091M4041H - Assignment 4
Algorithm Design and Analysis

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1 飞机降落问题(2)

根据题意得到的线性规划表示：

$$
\begin{aligned}
&max\ z \\
&s.t. \\
&s_i \le x_i \le t_i \ \ i = 1, 2, 3, ..., n \\
&x_i - x_{i-1} \ge z \ \ i = 2, 3, ..., n \\
&xi \ge 0 \ \ i = 1, 2, 3, ..., n \\
&z \ge 0
\end{aligned}
\tag{1}
$$

转化为标准形式：

$$
\begin{aligned}
&min\ -z \\
&s.t. \\
&-x_i \le -s_i \ \ i = 1, 2, 3, ..., n \\
&x_i \le t_i \ \ i = 1, 2, 3, ..., n \\
&-x_i + x_{i-1} \le -z \ \ i = 2, 3, ..., n \\
&xi \ge 0 \ \ i = 1, 2, 3, ..., n \\
&z \ge 0
\end{aligned}
\tag{2}
$$

尝试用glpk求解这个线性规划：

设定n=5,即5架飞机，其降落时间为0-23整数时间，时间窗口如下：[1,2] [3,6] [7,9] [13,15] [18,22]。则可写以下线性规划描述文件：

```
var x1 >= 0;
var x2 >= 0;
var x3 >= 0;
var x4 >= 0;
var x5 >= 0;
var x6 >= 0;
var d >= 0;

minimize z :d;
s.t. con1: x1 <= 15;
s.t. con2: x1 >= 8;
s.t. con3: x2 >= 23;
s.t. con4: x2 >= 21;
s.t. con5: x3 <= 53;
s.t. con6: x3 >= 28;
s.t. con7: x4 <= 66;
s.t. con8: x4 >= 59;
s.t. con9: x5 <= 99;
s.t. con10: x5 >= 72;
s.t. con11: x6 >= 105;
s.t. con12: d >= 13 ;
s.t. con13: d >= 7;
s.t. con14: d >= 31;
s.t. con15: d >= 13;
s.t. con16: d >= 33;


end;
```

glpk计算结果如下图：

```
Non-zeros:  23
Status:     OPTIMAL
Objective:  z = 4 (MAXimum)

  No.   Row name   St   Activity     Lower bound   Upper bound    Marginal
------ ------------ -- ------------ ------------- ------------- -------------
     1 z            B              4
     2 con1         NL             1             1                        -0.5
     3 con2         B              1                           2
     4 con3         B              5             3
     5 con4         B              5                           6
     6 con5         B              9             7
     7 con6         NU             9                           9           0.5
     8 con7         NL            13            13                       < eps
     9 con8         B             13                          15
    10 con9         NL            18            18                       < eps
    11 con10        B             18                          22
    12 con11        NL             0            -0                        -0.5
    13 con12        NL             0            -0                        -0.5
    14 con13        B              0            -0
    15 con14        B              1            -0

  No. Column name  St   Activity     Lower bound   Upper bound    Marginal
------ ------------ -- ------------ ------------- ------------- -------------
     1 x1           B              1             0
     2 x2           B              5             0
     3 x3           B              9             0
     4 x4           B             13             0
     5 x5           B             18             0
     6 d            B              4             0
```

由图可知各飞机降落时间结果为x1=1，x2=5，x3=9，x4=13，x5=18。最近的时间间隔为z=4。

2 安置加油站(4)

根据题意得到的线性规划表示：

$$
\begin{aligned}
&min\ z \\
&s.t. \\
&d_i - r \le x_i \le d_i + r \\
&x_{i+1} - x_i \le z \\
&x_i \ge 0 \\
&i = 1, 2, 3, ..., n \\
&z \ge 0
\end{aligned}
\tag{3}
$$

转化为标准形式：

$$
\begin{aligned}
&min\ z \\
&s.t. \\
&x_i \le d_i + r \\
&-x_i \le -d_i + r \\
&x_{i+1} - x_i \le z \\
&x_i \ge 0 \\
&i = 1, 2, 3, ..., n \\
&z \ge 0
\end{aligned}
\tag{4}
$$

尝试用glpk求解这个线性规划：

设n = 6,r = 3。d1 = 5,d2 = 18,d3 = 25,d4 = 56,d5 = 69,d6 = 102。

则可写以下线性规划描述文件：

```
var x1 >= 0;
var x2 >= 0;
var x3 >= 0;
var x4 >= 0;
var x5 >= 0;
var x6 >= 0;
var d >= 0;

minimize z :d;
s.t. con1: x1 <= 8;
s.t. con2: x1 >= 2;
s.t. con3: x2 <= 21;
s.t. con4: x2 >= 15;
s.t. con5: x3 <= 28;
s.t. con6: x3 >= 22;
s.t. con7: x4 <= 59;
s.t. con8: x4 >= 53;
s.t. con9: x5 <= 72;
s.t. con10: x5 >= 66;
s.t. con11: x6 <= 105;
s.t. con12: x6 >= 99;
s.t. con13: d >= x2-x1 ;
s.t. con14: d >= x3-x2;
s.t. con15: d >= x4-x3;
s.t. con16: d >= x5-x4;
s.t. con17: d >= x6-x5;


end;
```

glpk计算结果如下图：

```
Problem:    hw7
Rows:       18
Columns:    7
Non-zeros:  28
Status:     OPTIMAL
Objective:  z = 27 (MINimum)

  No.   Row name   St   Activity     Lower bound   Upper bound    Marginal
------ ------------ -- ------------ ------------- ------------- -------------
     1 z            B            27
     2 con1         B             2                           8
     3 con2         NL            2             2                       < eps
     4 con3         B            15                          21
     5 con4         NL           15            15                       < eps
     6 con5         B            26                          28
     7 con6         B            26            22
     8 con7         B            53                          59
     9 con8         NL           53            53                       < eps
    10 con9         NU           72                          72            -1
    11 con10        B            72            66
    12 con11        B            99                         105
    13 con12        NL           99            99                         1
    14 con13        B            14            -0
    15 con14        B            16            -0
    16 con15        NL            0            -0                       < eps
    17 con16        B             8            -0
    18 con17        NL            0            -0                         1

  No. Column name  St   Activity     Lower bound   Upper bound    Marginal
------ ------------ -- ------------ ------------- ------------- -------------
     1 x1           B             2             0
     2 x2           B            15             0
     3 x3           B            26             0
     4 x4           B            53             0
     5 x5           B            72             0
     6 x6           B            99             0
     7 d            B            27             0
```

由图可知6个安置加油站位置的结果为：x1 = 2，x2 = 15，x3 = 26，x4 = 53，x5 = 72，x6 = 99。此时目标z=27，为第五个和第六个加油站之间的距离。

3 Dual Simplex算法(7)

3.1 result analysis

使用glpk工具求解该线性规划，线性规划描述文件如下：

```
var x1 >= 0;
var x2 >= 0;
var x3 >= 0;
var x4 >= 0;
var x5 >= 0;
var x6 >= 0;

minimize z: -7*x1 + 7*x2 - 2*x3 - x4 -6*x5;

s.t. con1: 3*x1 - x2 + x3 - 2*x4 = -3;
s.t. con2: 2*x1 + x2 + x4 + x5= 4;
s.t. con3: -x1 + 3*x2 - 3*x4 + x6 = 12;

end;
```

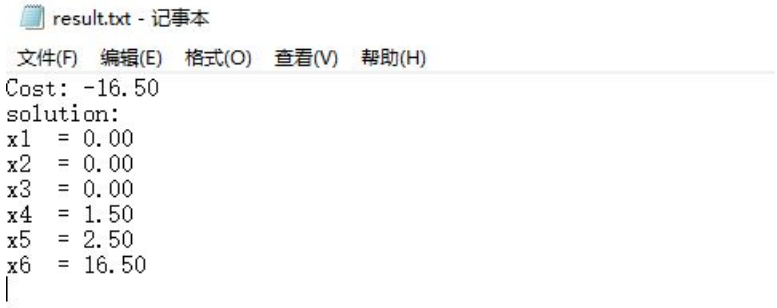
求解结果如下：

```
Problem:    test_hw7
Rows:       4
Columns:    6
Non-zeros:  17
Status:     OPTIMAL
Objective:  z = -16.5 (MINimum)

   No.   Row name   St   Activity     Lower bound   Upper bound    Marginal
------ ------------ -- ------------ ------------- ------------- -------------
     1 z            B          -16.5
     2 con1         NS            -3            -3             =          -2.5
     3 con2         NS             4             4             =            -6
     4 con3         NS            12            12             =         < eps

   No. Column name  St   Activity     Lower bound   Upper bound    Marginal
------ ------------ -- ------------ ------------- ------------- -------------
     1 x1           NL             0             0                       12.5
     2 x2           NL             0             0                       10.5
     3 x3           NL             0             0                        0.5
     4 x4           B            1.5             0
     5 x5           B            2.5             0
     6 x6           B           16.5             0
```

由上图可知结果为x1=0,x2=0,x3=0,x4=1.5,x5=2.5,x6=16.5。目标函数z的最小值为-16.5。Dual Simplex算法求解问题的结果如如下：



由图可知Dual Simplex算法求解结果和glpk工具一致。