Billion" strategy.

Cumulative Risk Profiles

We also can present the risk profile in cumulative form. Figure 4.24 shows the *cumulative risk profile* for "Counteroffer 5 Billion; Refuse Texaco Counteroffer." In this format, the vertical axis is the chance that the payoff is less than or equal to the corresponding value on the horizontal axis. This is only a matter of translating the information contained in the risk profile in Figure 4.19. There is no chance that the settlement will be less than zero. At zero, the chance jumps up to 24.9%, because there is a substantial chance that the court award will be zero. The graph continues at 24.9% across to $5 billion. (For example, there is a 24.9% chance that the settlement is less than or equal to $3.5 billion; that is, there is the 24.9% chance that the settlement is zero, and that is less than $3.5 billion.) At $5 billion, the line jumps up to 83.4% (which is 24.9% + 58.5%), because there is an 83.4% chance that the settlement is less than or equal to $5 billion. Finally, at $10.3 billion, the cumulative graph jumps up to 100%. The chance is 100% that the settlement is less than or equal to $10.3 billion.

Thus, you can see that creating a cumulative risk profile is just a matter of adding up, or accumulating, the chances of the individual payoffs. For any specific value

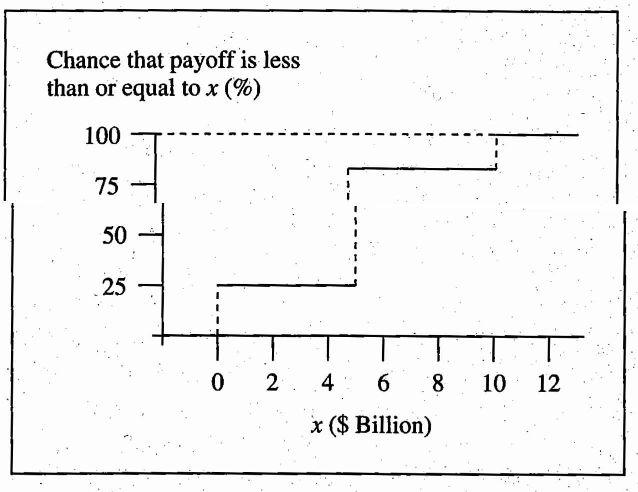


Figure 4.24
Cumulative risk profile for "Counteroffer $5 Billion; Refuse Texaco Counteroffer."

along the horizontal axis, we can read off the chance that the payoff will be less than or equal to that specific value. Later in this chapter, we show how to generate risk profiles and cumulative risk profiles in PrecisionTree. Cumulative risk profiles will be very helpful in the next section in our discussion of dominance.

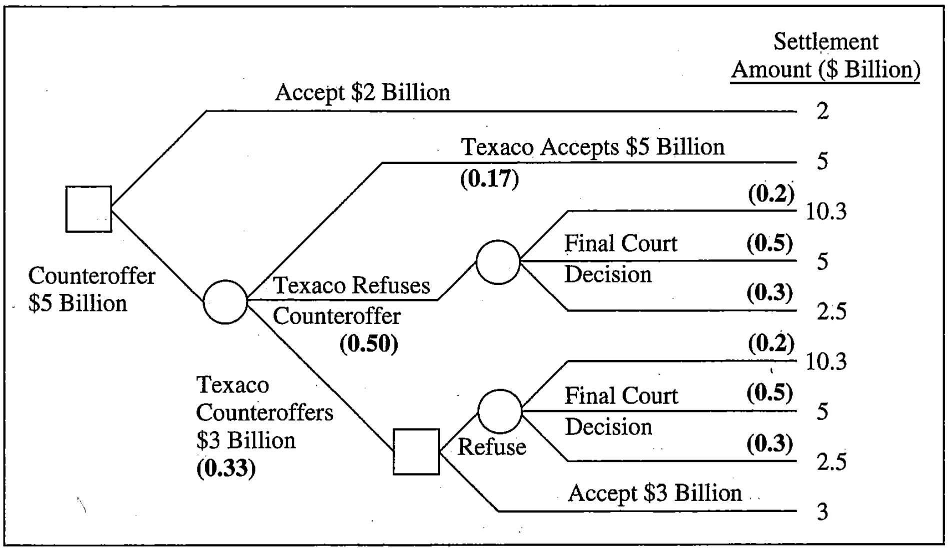
Dominance: An Alternative to EMV

Comparing expected values of different risky prospects is useful, but in many cases EMV inadequately captures the nature of the risks that must be compared. With risk profiles, however, we can make a more comprehensive comparison of the risks. But how can we choose one risk profile over another? Unfortunately, there is no clear answer that can be used in all situations. By using the idea of *dominance,* though, we can identify those profiles (and their associated strategies) that can be ignored. Such strategies are said to be *dominated,* because we can show logically, according to some rules relating to cumulative risk profiles, that there are better risks (strategies) available.

Suppose we modify Liedtke's decision as shown in Figure 4.2 so that \$2.5 billion is the minimum amount that he believes he could get in a court award. This decision is diagrammed in Figure 4.25. Now what should he do? It is rather obvious. Because he believes that he could do no worse than \$2.5 billion if he makes a counteroffer, he should clearly shun Texaco's offer of 2 billion. This kind of dominance is called *deterministic dominance,* signifying that the dominating alternative pays off at least as much as the one that is dominated.

We can show deterministic dominance in terms of the cumulative risk profiles displayed in Figure 4.26. The cumulative risk profile for "Accept \$2 Billion" goes from zero to 100% at \$2 billion, because the settlement for this alternative is bound to be \$2 billion. The risk profile for "Counteroffer \$5 Billion; Refuse Texaco

Figure 4.25
Hugh Liedtke's decision tree, assuming \$2.5 billion is minimum court award.



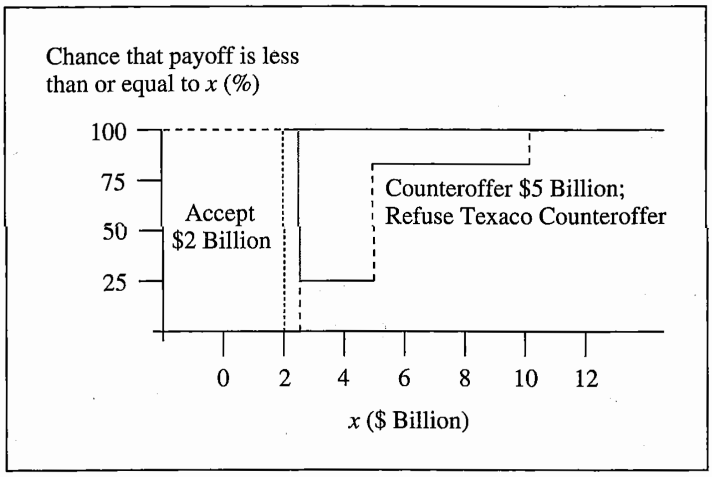


Figure 4.26
Cumulative risk profiles for alternatives in Figure 4.25.

Counteroffer" starts at $2.5 billion but does not reach 100% until $10.3 billion. Deterministic dominance can be detected in the risk profiles by comparing the value where one cumulative risk profile reaches 100% with the value where another risk profile begins. If there is a value x such that the chance of the payoff being less than or equal to x is 100% in alternative B, and the chance of the payoff being *less than* x is 0% in Alternative A, then A deterministically dominates B. Graphically, continue the vertical line where alternative A first leaves 0% (the vertical line at $2.5 billion for "Counteroffer $5 Billion"). If that vertical line corresponds to 100% for the other cumulative risk profile, then A dominates B. Thus, even if the minimum court award had been $2 billion instead of $2.5 billion, "Counteroffer $5 Billion" still would have dominated "Accept $2 Billion."

The following example shows a similar kind of dominance. Suppose that Liedtke is choosing between two different law firms to represent Pennzoil. He considers both law firms to be about the same in terms of their abilities to deal with the case, but one charges less in the event that the case goes to court. The full decision tree for this problem appears in Figure 4.27. Which choice is preferred? Again, it's rather obvious; the settlement amounts for choosing Firm A are the same as the corresponding amounts for choosing Firm B, except that Pennzoil gets more with Firm A if the case results in a damage award in the final court decision. Choosing Firm A is like choosing Firm B and possibly getting a bonus as well. Firm A is said to display *stochastic dominance* over Firm B. Many texts also use the term *probabilistic dominance* to indicate the same thing. (Strictly speaking, this is first-order stochastic dominance. Higher-order stochastic dominance comes into play when we consider preferences regarding risk.)

The cumulative risk profiles corresponding to Firms A and B (and assuming that Liedtke refuses a Texaco counteroffer) are displayed in Figure 4.28. The two cumulative risk profiles almost coincide; the only difference is that Firm A's profile is slightly to the right of Firm B's at $5 and $10 billion, which represents the possibility of Pennzoil having to pay less in fees. Stochastic dominance is represented in the cumulative risk profiles by the fact that the two profiles do not cross and that there is some space between them. That is, if two cumulative risk profiles are such that no part

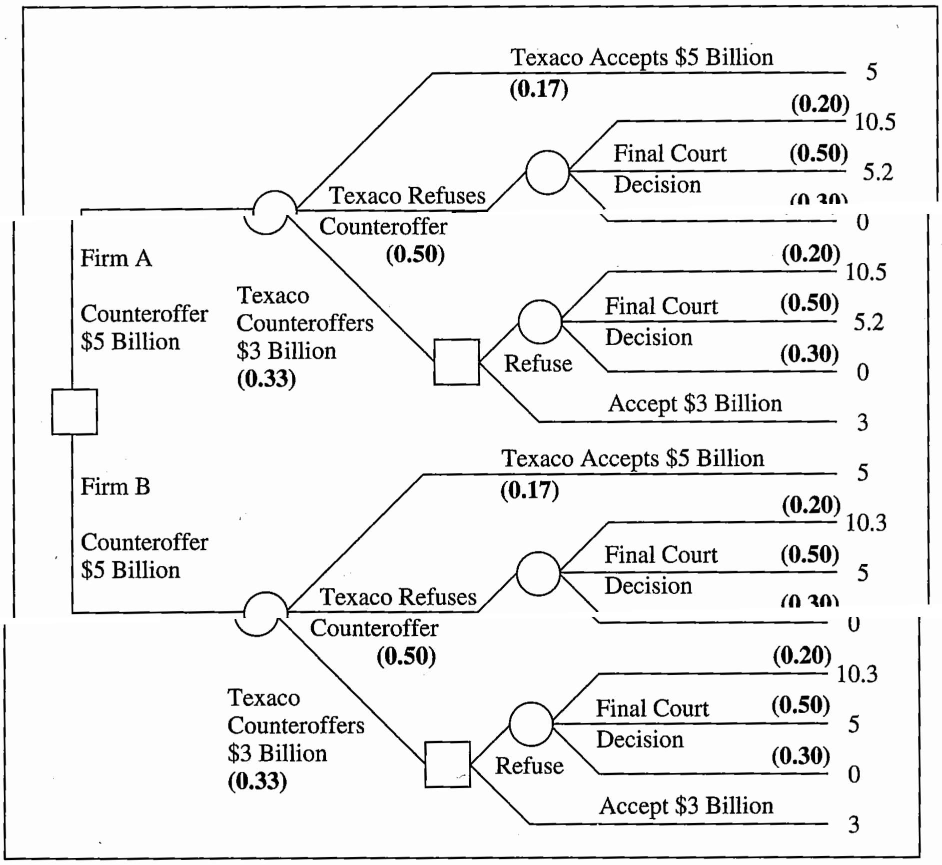


Figure 4.27
A decision tree comparing two law firms. Firm A charges less than Firm B if Pennzoil is awarded damages in court.

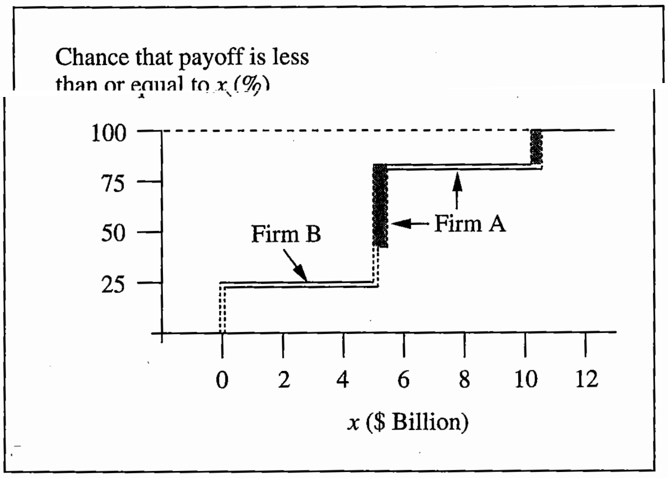


Figure 4.28
Cumulative risk profiles for two law firms in Figure 4.27. Firm A stochastically dominates Firm B.

of Profile A lies to the left of B, and at least some part of it lies to the right of B, then the strategy corresponding to Profile A stochastically dominates the strategy for Profile B.

The next example demonstrates stochastic dominance in a slightly different form. Instead of the consequences, the pattern of the probability numbers makes the preferred alternative apparent. Suppose Liedtke's choice is between two law firms that he considers to be of different abilities. The decision tree is shown in Figure 4.29. Carefully examine the probabilities in the branches for the final court decision. Which law firm is preferred? This is a somewhat more subtle situation than the preceding one. The essence of the problem is that for Firm C, the larger outcome values have higher probabilities. The settlement with Firm C is not bound to be at least as great or greater than that with Firm D, but with Firm C the settlement is more likely to be greater. Think of Firm C as being a better gamble if the situation comes down to a court decision. Situations like this are characterized by two alternatives that offer the same possible consequences, but the dominating alternative is more likely to bring a better consequence.

Figure 4.30 shows the cumulative risk profiles for the two law firms in this example. As in the last example, the two profiles nearly coincide, although space is found between the two profiles because of the different probabilities associated with the court award. Because Firm C either coincides with or lies to the right of Firm D, we can conclude that Firm C stochastically dominates Firm D.

Stochastic dominance can show up in a decision problem in several ways. One way is in terms of the consequences (as in Figure 4.27), and another is in terms of the probabilities (as in Figure 4.29). Sometimes stochastic dominance may emerge as a mixture of the two; both slightly better payoffs and slightly better probabilities may lead to one alternative dominating another.

Figure 4.29
Decision tree comparing two law firms. Firm C has a better chance of winning a damage award in court than does Firm D.

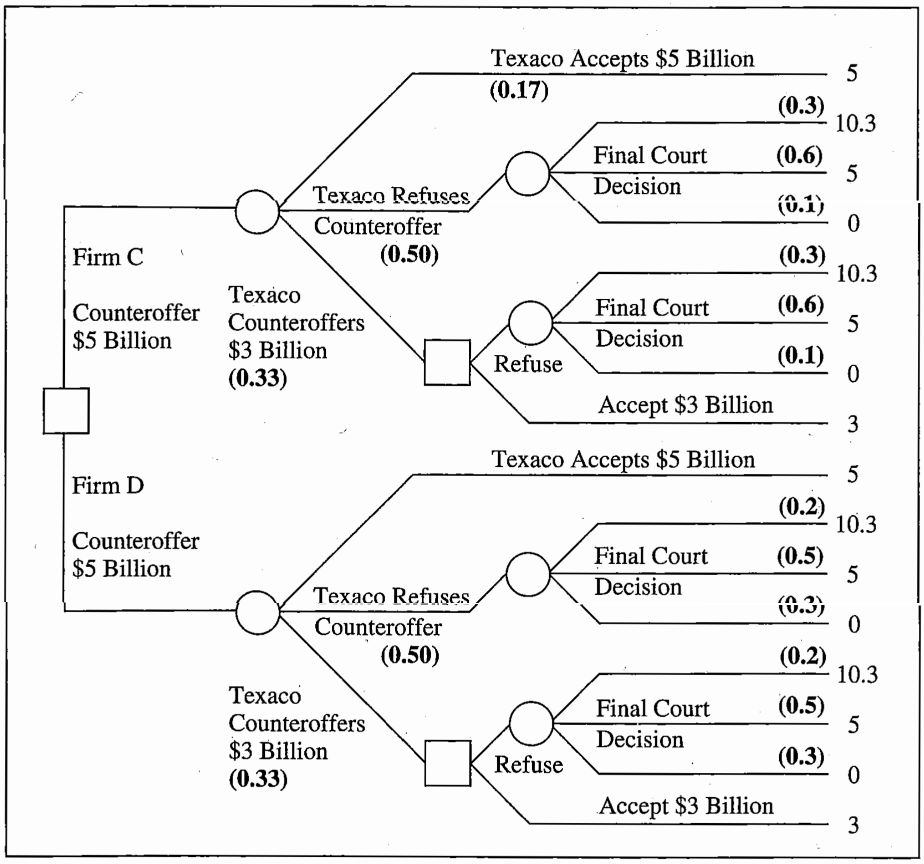
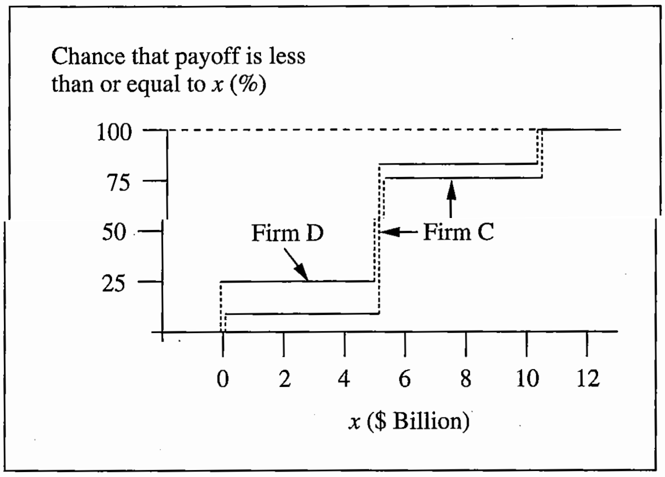


Figure 4.30
Cumulative risk profiles for law firms in Figure 4.29. Firm C stochastically dominates Firm D.



What is the relationship between stochastic dominance and expected value? It turns out that if one alternative dominates another, then the dominating alternative must have the higher expected value. This is a property of dominant alternatives that can be proven mathematically. To get a feeling for why it is true, think about the cumulative risk profiles, and imagine the EMV for a dominated Alternative B. If Alternative A dominates B, then its cumulative risk profile must lie at least partly to the right of the profile for B. Because of this, the EMV for A must also lie to the right of, and hence be greater than, the EMV for B.

Although this discussion of dominance has been fairly brief, one should not conclude that dominance is not important. Indeed, screening alternatives on the basis of dominance begins implicitly in the structuring phase of decision analysis, and, as alternatives are considered, they usually are at least informally compared to other alternatives. Screening alternatives formally on the basis of dominance is an important decision-analysis tool. If an alternative can be eliminated early in the selection process on that basis, considerable cost can be saved in large-scale problems. For example, suppose that the decision is where to build a new electric power plant. Analysis of proposed alternatives can be exceedingly expensive. If a potential site can be eliminated in an early phase of the analysis on the grounds that another dominates it, then that site need not undergo full analysis.

Making Decisions with Multiple Objectives

So far we have learned how to analyze a single-objective decision; in the Texaco-Pennzoil example, we have focused on Liedtke's objective of maximizing the settlement amount. How would we deal with a decision that involves multiple objectives? In this section, we learn how to extend the concepts of expected value and risk profiles to multiple-objective situations. In contrast to the grandiose Texaco-Pennzoil example, consider the following down-to-earth example of a young person deciding which of two summer jobs to take.

THE SUMMER JOB

Sam Chu was in a quandary. With two job offers in hand, the choice he should make was far from obvious. The first alternative was a job as an assistant at a local small business; the job would pay minimum wage ($5.25 per hour), it would require 25 to 35 hours per week, and the hours would be primarily during the week, leaving the weekends free. The job would last for three months, but the exact amount of work, and hence the amount Sam could earn, was uncertain. On the other hand, the free weekends could be spent with friends.

The second alternative was to work as a member of a trail-maintenance crew for a conservation organization. This job would require 10 weeks of hard work, 40 hours per week at $6.50 per hour, in a national forest in a neighboring state. The job would involve extensive camping and backpacking. Members of the maintenance crew would come from a large geographic area and spend the entire 10 weeks together, including weekends. Although Sam had no doubt about the earnings this job would provide, the real uncertainty was what the staff and other members of the crew would be like. Would new friendships develop? The nature of the crew and the leaders could make for 10 weeks of a wonderful time, 10 weeks of misery, or anything in between.

From the description, it appears that Sam has two objectives in this context: earning money and having fun this summer. Both are reasonable, and the two jobs clearly differ in these two dimensions; they offer different possibilities for the amount of money earned and the quality of summer fun.

The amount of money to be earned has a natural scale (dollars), and like most of us Sam prefers more money to less. The objective of having fun has no natural scale, though. Thus, a first step is to create such a scale. After considering the possibilities, Sam has created the scale in Table 4.5 to represent different levels of summer fun in the context of choosing a summer job. Although living in town and living in a forest camp pose two very different scenarios, the scale has been constructed in such a way that it can be applied to either job (as well as to any other prospect that might arise). The levels are numbered so that the higher numbers are more preferred.

Table 4.5
A constructed scale for summer fun.

5	(Best) A large, congenial group. Many new friendships made. Work is enjoyable, and time passes quickly.
4	A small but congenial group of friends. The work is interesting, and time off work is spent with a few friends in enjoyable pursuits.
3	No new friends are made. Leisure hours are spent with a few friends doing typical activities. Pay is viewed as fair for the work done.
2	Work is difficult. Coworkers complain about the low pay and poor conditions. On some weekends it is possible to spend time with a few friends, but other weekends are boring.
1	(Worst) Work is extremely difficult, and working conditions are poor. Time off work is generally boring because outside activities are limited or no friends are available.

With the constructed scale for summer fun, we can represent Sam's decision with the influence diagram and decision tree shown in Figures 4.31 and 4.32, respectively. The influence diagram shows the uncertainty about fun and amount of work, and that these have an impact on their corresponding consequences. The tree reflects Sam's belief that summer fun with the in-town job will amount to Level 3 in the constructed scale, but there is considerable uncertainty about how much fun the forest job will be. This uncertainty has been translated into probabilities based on Sam's uncertainty; how to make such judgments is the topic of Chapter 8. Likewise, the decision tree reflects uncertainty about the amount of work available at the in-town job.

Figure 4.31
Influence diagram for summer-job example.

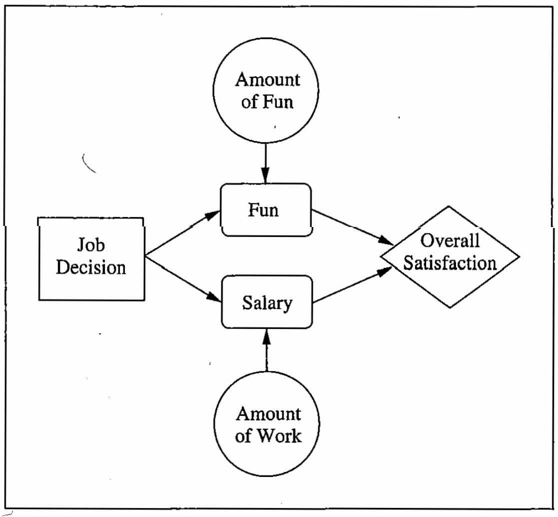
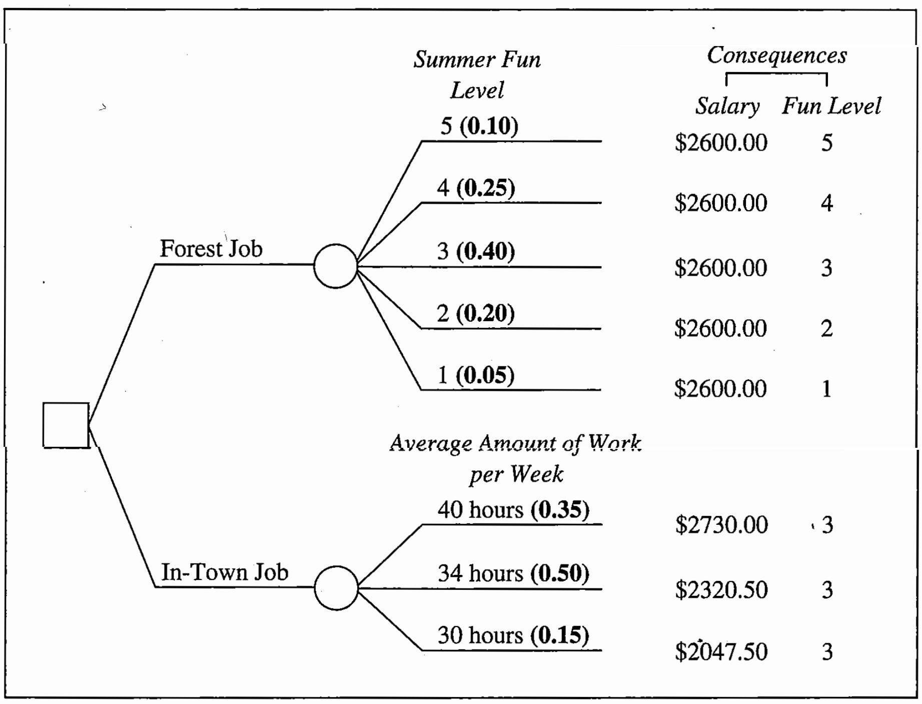


Figure 4.32
Decision tree for summer-job example.



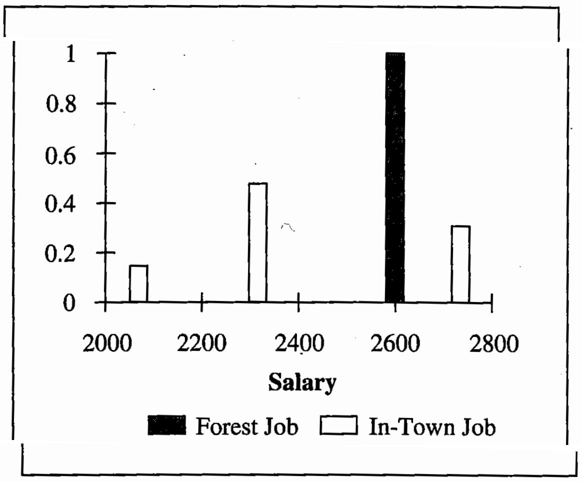
Analysis: One Objective at a Time

One way to approach the analysis of a multiple-objective decision is to calculate the expected value or create the risk profile for each individual objective. In the summer-job example, it is easy enough to do these things for salary. For the forest job, in which there is no uncertainty about salary, the expected value is \$2600, and the risk profile is a single bar at \$2600, as in Figure 4.33. For the in-town job, the expected salary is

$$\begin{aligned} E(\text{Salary}) &= 0.35(\$2730.00) + 0.50(\$2320.50) + 0.15(\$2047.50) \\ &= \$2422.88 \end{aligned}$$

The risk profile for salary at the in-town job is also shown in Figure 4.33.

Figure 4.33
Risk profiles for salary in the summer-job example.



Subjective Ratings for Constructed Attribute Scales

For the summer-fun constructed attribute scale, risk profiles can be created and compared (Figure 4.34), but expected-value calculations are not meaningful because no meaningful numerical measurements are attached to the specific levels in the scale. The levels are indeed ordered, but the ordering is limited in what it means. The labels do not mean, for example, that going from Level 2 to Level 3 would give Sam the same increase in satisfaction as going from Level 4 to Level 5. Thus, before we can do any meaningful analysis, Sam must *rate* the different levels in the scale, indicating how much each level is worth (to Sam) relative to the other levels. This is a subjective judgment on Sam's part. Different people with different preferences would be expected to give different ratings for the possible levels of summer fun.

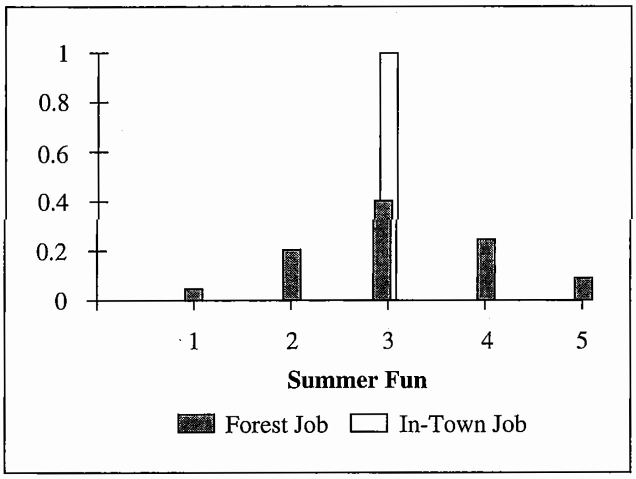


Figure 4.34
Risk profiles for summer fun in the summer-job example.

To make the necessary ratings, we begin by setting the endpoints of the scale. Let the best possible level (Level 5 in the summer-job example) have a value of 100 and the worst possible level (Level 1) a value of 0. Now all Sam must do is indicate how the intermediate levels rate on this scale from 0 to 100 points. For example, Level 4 might be worth 90 points, Level 3, 60 points, and Level 2, 25 points. Sam's assessments indicate that going from Level 3 to Level 4, with an increase of 30 points, is three times as good as going from Level 4 to Level 5 with an increase of only 10 points. Note that there is no inherent reason for the values of the levels to be evenly spaced; in fact, it might be surprising to find perfectly even spacing.

This same procedure can be used to create meaningful measurements for any constructed scale. The best level is assigned 100 points, the worst 0 points, and the decision maker must then assign rating points between 0 and 100 to the intermediate levels. A scale like this assigns more points to the preferred consequences, and the rating points for intermediate levels should reflect the decision maker's relative preferences for those levels.

With Sam's assessments, we can now calculate and compare the expected values for the amount of fun in the two jobs. For the in-town job, this is trivial because there is no uncertainty; the expected value is 60 points. For the forest job, the expected value is

$$\begin{aligned}\text{E(Fun Points)} &= 0.10(100) + 0.25(90) + 0.40(60) + 0.20(25) + 0.05(0) \\ &= 61.5\end{aligned}$$

With individual expected values and risk profiles, alternatives can be compared. In doing so, we can hope for a clear winner, an alternative that dominates all other alternatives on all attributes. Unfortunately, comparing the forest and in-town jobs does not produce a clear winner. The forest job appears to be better on salary, having no risk and a higher expected value. Considering summer fun, the news is mixed. The in-town job has less risk but a lower expected value. It is obvious that going from one job to the other involves trading risks. Would Sam prefer a slightly higher salary for sure and take a risk on how much fun the summer will be? Or would the in-town job be better, playing it safe with the amount of fun and taking a risk on how much money will be earned?

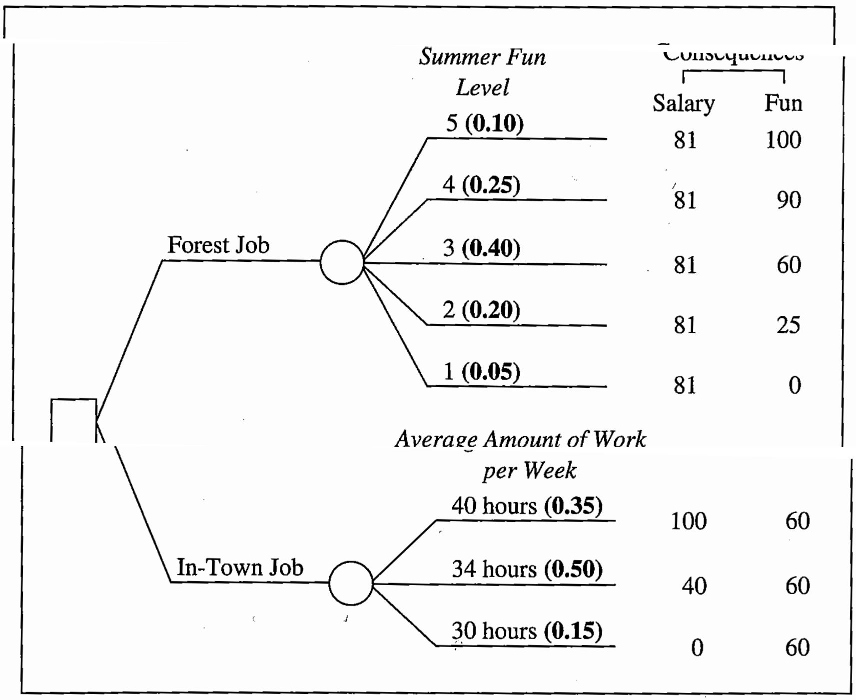
Assessing Trade-Off Weights

The summer-job decision requires Sam to make an explicit trade-off between the objectives of maximizing fun and maximizing salary. How can Sam make this trade-off? Although this seems like a formidable task, a simple thought experiment is possible that will help Sam to understand the relative value of salary and fun.

In order to make the comparison between salary and fun, it is helpful to measure these two on similar scales, and the most convenient arrangement is to put salary on the same 0 to 100 scale that we used for summer fun. As before, the best ($2730) and worst ($2047.50) take values of 100 and 0, respectively. To get the values for the intermediate salaries ($2320.50 and $2600), a simple approach is to calculate them proportionately. Thus, we find that $2320.50 is 40% of the way from $2047.50 to $2730, and so it gets a value of 40 on the converted scale. (That is, [$2320.50 – $2047.50]/[$2730 – $2047.50] = 0.40) Likewise, $2600 is 81% of the way from $2047.50 to $2730, and so it gets a value of 81. (In Chapter 15, we will call this approach *proportional scoring*.) With the ratings for salary and summer fun, we now can create a new consequence matrix, giving the decision tree in Figure 4.35.

Now the trade-off question can be addressed in a straightforward way. The question is how Sam would trade points on the salary scale for points on the fun scale. To do this we introduce the idea of *weights*. What we want to do is assign weights to salary and fun to reflect their relative importance to Sam. Call the weights k_s and k_f, where the subscripts s and f stand for salary and fun, respectively. We will use the weights to calculate a weighted average of the two ratings for any given consequence in order to get an overall score. For example, suppose that $k_s = 0.70$ and

Figure 4.35
Decision tree with ratings for consequences.



$k_f = 0.30$, reflecting a judgment that salary is a little more than twice as important as fun. The overall score (U) for the forest job with fun at Level 3 would be

$$\begin{aligned} U(\text{Salary: } 81,\ \text{Fun: } 60) &= 0.70(81) + 0.30(60) \\ &= 74.7 \end{aligned}$$

It is up to Sam to make an appropriate judgment about the relative importance of the two attributes. Although details on making this judgment are in Chapter 15, one important issue in making this judgment bears discussion here. Sam *must* take into consideration the ranges of the two attributes. Strictly speaking, the two weights should reflect the relative value of going from best to worst on each scale. That is, if Sam thinks that improving salary from $2047.50 to $2730 is three times as important as improving fun from Level 1 to Level 5, this judgment would imply weights $k_s = 0.75$ and $k_f = 0.25$.

Paying attention to the ranges of the attributes in assigning weights is crucial. Too often we are tempted to assign weights on the basis of vague claims that Attribute A (or its underlying objective) is worth three times as much as Attribute B. Suppose you are buying a car, though. If you are looking at cars that all cost about the same amount but their features differ widely, why should price play a role in your decision? It should have a low weight in the overall score. In the Texaco-Pennzoil case, we argued that we could legitimately consider only the objective of maximizing the settlement amount because its range was so wide; any other objectives would be overwhelmed by the importance of moving from worst to best on this one. In an overall score, the weight for settlement amount would be near 1, and the weight for any other attribute would be near zero.

Suppose that, after carefully considering the possible salary and summer-fun outcomes, Sam has come up with weights of 0.6 for salary and 0.4 for fun, reflecting a judgment that the range of possible salaries is 1.5 times as important as the range of possible summer-fun ratings. With these weights, we can collapse the consequence matrix in Figure 4.35 to get Figure 4.36. For example, if Sam chooses the forest job and the level of fun turns out to be Level 4, the overall score is $0.6(81) + 0.4(90) = 84.6$. The other endpoint values in Figure 4.36 can be found in the same way.

In these last two sections we have discussed some straightforward ways to make subjective ratings and trade-off assessments. These topics are treated more completely in Chapters 13, 15, and 16. For now you can rest assured that the techniques described here are fully compatible with those described in later chapters.

Analysis: Expected Values and Risk Profiles for Two Objectives

The decision tree in Figure 4.36 is now ready for analysis. The first thing we can do is fold back the tree to calculate expected values. Using the overall scores from Figure 4.36, the expected values are:

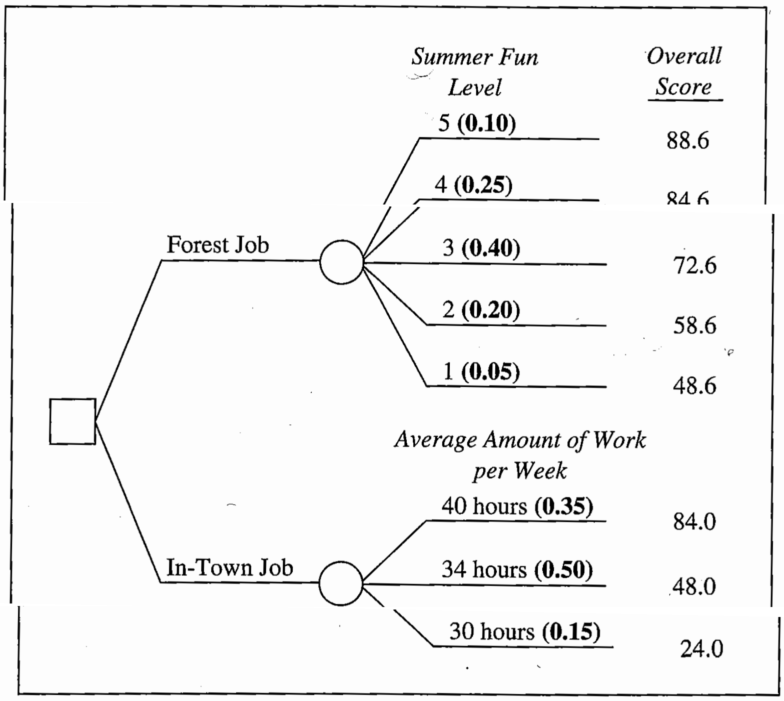


Figure 4.36
Decision tree with overall scores for summer-job example. Weights used are $k_s = 0.60$ and $k_f = 0.40$. For example, consider the forest job that has an outcome of Level 4 on the fun scale. The rating for salary is 81, and the rating for fun is 90. Thus, the overall score is 0.60(81) + 0.40(90) = 84.6.

$$
\begin{aligned}
\text{E(Score for Forest Job)} &= 0.10(88.6) + 0.25(84.6) \\
&\quad + 0.40(72.6) + 0.20(58.6) + 0.05(48.6) \\
&= 73.2 \\
\text{E(Score for In-Town Job)} &= 0.35(84) + 0.50(48) + 0.15(24) \\
&= 57
\end{aligned}
$$

Can we also create risk profiles for the two alternatives? We can; the risk profiles would represent the uncertainty associated with the overall weighted score Sam will get from either job. To the extent that this weighted score is meaningful to Sam as a measure of overall satisfaction, the risk profiles will represent the uncertainty associated with Sam's overall satisfaction. Figures 4.37 and 4.38 show the risk profiles and cumulative risk profiles for the two jobs. Figure 4.38 shows that, given the ratings and the trade-off between fun and salary, the forest job stochastically dominates the in-town job in terms of the overall score. Thus, the decision may be clear for Sam at this point; given Sam's assessed probabilities, ratings, and the trade-off, the forest job is a better risk. (Before making the commitment, though, Sam may want to do some *sensitivity analysis,* the topic of Chapter 5; small changes in some of those subjective judgments might result in a less clear choice between the two.)

Two final caveats are in order regarding the risk profiles of the overall score. First, it is important to understand that the overall score is something of an artificial outcome; it is an amalgamation in this case of two rating scales. As indicated above, Figures 4.37 and 4.38 only make sense to the extent that Sam is willing to interpret them as representing the uncertainty in the overall satisfaction from the two jobs.

Second, the stochastic dominance displayed by the forest job in Figure 4.38 is a relatively weak result; it relies heavily on Sam's assessed trade-off between the two attributes. A stronger result—one in which Sam could have confidence that the forest job is preferred regardless of his trade-off—requires that the forest job stochastically dominate the in-town job on each individual attribute. (Technically, however, even individual stochastic dominance is not quite enough: the risk profiles for the attributes must be combined into a single two-dimensional risk profile, or *bivariate probability distribution,* for each attribute. Then these two-dimensional risk profiles must be compared in much the same way we did with the single-attribute risk profiles. The good news is that as long as amount of work and amount of fun are *independent* (no arrow between these two chance nodes in the influence diagram in Figure 4.31), then finding that the same job stochastically dominates the other on each attribute guarantees that the same relationship holds in terms of the technically correct two-dimensional risk profile. Independence and stochastic dominance for multiple attributes will be discussed in Chapter 7.)

Figure 4.37
Risk profiles for summer jobs.

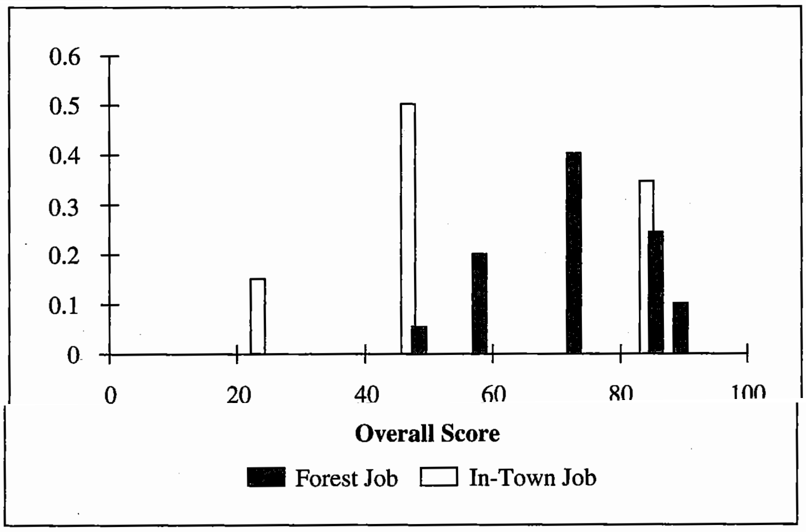
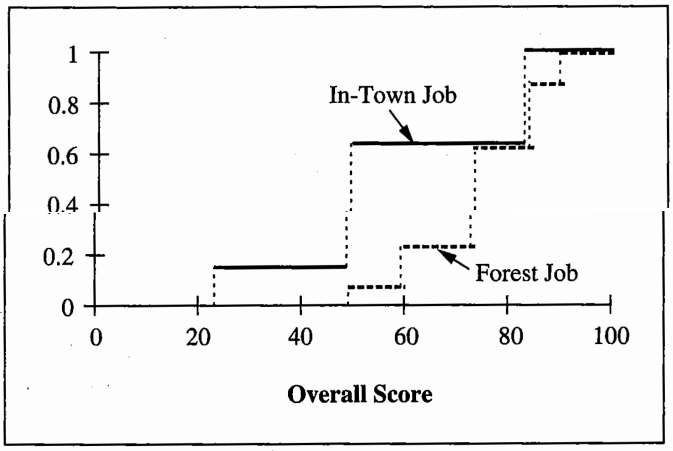


Figure 4.38
Cumulative risk profiles for summer jobs. The forest job stochastically dominates the in-town job.



Decision Analysis Using PrecisionTree

In this section, we will first learn how to analyze decision trees and influence diagrams, then we will learn how to model multiple-objective decisions in a spreadsheet. Although this chapter concludes our discussion of PrecisionTree's basic capabilities, we will revisit PrecisionTree in later chapters to introduce other features, such as sensitivity analysis (Chapter 5), simulation (Chapter 11), and utility curves (Chapter 14).

A major advantage of using a program like PrecisionTree is the ease with which it performs an analysis. With the click of a button, any tree or influence diagram can be analyzed, various calculations performed, and risk profiles generated. Because it is so easy to run an analysis, however, there can be a temptation to build a quick model, analyze it, and move on. As you learn the steps for running an analysis using PrecisionTree, keep in mind that the insights you are working toward come from careful modeling, then analysis, and perhaps iterating through the model-and-analysis cycle several times. We encourage you to "play" with your model, searching for different insights using a variety of approaches. Take advantage of the time and effort you've saved with the automated analysis by investing it in building a requisite model.

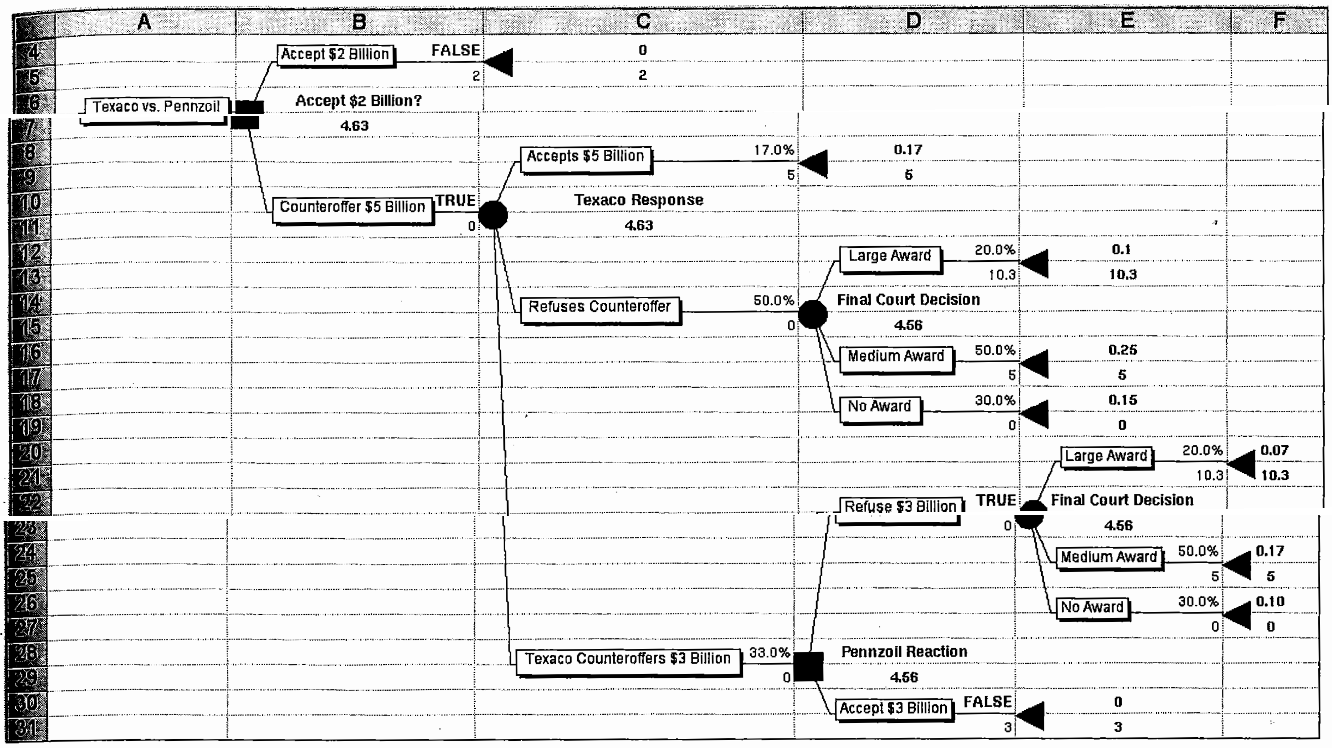
Decision Trees

Figure 4.39 shows the Texaco-Pennzoil decision tree. You can see the structure we developed early in the chapter: The first decision is whether to accept the $2 billion offer or to make a $5 billion counteroffer. Then there is a chance node indicating Kinnear's response, and so on through the tree. PrecisionTree automatically calculates expected values through the tree. The expected value for each chance node appears in the cell below the node's name. Likewise, at each decision node, the largest branch value (for the preferred alternative) can be seen in the cell below the decision node's name. The word "TRUE" identifies the preferred alternative for a decision, with all other alternatives labeled "FALSE" for that node. (You can define other decision criteria besides maximizing the expected value. See the on-line help manual, Chapter 5: Settings Command, for instructions.)

Before analyzing the Texaco-Pennzoil decision, we have to create the decision tree. You have two choices at this point. You may wish to build the tree from scratch, using the skills you learned in the last chapter. If so, be sure that your tree looks just like the one in Figure 4.39 before proceeding.

Alternatively, you may open the existing spreadsheet on the Palisade CD located at Examples\Chapter 4\TexPenDT.xls. If you use this spreadsheet, you will see that it is not complete; we want you to practice your tree-construction skills at least a little! In particular, you should notice two things about the tree at first glance. First, you will have to modify the values and probabilities from the default values supplied by the software. Make sure your expected values match those in Figure 4.39 before proceeding. Second, you will notice that the chance node "Final Court Decision" is

Figure 4.39
Texaco-Pennzoil decision tree.



missing following the "Refuse $3 Billion" branch. There are three possibilities for completing this part of the tree:

ALTERNATIVE 1: You can create the chance node using the techniques you learned in Chapter 3.

ALTERNATIVE 2: You can copy and paste the chance node by following these steps:

A2.1 Click on the **Final Court Decision** node (on the circle itself when the cursor changes to a hand).

A2.2 In the *Node Settings* dialog box click the **Copy** button.

A2.3 Now click on the end node (blue triangle) of the *Refuse $3 Billion* branch, and in the *Node Settings* dialog box click **Paste**.

PrecisionTree will re-create the "Final Court Decision" at the end of the branch.

ALTERNATIVE 3: The third possibility is to use the existing "Final Court Decision" node as a "reference node." In this case, you are instructing PrecisionTree to refer back to the "Final Court Decision" chance node (and all of its subsequent structure) at the end of the "Refuses Counteroffer" branch. To do this:

A3.1 Click on the end node of the *Refuse $3 Billion* branch and select the **Reference Node** option (gray diamond, fourth from the left).

A3.2 In the name box, type **Final Court Decision.**

A3.3 Click in the entry box that is situated to the right of the option button labeled *node of this tree.* Move the cursor outside the dialog box to the spreadsheet and click in the cell below the name *Final Court Decision* (Figure 4.39, **cell D15**). (Be sure to point to cell D15, which contains the value of the chance node, not cell D14, which contains the name of the node.)

A3.4 Click **OK.**

The dotted line that runs between the reference node (gray diamond at the end of the "Refuse $3 Billion" branch) and the "Final Court Decision" chance node indicates that the tree will be analyzed as if there was an identical "Final Court Decision" chance node at the position of the reference node. Reference nodes are useful as a way to graphically prune a tree without leaving out any of the mathematical details.

With the decision tree structured and the appropriate numbers entered, it takes only two clicks of the mouse to run an analysis.

STEP 1

1.1 Click on the **Decision Analysis** button (fourth button from the left on the PrecisionTree toolbar).

1.2 In the *Decision Analysis* dialog box that appears (Figure 4.40), choose the **Analyze All Choices** option located under the *Initial Decision* heading in the lower right-hand corner.

1.3 Click **OK.**

At this point, PrecisionTree creates a new workbook with several worksheets. The *Statistics Report* contains seven statistics on each of the two alternatives: "Accept $2 Billion" and "Counteroffer $5 Billion." (If we had not chosen *Analyze All Options,* only the optimal choice, "Counteroffer $5 Billion," would be reported.) The *Policy Suggestion Report* (Figure 4.41) clarifies the optimal strategy by trimming away all suboptimal choices at the decision nodes. A look at Figure 4.41 reveals that Liedtke should refuse the $2 billion offer and counter with an offer of $5 billion, and if Kinnear counters by offering $3 billion, then Liedtke should again refuse and go to court.

The remaining three worksheets all convey the same information, each in a different type of graph. For example, Figure 4.42 illustrates the *Cumulative Profile* (which we have called the "cumulative risk profile") for the two alternatives. (Again, choosing *Analyze All Options* forces PrecisionTree to include profiles for all of the

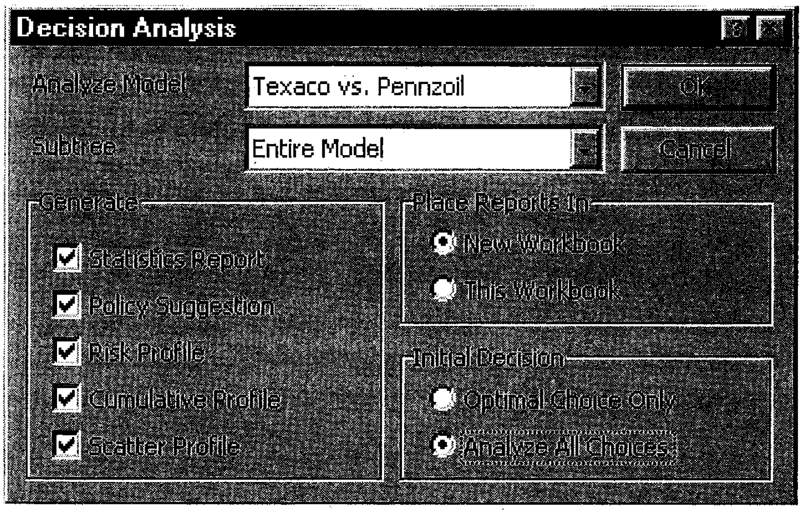


Figure 4.40 *Decision Analysis dialog box.*

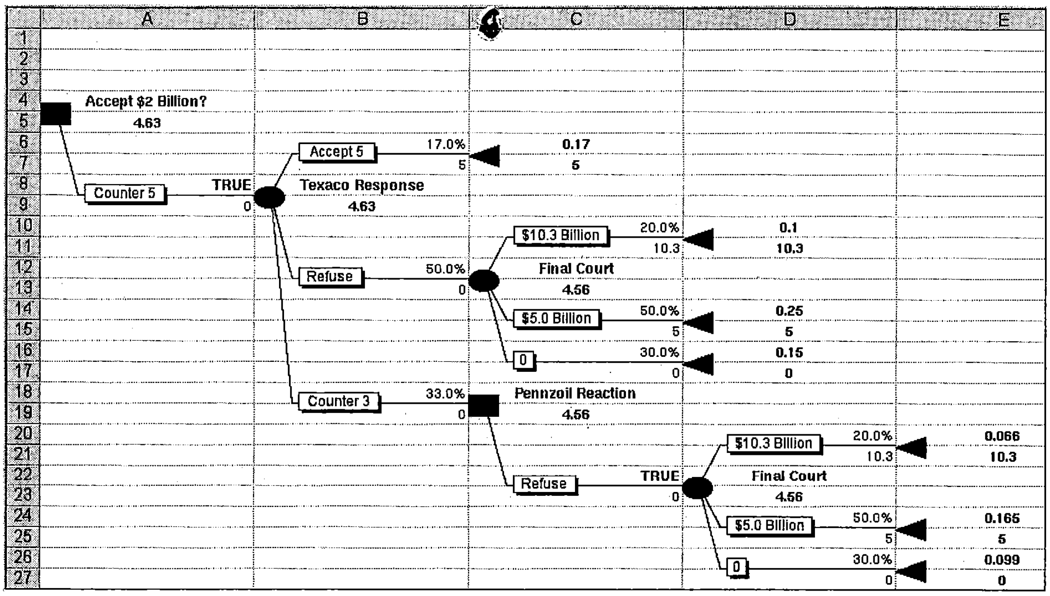


Figure 4.41 *Policy Suggestion for Texaco-Pennzoil problem.*

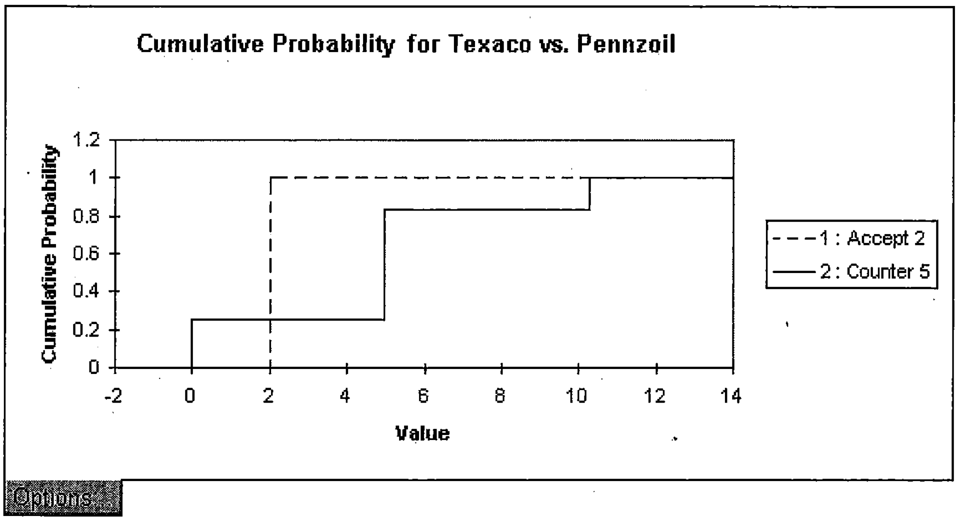


Figure 4.42 *Cumulative risk profiles for the "Accept $2 Billion" and the "Counteroffer $5 Billion" alternatives.*

initial options.) At the bottom left of the graph is an *Option* button that lets you modify the graph's characteristics, such as the minimum and maximum values on the two axes.

The *Decision Analysis* dialog box (Figure 4.40) offers several choices regarding which model to analyze or whether to analyze all or a portion of the tree. Because it is possible to analyze any currently open tree or influence diagram, ... the right model, especially if you have more than one on the same spreadsheet. Specify the tree or portion thereof that you wish to analyze using the pull-down list to the right of *Analyze Model.*

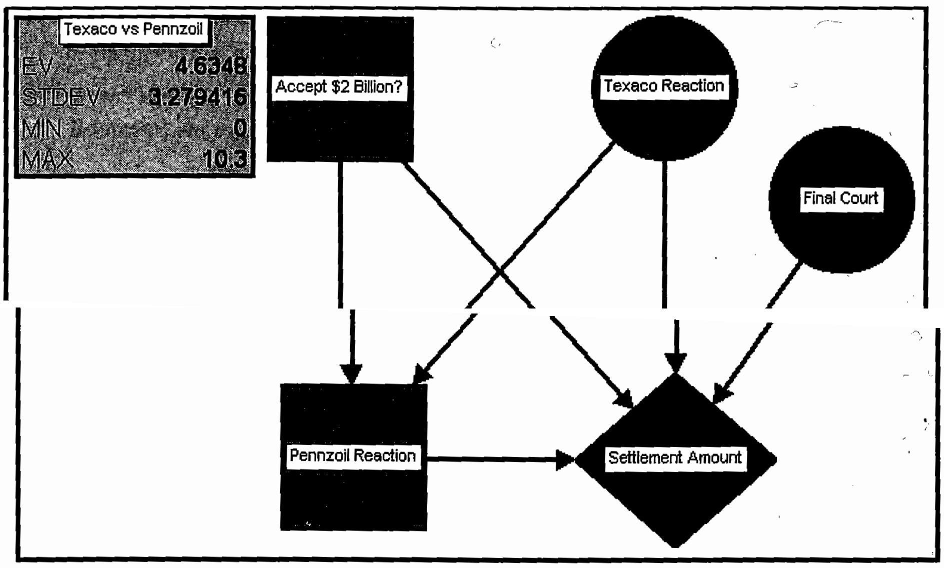
Influence Diagrams

PrecisionTree analyzes influence diagrams as readily as decision trees, which we demonstrate next with the Texaco-Pennzoil influence diagram. Because the numerical information of an influence diagram is contained in hidden tables, it is important that you carefully check that your diagram accurately models the decision before performing an analysis. One simple check is to verify that the values and probabilities are correctly entered. We will also see how PrecisionTree converts an influence diagram into a decision tree for a more thorough check. After you are satisfied that your influence diagram accurately models the decision, running an analysis is as simple as clicking two buttons.

We need to create the Texaco versus Pennzoil influence diagram, as shown in Figure 4.43, before analyzing it. One option would be to construct the influence diagram from scratch using the skills you learned in the last chapter. If so, be sure that the summary statistics box displays the correct expected value ($4.63 billion), standard deviation ($3.28 billion), minimum ($0 billion), and maximum ($10 billion).

Alternatively, you may open the existing spreadsheet on the Palisade CD located at Examples\Chapter 4\TexPenID.xls. Again, to encourage you to practice your

Figure 4.43
Influence diagram for Texaco versus Pennzoil.



influence-diagram construction skills, this spreadsheet contains a partially completed model. Step 2 below describes how to complete this model by adding the values and probabilities, Step 3 describes how to convert the influence diagram into a decision tree, and Step 4 describes how to analyze the influence diagram.

STEP 2 As a general rule, numbers are added to a node of an influence diagram by clicking on the name of the node, clicking the *Values* button in the *Influence Node Settings* dialog box, and entering the appropriate numbers in the *Influence Value Editor* dialog box. Remember to hit the *Return* button after each value is specified. (Refer to Chapter 3 for a more detailed review if necessary.)

2.1 Enter the numerical values and probabilities for the nodes "Accept $2 Billion?," "Texaco Response," and "Final Court Decision" using Figure 4.39 as a guide. For example, for the values of the "Accept $2 Billion?" node, enter 2 if accepted and enter 0 if $5 billion is counteroffered. Similarly, enter 5 if Texaco's response is to accept the $5 billion counteroffer and 0 if they refuse or counteroffer $3 billion. Enter 0 for *value if skipped* for these three nodes.

Adding values to the other two nodes ("Pennzoil Reaction" and "Settlement Amount") is slightly more complex:

2.2 Click on the decision node name **Pennzoil Reaction** and click on the **Values** button. A spreadsheet pops up titled *Influence Value Editor,* in which we use the entries on the right to specify the values in the *Value* column (Figure 4.44). Essentially, the *Influence Value Editor* is a consequence table similar to Table 4.1 except the columns are read from right to left.

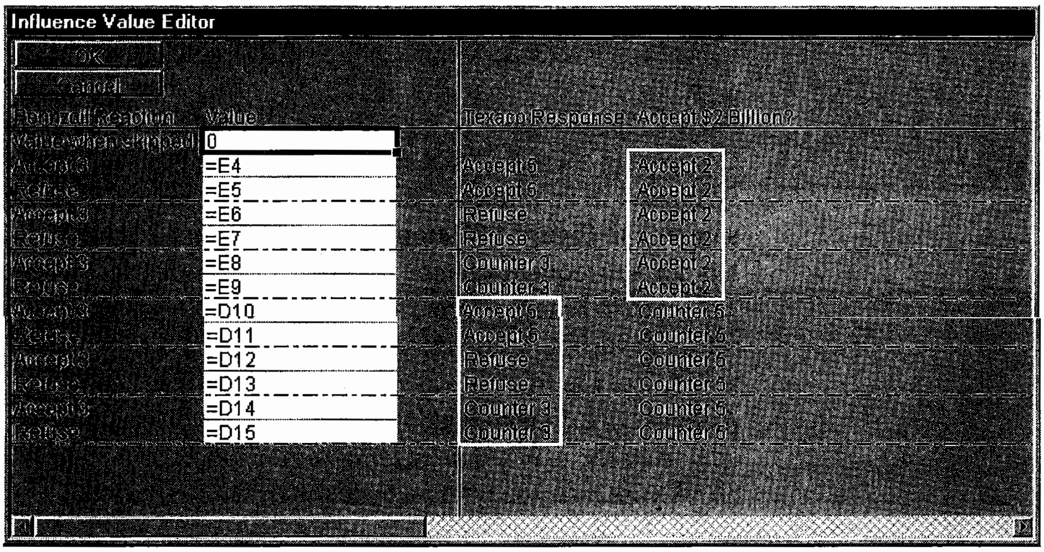


Figure 4.44
Influence Value Editor for "Pennzoil Reaction" decision node.

2.3 Start by typing **0** in the first row (*Value when skipped*) and hit **Enter.**

2.4 Type the equals sign (=) in the second row. For this row, the alternative we are entering is found by reading the rightmost entry—accept the $2 billion offer. Hence its value is 2, which is accessed by clicking on the first **Accept 2** in the column below the *Accept $2 Billion* heading. Row 2 should now read: =E4. Hit **Enter.**

2.5 The next five entries also involve accepting the $2 billion, so repeat Step 2.4 five times, and for each row click on the corresponding **Accept 2** in the rightmost column. Alternatively, because this is an Excel spreadsheet, you can use the fill-down command. As a guide in completing this node, we have outlined the outcomes that you are to reference with boxes in Figure 4.44.

2.6 Continuing onto rows 7 –12, the rightmost alternative is to counteroffer $5 billion. Since Pennzoil has counteroffered, the values are now determined by Texaco's response. Specify the appropriate value by clicking on the corresponding row under the *Texaco Response* heading.

2.7 Click **OK** when your *Influence Value Editor* matches Figure 4.44.

2.8 Following the same procedure, open the *Influence Value Editor* for **Settlement Amount** and enter the appropriate values as you read from left to right. Figure 4.45 highlights the cells to click on.

When finished entering the values, the summary statistics box should display the correct expected value ($4.63 billion), standard deviation ($3.28 billion), minimum ($0 billion), and maximum ($10 billion).

We can also convert the influence diagram into a decision tree.

STEP 3

3.1 Click on the name of the influence diagram—**Texaco versus Pennzoil.**

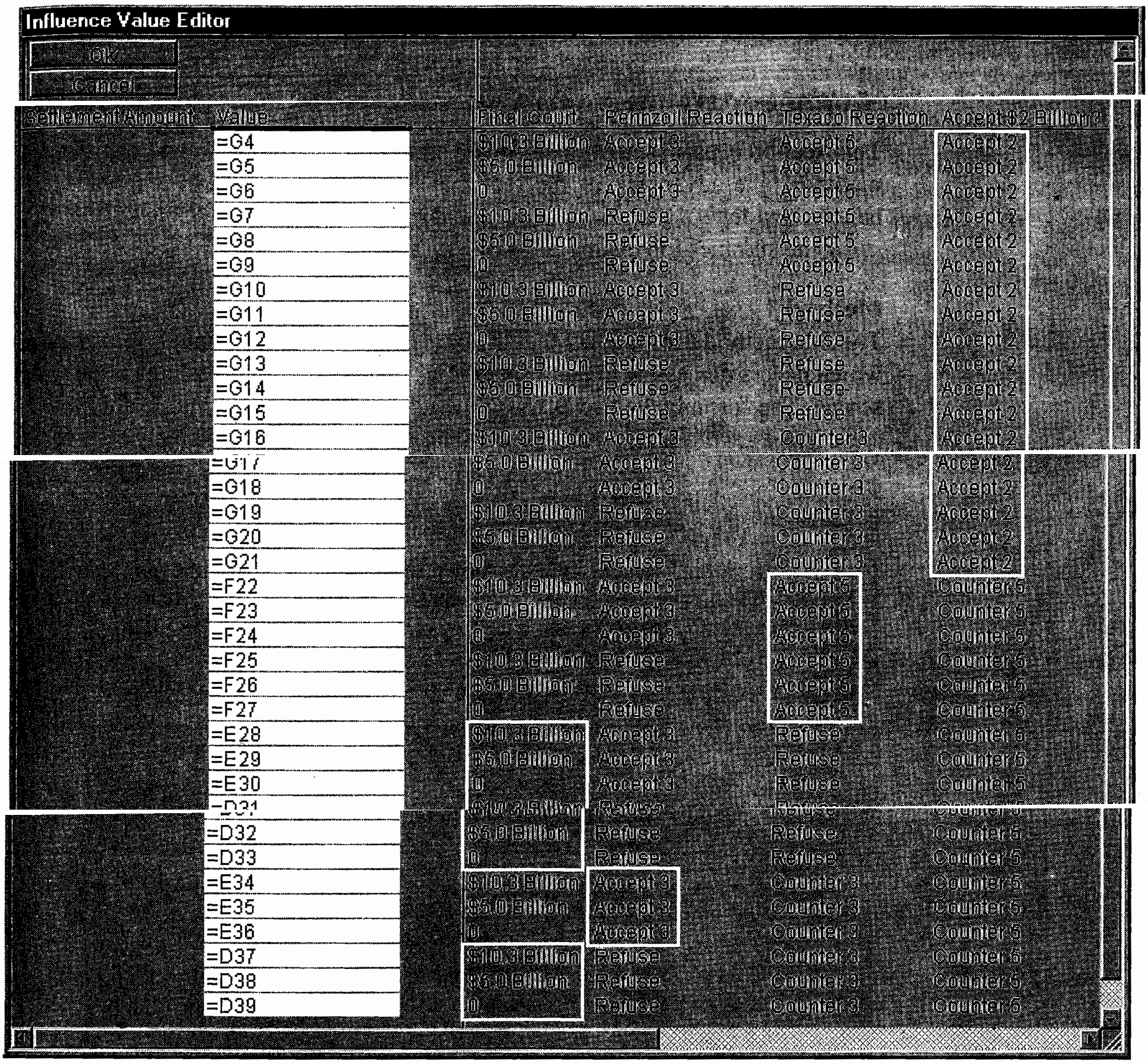
3.2 In the *Influence Diagram Settings* dialog box, click on **Convert to Tree.**

A new spreadsheet is added to your workbook containing the converted tree. Comparing the converted tree to Figure 4.39 clearly shows the effect of assuming symmetry in an influence diagram, as discussed on page 120. For example, the decision tree in Figure 4.39 stops after the "Accept 2" branch, whereas the converted tree has all subsequent chance and decision nodes following the "Accept 2" branch. These extra branches are due to the symmetry assumption and explain why the payoff table for the influence diagram required so many entries. PrecisionTree has the ability to incorporate asymmetry into influence diagrams via structure arcs. Although we do not cover this feature of PrecisionTree, you can learn about structure arcs from the user's manual and on-line help.

We are now ready to analyze the influence diagram.

Figure 4.45

Settlement Amount Values dialog box for Texaco-Pennzoil model.



Influence Value Editor

OK

Cancel

Settlement Amount	Value	Final Court	Pennzoil Reaction	Texaco Reaction	Accept $2 Billion?
	=G4	$10.3 Billion	Accept 3	Accept 5	Accept 2
	=G5	$5.0 Billion	Accept 3	Accept 5	Accept 2
	=G6	0	Accept 3	Accept 5	Accept 2
	=G7	$10.3 Billion	Refuse	Accept 5	Accept 2
	=G8	$5.0 Billion	Refuse	Accept 5	Accept 2
	=G9	0	Refuse	Accept 5	Accept 2
	=G10	$10.3 Billion	Accept 3	Refuse	Accept 2
	=G11	$5.0 Billion	Accept 3	Refuse	Accept 2
	=G12	0	Accept 3	Refuse	Accept 2
	=G13	$10.3 Billion	Refuse	Refuse	Accept 2
	=G14	$5.0 Billion	Refuse	Refuse	Accept 2
	=G15	0	Refuse	Refuse	Accept 2
	=G16	$10.3 Billion	Accept 3	Counter 3	Accept 2
	=G17	$5.0 Billion	Accept 3	Counter 3	Accept 2
	=G18	0	Accept 3	Counter 3	Accept 2
	=G19	$10.3 Billion	Refuse	Counter 3	Accept 2
	=G20	$5.0 Billion	Refuse	Counter 3	Accept 2
	=G21	0	Refuse	Counter 3	Accept 2
	=F22	$10.3 Billion	Accept 3	Accept 5	Counter 5
	=F23	$5.0 Billion	Accept 3	Accept 5	Counter 5
	=F24	0	Accept 3	Accept 5	Counter 5
	=F25	$10.3 Billion	Refuse	Accept 5	Counter 5
	=F26	$5.0 Billion	Refuse	Accept 5	Counter 5
	=F27	0	Refuse	Accept 5	Counter 5
	=E28	$10.3 Billion	Accept 3	Refuse	Counter 5
	=E29	$5.0 Billion	Accept 3	Refuse	Counter 5
	=E30	0	Accept 3	Refuse	Counter 5
	=D31	$10.3 Billion	Refuse	Refuse	Counter 5
	=D32	$5.0 Billion	Refuse	Refuse	Counter 5
	=D33	0	Refuse	Refuse	Counter 5
	=E34	$10.3 Billion	Accept 3	Counter 3	Counter 5
	=E35	$5.0 Billion	Accept 3	Counter 3	Counter 5
	=E36	0	Accept 3	Counter 3	Counter 5
	=D37	$10.3 Billion	Refuse	Counter 3	Counter 5
	=D38	$5.0 Billion	Refuse	Counter 3	Counter 5
	=D39	0	Refuse	Counter 3	Counter 5

STEP 4 The procedure for analyzing an influence diagram is the same as analyzing a decision tree.

4.1 Click on the **Decision Analysis** button (fourth button from the left on the PrecisionTree toolbar) and click **OK.**

The *Analyze All Choices* option that we instructed you to choose for decision trees is not available for influence diagrams. Thus, PrecisionTree's output for influence diagrams reports only on the optimal alternative via one set of statistics and one risk profile. The output for influence diagrams is interpreted the same as for decision trees.

Multiple-Attribute Models

This section presents two methods for modeling multiple-attribute decisions. Both methods take advantage of the fact that PrecisionTree runs in a spreadsheet and hence provides easy access to side calculations. We will explore these two methods using the summer-job example for illustration.

Method 1

The first method uses the fact that the branch value is entered into a spreadsheet cell, which makes it possible to use a specific formula in the cell to calculate the weighted scores. The formula we use is $U(s, f) = k_s{*}s + k_f{*}f$, where k_s and k_f are the weights and s and f are the scaled values for "Salary" and "Fun," respectively.

STEP 5

5.1 Build the decision tree, as shown in Figure 4.46, using the given probabilities. Leave the values (the numbers below the branches) temporarily at zero. Alternatively, the base tree can be found in Palisade's CD, Examples\Chapter 4\Method1.xls.

STEP 6

6.1 Create the weights table by typing **0.6** in cell B3 and **=1-B3** in cell C3.

STEP 7

7.1 Create the consequence table by entering the appropriate scaled salary and scaled fun scores corresponding to the branch values in columns F and G, as shown in Figure 4.46.

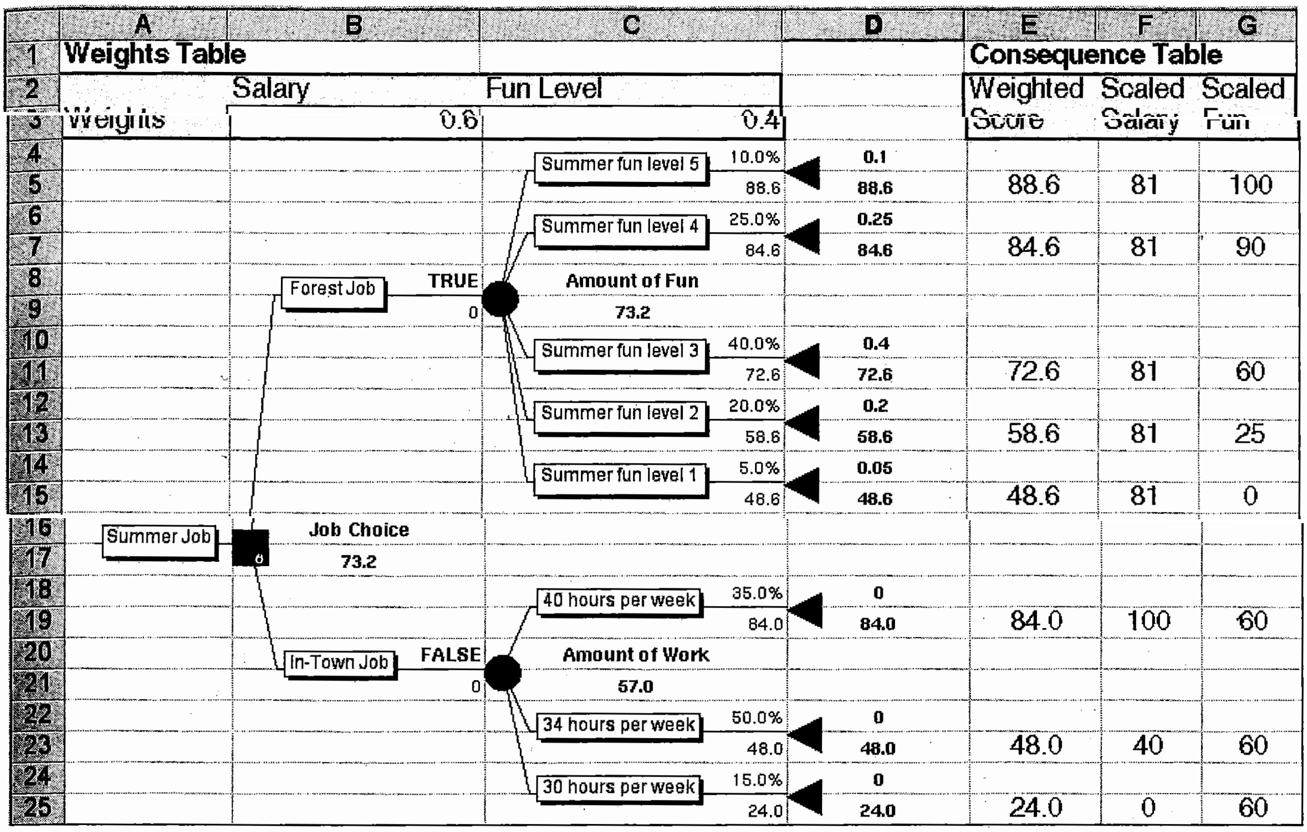
7.2 Compute the weighted scores for the top branch by clicking in cell **E5** and type **=$B$3*F5+$C$3*G5**. A score of 88.6 should appear in E5.

7.3 Click on cell **E5** and copy (either Ctrl-C or Edit-Copy).

7.4 Click into cell **E7** and paste. A score of 84.6 should appear in E7. Continue pasting the formula into the cells corresponding to each of the branches. You can streamline the process by copying the formula once, then holding down the control key while highlighting each cell into which you want to paste the formula. Now choose paste, and the formula is inserted into each highlighted cell. The weighted scores should be the same as those in Figure 4.46.

Figure 4.46

Modeling multiattribute decisions using the spreadsheet for calculations.



STEP 8

8.1 We now place the weighted scores into the decision tree. Click in cell **C5** and type **=E5**.

8.2 Copy the contents of **C5** and paste into the cells below each branch of the decision tree. At this point, your tree should be identical to the one in Figure 4.46.

Solving the tree and examining the risk profiles demonstrates that the "Forest Job" stochastically dominates the "In-Town Job."

Method 2

The second method is more sophisticated and eliminates the need to enter a separate formula for each outcome. Instead, a link is established between the decision tree and an Excel table along which input values pass from the tree to the table. The end-point value is calculated for the given input values, and passed back to the corresponding end node. Specifically, for the summer-job example, we will import the salary and fun levels from the tree, compute the weighted score, and export it back into the tree. The table acts as a template for calculating end-node values. Let's try it out.

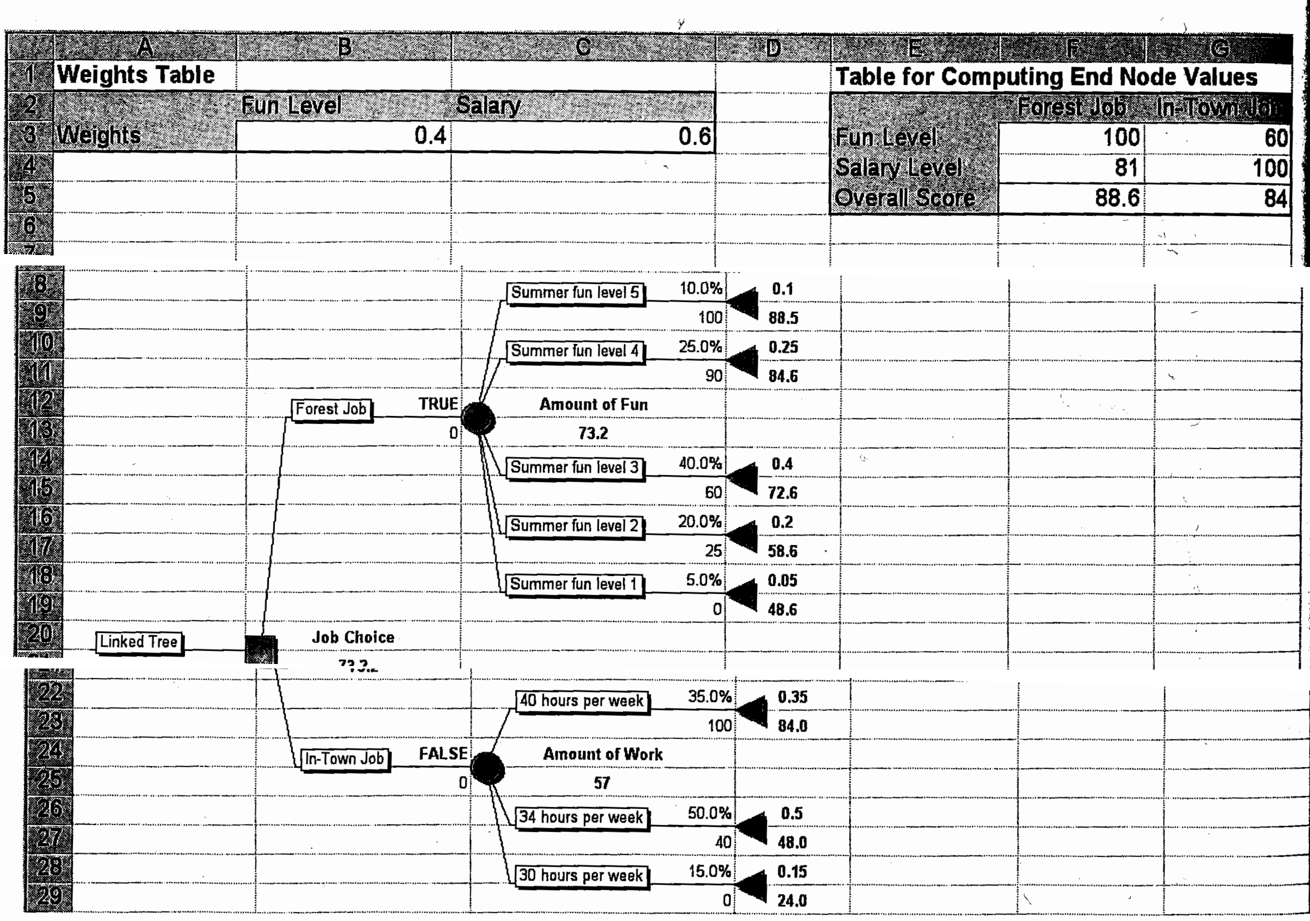
STEP 9

9.1 Construct the "Summer Job" decision tree as shown in Figure 4.47 and name the tree **Linked Tree.** The base tree can also be found on the Palisade CD, Examples\Chapter 4\LinkSummer.xls.

STEP 10 Next, we construct the tables as shown at the top of Figure 4.47. The table for computing the end-node values has one column for each alternative. When we are finished, this table will import the specific values from each branch and then export the overall weighted score to each end node.

Figure 4.47

A decision tree linked to a table that computes the end-node values.



10.1 Enter **0.40** in B3 and **= 1 – B3** in C3. These are the weights for "Fun" and "Salary," respectively.

10.2 Enter the values **100, 81, 60,** and **100,** respectively, in cells F3, F4, G3, and G4 as shown in Figure 4.47.

10.3 Enter the overall score formula $U(s, f) = k_f{*}f + k_s{*}s$ in cell F5 as **= F3*$B$3+F4*$C$3.**

10.4 Enter the corresponding formula in C5 as **= G3*$B$3+G4*$C$3** (or use the fill-right command). This table will be used to compute the values for each node. Currently, the end-node values are the scaled fun or salary levels, such as 100 for the top branch of the "Forest Job" alternative. These values need to be changed to reflect the overall score.

STEP 11 Now that we have built the tree and the table, we will link them together. We first link each of the end nodes to the formulas for the overall score. This tells PrecisionTree what formula to use when calculating the end-node values.

11.1 Click directly on the tree's name: **Linked Tree.**

11.2 Under the *Payoff Calculation* heading in the *Tree Settings* dialog box, choose the **Link to Spreadsheet Model** option button and choose the **Automatically Update Link** box.

11.3 Click **OK.** The nodes have turned white, indicating that the links can now be established.

11.4 One by one, click on each end node (the white triangle) of the "Forest Job" alternative, choose the option button **Cell** under *Link Payoffs Values From,* and either type **F5** in the text box or click on **F5** in the spreadsheet. After you click **OK,** two changes occur: The end node turns blue, indicating the link has been established, and the end-node value changes to 88.6, which is the "Forest Job" overall score when $s = 81$ and $f = 100$. Be sure to change all five end nodes.

11.5 Notice that each end node has a value of 88.6, which is correct only for the top branch. Type **90** into F3. Now all the end-node values are 84.6. Because we have not yet linked cell F3 to the chance node *Amount of Fun,* PrecisionTree does not know to input the different fun level scores from the tree into the formula. We link the chance nodes to the formula in Step 12 below.

11.6 Now move to the "In-Town" alternative. One by one, click on each end node (white triangles) of the "In-Town" alternative, choose the option button **Cell** under *Link Payoffs Values From,* and either type **G5** in the text box or click on **G5** in the spreadsheet. This changes the end-node value to 84.0, which is the "In-Town" overall score when $s = 100$ and $f = 60$. Be sure to change all three end nodes. Again, all end-node values are 84 because we have not linked the chance node *Amount of Work* to the formula.

STEP 12 We have connected the table output to the end-node values. Now we need to link the two chance nodes with the table so that the spreadsheet calculates the overall scores at the end nodes using the appropriate inputs.

12.1 To link the *Amount of Fun* chance node to the *Fun Level* in the table, click on the **Amount of Fun** node (the circle, not the name) to access the *Node Settings* dialog box.

12.2 Under the heading *Link Branch Value To,* click the **Cell** option button, click in the text box to the right, type **F3,** and click **OK.** Thus, for each branch, PrecisionTree calculates the end-node value by first sending the branch value to the table, then calculating the overall score, and finally sending that value back to the tree.

The end-node values for the "Forest Job" have been recalculated and now match Figure 4.47. Color has returned to the "Amount of Fun" chance node.

STEP 13

13.1 Finally, to link the *Amount of Work* chance node, click directly on the **Amount of Work** node.

13.2 Click the option button **Cell,** click in the text box to the right, and type **G4.**

13.3 Click **OK.** Your worksheet should match Figure 4.47.

This completes the construction of the linked tree. Analysis of this tree would proceed as described above. The linked-tree method becomes more advantageous as the decision tree grows in size and complexity. Also, it often happens [illegible] is first modeled in a spreadsheet, and later developed into a decision tree. It may be natural in such a case to use the linked-tree method where the end-node values are computed using the existing spreadsheet calculations.

SUMMARY

This chapter has demonstrated a variety of ways to use quantitative tools to make choices in uncertain situations. We first looked at the solution process for decision trees using expected value [or expected monetary value (EMV) when consequences are dollars]. This is the most straightforward way to analyze a decision model; the algorithm for solving a decision tree is easy to apply, and expected values are easy to calculate.

We also explored the process of solving influence diagrams using expected values. To understand the solution process for influence diagrams, we had to look at their internal structures. In a sense, we had to fill in certain gaps left from Chapter 3 about how influence diagrams work. The solution procedure works out easily once we understand how the problem's numerical details are represented internally. The procedure for reducing nodes involves calculating expected values in a way that parallels the solution of a decision tree.

Risk profiles can be used to compare the riskiness of strategies and give comprehensive views of risks faced by a decision maker. Thus, risk profiles provide additional information to the decision maker trying to gain insight into the decision situation and the available alternatives. We also showed how cumulative risk profiles can be used to identify dominated alternatives.

The chapter ended with a set of detailed instructions on how to solve both decision trees and influence diagrams in PrecisionTree. We also described two different ways to model multiple-objective decisions in PrecisionTree. Both methods made use of the fact that PrecisionTree works within a spreadsheet, allowing us to input formulas that combine multiple-objective scores into a single score. The first method required that we define a separate formula for each end branch of the decision tree. The second method linked the end branches to one formula whose inputs came from the branch values of the tree.

EXERCISES

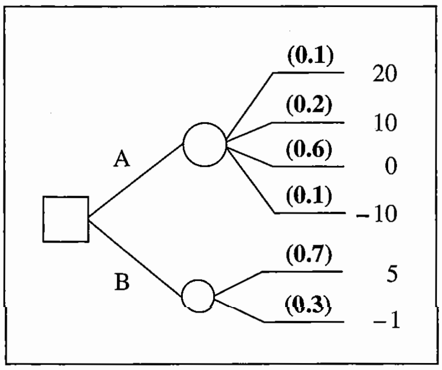
4.1 Is it possible to solve a decision-tree version of a problem and an equivalent influence-diagram version and come up with different answers? If so, explain. If not, why not?

4.2 Explain in your own words what it means when one alternative stochastically dominates another.

4.3 The analysis of the Texaco-Pennzoil example shows that the EMV of counteroffering with \$5 billion far exceeds \$2 billion. Why might Liedtke want to accept the \$2 billion anyway? If you were Liedtke, what is the smallest offer from Texaco that you would accept?

4.4 Solve the decision tree in Figure 4.48.

Figure 4.48
Generic decision tree for Exercise 4.4.



4.5 Draw and solve the influence diagram that corresponds to the decision tree in Figure 4.48.

4.6 Solve the decision tree in Figure 4.49. What principle discussed in Chapter 4 is illustrated by this decision tree?

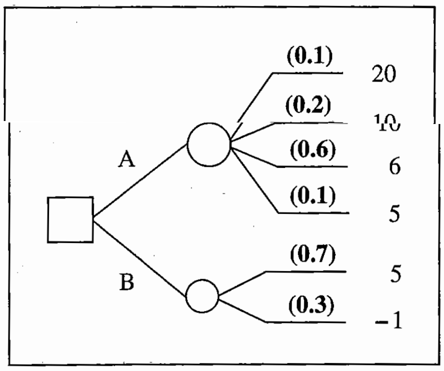


Figure 4.49
Generic decision tree for Exercise 4.6.

4.7 Which alternative is preferred in Figure 4.50? Do you have to do any calculations? Explain.

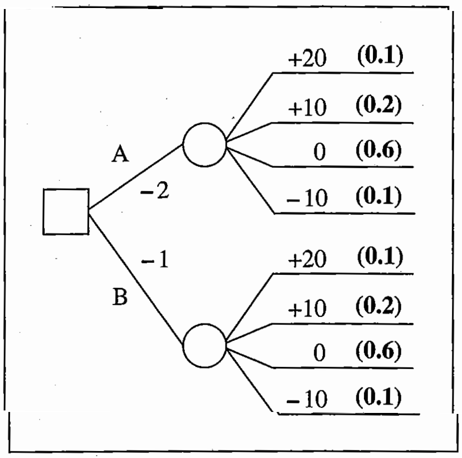


Figure 4.50
Generic decision tree for Exercise 4.7.

4.8 Solve the decision tree in Figure 4.51.

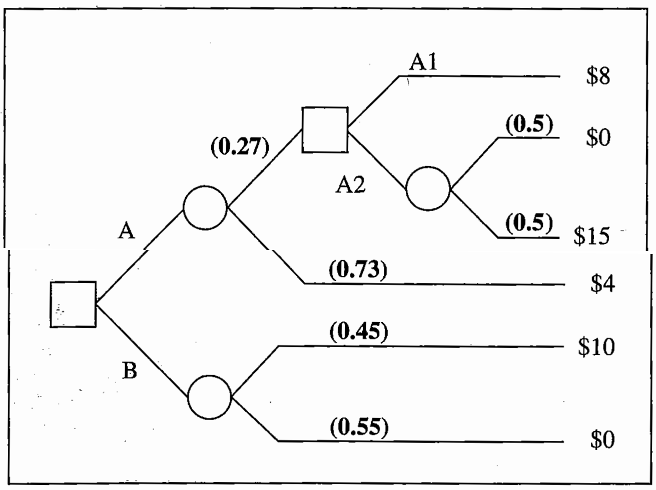


Figure 4.51
Generic decision tree for Exercise 4.8.

4.9 Create risk profiles and cumulative risk profiles for all possible strategies in Figure 4.51. Is one strategy stochastically dominant? Explain.

4.10 Draw and solve the influence diagram that corresponds to the decision tree in Figure 4.51.

4.11 Explain why deterministic dominance is a special case of stochastic dominance.

4.12 Explain in your own words why it is important to consider the ranges of the consequences in determining a trade-off weight.

4.13 Solve the influence diagram for the umbrella problem shown in Figure 4.10.

QUESTIONS AND PROBLEMS

4.14 A real-estate investor has the opportunity to purchase an apartment complex. The apartment complex costs \$400,000 and is expected to generate net revenue (net after all operating and finance costs) of \$6000 per month. Of course, the revenue could vary because the occupancy rate is uncertain. Considering the uncertainty, the revenue could vary from a low of -\$1000 to a high of \$10,000 per month. Assume that the investor's objective is to maximize the value of the investment at the end of 10 years.

a Do you think the investor should buy the apartment complex or invest the \$400,000 in a 10-year certificate of deposit earning 9.5%? Why?

b The city council is currently considering an application to rezone a nearby empty parcel of land. The owner of that land wants to build a small electronics-assembly plant. The proposed plant does not really conflict with the city's overall land use plan, but it may have a substantial long-term negative effect on the value of the nearby residential district in which the apartment complex is located. Because the city council currently is divided on the issue and will not make a decision until next month, the real-estate investor is thinking about waiting until the city council makes its decision.

If the investor waits, what could happen? What are the trade-offs that the investor has to make in deciding whether to wait or to purchase the complex now?

c Suppose the investor could pay the seller \$1000 in earnest money now, specifying in the purchase agreement that if the council's decision is to approve the rezoning, the investor can forfeit the \$1000 and forego the purchase. Draw and solve a decision tree showing the investor's three options. Examine the alternatives for dominance. If you were the investor, which alternative would you choose? Why?

4.15 A stock market investor has \$500 to spend and is considering purchasing an option contract on 1000 shares of Apricot Computer. The shares themselves are currently selling for \$28.50 per share. Apricot is involved in a lawsuit, the outcome of which will be known within a month. If the outcome is in Apricot's favor, analysts expect Apricot's stock price to increase by \$5 per share. If the outcome is unfavorable, then the price is expected to drop by \$2.75 per share. The option costs \$500, and owning the option would allow the investor to purchase 1000 shares of Apricot stock for \$30 per share. Thus, if the investor buys the option and Apricot prevails in the lawsuit, the investor would make an immediate profit. Aside from purchasing the option, the investor could (1) do nothing and earn about 8% on his money, or (2) purchase \$500 worth of Apricot shares.

a Construct cumulative risk profiles for the three alternatives, assuming Apricot has a 25% chance of winning the lawsuit. Can you draw any conclusions?

b If the investor believes that Apricot stands a 25% chance of winning the lawsuit, should he purchase the option? What if he believes the chance is only 10%? How large does the probability have to be for the option to be worthwhile?

4.16 Johnson Marketing is interested in producing and selling an innovative new food processor. The decision they face is the typical "make or buy" decision often faced by manufacturers. On one hand, Johnson could produce the processor itself, subcontracting different subassemblies, such as the motor or the housing. Cost estimates in this case are as follows:

Alternative: Make Food Processor

Cost per Unit ($)	Chance (%)
35.00	25
42.50	25
45.00	37
49.00	13

The company also could have the entire machine made by a subcontractor. The subcontractor, however, faces similar uncertainties regarding the costs and has provided Johnson Marketing with the following schedule of costs and chances:

Alternative: Buy Food Processor

Cost per Unit ($)	Chance (%)
37.00	10
43.00	40
46.00	30
50.00	20

If Johnson Marketing wants to minimize its expected cost of production in this case, should it make or buy? Construct cumulative risk profiles to support your recommendation. (*Hint:* Use care when interpreting the graph!)

4.17 Analyze the difficult decision situation that you identified in Problem 1.9 and structured in Problem 3.21. Be sure to examine alternatives for dominance. Does your analysis suggest any new alternatives?

4.18 Stacy Ennis eats lunch at a local restaurant two or three times a week. In selecting a restaurant on a typical workday, Stacy uses three criteria. First is to minimize the amount of travel time, which means that close-by restaurants are preferred on this attribute. The next objective is to minimize cost, and Stacy can make a judgment of the average lunch cost at most of the restaurants that would be considered. Finally, variety comes into play. On any given day, Stacy would like to go someplace different from where she has been in the past week.

Today is Monday, her first day back from a two-week vacation, and Stacy is considering the following six restaurants:

	Distance (Walking Time)	Average Price ($)
Sam's Pizza	10	3.50
Sy's Sandwiches	9	2.85
Bubba's Italian Barbecue	7	6.50
Blue China Cafe	2	5.00
The Eating Place	2	7.50
The Excel-Soaring Restaurant	5	9.00

a If Stacy considers distance, price, and variety to be equally important (given the range of alternatives available), where should she go today for lunch? (*Hints:* Don't forget to convert both distance and price to similar scales, such as a scale from 0 to 100. Also, recall that Stacy has just returned from vacation; what does this imply for how the restaurants compare on the variety objective?)

b Given your answer to part a, where should Stacy go on Thursday?

4.19 The national coffee store Farbucks needs to decide in August how many holiday-edition insulated coffee mugs to order. Because the mugs are dated, those that are unsold by January 15 are considered a loss. These premium mugs sell for $23.95 and cost $6.75 each. Farbucks is uncertain of the demand. They believe that there is a 25% chance that they will sell 10,000 mugs, a 50% chance that they will sell 15,000, and a 25% chance that they will sell 20,000.

a Build a linked-tree in PrecisionTree to determine if they should order 12,000, 15,000, or 18,000 mugs. Be sure that your model does not allow Farbucks to sell more mugs than it ordered. You can use the IF( ) command in Excel. If demand is less than the order quantity, then the amount sold is the demand. Otherwise, the amount sold is the order quantity. (See Excel's help or function wizard for guidance.)

b Now, assume that any unsold mugs are discounted and sold for $5.00. How does this affect the decision?

CASE STUDIES

GPC'S NEW PRODUCT DECISION

The executives of the General Products Company (GPC) have to decide which of three products to introduce, A, B, or C. Product C is essentially a risk-free proposition, from which the company will obtain a net profit of $1 million. Product B is considerably more risky. Sales may be high, with resulting net profit of $8 million, medium with net profit of $4 million, or low, in which case the company just breaks even. The probabilities for these outcomes are

$$P(\text{Sales High for B}) = 0.38$$
$$P(\text{Sales Medium for B}) = 0.12$$
$$P(\text{Sales Low for B}) = 0.50$$

Product A poses something of a difficulty; a problem with the production system has not yet been solved. The engineering division has indicated its confidence in solving the problem, but there is a slight (5%) chance that devising a workable solution may take a long time. In this event, there will be a delay in introducing the product, and that delay will result in lower sales and profits. Another issue is the price for Product A. The options are to introduce it at either high or low price; the price would not be set until just before the product is to be introduced. Both of these issues have an impact on the ultimate net profit.

Finally, once the product is introduced, sales can be either high or low. If the company decides to set a low price, then low sales are just as likely as high sales. If the company sets a high price, the likelihood of low sales depends on whether the product was delayed by the production problem. If there was no delay and the company sets a high price, the probability is 0.4 that sales will be high. However, if there is a delay and the price is set high, the probability is only 0.3 that sales will be high. The following table shows the possible net profit figures (in millions) for Product A:

	Price	High Sales ($ Million)	Low Sales ($ Million)
Time delay	High	5.0	(0.5)
	Low	3.5	1.0
No delay	High	8.0	0.0
	Low	4.5	1.5

Questions

1. Draw an influence diagram for GPC's problem. Specify the possible outcomes and the probability distributions for each chance node. Specify the possible alternatives for each decision node. Write out the complete table for the consequence node. (If possible, use a computer program for creating influence diagrams.)

2. Draw a complete decision tree for GPC. Solve the decision tree. What should GPC do? (If possible, do this problem using a computer program for creating and solving decision trees.)

3. Create cumulative risk profiles for each of the three products. Plot all three profiles on one graph. Can you draw any conclusions?

4. One of the executives of GPC is considerably less optimistic about Product B and assesses the probability of medium sales as 0.3 and the probability of low sales as

0.4. Based on expected value, what decision would this executive make? Should this executive argue about the probabilities? Why or why not? (*Hint:* Don't forget that probabilities have to add up to 1!)

5 Comment on the specification of chance outcomes and decision alternatives. Would this specification pass the clarity test? If not, what changes in the problem must be made in order to pass the clarity test?

SOUTHERN ELECTRONICS, PART I

Steve Sheffler is president, CEO, and majority stockholder of Southern Electronics, a small firm in the town of Silicon Mountain. Steve faces a major decision: Two firms, Big Red Business Machines and Banana Computer, are bidding for Southern Electronics.

Steve founded Southern 15 years ago, and the company has been extremely successful in developing progressive computer components. Steve is ready to sell the company (as long as the price is right!) so that he can pursue other interests. Last month, Big Red offered Steve $5 million and 100,000 shares of Big Red stock (currently trading at $50 per share and not expected to change substantially in the future). Until yesterday, Big Red's offer sounded good to Steve, and he had planned on accepting it this week. But a lawyer from Banana Computer called last week and indicated that Banana was interested in acquiring Southern Electronics. In discussions this past week, Steve has learned that Banana is developing a new computer, code-named EYF, that, if successful, will revolutionize the industry. Southern Electronics could play an important role in the development of the machine.

In their discussions, several important points have surfaced. First, Banana has said that it believes the probability that the EYF will succeed is 0.6, and that if it does, the value of Banana's stock will increase from the current value of $30 per share. Although the future price is uncertain, Banana judges that, conditional on the EYF's success, the expected price of the stock is $50 per share. If the EYF is not successful, the price will probably decrease slightly. Banana judges that if the EYF fails, Banana's share price will be between $20 and $30, with an expected price of $25.

Yesterday Steve discussed this information with his financial analyst, who is an expert regarding the electronics industry and whose counsel Steve trusts completely. The analyst pointed out that Banana has an incentive to be very optimistic about the EYF project. "Being realistic, though," said the analyst, "the probability that the EYF succeeds is only 0.4, and if it does succeed, the expected price of the stock would be only $40 per share. On the other hand, I agree with Banana's assessment for the share price if the EYF fails."

Negotiations today have proceeded to the point where Banana has made a final offer to Steve of $5 million and 150,000 shares of Banana stock. The company's representative has stated quite clearly that Banana cannot pay any more than this in a straight transaction. Furthermore, the representative claims, it is not clear why Steve will not accept the offer because it appears to them to be more valuable than the Big Red offer.

Questions

1. In terms of expected value, what is the least that Steve should accept from Banana? (This amount is called his *reservation price.*)
2. Steve obviously has two choices, to accept the Big Red offer or to accept the Banana offer. Draw an influence diagram representing Steve's decision. (If possible, do this problem using a computer program for structuring influence diagrams.)
3. Draw and solve a complete decision tree representing Steve's decision. (If possible, do this problem using a computer program for creating and solving decision trees.)
4. Why is it that Steve cannot accept the Banana offer as it stands?

SOUTHERN ELECTRONICS, PART II

Steve is well aware of the difference between his probabilities and Banana's, and he realizes that because of this difference, it may be possible to design a contract that benefits both parties. In particular, he is thinking about put options for the stock. A put option gives the owner of the option the right to sell an asset at a specific price. (For example, if you own a put option on 100 shares of General Motors (GM) with an exercise price of $75, you could sell 100 shares of GM for $75 per share before the expiration date of the option. This would be useful if the stock price fell below $75.) Steve reasons that if he could get Banana to include a put option on the stock with an exercise price of $30, then he would be protected if the EYF failed.

Steve proposes the following deal: He will sell Southern Electronics to Banana for $530,000 plus 280,000 shares of Banana stock and a put option that will allow him to sell the 280,000 shares back to Banana for $30 per share any time within the next year (during which time it will become known whether the EYF succeeds or fails).

Questions

1. Calculate Steve's expected value for this deal. Ignore tax effects and the time value of money.
2. The cost to Banana of their original offer was simply

$$\$5,000,000 + 150,000(\$30) = \$9,500,000$$

Show that the expected cost to Banana of Steve's proposed deal is less than $9.5 million, and hence in Banana's favor. Again, ignore tax effects and the time value of money.

STRENLAR

Fred Wallace scratched his head. By this time tomorrow he had to have an answer for Joan Sharkey, his former boss at Plastics International (PI). The decision was difficult to make. It involved how he would spend the next 10 years of his life.

Four years ago, when Fred was working at PI, he had come up with an idea for a revolutionary new polymer. A little study—combined with intuition, hunches, and educated guesses—had convinced him that the new material would be extremely strong for its weight. Although it would undoubtedly cost more than conventional materials, Fred discovered that a variety of potential uses existed in the aerospace, automobile manufacturing, robotics, and sporting goods industries.

When he explained his idea to his supervisors at PI, they had patiently told him that they were not interested in pursuing risky new projects. His appeared to be even riskier than most because, at the time, many of the details had not been fully worked out. Furthermore, they pointed out that efficient production would require the development of a new manufacturing process. Sure, if that process proved successful, the new polymer could be a big hit. But without that process the company simply could not provide the resources Fred would need to develop his idea into a marketable product.

Fred did not give up. He began to work at home on his idea, consuming most of his evenings and weekends. His intuition and guesses had proven correct, and after some time he had worked out a small-scale manufacturing process. With this process, he had been able to turn out small batches of his miracle polymer, which he dubbed Strenlar. At this point he quietly began to assemble some capital. He invested $100,000 of his own, managed to borrow another $200,000, and quit his job at PI to devote his time to Strenlar.

That was 15 months ago. In the intervening time he had made substantial progress. The product was refined, and several customers eagerly awaited the first production run. A few problems remained to be solved in the manufacturing process, but Fred was 80% sure that these bugs could be worked out satisfactorily. He was eager to start making profits himself; his capital was running dangerously low. When he became anxious, he tried to soothe his fears by recalling his estimate of the project's potential. His best guess was that sales would be approximately $35 million over 10 years, and that he would net some $8 million after costs.

Two weeks ago, Joan Sharkey at PI had surprised him with a telephone call and had offered to take Fred to lunch. With some apprehension, Fred accepted the offer. He had always regretted having to leave PI, and was eager to hear how his friends were doing. After some pleasantries, Joan came to the point.

"Fred, we're all impressed with your ability to develop Strenlar on your own. I guess we made a mistake in turning down your offer to develop it at PI. But we're interested in helping you out now, and we can certainly make it worth your while. If you will grant PI exclusive rights to Strenlar, we'll hire you back at, say $40,000 a year, and we'll give you a 2.5 percent royalty on Strenlar sales. What do you say?"

Fred didn't know whether to laugh or become angry. "Joan, my immediate reaction is to throw my glass of water in your face! I went out on a limb to develop the product, and now you want to capitalize on my work. There's no way I'm going to sell out to PI at this point!"

The meal proceeded, with Joan sweetening the offer gradually, and Fred obstinately refusing. After he got back to his office, Fred felt confused. It would be nice to work at PI again, he thought. At least the future would be secure. But there would never be the potential for the high income that was possible with Strenlar. Of course, he thought grimly, there was still the chance that the Strenlar project could fail altogether.

At the end of the week, Joan called him again. PI was willing to go either of two ways. The company could hire him for $50,000 plus a 6% royalty on Strenlar gross sales. Alternatively, PI could pay him a lump sum of $500,000 now plus options to purchase up to 70,000 shares of PI stock at the current price of $40 any time within the next three years. No matter which offer Fred accepted, PI would pay off Fred's creditors and take over the project immediately. After completing development of the manufacturing process, PI would have exclusive rights to Strenlar. Furthermore, it turned out that PI was deadly serious about this game. If Fred refused both of these offers, PI would file a lawsuit claiming rights to Strenlar on the grounds that Fred had improperly used PI's resources in the development of the product.

Consultation with his attorney just made him feel worse. After reviewing Fred's old contract with PI, the attorney told him that there was a 60% chance that he would win the case. If he won the case, PI would have to pay his court costs. If he lost, his legal fees would amount to about $20,000.

Fred's accountant helped him estimate the value of the stock options. First, the exercise date seemed to pose no problem; unless the remaining bugs could not be worked out, Strenlar should be on the market within 18 months. If PI were to acquire the Strenlar project and the project succeeded, PI's stock would go up to approximately $52. On the other hand, if the project failed, the stock price probably would fall slightly to $30.

As Fred thought about all of the problems he faced, he was quite disturbed. On one hand, he yearned for the comradery he had enjoyed at PI four years ago. He also realized that he might not be cut out to be an entrepreneur. He reacted unpleasantly to the risk he currently faced. His physician had warned him that he may be developing hypertension and had tried to persuade him to relax more. Fred knew that his health was important to him, but he had to believe that he would be able to weather the tension of getting Strenlar onto the market. He could always relax later, right? He sighed as he picked up a pencil and pad of paper to see if he could figure out what he should tell Joan Sharkey.

Question

1 Do a complete analysis of Fred's decision. Your analysis should include at least structuring the problem with an influence diagram, drawing and solving a decision tree, creating risk profiles, and checking for stochastic dominance. What do you think Fred should do? Why? (*Hint:* This case will require you to make certain assumptions in order to do a complete analysis. State clearly any assumptions you make, and be careful that the assumptions you make are both reasonable and consistent with the information given in the case. You may want to analyze your decision model under different sets of assumptions. Do not forget to consider issues such as the time value of money, riskiness of the alternatives, and so on.)

JOB OFFERS

Robin Pinelli is considering three job offers. In trying to decide which to accept, Robin has concluded that three objectives are important in this decision. First, of course, is to maximize disposable income—the amount left after paying for housing, utilities, taxes, and other necessities. Second, Robin likes cold weather and enjoys winter sports. The third objective relates to the quality of the community. Being single, Robin would like to live in a city with a lot of activities and a large population of single professionals.

Developing attributes for these three objectives turns out to be relatively straightforward. Disposable income can be measured directly by calculating monthly take-home pay minus average monthly rent (being careful to include utilities) for an appropriate apartment. The second attribute is annual snowfall. For the third attribute, Robin has located a magazine survey of large cities that scores those cities as places for single professionals to live. Although the survey is not perfect from Robin's point of view, it does capture the main elements of her concern about the quality of the singles community and available activities. Also, all three of the cities under consideration are included in the survey.

Here are descriptions of the three job offers:

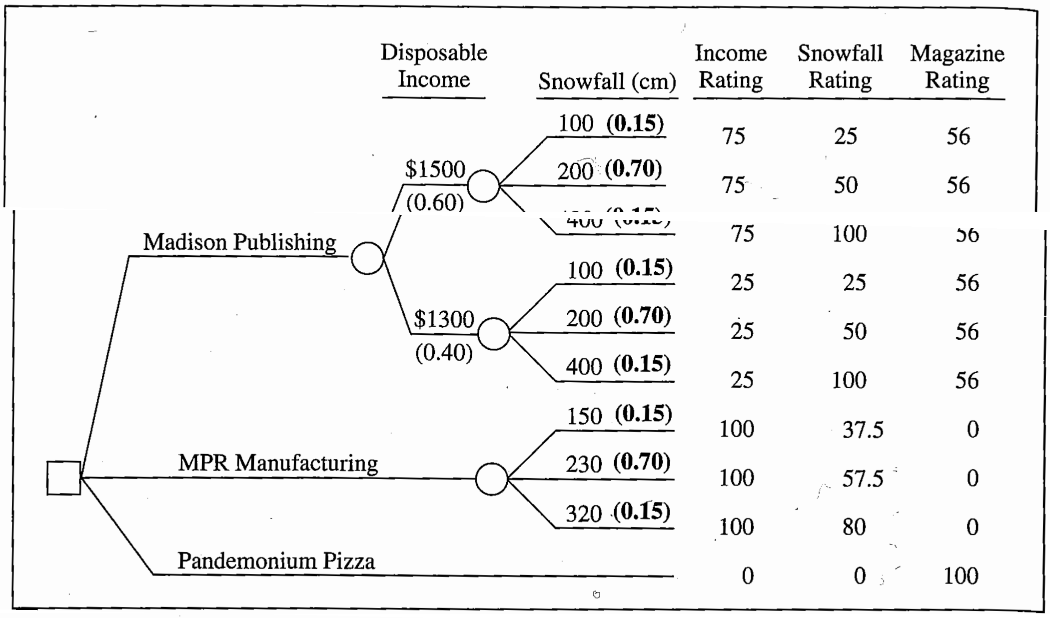
1. MPR Manufacturing in Flagstaff, Arizona. Disposable income estimate: $1600 per month. Snowfall range: 150 to 320 cm per year. Magazine score: 50 (out of 100).
2. Madison Publishing in St. Paul, Minnesota. Disposable income estimate: $1300 to $1500 per month. (This uncertainty here is because Robin knows there is a wide variety in apartment rental prices and will not know what is appropriate and available until spending some time in the city.) Snowfall range: 100 to 400 cm per year. Magazine score: 75.
3. Pandemonium Pizza in San Francisco, California. Disposable income estimate: $1200 per month. Snowfall range: negligible. Magazine score: 95.

Robin has created the decision tree in Figure 4.52 to represent the situation. The uncertainty about snowfall and disposable income are represented by the chance nodes as Robin has included them in the tree. The ratings in the consequence matrix are such that the worst consequence has a rating of zero points and the best has 100.

Questions

1. Verify that the ratings in the consequence matrix are proportional scores (that is, that they were calculated the same way we calculated the ratings for salary in the summer-fun example in the chapter).

Figure 4.52
Robin Pinelli's decision tree.



2 Comment on Robin's choice of annual snowfall as a measure for the cold-weather–winter-sports attribute. Is this a good measure? Why or why not?

3 After considering the situation, Robin concludes that the quality of the city is most important, the amount of snowfall is next, and the third is income. (Income is important, but the variation between $1200 and $1600 is not enough to make much difference to Robin.) Furthermore, Robin concludes that the weight for the magazine rating in the consequence matrix should be 1.5 times the weight for the snowfall rating and three times as much as the weight for the income rating. Use this information to calculate the weights for the three attributes and to calculate overall scores for all of the end branches in the decision tree.

4 Analyze the decision tree using expected values. Calculate expected values for the three measures as well as for the overall score.

5 Do a risk-profile analysis of the three cities. Create risk profiles for each of the three attributes as well as for the overall score. Does any additional insight arise from this analysis?

6 What do you think Robin should do? Why?

SS *KUNIANG*, PART II

This case asks you to find the optimal amount for NEES to bid for the SS *Kuniang* (page 107). Before doing so, though, you need additional details. Regarding the Coast Guard's (CG) salvage judgment, NEES believes that the following probabilities are an appropriate representation of its uncertainty about the salvage-value judgment:

$$P(\text{CG judgment} = \$9 \text{ million}) = 0.185$$
$$P(\text{CG judgment} = \$4 \text{ million}) = 0.630$$
$$P(\text{CG judgment} = \$1.5 \text{ million}) = 0.185$$

The obscure-but-relevant law required that NEES pay an amount (including both the winning bid and refitting cost) at least 1.5 times the salvage value for the ship in order to use it for domestic shipping. For example, if NEES bid $3.5 million and won, followed by a CG judgment of $4 million, then NEES would have to invest at least $2.5 million more: $\$3.5 + \$2.5 = \$6 = \$4 \times 1.5$. Thus, assuming NEES submits the winning bid, the total investment amount required is either the bid or 1.5 times the CG judgment, whichever is greater.

As for the probability of submitting the highest bid, recall that winning is a function of the size of the bid; a bid of $3 million is sure to lose, and a bid of $10 million is sure to win. For this problem, we can model the probability of winning (P) as a linear function of the bid: $P = (\text{Bid} - \$3 \text{ million})/(\$7 \text{ million})$.

Finally, NEES's values of $18 million for the new ship and $15 million for the tug-barge alternatives are adjusted to reflect differences in age, maintenance, operating costs, and so on. The two alternatives provide equivalent hauling capacity. Thus, at $15 million, the tug-barge combination appears to be the better choice.

Questions

1 Reasonable bids may fall anywhere between $3 and $10 million. Some bids, though, have greater expected values and some less. Describe a strategy you can use to find the optimal bid, assuming that NEES's objective is to minimize the cost of acquiring additional shipping capacity. (*Hint:* This question just asks you to describe an approach to finding the optimal bid.)

2 Use your structure of the problem (or one supplied by the instructor), along with the details supplied above, to find the optimal bid.

REFERENCES

The solution of decision trees as presented in this chapter is commonly found in textbooks on decision analysis, management science, and statistics. The decision-analysis texts listed at the end of Chapter 1 can provide more guidance in the solution of decision trees if needed. In contrast, the material presented here on the solution of influence diagrams is relatively new. For additional basic instruction in the construction and analysis of decisions using influence diagrams, the user's manual for PrecisionTree and other influence-diagram programs can be helpful.

The solution algorithm presented here is based on Shachter (1986). The fact that this algorithm deals with a decision problem in a way that corresponds to solving a symmetric decision tree means that the practical upper limit for the size of an influence diagram that can be solved using the algorithm is relatively small. Recent work has explored a variety of ways to exploit asymmetry in decision models and to solve influence diagrams and related representations more efficiently (Call and Miller 1990; Covaliu and Oliver 1995; Smith et al. 1993; Shenoy 1993).

An early and quite readable article on risk profiles is that by Hertz (1964). We have developed them as a way to examine the riskiness of alternatives in a heuristic way and also as a basis for examining alternatives in terms of deterministic and stochastic dominance. Stochastic dominance itself is an important topic in probability. Bunn (1984) gives a good introduction to stochastic dominance. Whitmore and Findlay (1978) and Levy (1992) provide thorough reviews of stochastic dominance.

Our discussion of assigning rating points and trade-off rates is necessarily brief in Chapter 4. These topics are covered in depth in Chapters 13 to 16. In the meantime, interested readers can get more information from Keeney (1992) and Keeney and Raiffa (1976).

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EPILOGUE

What happened with Texaco and Pennzoil? You may recall that in April of 1987 Texaco offered a $2 billion settlement. Hugh Liedtke turned down the offer. Within days of that decision, and only one day before Pennzoil began to file liens on Texaco's assets, Texaco filed for protection from creditors under Chapter 11 of the federal bankruptcy code, fulfilling its earlier promise. In the summer of 1987, Pennzoil submitted a financial reorganization plan on Texaco's behalf. Under their proposal, Pennzoil would receive approximately $4.1 billion, and the Texaco shareholders would be able to vote on the plan. Finally, just before Christmas 1987, the two companies agreed on a $3 billion settlement as part of Texaco's financial reorganization.