Digital Video Processing
Homework II

İrem Beyza ONUR

April 5, 2022

In this homework, Horn and Schunk algorithm is performed for motion estimation on a video sequence of 238 frames. For each frame pair, the first pair is defined as the anchor, and the second one is defined as the target frame. According to Horn and Schunk algorithm introduces the smoothness constraint to the well-known aperture problem given in Equation (1).

$$I_x \times u + I_y \times v + I_t = 0 \tag{1}$$

As seen in Equation (1) there are 2 unknowns, whereas only one equation. Horn and Schunk algorithm assumes the existence of smoothness in the whole image and tries to minimize the brightness error as well as the smoothness error.
The total error is given in Equation (2).

$$E = \int\int [(I_x \times u + I_y \times v + I_t)^2 + \alpha^2 \times (\nabla \mid u \mid^2 + \nabla \mid v \mid^2)] dxdxy \tag{2}$$

The error function is iteratively minimized by the updating process given in Figure 1.

$$u^{k+1} = \overline{u}^k - \frac{I_x(I_x\overline{u}^k + I_y\overline{v}^k + I_t)}{4\alpha^2 + I_x^2 + I_y^2}$$
$$v^{k+1} = \overline{v}^k - \frac{I_y(I_x\overline{u}^k + I_y\overline{v}^k + I_t)}{4\alpha^2 + I_x^2 + I_y^2}$$

Figure 1: Minimizing the error function.

In the equations above, I_x, I_y, and I_t represent the partial derivatives of brightness

with respect to x,y, and t. On the other hand, u and v represent the optical flow vectors, whereas α denotes the smoothness parameter which is set to 1 in this implementation. Note that, the error is calculated as the difference between the previous flow and estimated flow in this implementation, despite the theoretical explanation given above.
Figure 2 shows the distribution of error versus 25 iterations.

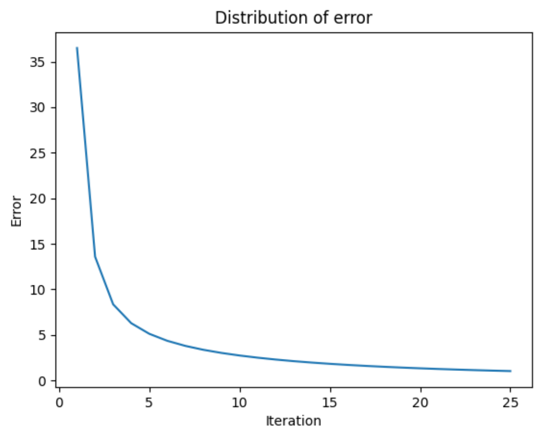

Figure 2: The distribution of error (α = 1).

As seen in Figure 2, the error shows a sharp decrease for the first 10 iterations, whereas it smooths afterward. The error function takes the value of 2.65 after iteration 10, and 1.02 at the end of the iterations.

The experiment is repeated with the smoothness parameter which is ten times as the previous one and Figure 3 is obtained.

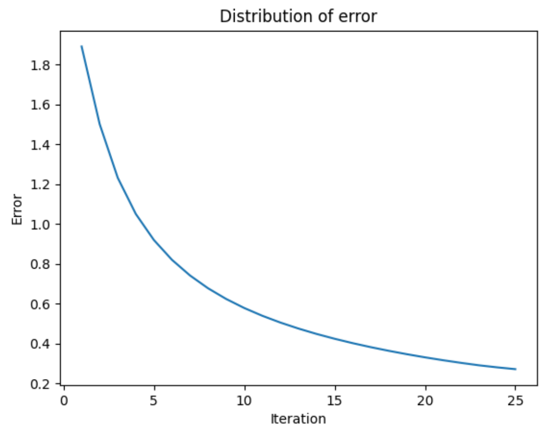

Figure 3: The distribution of error ($\alpha = 10$).

As seen in Figure 3, the characteristics of loss distributions vary with the smoothness parameter. The error reaches 0.5 after iteration 10, whereas 0.26 at the end of the process.
Another experiment is conducted with the smoothness parameter of 0 and Figure 4 is obtained.

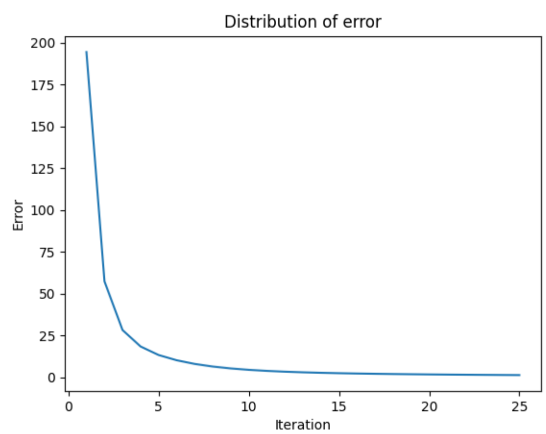

Figure 4: The distribution of error ($\alpha = 0$).

As seen in Figure 4, the error reaches the highest value with the parameter of 0. The error takes the value of 4 after iteration 10, whereas it reaches 1.5 at the end of the iterations. The experiments conducted with different smoothness parameters show the effect of the smoothness parameter on the accuracy of the optical flow estimation. As the smoothness parameter increases, better performance of the motion estimation is achieved.
The estimated optical flow vectors for the first 4 frames are reported with various smoothness parameters as in the Figures below. Note that, forward optical flow vectors are reported on the frames.

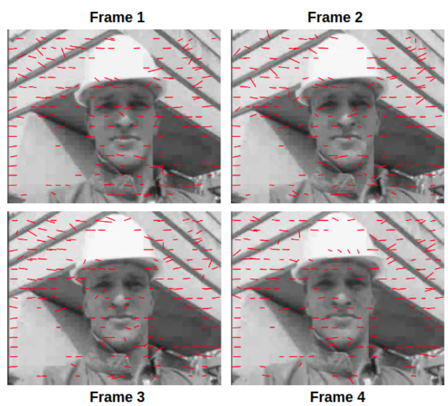

Figure 5: The estimated optical flow vectors ($\alpha = 1$).

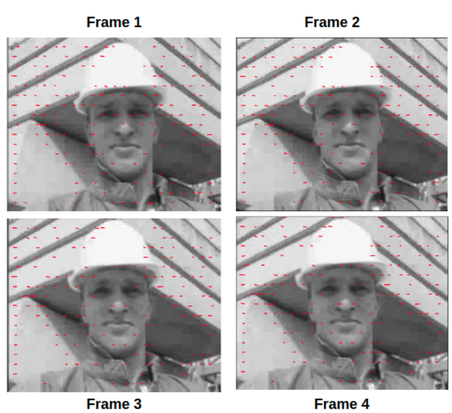

Figure 6: The estimated optical flow vectors ($\alpha = 5$).

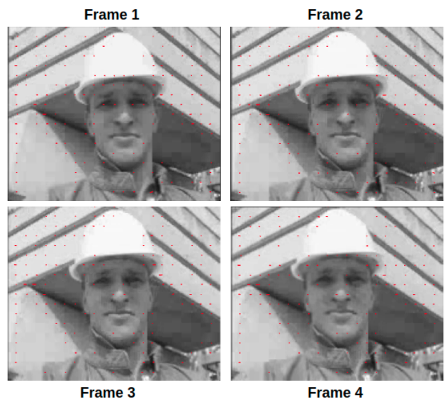

Figure 7: The estimated optical flow vectors ($\alpha = 10$).

As seen in the Figures above, as the smoothness parameter increases, the magnitude of the estimated optical flow vectors decreases. Increasing the smoothness parameter means assuming smoother motion transitions, resulting in obtaining vectors with smaller magnitudes. Using these optical flow vectors, corresponding frames are estimated as in Figure 8.

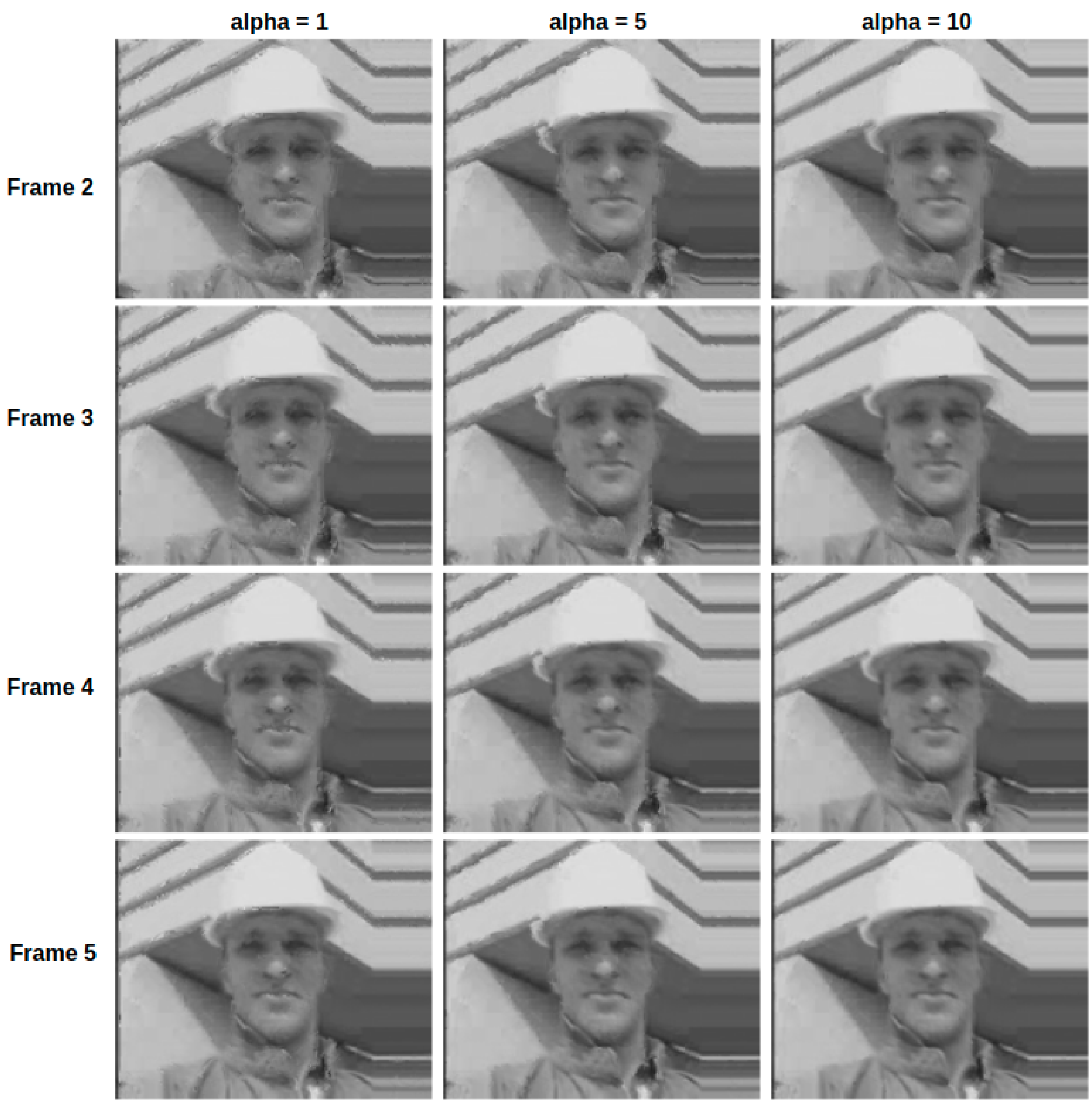

Figure 8: The estimated video frames.

As seen in Figure 8, the transitions across the frames are much smoother with the higher smoothness parameters, resulting in better estimations. The considerably sharp transitions across the frames are indicators of the optical flow vectors with higher magnitudes achieved with smaller smoothness parameters as mentioned in earlier Figures.

References

[1] K. Reinhard (2014). Concise Computer Vision.Springer.

[2] G. Guido. CS 6320. Class Lecture, Topic: “Optical Flow I.” Computer Engineering Department, University of Utah, Utah, Spring, 2012.

[3] Vineeth,S(2021)Video-Interpolation-using-Optical-Flow [https://github.com/vineeths96/Video-Interpolation-using-Optical-Flow].