

Among several plausible explanations for a phenomenon, the simplest is the best.

Variable Selection

select the best subset of predictors to explain the data in the simplest way
unnecessary predictors add noise to the estimation of other quantities that interest us, wasting degrees of freedom

collinearity is caused by having too many variables provide the same information
save time any money by not measuring redundant predictors

Model Hierarchy

lower order terms should not be removed before higher order terms of the same variable

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \varepsilon$$

x is not significant but x^2 is
remove x term to get reduced model

$$y = \beta_0 + \beta_2 x^2 + \varepsilon$$

if you make a scale change from x to $(x + a)$, model becomes

$$y = \beta_0 + \beta_2 a^2 + 2\beta_2 ax + \beta_2 x^2 + \varepsilon$$

the first order x term reappears
scale changes shouldn't make any important changes to the model

Stepwise Testing

compares successive models

$\alpha_{criteria}$ doesn't have to be 5%, 15-20% cutoff may work best

possible to miss the optimal model because adding/dropping one variable at a time

no multiple testing correction so p-values should not be assumed to be the Type I Error

procedures are not directly linked to final objectives of prediction or explanation so may not help solve the problem of interest

Backward Elimination/Top Down

- Step 1 start with all the predictors in the model
- Step 2 remove the predictor with the highest p-value $> \alpha_{criteria}$
- Step 3 refit the model
- Step 4 repeat Steps 2-3 until all non-significant predictors are removed

Forward Selection/Bottom Up

- Step 1 start with no predictors
- Step 2 add the predictor with the lowest p-value $< \alpha_{criteria}$
- Step 3 refit the model
- Step 4 repeat Step 2-3 until no new predictors can be added

Stepwise Regression

combination of backward elimination and forward selection

at each stage, a variable may be added or removed

top down stepwise regression alternate drop step and add step

bottom up stepwise regression alternate add step with drop step

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Criterion Testing

find the model that optimizes the measure of goodness of fit and reduces the model's complexity

prefer small values of AIC and BIC with smaller RSS and a small number of parameters

larger models fit better so will have smaller RSS but use more parameters

need to find a balance between RSS and number of parameters

BIC penalizes larger models more heavily and tend to prefer smaller models compared to AIC

$$\text{Akaike Information Criterion (AIC)} \quad n \times \log\left(\frac{RSS}{n}\right) + 2p'$$

$$\text{Bayes Information Criterion (BIC)} \quad n \times \log\left(\frac{RSS}{n}\right) + \log(n)p'$$

Adjusted R^2

$$R^2 = \frac{SS_{reg}}{S_{YY}} = 1 - \frac{RSS}{S_{YY}}$$

adding variables can only decrease RSS and increase R^2

R^2 is not a good criterion because it always prefers the larger model

use significant changes of RSS and the criterion

$$R_a^2 = 1 - \frac{\frac{RSS}{n-p-1}}{\frac{S_{YY}}{n-1}} = 1 - \left(\frac{n-1}{n-p-1}\right)(1-R^2) = 1 - \frac{\hat{\sigma}_{Model}^2}{\hat{\sigma}_{Null}^2}$$

adding a predictor will only increase R_a^2 if it has some predictive value

Mallow's C_p Statistics

average mean square error of prediction = $\frac{1}{\sigma^2} \sum E(\hat{y}_i - E(y_i))^2$

$$C_p = \frac{RSS_p}{\hat{\sigma}^2} + 2p - n$$

$\hat{\sigma}^2$ is from the model with all the predictors

RSS_p is from a model with p parameters

for the full model, $C_p = p$

if a p -predictor model fits, then $E[RSS_p] = (n-p)\sigma^2$ and $E(C_p) \approx p$

if a model has a bad fit, C_p will be much larger than p

models with a good fit will have small p and a C_p around or less than p