

Master Degree in Computational Social Science
2024-2025

Master Thesis

“From Votes to Seats: Understanding Proportionality in the Spanish Electoral System (1982–2023)”

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Madrid, 31/08/2025

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SUMMARY

This thesis investigates the extent to which the Spanish electoral system translates votes into seats proportionally and how alternative institutional configurations might reshape patterns of representation. Covering all general elections between 1982 and 2023, the analysis systematically compares apportionment methods, legal thresholds, and constituency designs using computational simulations in R and established disproportionality indices. The study shows that the Spanish system, though formally proportional, systematically distorts representation and favors certain parties, revealing that institutional choices, especially district size and allocation rules, have decisive political consequences. More broadly, the results highlight a fundamental trade-off between proportionality and stability: rules that deliver fairer representation often prove more volatile, while more robust rules introduce systematic bias. Over time, disproportionality has fluctuated with the evolution of the party system, underscoring that electoral institutions cannot be assessed in isolation but must be understood in interaction with broader political dynamics. In doing so, the thesis contributes to debates on electoral reform by clarifying both the structural biases of the Spanish system and the conditions under which alternative designs would perform differently.

Keywords: Electoral systems, Spain, Disproportionality, Apportionment Methods, Robustness

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1. INTRODUCTION

Electoral systems, often presented as neutral mechanisms, play a central role in shaping political representation and influencing election outcomes. By transforming votes into seats, the design of an electoral system can decisively determine who is elected and which parties come to power: the same distribution of votes might yield a coalition in one system or an outright majority in another. For this reason, electoral rules are not merely technical details but fundamental components of democratic governance.

The Spanish case illustrates this tension particularly well. Despite being formally classified as a proportional representation system, the Spanish electoral design has been repeatedly criticized for generating significant distortions between votes and seats. Several authors have long pointed to its biases: Rose (1983) argued that it is “more disproportional than majoritarian systems themselves,” while Soriano et al. (2001) went so far as to describe it as “the worst electoral system in Europe”. These critiques highlight the need to study not only the level of proportionality achieved under the Spanish system but also the institutional rules that drive its outcomes.

This thesis therefore focuses on the Spanish general elections and seeks to analyze how the translation of votes into seats is shaped by three core institutional dimensions: the apportionment method, the legal threshold, and the design of electoral constituencies. The central objective is to assess the extent to which the current system (based on the D’Hondt method and provincial electoral districts) deviates from proportionality, and to evaluate how alternative configurations might alter patterns of representation.

The analysis is structured around three guiding questions.

1. To what extent does the current Spanish system is a proportional system?
2. How would parliamentary representation and proportionality change under different apportionment methods, thresholds, or constituency designs?
3. How sensitive are seat allocations to small variations in vote shares?

Methodologically, the study adopts a computational approach. A set of modular functions in R has been developed to simulate electoral outcomes under a variety of rules, enabling systematic comparisons across all general elections held between 1982 and 2023. This makes it possible not only to isolate the effects of each electoral rule, but also to observe how these

rules perform under different electoral contexts, from the relative stability of two-party competition to the more fragmented multiparty configurations of recent years.

Ultimately, the empirical analysis relies primarily on official electoral results from the Spanish Ministry of the Interior, complemented with demographic and territorial data from the National Institute of Statistics (INE). To facilitate access to and processing of these sources, the study draws on a dedicated R package developed within a collaborative project supervised by Javier Álvarez Liébana. Within this framework, David Pereiro Pol was primarily responsible for shaping the package, while Mafalda González González and the author of this thesis contributed to its development and use in their respective analyses. The package itself constitutes a continuation and extension of the one created by Mikaela De Smedt in her Master's Thesis (see also the `{infoelectoral}` package by Héctor Meleiro (2018)).

2. LITERATURE REVIEW

2.1. Previous research on the Spanish case

A considerable body of authors have examined how the institutional design of the Spanish electoral system shapes disproportionality. Early contributions focused on counterfactual simulations of single elections to illustrate the weight of specific rules. For instance, Montero and Riera (2009), analyzing the 2008 general election, showed that modifying either the minimum seat allocation per province or the apportionment formula significantly altered patterns of representation. Their study demonstrated both the potential of computational simulations to inform debates on electoral reform and the limitations of static comparisons based on fixed vote distributions.

More recent work has extended this approach to subsequent contests. Sevilla Duro (2021), using the 2019 election, combined visualizations and reform scenarios to highlight how provincial magnitudes and electoral thresholds push outcomes toward majoritarian dynamics, even under fragmented multiparty competition.

Other studies have specifically targeted the role of constituencies. Mediavilla Retuerto (2016), using the 2015 election, recalculated seat distributions under a single national district and under autonomous community constituencies. His findings pointed to district magnitude as the main source of disproportionality. While a single national district without thresholds maximizes proportionality and small-party representation, it makes absolute majorities unlikely, reinforcing coalition dynamics. At the regional level, Bustos and Caballero (2023) simulated alternative formulas in Catalonia, showing that Sainte-Laguë or larger districts would have enabled additional parties (e.g., PDeCAT, Recortes Cero) to gain representation, underlining the sensitivity of outcomes to both formula and district design.

Taken together, this body of research identifies key drivers of disproportionality but remains limited in scope. Most studies focus on a single election or on isolated institutional parameters, and many present simulation outcomes without systematically analyzing the disproportionality underlying those results. What is still lacking is a systematic, longitudinal evaluation of Spanish general elections that compares alternative apportionment methods, thresholds, and constituency designs across all contests since 1982, using standardized indices of disproportionality to assess their effects.

2.2. Apportionment methods in comparative perspective

To address this gap, the present study applies eight widely used apportionment methods, covering the two principal families of proportional seat allocation: divisor methods (highest averages) and quota–remainder methods. Within the divisor family, *D’Hondt* (Jefferson) and *Webster/Sainte-Laguë* stand as the most common comparative standards, while *Adams*, *Dean*, and *Hill–Huntington* complete the classical set of rounding rules (ceiling, harmonic mean, and geometric mean, respectively) (Balinski & Young, 2001). In the quota–remainder family, the analysis includes *Hamilton* (Hare–Niemeyer) and *Hagenbach–Bischoff*, both widely regarded as reference variants. As a majoritarian contrast *First-Past-the-Post* (FPTP) is also incorporated. This selection follows established syntheses in both the mathematics of representation and comparative political science. A detailed overview of the functioning of these apportionment methods is provided in the Appendix.

2.3. Measuring proportionality

Finally, to evaluate the performance of these rules it is necessary to rely on standardized criteria of proportionality. Proportionality is assessed through established indices which make it possible to rank scenarios objectively by quantifying the distance between vote and seat shares. Two main types of indices are applied in this study. First, absolute difference measures, which capture the percentage of seats “misallocated” in aggregate terms (Karpov, 2008). Within this category, we include the Gallagher index (one of the most widely used measures in comparative literature) together with the Loosemore–Hanby index, Rae’s index, and the Lijphart index. Second, relative ratio measures, which assess over- and under-representation in proportion to vote shares and are particularly sensitive to the treatment of smaller parties that nonetheless gain representation (Karpov, 2008). Here we consider the Sainte-Laguë index and the D’Hondt index. Together, these indices provide a robust framework for comparing the proportionality of both the current system and the counterfactual simulations. A detailed description of the logic, properties, and interpretative logic of each index is provided in the Appendix.

3. METHODOLOGY

To address the research objectives, the methodological design relied on the creation of a set of modular functions in R that allow electoral outcomes to be simulated under a wide range of allocation methods, and subsequently evaluated in terms of proportionality using established disproportionality indices. The data were drawn from the `{pollspain}` package, through the function `summary_election_data()`, which retrieves aggregate electoral data for a given election and level of aggregation. For the purposes of this thesis, datasets were downloaded for all general elections between 1982 and 2023 and for all levels of aggregation (all, prov, and ccaa), depending on the constituency design required for each simulation. As previously noted, this package, and specifically this function, compiles information from the Spanish Ministry of the Interior.

3.1. Implementation of Apportionment Methods

The first step was the development of modular functions in R to implement the apportionment methods analyzed in this study. Each function was designed with the same structure of inputs and outputs, which guarantees comparability across elections and makes them adaptable to any dataset with equivalent information. The main inputs are party identifiers, vote totals, blank ballots (for thresholds and quotas), the number of seats in the district, and an optional electoral threshold (default 3%).

Regarding outputs, two formats are available. The short version provides a straightforward allocation, returning each party's votes and seats. The long version incorporates method-specific details: divisors and quotients in divisor methods (D'Hondt, Webster, Adams, etc.) or quotas and remainders in largest remainder methods (Hamilton, Hagenbach–Bischoff). This dual output structure allows both general comparisons across scenarios and more detailed inspections of how distortions arise within each method. These functions were developed in collaboration with David Pereiro Pol, who introduced modifications that enhanced their performance and reliability.

3.2. General simulation framework

Once the method-specific functions were defined, the next step was to design functions capable of applying them systematically to the different simulation scenarios considered in this study.

The central function for this purpose is `allocate_seats()`, which acts as a wrapper that delegates the apportionment logic to the appropriate method-specific function, depending on the argument specified. This function accepts the same parameters as the allocation methods, with the addition of the apportionment method to be applied, and returns a standardized result, allowing the same analytical framework to be applied regardless of the method selected.

This standardized function serves as the foundation for all the simulations carried out in the thesis, which were organized into four main groups:

- 1. Replication of the current Spanish system:** The first step was to reproduce the existing institutional design.. Since `allocate_seats()` only distributes a fixed number of seats (350) without accounting for their distribution across provinces, an additional function was developed: `allocate_seats_by_prov()`. This function automates the allocation process within each province, using the number of seats assigned to each district. To do so, it draws on a manually compiled table (`total_seats`) that specifies the number of seats corresponding to each province in every election. The function groups electoral data by province and year, calls `allocate_seats()` for each group, and then aggregates results to the national level by candidacy (`id_candidacies_nat`).
- 2. Threshold simulations:** To test the effect of electoral thresholds, the same procedure was applied using `allocate_seats_by_prov()` across provincial datasets, but varying the threshold parameter from 0% to 10%.
- 3. Constituency design:** Alternative district configurations were also tested while maintaining D'Hondt and the 3% threshold. For the national constituency, no additional function was required, and `allocate_seats()` was applied directly to the national-level dataset. For the regional constituency, a new function (`allocate_seats_by_ccaa()`) was created, mirroring the logic of `allocate_seats_by_prov()` but grouping data at the level of the autonomous communities. In this case, a table (`total_seats_ccaa`) was constructed to map provinces to their corresponding regions and compute the number of seats available in each autonomous community for each election year.

4. Alternative apportionment methods: Finally, all seat allocation formulas were applied while holding constant the provincial constituencies and the 3% threshold. This was done through `allocate_seats_by_prov()`, specifying the method parameter.

To deepen the analysis, additional cross-simulations were also performed, combining different institutional rules through the functions described above.

At this stage, the results are stored in lists, with each list corresponding to a different simulation scenario. Within each list, the outcomes are organized by election year, and each element follows the same standardized structure. An example of this output format is shown below.

Figure 1

Example of standardized output structure for seat allocation results (Hamilton method, 2023 election, provincial constituencies).

A tibble: 59 × 4

<code>id_candidacies_nat</code> <chr>	<code>abbrev_candidacies</code> <chr>	<code>total_ballots</code> <dbl>	<code>total_seats</code> <dbl>
000005	PP	8161117	121
000002	PSOE	7821777	111
000006	VOX	3057068	48
000010	SUMAR	3044983	38
000050	ERC	466020	7
000057	JXCAT-JUNTS	395429	6
000071	EH-BILDU	335129	6
000075	PNV	277289	4
000007	PACMA	169240	0
000065	BNG	153995	4

3.3. Calculation of disproportionality indices

The second stage of the methodology focused on evaluating these outputs through widely used indices of disproportionality. For this purpose, a dedicated function (`calculate_disproportionality_indices()`) was implemented. This function operates by:

- Converting both votes and seats into percentages.
- Excluding parties with zero votes to avoid distortions in ratio-based measures.
- Computing the six established indices

The function returns a list of values corresponding to each index, which is then applied systematically to every simulation dataset across all elections. The outputs are subsequently merged into a single long-format tibble, `all_results_long`, as shown in the image below:

Figure 2

Example of long-format output (`all_results_long`) storing disproportionality indices by election, simulation, and index type.

A tibble: 3,276 × 4

election <chr>	simulation <chr>	index <chr>	value <dbl>
2023	actual_system	Gallagher	5.66450857
2023	actual_system	Loosemore–Hanby	9.64153866
2023	actual_system	Sainte–Laguë	6.03322997
2023	actual_system	Rae's	0.32683182
2023	actual_system	Lijphart	5.81529272
2023	actual_system	D'Hondt	1.34062505
2019N	actual_system	Gallagher	6.35709568
2019N	actual_system	Loosemore–Hanby	11.45258167
2019N	actual_system	Sainte–Laguë	9.16918944
2019N	actual_system	Rae's	0.34186811

3.4. Robustness analysis

The final part of the methodology focused on testing the robustness of apportionment methods, that is, their stability when confronted with small changes in vote shares.

To test this dimension, the analysis builds on the actual provincial-level results of each election and applies a statistical simulation procedure. Based on the observed vote shares of each party (`porc_candidacies_valid`), alternative vote distributions are generated using the Dirichlet distribution. This choice is particularly suitable because it guarantees two essential properties of simulated electoral data: values always remain between 0 and 1, and their sum is exactly 100 percent. In practice, this means that the simulated votes represent plausible perturbations of the real results, capturing the inherent uncertainty of any electoral process.

From these simulated proportions, ballots are reconstructed and processed under a subset of seat-allocation functions defined earlier (D'Hondt, Webster, Adams, Hagenbach–Bischoff, and Hamilton). These methods were selected because, as shown in the results analysis, they consistently rank among the least disproportional options. The wrapper function

`assign_seats()` transforms each simulated vector into ballots and applies the selected apportionment rule, while `process_province_simulations()` iterates this procedure across provinces and repetitions, producing matrices of seat allocations by party. For each election and province, 1,000 simulated scenarios were generated (a number chosen due to computational constraints, as the procedure required more than 15 hours to complete; ideally, 10,000 would provide a finer picture of variability).

This framework makes it possible to examine whether seat allocations remain consistent (meaning that parties' seat totals are relatively stable across simulations) or whether they fluctuate substantially in response to small changes in vote shares.

All of the code mentioned can be found in the following GitHub repository:

https://github.com/irenebosque25/Master_Thesis/

4. RESULTS ANALYSIS

This section discusses the results of the simulations. It should be noted that a substantial amount of information has been condensed in order to comply with the space constraints of this thesis. For this reason, in cases where simulations generated an excessive number of components for comparison, we focused exclusively on one of the indices under study, specifically, the Gallagher index, given its widespread use in the literature.

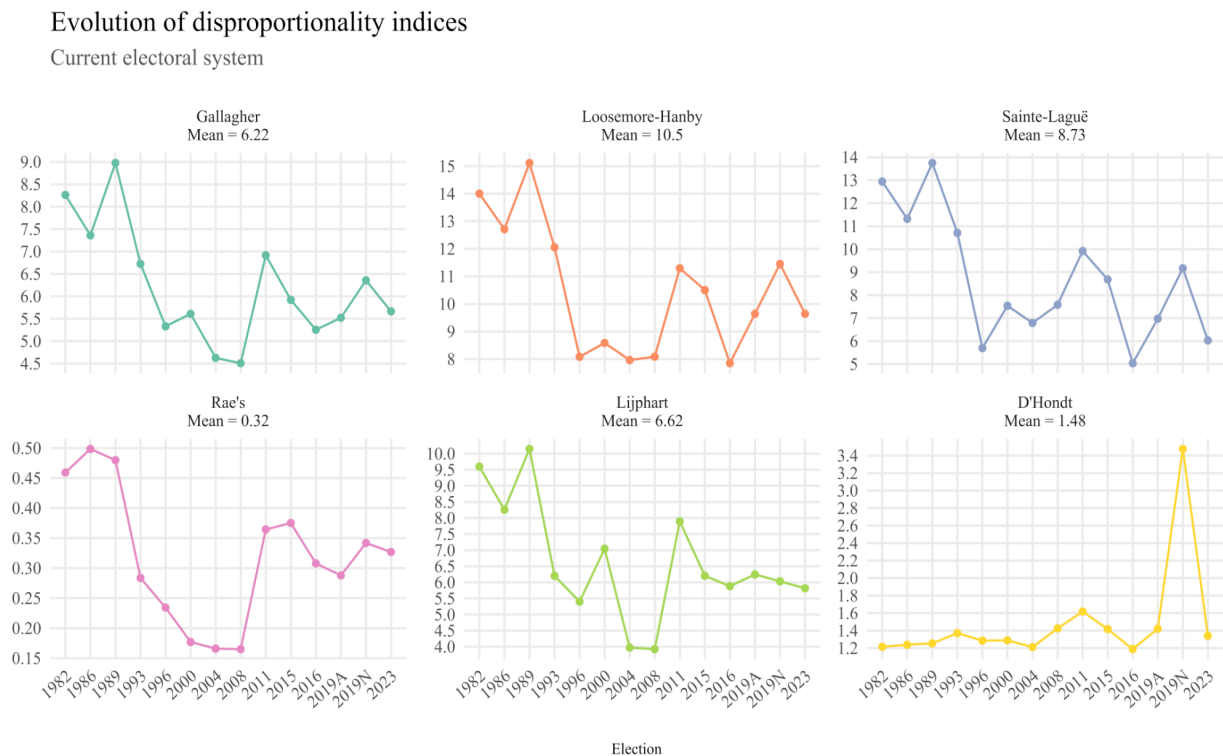
4.1. Proportionality of the Current Spanish Electoral System

The first step in the empirical analysis is to examine how the Spanish electoral system functions in practice. The objective of this section is twofold: to evaluate the extent to which the institutional design ensures a proportional translation of votes into seats across the entire history of democratic elections in Spain, and to identify the main distortions that emerge from this process.

The evolution of disproportionality across elections is summarized in Figure 3.

Figure 3

Evolution of disproportionality indices under the current electoral system (1982–2023).



The Gallagher index, one of the most widely used measures in the literature, yields an average value of around six. This places Spain far from the most proportional democracies, such as Sweden (≈ 0.64), Denmark (≈ 1.16), or Switzerland (≈ 3.6), but not among the most disproportional either, since its score remains slightly below the European average (≈ 7.88) (Gallagher, 2024).

Other indices, however, reveal more pronounced distortions. The Loosemore–Hanby index, averaging 10.5, and the Sainte-Laguë index, close to nine, situate Spain well above countries like Germany (≈ 4.93 and 5.37), Sweden (≈ 3.67 and 3.73), or Portugal (≈ 7.74 and 5.91) (Martínez-Panero et al., 2019). Their higher values suggest that disproportionality in Spain is not only persistent but also more severe when measured in terms of aggregate deviations or relative ratios, particularly affecting smaller or regionally concentrated parties.

The evolution of disproportionality in Spain follows a three-phase trajectory. Between 1982 and 1996, all indices register consistently high levels. This reflects the predominance of small district magnitudes combined with a party system in which PSOE and PP were the dominant forces but coexisted with several medium and minor state-wide parties, such as IU and CDS, whose dispersed vote shares translated poorly into seats. From 2000 to 2008, disproportionality decreases markedly. During this period of stable two-party competition, the provincial application of the D’Hondt method appeared unusually proportional. Since the vast majority of votes were concentrated in PSOE and PP, their conversion into seats was highly accurate. From 2011 onwards, disproportionality rises again and becomes more volatile. A noticeable increase occurs in 2011, and from 2015 the return of a fragmented multiparty system drives disproportionality upwards once more as new medium and small parties re-enter the electoral arena.

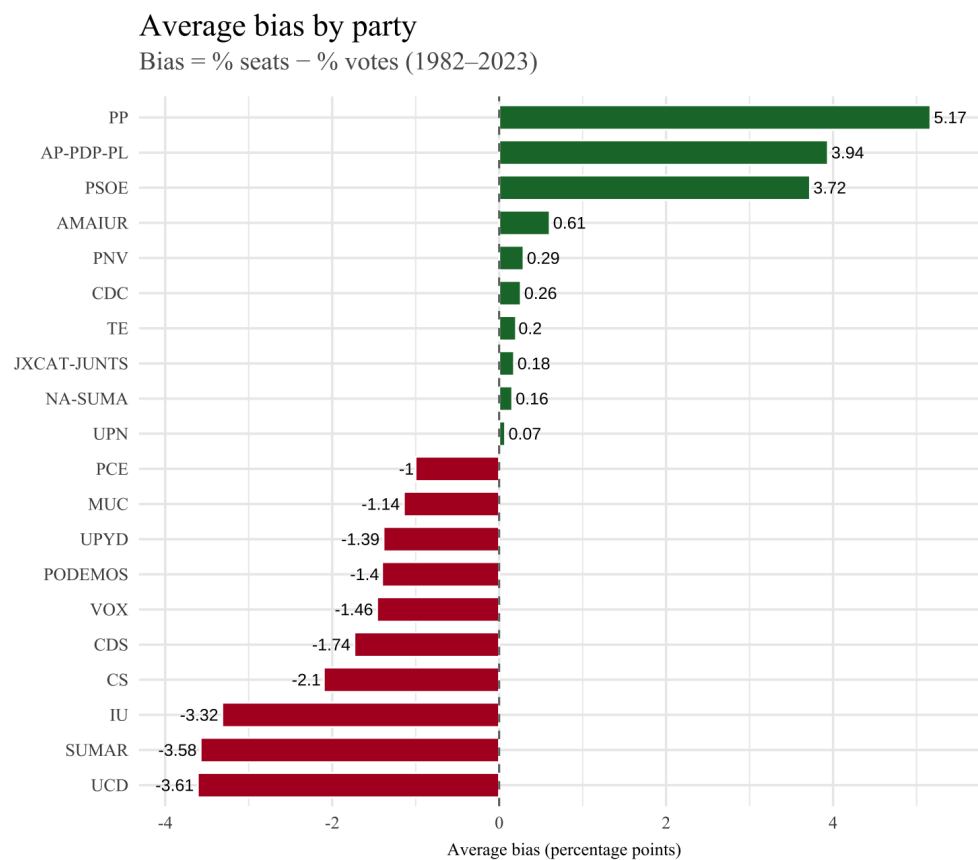
Differences in the sensitivity and interpretation of the indices are particularly evident in the case of D’Hondt. Although this measure usually displays limited variation across elections, it highlights an exceptional outlier in November 2019, when *Teruel Existe* obtained a seat with just 19,761 votes. By contrast, earlier cases of overrepresentation required substantially higher vote totals, such as the Regionalist Party of Cantabria in April 2019 (52,266 votes) or the PNV in 2016 (57,402 votes). This illustrates the structural disproportionality of the system: in the same 2019 election, Ciudadanos required approximately 165,000 votes per seat.

4.1.1. Winners and losers under the current system

To move beyond the aggregate indices, Figure 4 focuses on the parties most consistently advantaged and disadvantaged by the system between 1982 and 2023.

Figure 4

Average seat–vote bias by party under the current electoral system (1982–2023), showing the ten parties with the highest and lowest mean bias.



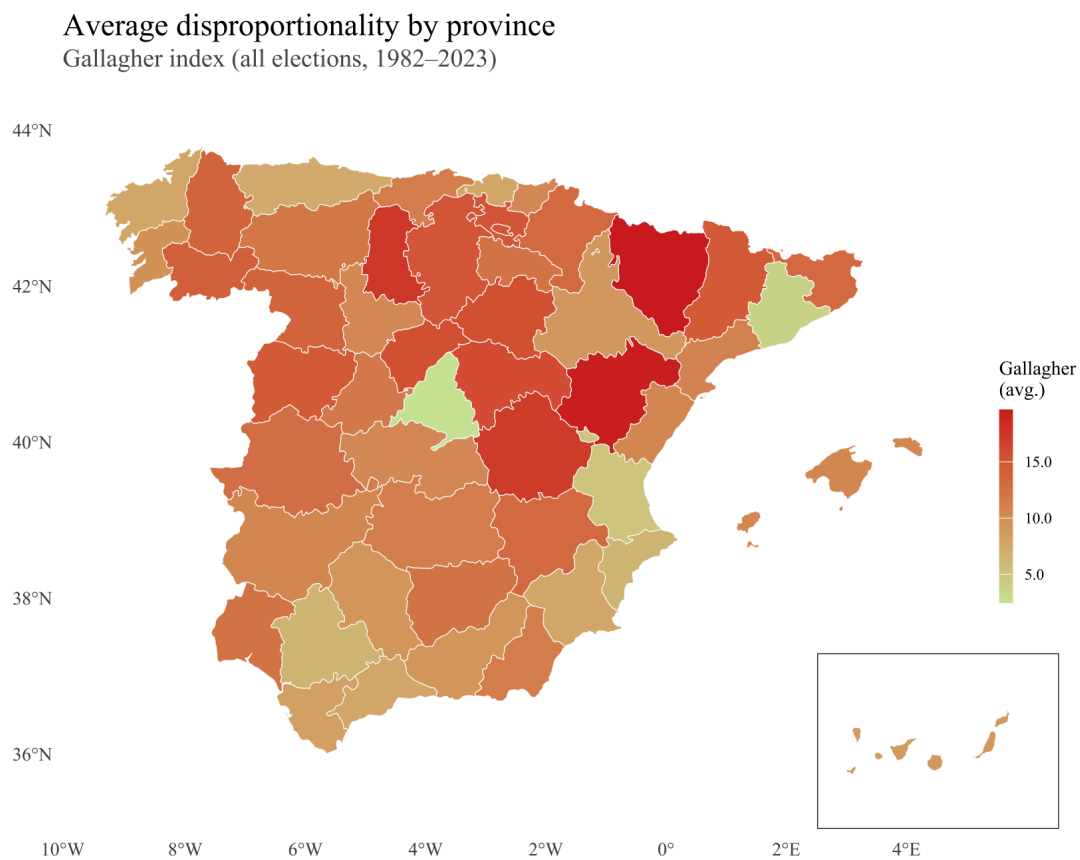
The results reveal a consistent pattern. The two major state-wide parties, PP with an average bias of 5.2 points and PSOE with 3.7, are systematically overrepresented, gaining seats beyond their vote share. Several regionally based parties with territorially concentrated support, such as PNV, CiU/CDC, or JxCAT in specific periods, also enjoy modest advantages since geographically clustered votes can be more effectively transformed into representation. In contrast, medium-sized state-wide formations are systematically penalized: IU, UCD, Ciudadanos, Vox, and Podemos all recorded negative mean biases, reflecting how their relatively dispersed support made it difficult to secure proportional representation.

4.1.2. Disproportionality by constituency

To capture the territorial variation embedded in the Spanish electoral system, Figure 5 maps average disproportionality across provinces using the Gallagher index for all general elections held between 1982 and 2023. Ceuta and Melilla are excluded, since their single-member nature makes proportionality unattainable and would distort comparisons.

Figure 5

Average disproportionality by province (Gallagher index, 1982–2023).



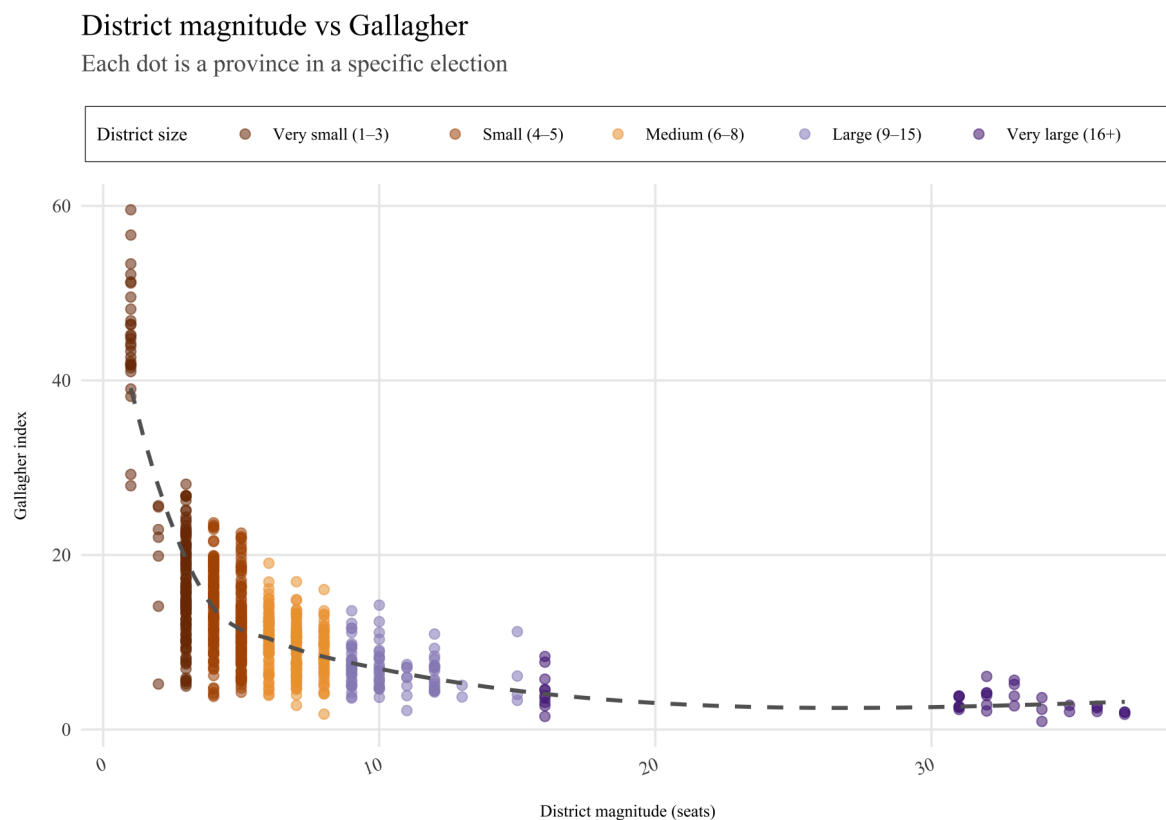
The highest disproportionality is concentrated in small provinces of the interior, such as Huesca, Teruel, Cuenca, Palencia, Ávila or Guadalajara,. These constituencies register average Gallagher values above 10 points, i.e., winning representation requires very high vote share, reflecting almost majoritarian conditions. By contrast, the lowest disproportionality is found in the largest provinces, especially Madrid, Barcelona, Valencia and Sevilla, which

remain around or below 5 points on average and allow a much closer translation of votes into seats.

However, with the exception of these outliers, most medium-sized provinces cluster around relatively similar values of disproportionality. To examine how district size shapes these differences, the following figure plots the relationship between district magnitude (the number of seats at stake) and disproportionality.

Figure 6

Relationship between district magnitude and disproportionality (Gallagher index, provincial level, 1982–2023)



We observe that the shape of the curve is non-linear. The decline in disproportionality is much steeper when moving from very small districts to medium-sized ones, while the additional gains in proportionality diminish beyond a certain threshold (around 10–12 seats). In other words, the relationship resembles an exponential decay rather than a constant proportional change.

4.2. Simulations

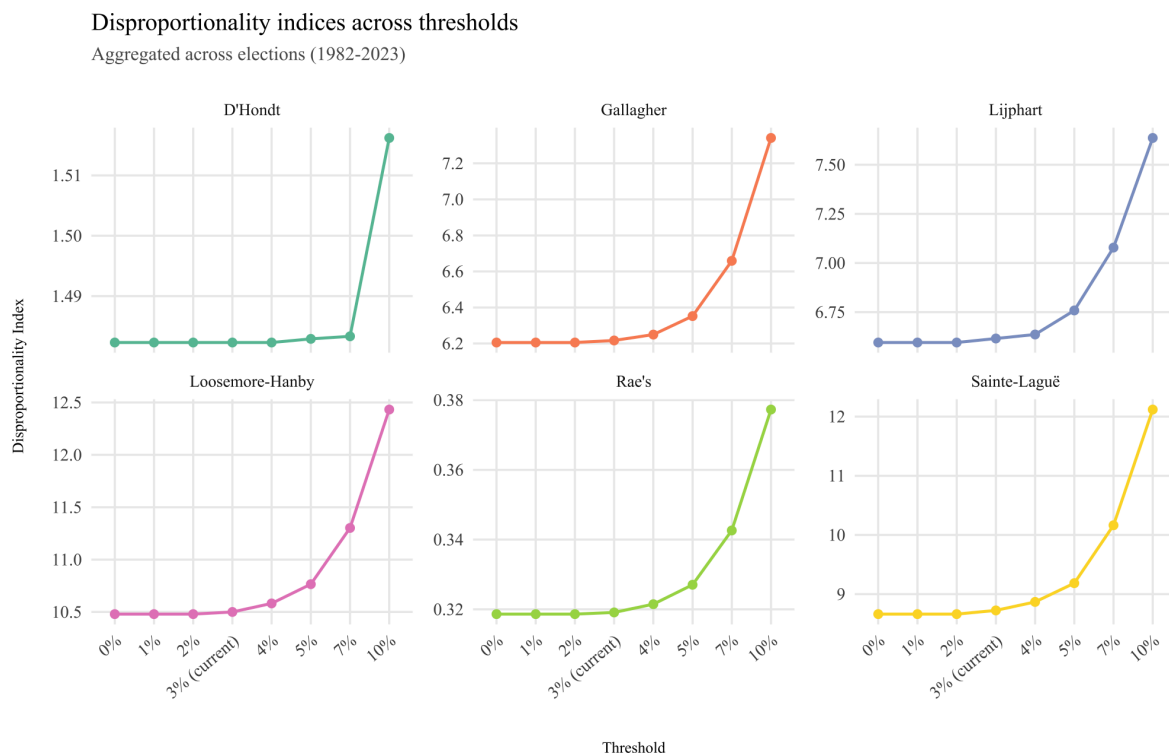
Having examined the current system under its existing rules, the analysis now turns to a series of simulations that test how outcomes would change if key elements of the electoral design were altered.

4.2.1. Threshold Effect

This simulation explores how different legal thresholds influence disproportionality, varying the cutoff between 0% and 10% while keeping both the d'Hondt formula and provincial constituencies constant. Figure 7 plots the evolution of the main indices under these scenarios, allowing us to assess whether thresholds play a decisive role.

Figure 7

Disproportionality indices across electoral thresholds

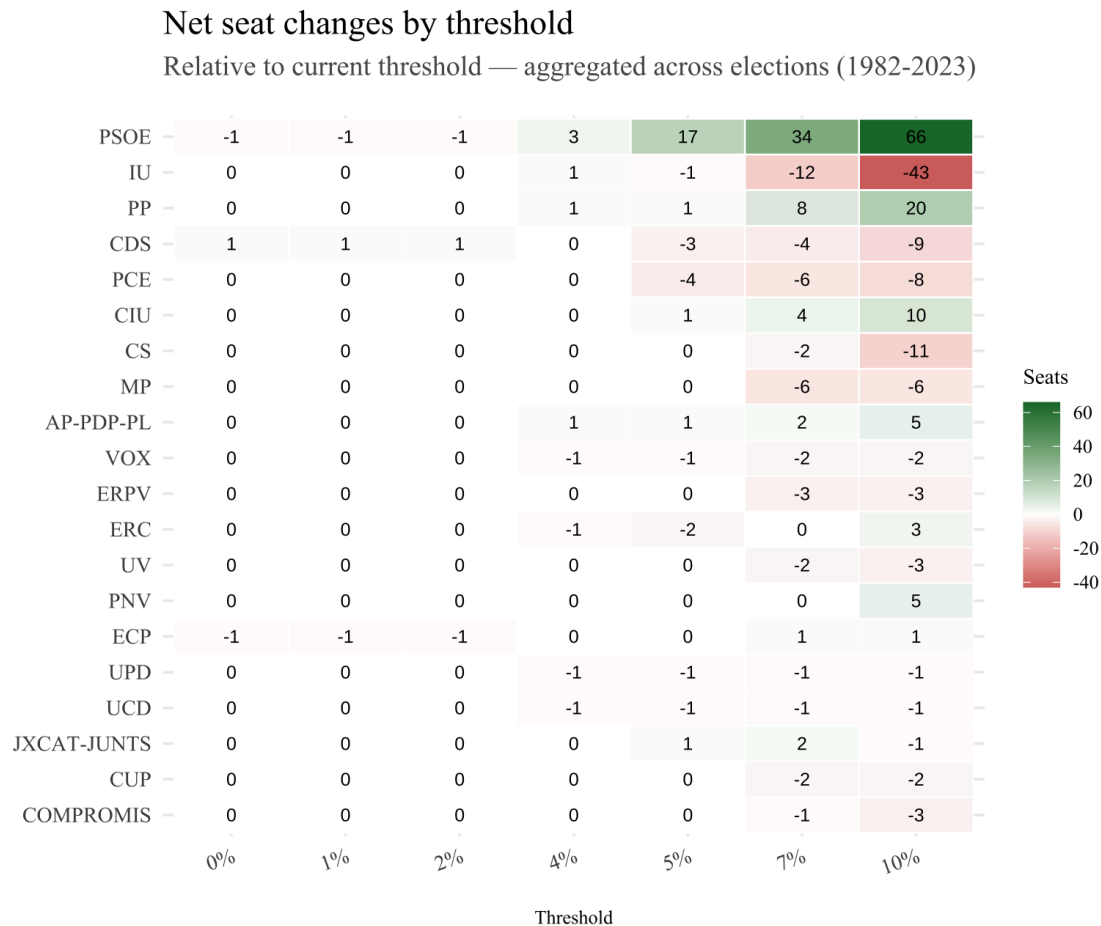


The results show remarkable stability up to the 5% level across all indices. Clear increases are only visible at very high thresholds, confirming that moderate changes have little influence on overall disproportionality.

To complement the previous analysis, Figure 8 illustrates the distributional consequences by party, showing net seat changes relative to the current 3% rule¹.

Figure 8

Net seat changes by party under alternative thresholds, relative to the current 3% rule (1982–2023).



As observed in the figure, thresholds below 5% have virtually no effect, while higher ones disproportionately benefit PSOE and PP at the expense of medium-sized state-wide such as IU or Ciudadanos.

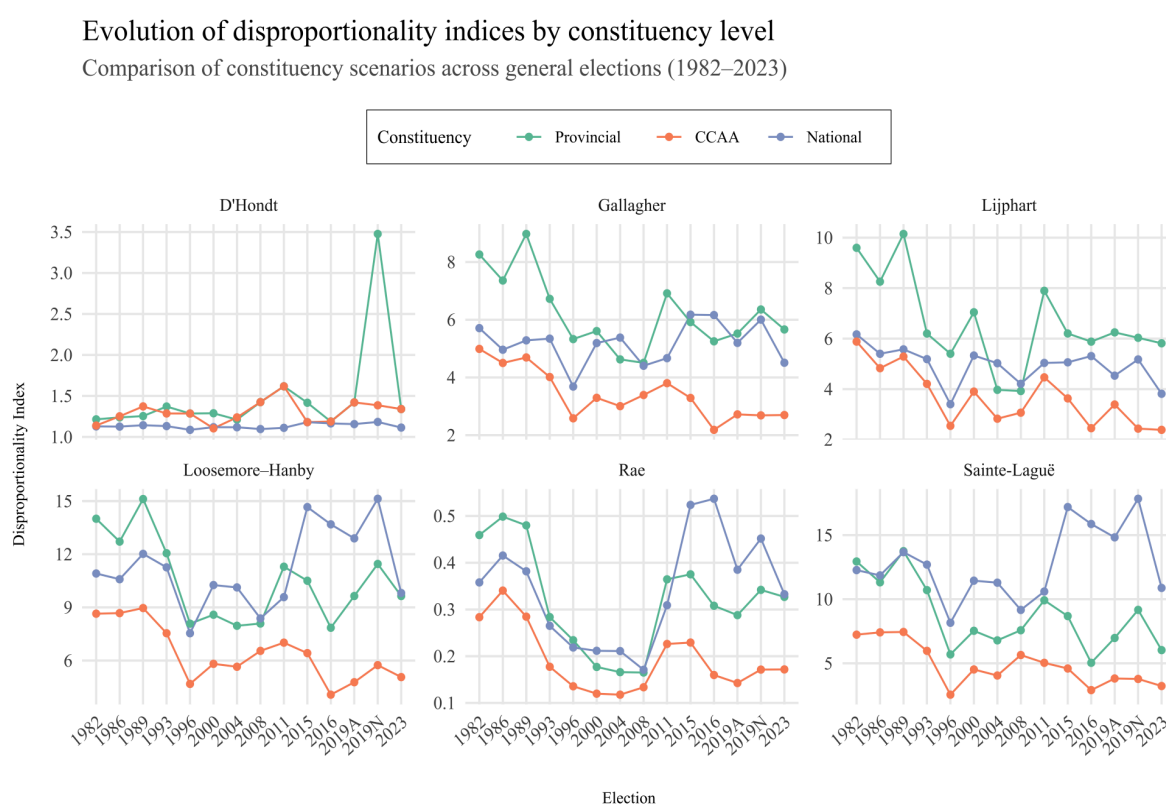
¹ For clarity, only the twenty parties most affected are shown.

4.2.2. The effect of constituency type

To observe how election results would change under different constituency designs, Figure 9 shows the evolution of disproportionality indices between 1982 and 2023 in three scenarios: provincial constituencies (the current system), regional constituencies at the level of the autonomous communities (CCAA), and a single nationwide constituency.

Figure 9

Evolution of disproportionality indices under alternative constituency designs (1982–2023).



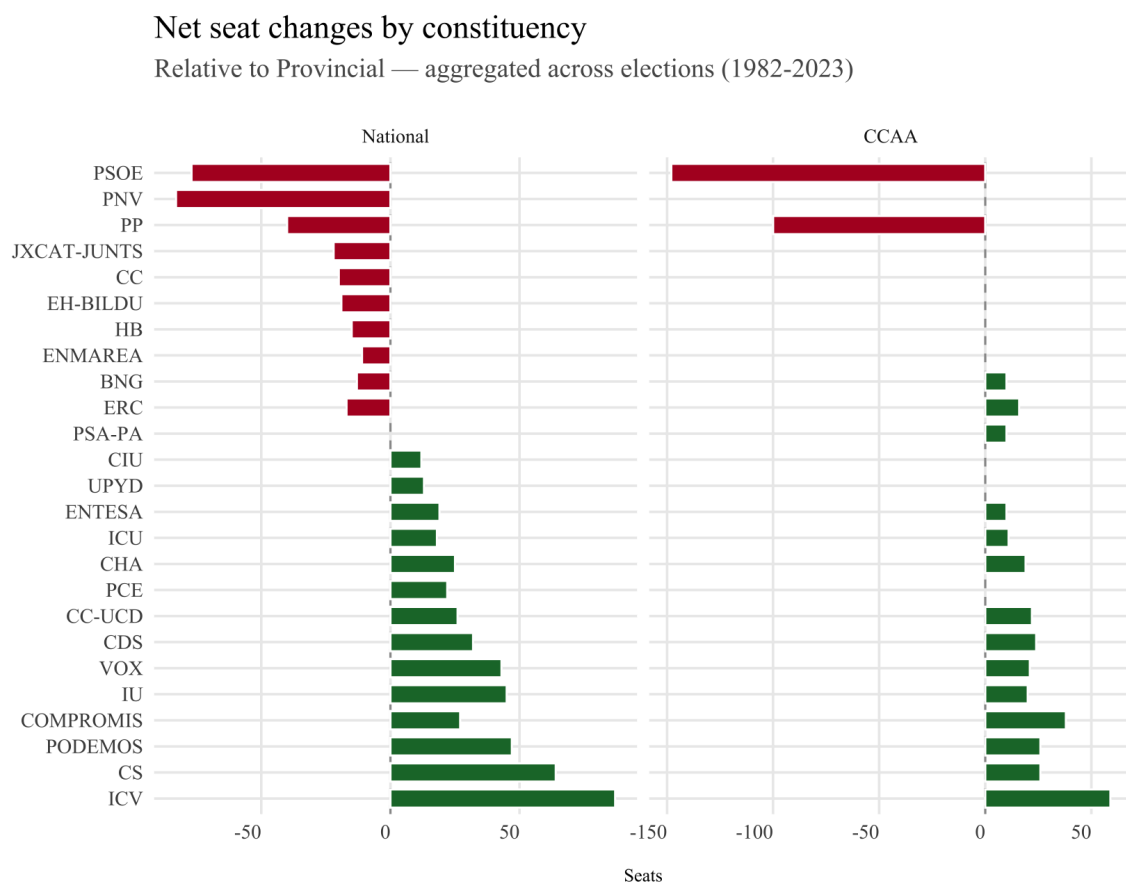
The results show that regional constituencies consistently yield the lowest levels of disproportionality across all measures. By contrast, the relative position of provincial and national constituencies varies depending on the index and the historical period. For Gallagher and Lijphart, the provincial model produces higher disproportionality in almost every election. In other indices, however, the comparison shifts over time: during the first democratic decades up to 1996, the simulations suggest that disproportionality is higher under the provincial system than under a national constituency, whereas after 2015 the pattern

reverses: in a fragmented multiparty context, the nationwide district generates more distortional outcomes, as it increasingly penalizes smaller forces.

To complement the index-based analysis, Figure 10 illustrates how seat allocations would shift under the constituency simulations².

Figure 10

Distributional effects of alternative constituency designs: net seat changes by party (1982-2023)



Under a national constituency, the major state-wide parties (PSOE and PP) lose a substantial number of seats, as their structural advantage in provincial districts disappears once votes are aggregated nationwide. The most severely affected, however, are regionalist forces such as PNV, Junts, CC, or EH Bildu, which lose the territorial premium that provincial districts provide. By contrast, medium and small state-wide parties such as IU, Podemos, Ciudadanos, or Vox emerge as the main winners, as dispersed support can be more effectively converted into representation once territorial barriers are removed.

² For clarity, only the parties showing the largest changes relative to the provincial system are included.

In the autonomous communities constituency scenario, the dynamics differ. Regionalist parties are largely unaffected (and in some cases even benefit, as with ERC or BNG) because their support remains concentrated within the boundaries of their home regions. Instead, the main losers are PSOE and PP, which across all elections would forfeit more than 100 seats. The beneficiaries, once again, are medium and small state-wide parties, whose dispersed vote shares translate into significantly higher levels of representation.

Having examined thresholds and constituency design separately, the next step is to consider their interaction. Figure 11 combines both dimensions, showing how different threshold levels operate under alternative constituency types. This allows us to evaluate whether their effects simply add up or whether they generate distinct patterns of disproportionality when applied together.

Figure 11

Gallagher Index by Threshold and Constituency			
Aggregated across all elections (1982–2023)			
Threshold	Gallagher Index (↓ better)		
	Provincial	CCAA	National
0%	6.22	3.42	5.19
1%	6.21	3.32	2.71
2%	6.21	3.35	4.30
4%	6.25	3.53	6.40
5%	6.35	3.97	8.01
7%	6.66	4.89	9.64
10%	7.34	6.58	11.86

While earlier sections showed that thresholds alone have only limited effects, their impact varies sharply depending on the territorial framework. In both provincial and regional scenarios, disproportionality increases only moderately even at high thresholds and never reaches critical levels. By contrast, under a national constituency thresholds have a much stronger effect: at 1% the system is extremely proportional, but from 4% onwards

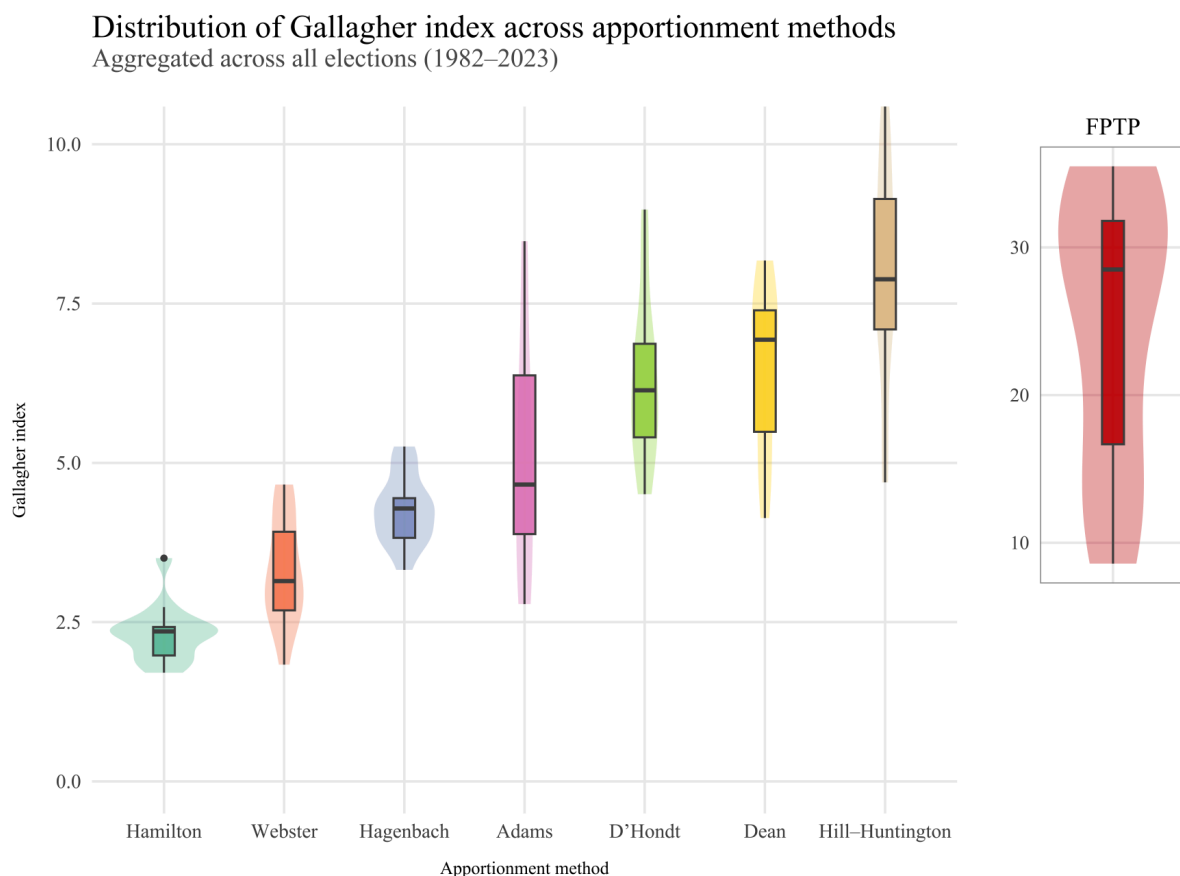
disproportionality rises steeply. With thresholds of 7% and 10%, a substantial number of parties would be excluded from representation making the national model far more sensitive to this rule.

4.2.3. The effect of apportionment method

Lastly, we turn to the role of apportionment methods. To assess how alternative seat allocation formulas shape electoral outcomes, Figure 12 presents the distribution of the Gallagher index under different methods, while keeping the provincial district design and the 3% threshold constant.

Figure 12

Distribution of disproportionality across apportionment methods (Gallagher index, 1982–2023).



The results reveal notable differences. Hamilton and Webster emerge as the most proportional methods, with average Gallagher values around 2–3 points. Hagenbach-Bischoff and Adams occupy an intermediate level (4–5 points), still providing relatively high proportionality. By

contrast, d'Hondt, Dean, and Hill–Huntington register the highest values within the proportional group (5–7 points on average), reflecting their systematic bias in favor of the largest parties.

The case of First Past the Post (FPTP) is radically different: with Gallagher values above 30, its disproportionality is so extreme that it must be represented on a separate scale to avoid distorting the rest of the graph. This outcome confirms the majoritarian nature of FPTP, in which the relationship between votes and seats is almost entirely broken compared to proportional methods.

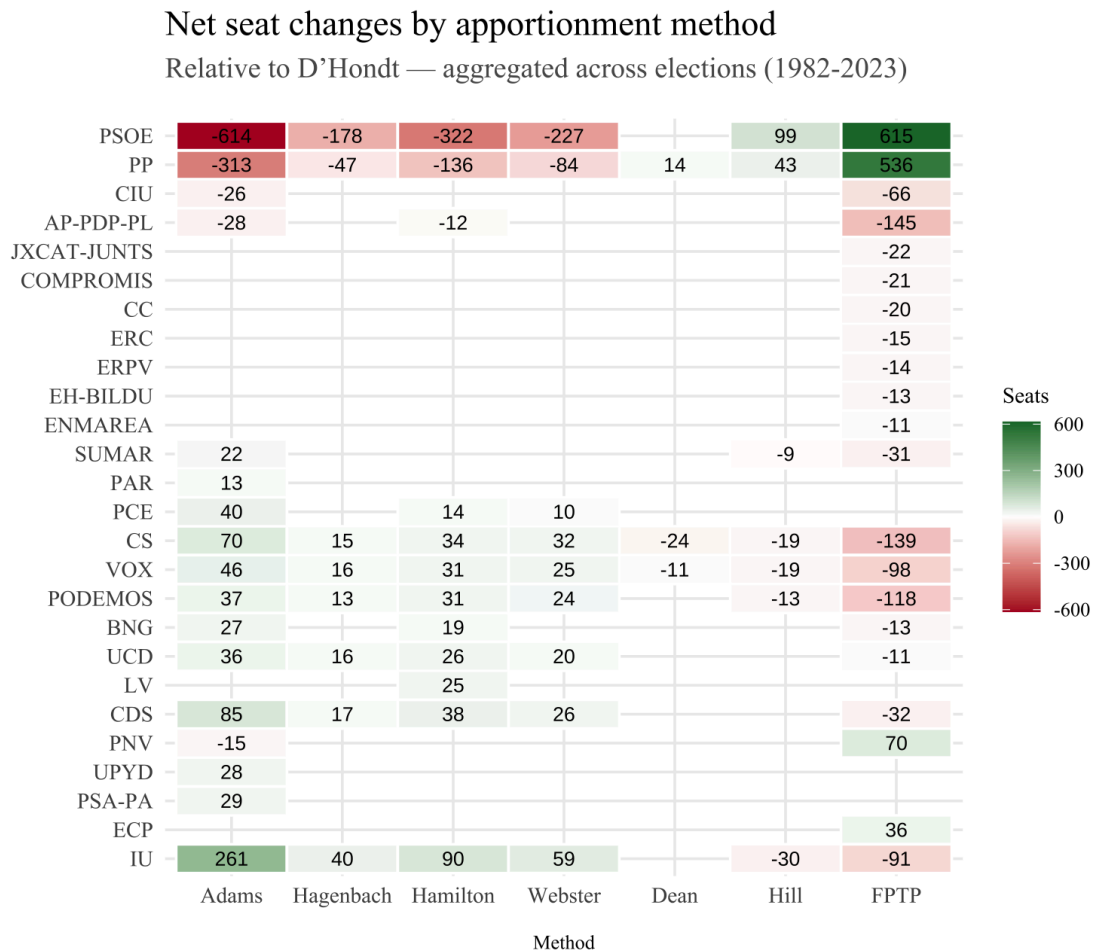
It is also worth noting the differences in the length of the distributions: methods such as Adams, Dean, and Hill–Huntington display wider tails, indicating greater sensitivity to specific political contexts. In contrast, Hamilton and Hagenbach-Bischoff exhibit more compact distributions, suggesting that their performance remains more stable across different electoral scenarios.

Figure 13 illustrates these dynamics by showing the net seat changes for each party between 1982 and 2023 under alternative formulas, compared to d'Hondt³.

³ For clarity, only the parties with the largest differences are included.

Figure 13

Net seat changes by party under alternative apportionment methods, relative to D'Hondt (1982–2023).



The major state-wide parties (PSOE and PP) are the principal losers under the more proportional methods. Under Adams, PSOE loses more than 600 seats in total and PP more than 300, with similarly large reductions under Hamilton, Hagenbach, and Webster. By contrast, Dean produces more moderate effects, while Hill–Huntington and especially FPTP work in the opposite direction, generating substantial seat gains for the largest parties. Notably, although Adams does not appear as the most proportional method in the Gallagher index distribution, it is by far the formula that most penalizes PSOE and PP while simultaneously boosting medium-sized competitors.

The main beneficiaries of the proportional methods are medium and small state-wide parties. Izquierda Unida (IU) is the most striking case, with over 260 additional seats under Adams and significant gains under Hamilton, Hagenbach, and Webster. Podemos, Ciudadanos, and Vox also benefit markedly from these methods. Dean and Hill–Huntington, however, limit or even reverse these gains, particularly for Ciudadanos and Vox, and in the case of Hill also for left-wing forces such as IU and Podemos. FPTP once again stands out for its extreme effects, depriving state-wide minor parties, especially Ciudadanos, Vox, Podemos, and IU, of virtually all their representation.

By contrast, regionalist parties are far less affected by changes in apportionment method. Their seat shares remain relatively stable across formulas, suggesting that their representation is shaped more decisively by other institutional rules than by the choice of apportionment method.

It is also important to examine how the apportionment methods interact with the rest of the system’s rules, particularly the choice of constituency design which is the most impactful rule as we’ve seen in previous analysis. The methods included are those that rank among the most proportional overall.

Figure 14

Gallagher Index by Mixed Rules (Method × Constituency)						
Aggregated across all elections (1982–2023)						
System	Level			Improvement vs D’Hondt		
	Mean Gallagher	Median Gallagher	SD	Mean Δ vs D’Hondt	Best Δ	Worst Δ
National – Hagenbach–Bischoff	0.43	0.41	0.10	–5.84	–8.58	–3.95
National – Hamilton	0.48	0.44	0.15	–5.80	–8.53	–3.83
CCAA – Hamilton	1.09	1.22	0.30	–5.19	–7.46	–3.39
CCAA – Adams	1.69	1.62	0.35	–4.59	–6.75	–3.05
CCAA – Hagenbach–Bischoff	1.70	1.66	0.29	–4.58	–6.80	–2.86
CCAA – Webster	2.08	1.92	0.62	–4.20	–5.94	–2.26
National – Adams	5.01	5.08	0.73	–1.27	–3.89	0.77
National – Webster	5.09	5.12	0.74	–1.19	–3.76	0.91
Lower = more proportional. Δ < 0 improves over D’Hondt.						

The simulations highlight that a national constituency combined with Hagenbach–Bischoff or Hamilton produces the lowest levels of disproportionality, approaching perfect proportionality and remaining highly consistent across elections. Regional constituencies also generate substantial improvements, though with greater variation depending on the method. By contrast, some combinations, such as Adams or Webster at the national level, perform less well, occasionally even matching or exceeding the disproportionality of d’Hondt.

Even so, all alternatives represent a marked improvement over the current provincial d’Hondt system. National–Hagenbach–Bischoff and National–Hamilton reduce the Gallagher index by nearly six points on average, while regional alternatives cut it by four to five.

4.2.3.1. The robustness of different appointment methods

As a final step, the analysis turns to the robustness of different apportionment methods, understood as the stability of seat allocations when subjected to small changes in vote shares. This dimension is important because proportionality alone does not capture how resilient an electoral rule is to electoral volatility: a system may translate votes into seats in a proportional way, but if small fluctuations in support lead to large swings in representation, its performance can be considered fragile.

To examine this, 1,000 vote-perturbation simulations were carried out for a selection of Spanish general elections: two from the more bipartisan context of 1989 and 2000, and two from the fragmented multiparty context of 2016 and November 2019. In all cases, the provincial constituency design and a 3% threshold were held constant, so as to isolate the effect of the apportionment method.

Figure 15

Aggregate Robustness by Apportionment Method				
Lower dispersion ⇒ higher robustness (1,000 vote-perturbation simulations per election)				
Method	Total SD	Mean SD	Max SD	Total IQR (p95–p05)
D'Hondt	45.790	0.153	2.377	134.00
Webster	46.681	0.156	2.194	137.00
Adams	49.931	0.166	2.020	145.00
Hagenbach–Bischoff	50.959	0.170	2.054	143.05
Hamilton	54.214	0.181	2.129	155.05
SD = standard deviation of seats; IQR = width between the 95th and 5th percentiles.				

Grounding the exercise in historical electoral data allows us to place robustness in perspective. Previous analyses showed that D'Hondt is among the least proportional methods, and that almost any alternative combination of constituency and formula outperforms it. Yet, when applied to four of the key elections in Spain's democratic history, D'Hondt emerges as the most robust.

The analysis highlights a clear trade-off: methods such as Hamilton, Hagenbach–Bischoff, and Adams increase proportionality but reduce robustness, while D'Hondt provides greater stability at the expense of less proportional outcomes. Webster, in particular, offers an intermediate solution, combining relatively high proportionality with a strong degree of resilience.

5. CONCLUSION

This thesis has provided a comprehensive assessment of how Spain's electoral rules shape the translation of votes into seats. Although the system is formally proportional, evidence from all general elections between 1982 and 2023 shows persistent distortions. These systematically benefit the two main state-wide parties (PSOE and PP) and, though less consistently, regionalist parties with concentrated support, while disadvantaging medium-sized state-wide forces (IU, Ciudadanos, Podemos, Vox) whose dispersed electorates convert poorly into seats. The decisive driver of these imbalances is district magnitude: small provinces operate almost like majoritarian districts, with Gallagher values often above 10, and proportionality gains taper off once magnitudes exceed 10–12 seats.

Beyond this structural feature, the simulations highlight important interactions among institutional rules. Thresholds matter little in provincial or regional frameworks but become highly consequential under a national constituency, where disproportionality rises sharply once thresholds exceed 4–5%. Apportionment methods also reveal a proportionality–robustness trade-off: quota–remainder rules such as Hamilton and Hagenbach–Bischoff approach near-perfect proportionality, especially at the national level, but are prone to volatility. D'Hondt, by contrast, consistently ranks among the least proportional methods yet proves the most robust when applied to Spanish electoral history. Webster emerges as the best compromise, curbing over-representation while preserving stability.

A key contribution of this thesis lies in its longitudinal scope. By systematically applying disproportionality indices to all general elections between 1982 and 2023, the analysis shows how institutional rules interact with party-system dynamics: high disproportionality in the 1980s, a decline and stabilization during the two-party era of the 1990s and 2000s, renewed increases after 2011, and oscillations in the fragmented multiparty system since 2015. In this perspective, constituency design proves especially sensitive to political context: regional constituencies consistently reduce disproportionality without undermining regionalist parties, while a single national district improved proportionality in the earlier bipartisan decades but, in today's fragmented context, tends to exacerbate distortions by penalizing smaller forces. These findings underline that institutional reforms cannot be evaluated in isolation; their effects are conditional on the broader sociopolitical environment.

Nonetheless, some limitations remain. Robustness checks were restricted to a subset of methods-design combinations and to a limited number of simulated scenarios due to computational constraints. These restrictions do not alter the overall rankings of apportionment methods or constituency designs, but they do narrow the scope of the sensitivity tests and prevent a fully exhaustive evaluation of volatility under all possible configurations. Moreover, by privileging the Gallagher index as the main benchmark, the study has offered a standardized basis for comparison but left less room to explore in depth the specific interpretive nuances of alternative disproportionality indices.

Future research could build on these contributions in several ways. First, expanding robustness analyses to cover the full set of electoral rules and all general elections would provide a more comprehensive picture of how stability interacts with proportionality across contexts. In addition, the simulations used here are static in that they assume fixed party strategies. Future work could integrate behavioral responses such as strategic voting, party entry, or coalition formation, thereby producing counterfactuals that more closely approximate the dynamics of real electoral competition.

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APPENDIX A. APPORTIONMENT METHODS

This appendix provides a detailed overview of the apportionment methods applied in the study. Eight formulas are considered, grouped into two main families of proportional seat allocation together with a majoritarian benchmark for contrast. The following sections summarize the logic and procedure with illustrative references to their practical use in different countries.

A.1 Divisor methods (highest averages)

D'Hondt (Jefferson)

This method assigns seats by dividing each party's vote total by a sequence of divisors (1, 2, 3, ...) and awarding seats to the highest resulting quotients until all seats are filled (Kotanidis, 2019). Equivalently, it can be understood as using a common divisor and rounding down (floor). The D'Hondt formula systematically favors larger parties, especially in small-magnitude districts, but is widely adopted due to its monotonicity and consistency properties (Schuster et al., 2003). It is one of the most widely used apportionment methods worldwide, applied in countries such as Belgium, Portugal, and Finland.

Webster / Sainte-Laguë

The Webster (or Sainte-Laguë) method follows the same logic as D'Hondt, but instead of dividing by 1, 2, 3, 4, ... it uses odd numbers as divisors: 1, 3, 5, 7, This reduces the structural advantage of large parties and is generally regarded as the most "size-neutral" divisor rule (Schuster et al., 2003). Countries applying this method include Germany, Sweden, and New Zealand.

Adams

Adams's method divides each party's votes by a modified divisor and then rounds all quotas upward to the next integer (ceiling rule). In practice, the divisor is adjusted iteratively, just as in Jefferson's method, until the sum of seats equals the house size. Because every quota is rounded up, the standard divisor initially yields a total that is too large, requiring recalibration (LibreTexts, 2022). It is rarely applied in practice but remains relevant in theoretical and comparative analyses.

Dean

Dean's method uses the harmonic mean between two consecutive integers as the rounding threshold. If a party's quota exceeds this harmonic mean, the seat is rounded up; if not, it is rounded down (Curley, 2020). Like Adams, Dean is not used in practice but remains of academic interest for comparative analyses of divisor rules.

Hill–Huntington

The Huntington–Hill method, also called “equal proportions,” uses the geometric mean of two consecutive integers as the rounding threshold. If the quota exceeds the geometric mean, it is rounded up; otherwise, it is rounded down (U.S. Census Bureau, 2024). Adopted in the United States since 1941 for the apportionment of House seats among the states, it remains specific to the U.S. context, where a fixed chamber size of 435 representatives has applied since 1929 (Feature Column, 2020).

A.2. Quota–remainder methods (largest remainders)

Hamilton (Hare–Niemeyer)

Hamilton's rule first assigns each party the integer part of its Hare quota, defined as total votes divided by total seats, and then allocates remaining seats to the parties with the largest fractional remainders. This method strongly adheres to quotas and benefits medium and small parties, but it can generate paradoxes such as the Alabama paradox, in which a party may lose seats if the size of the assembly increases (Balinski & Young, 2001). Historically, it has been used in the United States and in some Russian elections.

Hagenbach–Bischoff

The Hagenbach–Bischoff system applies a modified version of the quota method. Each party is first allocated seats based on how many times its vote total contains this quota, keeping only the integer part of the result. Any remaining seats are then distributed to the parties with the largest unused vote remainders (Pukelsheim, 2017). Switzerland is one of the few countries that has applied this rule in practice.

A.3 Majoritarian benchmark

First-Past-the-Post (FPTP).

As a contrast, the study also considers the plurality rule applied in single-member districts. The candidate or party with the most votes wins the seat, regardless of whether they secure a majority. FPTP is not proportional and systematically favors geographically concentrated support, typically inflating representation of the largest parties while penalizing dispersed minorities (*First Past The Post*, s. f.). It is applied in countries such as the United Kingdom, Canada, and India.

APPENDIX B. DISPROPORTIONALITY INDICES

To evaluate the performance of different apportionment rules, this study relies on standardized indices of disproportionality. These indices quantify the distance between vote shares and seat shares, making it possible to rank scenarios objectively and compare outcomes across time and institutional designs. Two main families can be distinguished: absolute difference measures, which summarize deviations in percentage-point terms, and relative ratio measures, which compare seat-to-vote ratios (Karpov, 2008). The following sections describe the main indices employed in this study, their logic, properties, and main limitations.

B.1 Absolute difference measures

Gallagher Index (Least Squares Index)

Introduced by Michael Gallagher (1991), the index calculates disproportionality by squaring the differences between each party's vote share and seat share, summing across all parties, and then taking the square root. This procedure downweights minor deviations and highlights major distortions, making it one of the most robust and widely used measures in comparative political science. Substantively, it answers the question: *how severe are the main distortions between votes and seats once minor differences are discounted?*

Loosemore–Hanby Index

Proposed by Jean Loosemore and Victor Hanby (1971), this index sums the absolute differences between votes and seats and divides by two, yielding the share of seats that would need to change hands to achieve perfect proportionality. Its interpretation is straightforward: if the index equals eight, 8% of seats would have to be reassigned to eliminate disproportionality. The main limitation is that it treats all deviations equally, regardless of whether they concern large or small parties (Kalogirou, 2017).

Rae Index

Douglas Rae proposed this index as the average absolute deviation between each party's vote share and seat share. It answers the question: *on average, by how many percentage points*

does each party's seat share diverge from its vote share? The simplicity of Rae's index makes it easy to compare across systems with different numbers of parties. However, it is sensitive to the inclusion of many small or irrelevant parties, which can artificially lower the average level of disproportionality (Kalogirou, 2017).

Lijphart Index

Arend Lijphart introduced this measure as one of the simplest indicators of disproportionality. Instead of averaging across parties, it captures only the largest single deviation between votes and seats. Its strength lies in clarity: it highlights the most distorted case in an election. The drawback is that it ignores the rest of the distribution, so two elections with very different systemic patterns can have the same score if their maximum distortion is identical (Gallagher, 1991).

B.2 Relative ratio measures

Sainte-Laguë Index.

Developed by André Sainte-Laguë, this index evaluates disproportionality in relative rather than absolute terms. It compares seat-to-vote ratios across parties, weighting results by vote share. Substantively, it asks: *to what extent are parties rewarded or penalized relative to their electoral support?* The index is particularly sensitive to cases in which small parties obtain representation disproportionate to their vote share, while being more forgiving of minor over-representation of large parties. A limitation is that it has no natural upper bound, making it harder to interpret absolute values than indices such as Gallagher or Loosemore–Hanby (Kalogirou, 2017).

D'Hondt Index.

Based on the work of Victor D'Hondt, this index also focuses on relative disproportionality, but in a different way: it identifies the most over-represented party and expresses disproportionality as the ratio of its seat share to its vote share. The advantage is its clear diagnostic of systemic bias toward dominant parties; the drawback is that it considers only one party and disregards overall disproportionality (Gallagher, 1991).

APPENDIX C. DECLARATION OF USE OF GENERATIVE IA IN THE MASTER FINAL PROJECT

I have used Generative AI in this work

Check all that apply:

YES	NO
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If you have ticked YES, please complete the following 3 parts of this document:

Part 1: Reflection on ethical and responsible behaviour

Please be aware that the use of Generative AI carries some risks and may generate a series of consequences that affect the moral integrity of your performance with it. Therefore, we ask you to answer the following questions honestly (*please tick all that apply*):

Question		
In my interaction with Generative AI tools, I have submitted sensitive data with the consent of the data subjects.		
YES, I have used this data with permission	NO, I have used this data without authorisation	NO, I have not used sensitive data
In my interaction with Generative AI tools, I have submitted copyrighted materials with the permission of those concerned.		
YES, I have used these materials with permission	NO, I have used these materials without permission	NO, I have not used protected materials
In my interaction with Generative AI tools, I have submitted personal data with the consent of the data subjects.		

YES, I have used this data with permission	NO, I have used this data without authorisation	NO, I have not used personal data
My use of the Generative AI tool has respected its terms of use , as well as the essential ethical principles, not being maliciously oriented to obtain an inappropriate result for the work presented, that is to say, one that produces an impression or knowledge contrary to the reality of the results obtained, that supplants my own work or that could harm people.		
YES		NO

Part 2: Declaration of technical use

Use the following model statement as many times as necessary, in order to reflect all types of iteration you have had with Generative AI tools. Include one example for each type of use where indicated: *[Add an example]*.

I declare that I have made use of the Generative AI system Chat GPT 5 for:

Documentation and drafting:

- *Revision or rewriting of previously drafted paragraphs*

I used ChatGPT selectively to clarify certain paragraphs that I felt required greater fluency, always maintaining their original meaning. In some cases, I also relied on it to condense overly long sections into more concise formulations.

Develop specific content

Generative AI has been used as a support tool for the development of the specific content of the dissertation (MFP), including:

- *Assistance in the development of lines of code (programming)*

ChatGPT was particularly helpful in identifying and correcting errors, as well as in generating specific components of visualizations, especially those related to aesthetics.

- *Optimisation processes*

I relied on ChatGPT mainly to optimise code, both in terms of efficiency and readability. It was especially useful in reducing redundancy that initially required many lines.

Part 3: Reflection on utility

Please provide a personal assessment (free format) of the strengths and weaknesses you have identified in the use of Generative AI tools in the development of your work. Mention if it has helped you in the learning process, or in the development or drawing conclusions from your work.

In my view, the main strength of using generative AI lies in its ability to provide quick solutions for specific tasks. For instance, it is very effective in improving the clarity of sentences that are otherwise difficult to rephrase, and it considerably reduces trial-and-error time when working with code, as it identifies mistakes almost instantly and suggests optimised alternatives.

At the same time, I found that AI tools are less reliable for more creative or conceptual tasks. The outputs often require careful revision, as errors or inaccuracies are frequent when relying on them without verification. In this sense, while AI can save significant time in debugging or polishing text, it may also generate additional workload if one relies too heavily on it without proper oversight.