Course “Basics of soft computing theory”

Tutorial for the laboratory work #3

Topic: “Fuzzy inference”

Theory part:

Fuzzy implication is expressed in the following form:

$$A \to B = \overline{A \cup B} \text{ и } \mu_{A \to B}(x) = \max\{1 - \mu_A(x), \mu_B(x)\}.$$

The basis for the operation of fuzzy logical inference is the rules database containing fuzzy statements in the "If-Then" form and membership functions for the corresponding linguistic terms.

In this case, the following conditions must be met:

- there is at least one rule for each linguistic term in the output variable;
- for any term of an input variable, there is at least one rule in which this term is used as a precondition (left part of the rule).

Otherwise, there is an incomplete fuzzy rules database.

Statements are called fuzzy if :

1. Statements $<\beta$ is $\beta'>$, where β is the name of the linguistic variable, β' is its value, which corresponds to a fuzzy set on the universal set X.
2. Statements $<\beta$ is $m\beta'>$, where m is a modifier that corresponds to the words "very," "more or less," "much more," etc.
3. Compound statements formed from statements of types 1 and 2 and conjunctions "AND", "OR", "IF ..., THEN ..., ELSE".

A linguistic variable is a set of $<\beta, T, X, G, M>$, where β is the name of the linguistic variable, T is the set of its values (term-set), which are the name of fuzzy variables, the domain of each of which is the set X (the set T is called the base term-set of the linguistic variable), G is a syntactic procedure that allows you to operate with the elements of the term set T, in particular, to generate new terms (values), M is a semantic procedure that allows you to turn each new value of a linguistic variable formed by the procedure G into a fuzzy variable, i.e. form the corresponding fuzzy set.

A fuzzy variable is characterized by a triple $<\alpha, X, A>$, where α is the name of the variable, X is a universal set (domain of definition of α), A

is a fuzzy set on X, which describes the restrictions (i.e. $\mu_A(x)$) on the value of the fuzzy variable A.

An example of solving the problem

Problem. Build a fuzzy knowledge base (use at least three linguistic variables) for the task of determining the time by a student spent on solving the problems of this manual (take into account the student's progress and the number of variants to be solved), check it for completeness and produce a fuzzy conclusion for specific values (choose randomly).

Description of the solution process. To implement logical inference, you must do the following:

1. Formulate in natural language in the form of a sentence "if ..., then" the laws of the subject area.

2. Select from these sentences linguistic variables, their values (construct their membership functions), statements of various types, formalize fuzzy rules.

3. Check the resulting knowledge base for completeness.

4. Conduct fuzzification (we select the input data at random).

5. Carry out accumulation.

6. Carry out defuzzification.

Solution

1) The sentences describing the task are as follows:

   - *If a student's academic performance is high or good and he solves a small number of variants, then he needs a little time.*
   - *If a student's academic performance is high or good and he solves many variants, then he needs a rather long period of time.*
   - *If a student's performance is low and he solves many variants, then he needs a lot of time.*
   - *If a student's academic performance is average and he solves a sufficiently large number of variants, then he needs a rather long period of time.*

   Select linguistic variables from these sentences (we define them through a formal notation $<\beta, T, X, G, M>$):

   1. β=student academic performance, T = ("high", "average", "low"), X = [2, 5] (a five-point system is used), G = ("very low", "high or

average"), M - decrease per unit degree, the membership of a fuzzy variable "high", the operation of combining fuzzy sets;

2. β=number of options, T = ("few", "enough", "a lot"), X = [1, 20] (number of options is 20 in each topic), G = ("a lot", "enough or not enough"), M - increase per unit degree, the membership of a fuzzy variable "many", the operation of combining fuzzy sets;

3. β=amount of time, T = ("little", "enough", "a lot"), X = [1, 7] (number of hours per week devoted to the subject of study), G = ("a lot", "enough or not enough"), M - increase per unit degree, the membership of a fuzzy variable "many", the operation of combining fuzzy sets;

For a complete assignment of the linguistic variable, it is necessary to determine the fuzzy variables included in T:

Academic performance (low, average, high):

Number of variants (few, enough, a lot):

Amount of time (little, enough, a lot):

Taking into account the selected linguistic variables, the fuzzy rules are as follows:

1. If Academic Performance = "high" or Academic Performance = "average" and Number of variants = "few", then Amount of time = "little".

2. If Academic Performance = "high" or Academic Performance = "average" and Number of variants= "many", by Amount of Time = "enough".

3. If Academic Performance = "low" and Number of variants = "a lot", then Amount of Time = "a lot".

4. If Academic Performance = "average" and Number of variants = "enough", then Amount of time = "a lot".

2) Let's check the resulting base for completeness:

- there is at least one rule for each linguistic term of the output variable - the output variable "Amount of time" has 3 terms: "little" is used in 1 rule, "enough" in 2 and 4, "a lot" - in 3;
- for any term in the input variable, there is at least one rule in which this term is used as a precondition - there are two input variables "Academic performance" and "Number of variants" each of them has three terms: "high" is used in 1 and 2 rules, " average" - 1, 2 and 4, "low" - 3, "little" - 1, "enough" - 4, "a lot" - 3 and 2.

This means that the resulting fuzzy rule base is complete.

3) Let there be a student Ivanov A.A. having an average score of 3.5 and deciding to solve 9 options, you need to determine how much time it will take.

Let us determine the degree of confidence of the simplest statements:

Academic performance = "high" - 0;

Academic performance = "average" - 0,5;

Academic performance = "low" - 1;

Number of variants = "few" - 0;

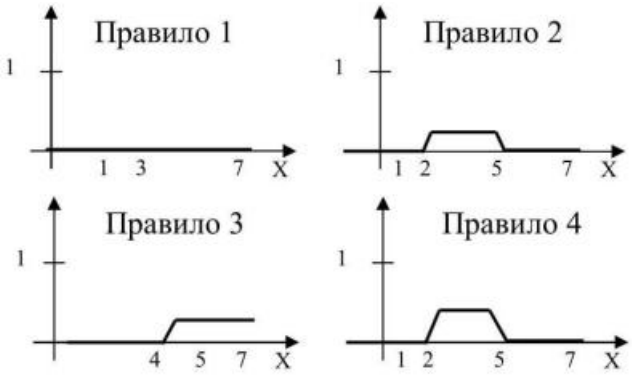
Number of variants = "enough" - 0,5;

Number of variants = "a lot" - 0,125.

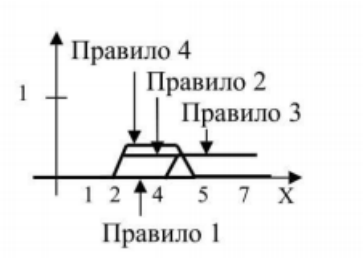
Let us determine the degree of confidence of the hypothesis of the rules.

1: min (max (0, 0.5), 0) = 0;
2: min (max (0, 0.5), 0.125) = 0.125;
3: min (1, 0.125) = 0.125;
4: min (0.5, 0.5) = 0.5.

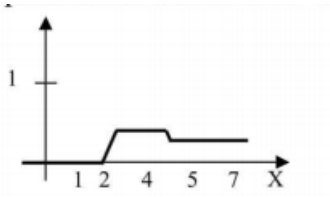
Let's construct a new output fuzzy variable using the obtained degrees of confidence:



4) Accumulation:



New term of output variable Number of hours:



5) Based on the obtained graph of the membership degree of the output term, we can say that A.A. Ivanov, who has an average grade of 3.5, will need at least 2.75 hours to solve 9 variants of tasks (the degree of confidence of this statement is 0.5).

Problems

The list of tasks will be sent for each student by the lecturer.