“Basics of a soft computing theory”
Laboratory Work №2

Approximation of a function with one variable with the use of a fuzzy system

Objectives

1. Acquire the technique of constructing fuzzy systems
2. Learn to approximate a function with one variable with the use of a fuzzy system

Tasks

It is necessary to construct a fuzzy system needed for the approximation of the table function $y_i = f(x_i)$, $i = \overline{1.10}$. The system must be constructed using the interface program of the fuzzy logic package of the MATLAB software environment. Tasks variants are presented in Table 1.

Task variants

The values $x_i = i \times 0.1$, $i = \overline{1.10}$ are the same for each variant.

Table 1

i	Value $y_i = y_i(x_i)$									
	№1	№2	№3	№4	№5	№6	№7	№8	№9	№10
1	1.50	2.09	2.02	1.99	2.23	2.07	2.18	-0.10	-0.16	2.09
2	1.26	2.05	1.98	2.03	2.29	2.17	2.43	-0.21	0.01	2.31
3	0.99	2.19	1.67	2.20	2.27	2.21	2.40	0.01	0.10	2.72
4	0.97	2.18	1.65	2.39	2.62	2.31	2.43	0.05	0.16	2.77
5	0.91	2.17	1.57	2.19	2.72	2.10	2.65	-0.13	0.05	2.78
6	0.71	2.27	1.42	2.61	2.82	2.09	2.75	-0.23	0.35	2.97
7	0.43	2.58	1.37	2.35	3.13	2.12	2.67	-0.21	0.19	3.00
8	0.54	2.73	1.07	2.60	3.49	1.63	2.66	-0.43	0.50	3.51
9	0.19	2.82	0.85	2.55	3.82	1.78	2.63	-0.57	0.74	3.43
10	0.01	3.04	0.48	2.49	3.95	1.52	2.75	-0.44	1.03	3.58

Example:

The dependency between the variables y and x is given in Table 2. It is required to build a fuzzy system with which makes it is possible to approximate the proposed in the table function.

Table 2

i	x_i	$y_i(x_i)$
1	0.1	2.05
2	0.2	1.94
3	0.3	1.92
4	0.4	1.87
5	0.5	1.77
6	0.6	1.74
7	0.7	1.71
8	0.8	1.60
9	0.9	1.56
10	1	1.40

For the approximation of function with one variable, we use the MATLAB environment. The **fuzzy** command from the command line mode launches the interface program of the Fuzzy Logic package that is the editor of the fuzzy interface system FIS (Fuzzy Inference System).
Choose the option **New Sugeno FIS** (a new system of the Sugeno type) the in **File** menu. The editor window will open (fig. 1). In the window, there are fields for the input, output function and rules editor. It is necessary to rename the blocks input1 into **x**, output1 into **y**.

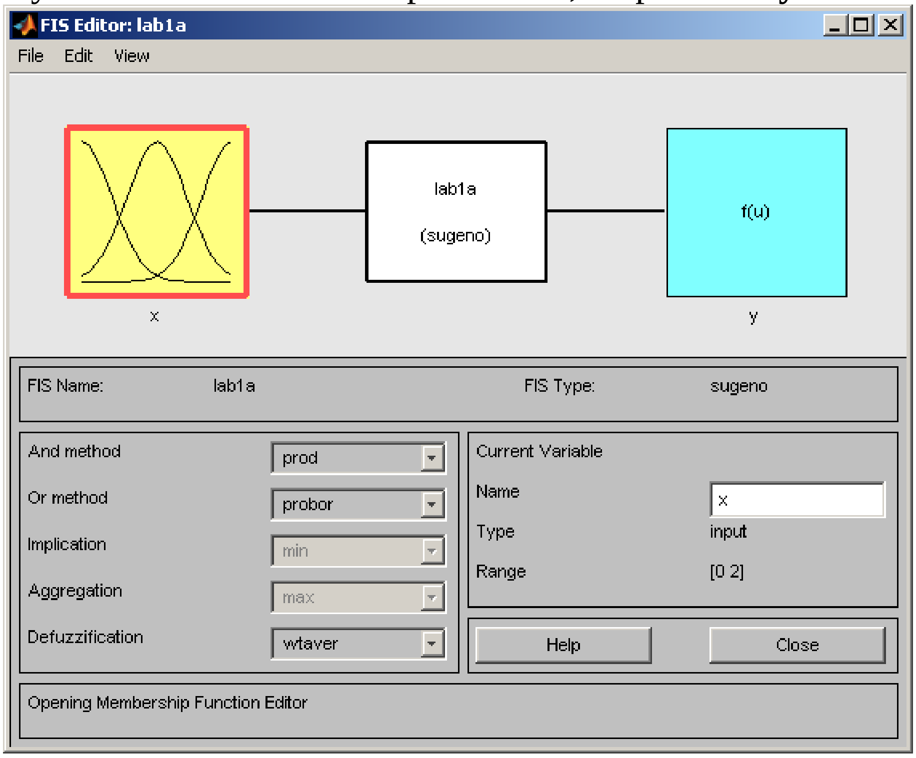

Figure 1 - Function editor window

The following values are set in the block of parameters of the fuzzy system of the Sugeno type:

- Menu **And method** allows to set the implementation of the logical operation AND: ***min*** - minimum and ***prod*** - algebraic product;
- Menu **Or method** allows to set the implementation of the logical operation OR: ***max*** - maximum and ***probor*** - algebraic sum;
- Menu **Defuzzifunction** allows to set one of the defuzzification methods:

  wtaver - weighted average method in which $z_0 = \frac{\sum_{i=1}^{\eta} \alpha_i z_i^*}{\sum_{i=1}^{n} \alpha_i}$

  or ***wtsum*** - weighted sum method in which $z_0 = \sum_{i=1}^{\eta} \alpha_i z_i^*$,

  where η is the number of rules in the knowledgebase.

Open the membership function editor window by clicking on the block x (input1) (fig. 2). It is necessary to add the membership functions. For this, choose the option **Add MFs** in the **Edit** menu (fig. 3). In the opened dialog box, set the type and number of membership functions (9, then add 1 more, to get 10 membership functions total). Choose Gaussian functions (**gaussmf**) as membership functions. Then set the range of x from 0 to 1 (the range corresponding to Table 2) in the window of the membership functions editor in the **Range** field. Then it is necessary to move the graphs of the given membership functions so that the ordinates of their maximums coincide with the given values of x. For this, set the range of the curve and its center position in the **Params** field.

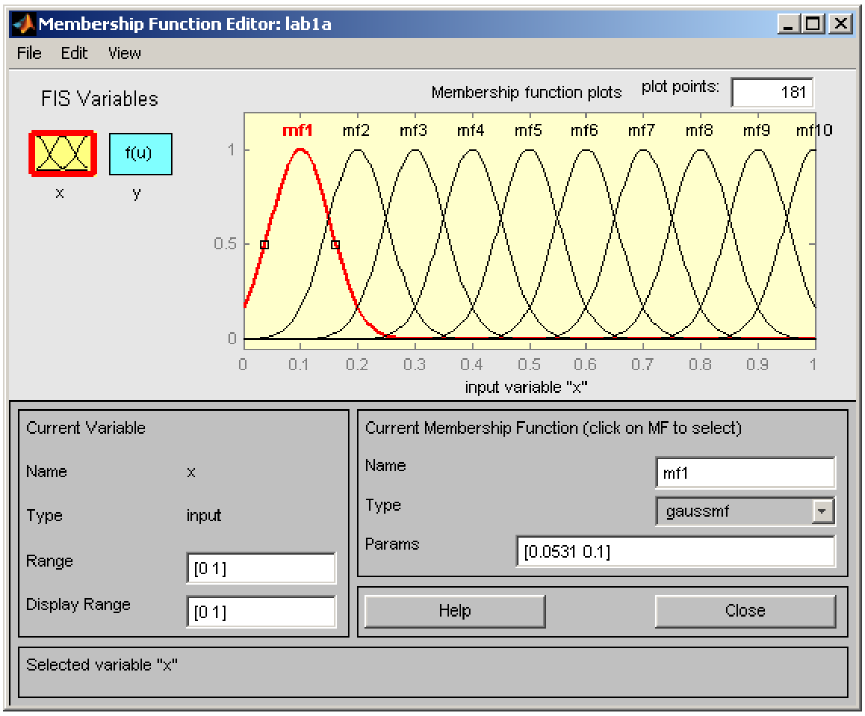

Figure 2 - Membership function editor window for x

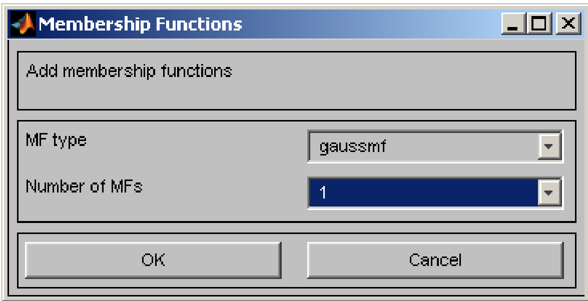

Figure 3 - Adding a membership function window

Next, it is necessary to set the membership functions for *y*, by clicking on the y block and going to the membership function editor (fig. 4). Add the membership functions (menu **Edit\Add MFs**). As the membership function, select 10 constants according to the number of different values of *yi* in Table 2. Thus, the constructed system is the Sugeno type fuzzy system of a 0-order. Set the values of the function *y* in the **Params** field of the membership function editor window.

In the FIS-editor window (fig. 4), open the rules editor window by clicking on the middle block (fig. 5).

When entering a rule, it is necessary to indicate the correspondence between each membership function of the argument *x* and the numerical value of *y*.

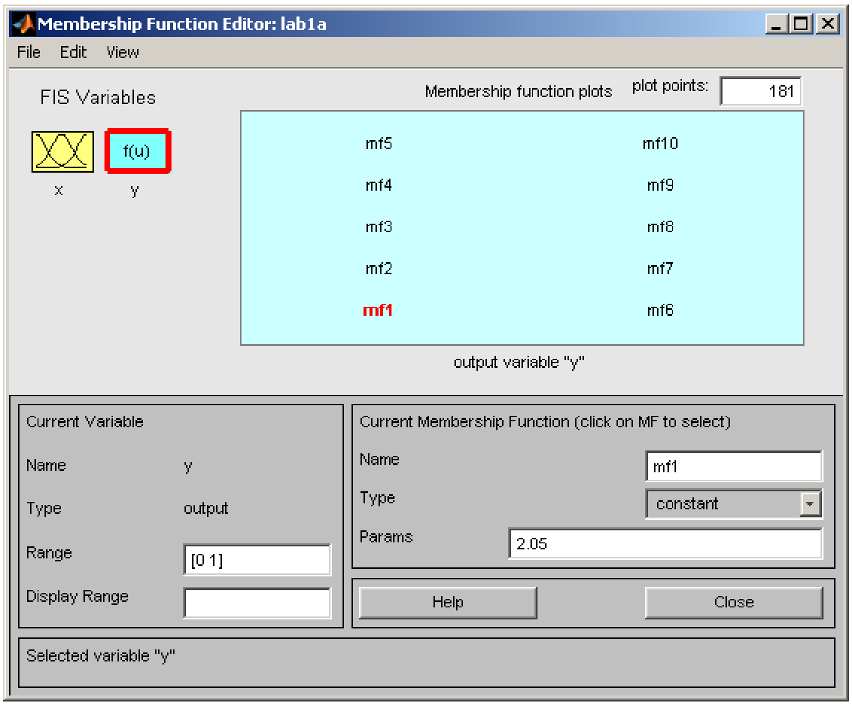

Figure 4 - Membership function editor window for y

There is an opportunity to view the rules (fig. 6). For this choose the **View - rules** menu option. In the left part of the window, the membership functions of the argument *x* are presented in graphical form. In the right one - the output variable *y* with the explanations of the decision-making mechanism. The vertical line crossing the graphs in the left part of the window allows you to change the values of the input values while the values of *y* on the right side change accordingly.

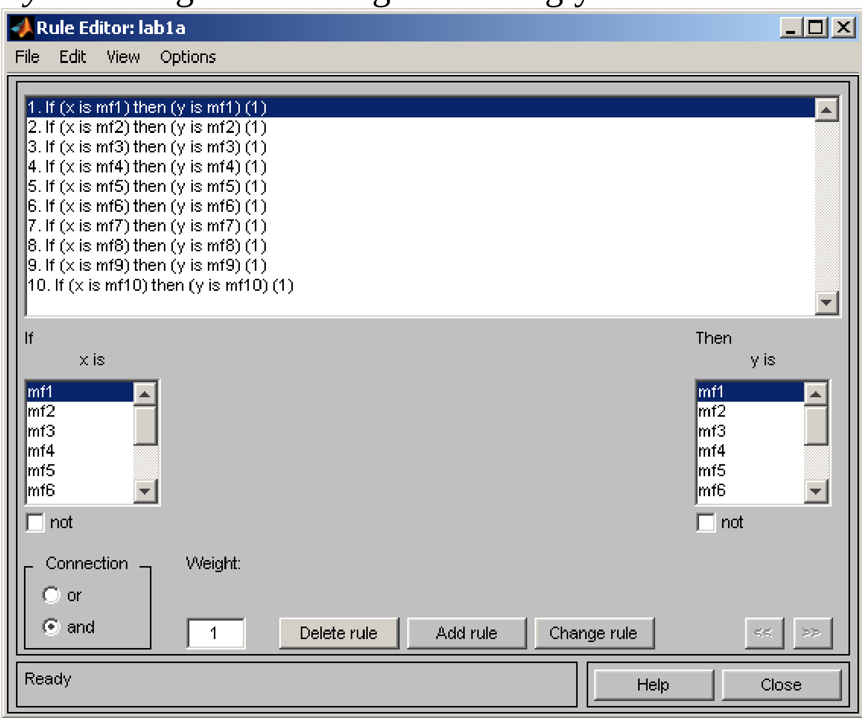

Figure 5 - Rules editor window

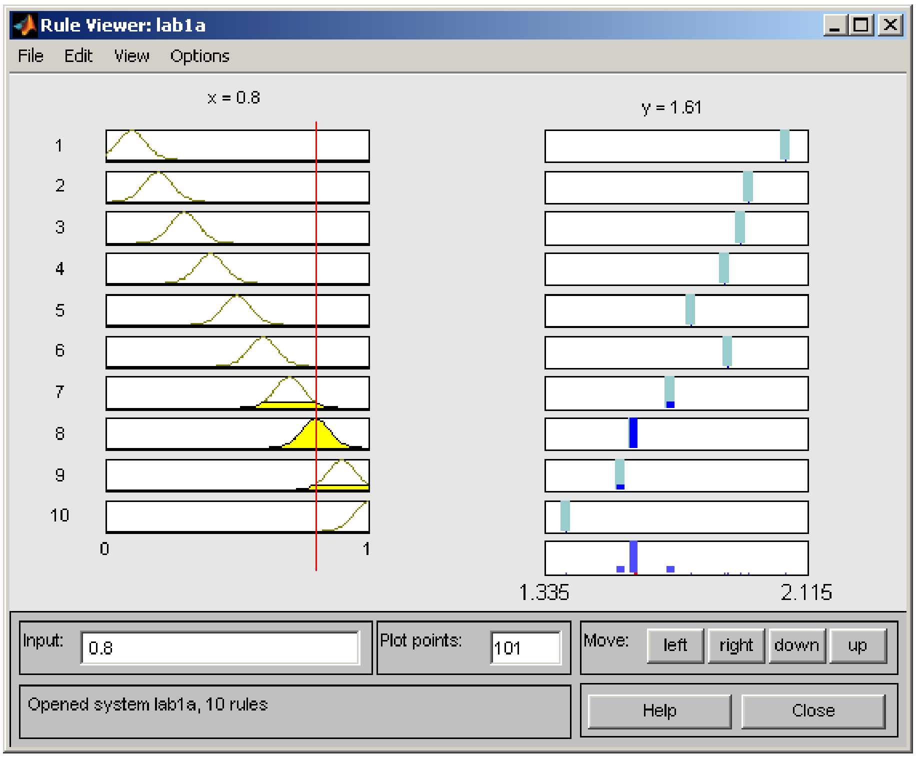

Figure 6 - Rules viewer

As the result, a fuzzy system that solved the problem of approximation of a given table function was constructed. Table 3 shows a comparison of the outputs of the fuzzy system with the initial values of *y* (the fourth column in the table shows the outputs of the fuzzy system).

It is seen from Table 3 that the maximum deviation $\Delta i = |\tilde{y}_i(x_i) - y_i(x_i)|$ of the outputs of the fuzzy system from the initial data is 0,02.

Choose **View-Surface** from the menu and open the window for viewing the output surface. In this case, go to viewing the curve *y(x)* (fig. 7).

Table 3

i	x_i	$y_i(x_i)$	$\tilde{y}_i(x_i)$	$\Delta i = \|\tilde{y}_i(x_i) - y_i(x_i)\|$
1	0.1	2.05	2.03	0.02
2	0.2	1.94	1.95	0.01
3	0.3	1.92	1.92	0
4	0.4	1.87	1.86	0.01
5	0.5	1.77	1.78	0.01
6	0.6	1.74	1.74	0
7	0.7	1.71	1.72	0.01

8	0.8	1.60	1.61	0.01
9	0.9	1.56	1.54	0.02
10	1	1.40	1.42	0.02

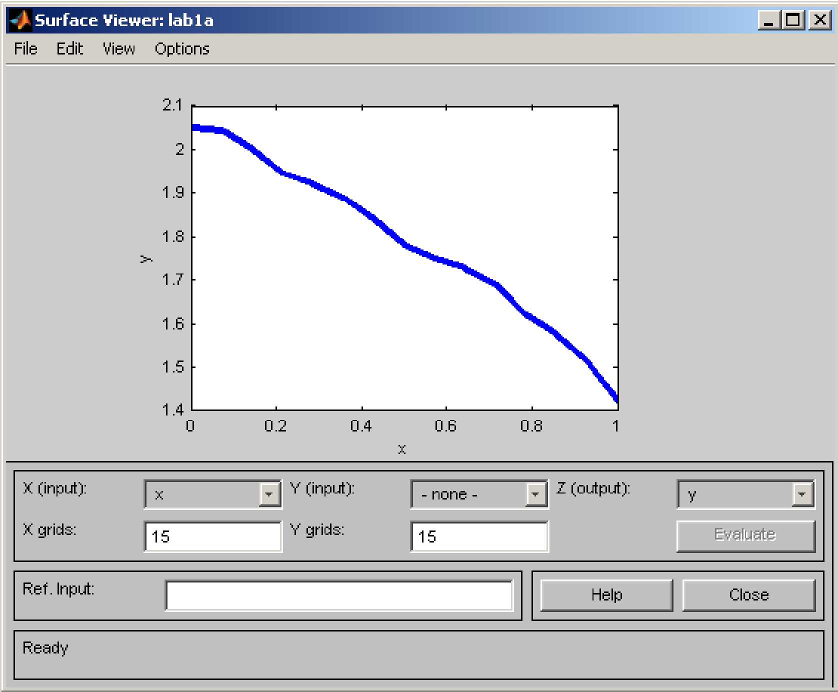

Figure 7 - Graph of the function

In the conclusion, it should be pointed out that with the help of the above editor it is possible to make the necessary adjustments to the fuzzy system at any stage of its designing up to the setting of any special user-defined membership function.

Conclusions (example):

As a result of the laboratory work, I got acquainted with the theoretical information about fuzzy sets, their properties, knowledgebases, fuzzy logic inference. During the laboratory work, I constructed the fuzzy system for the approximation of a given table function with the help of the Fuzzy Logic environment MATLAB. I made a comparison of the outputs of the fuzzy system with the given values of *y*. As the result, it was found that the maximum deviation was 0,02 (for example at x = 0,1), which is 1,025%.

Test questions.

1. Give the definition of a fuzzy set.
2. Describe the stages of the fuzzy inference.
3. Which inference algorithm was used during the laboratory training? Describe the given algorithm.